LOBACHEVSKY STATE UNIVERSITY
of NIZHNI NOVGOROD
National Research University

Problems in creating adequate stochastic model of memristors

Alexander Dubkov (Russia)

StoLab

OUTLINE

- what is the memristor
- possible applications and motivation to investigate
- existing models of memristors
- laboratory of stochastic multistable systems in NN
- theoretical findings
- experimental results
- conclusions

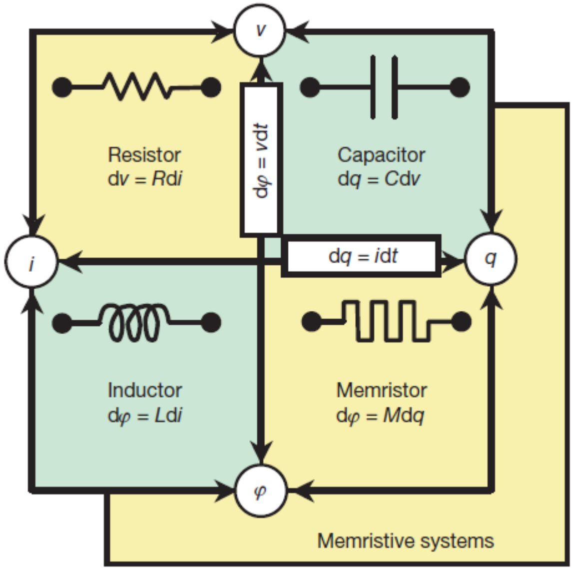
Memristor = Memory + Resistor

<u>Memristors</u> are considered as

- ❑ **the basis for <u>a new generation of electronic synaptic devices</u> designed to mimic the adaptive behavior of biological systems and**
- ❑ **the basis for <u>a new generation of nonvolatile memory</u>.**

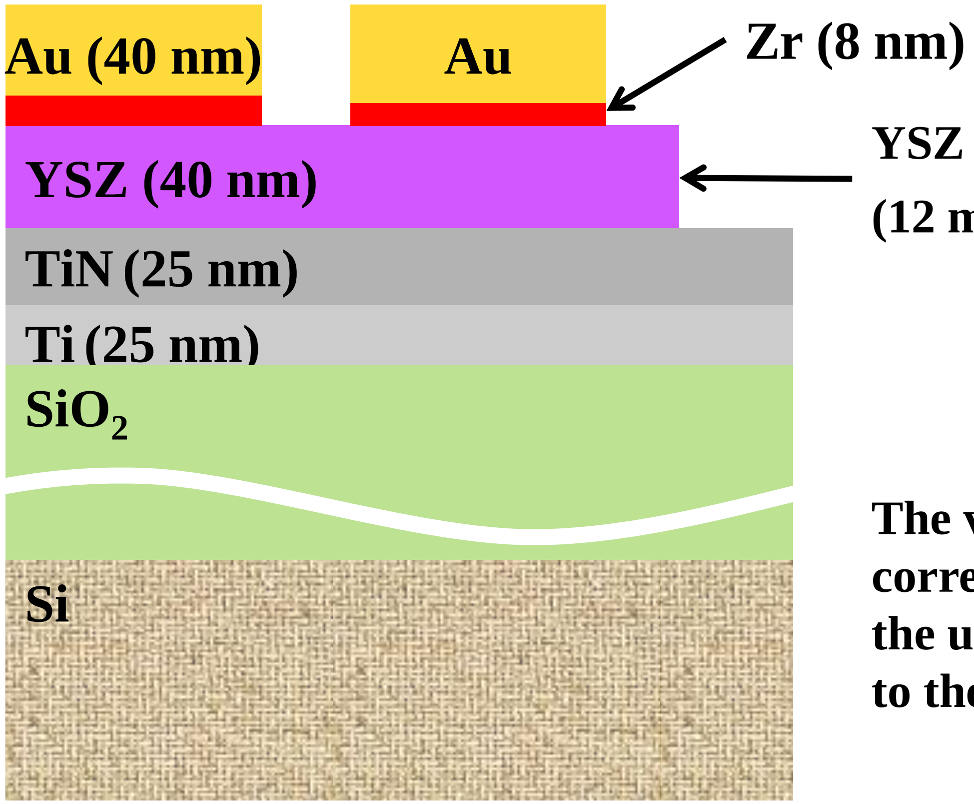
The four fundamental two-terminal circuit elements: resistor, capacitor, inductor and memristor

L. Chua et al, Proc. IEEE Trans. Circuit Theory, 1971;
Appl. Phys. A, V.124, 563, 2018



Memristive structure

Device area: 20 μm × 20 μm

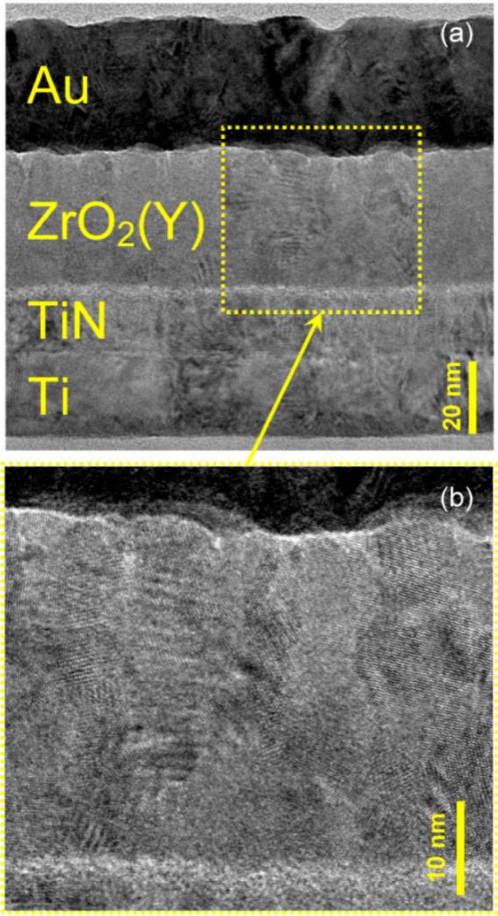


YSZ - yttria stabilized zirconia (12 mol. % Y_2O_3)

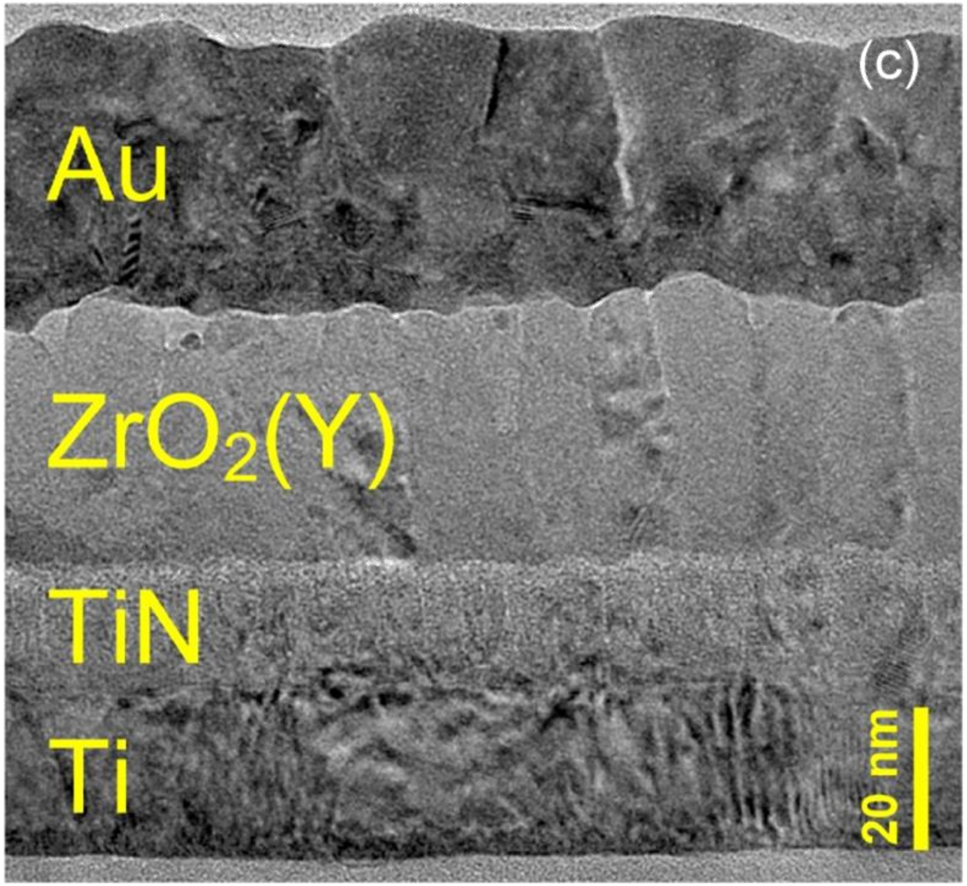
The voltage on the structures corresponded to the potential of the upper electrode with respect to the potential of the substrate

HR TEM image of a transverse cross section of the memristive structure

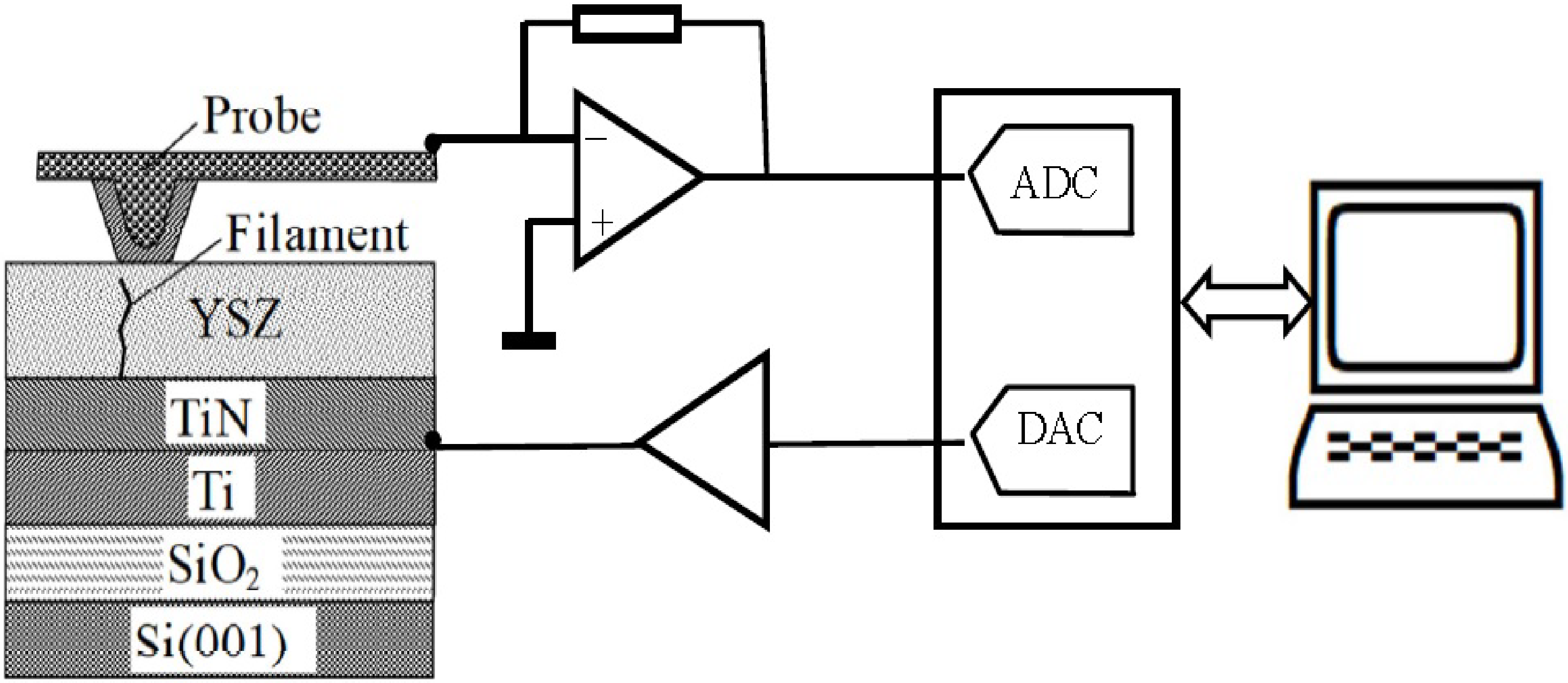
Before forming



After forming

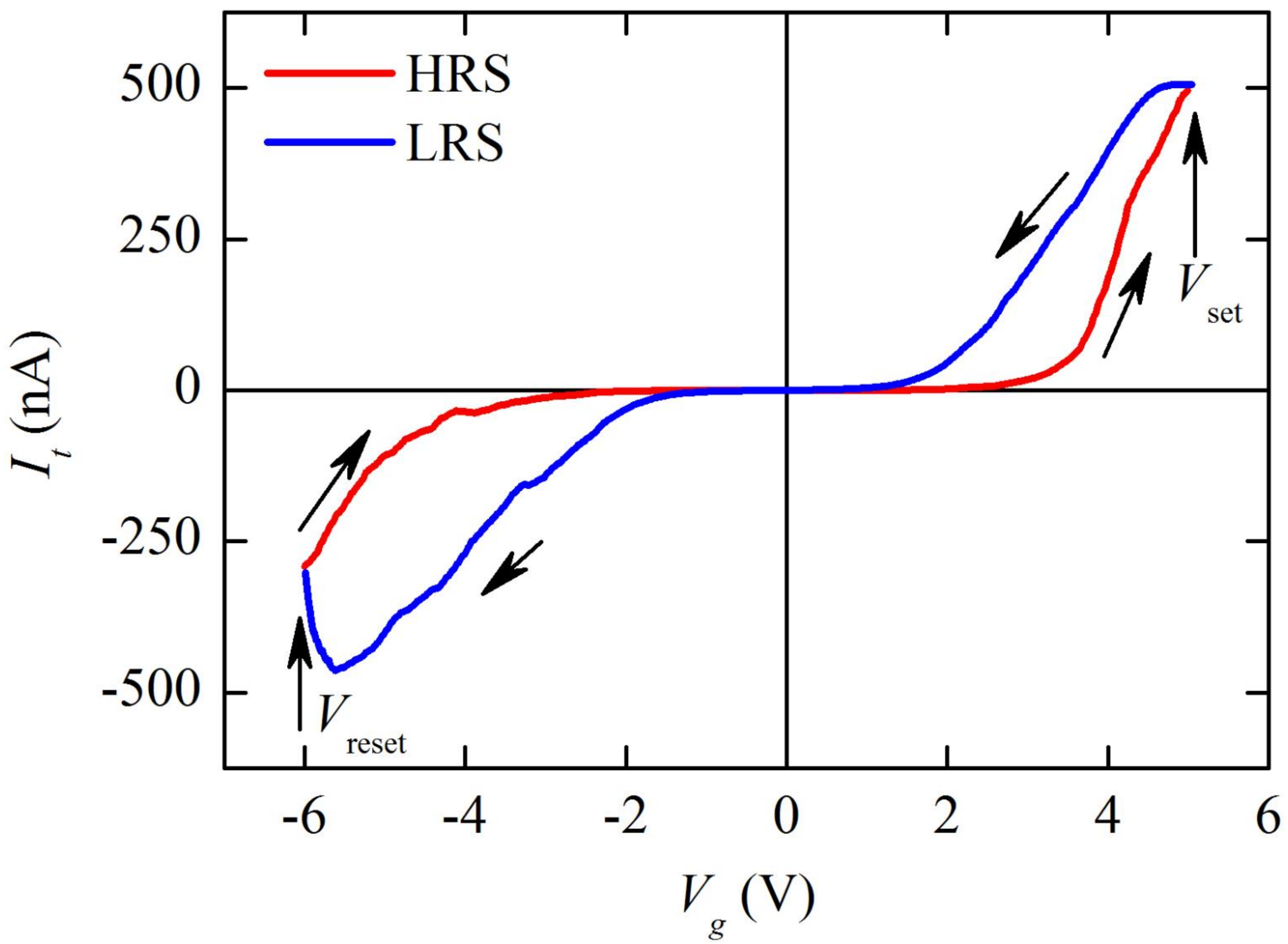


Conductive Atomic Force Microscopy (CAFM)

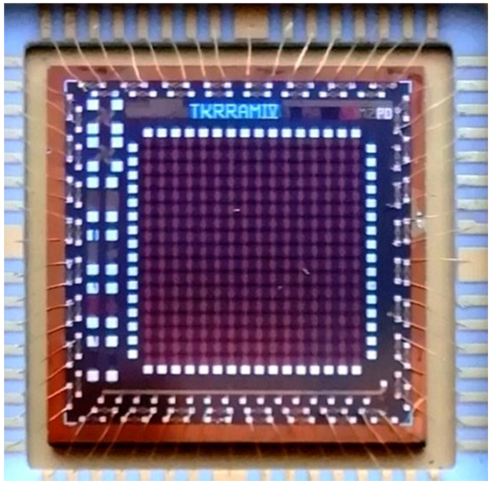


Experimental Details

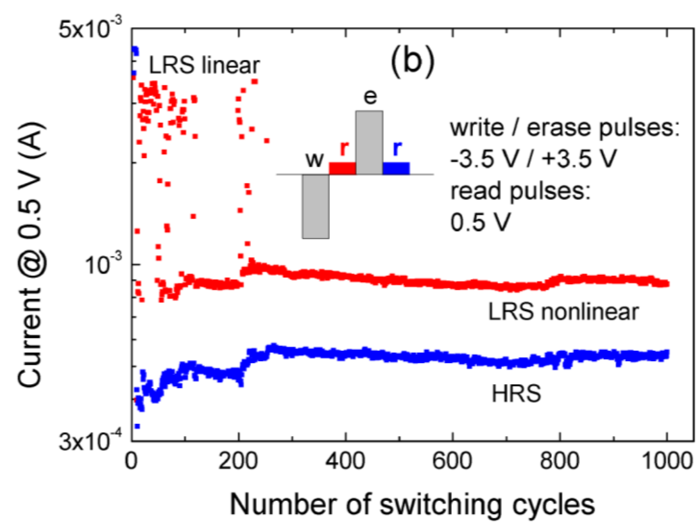
Current-Voltage (I-V) Curve of the CAFM Probe-to-Sample Contact



Memristors for Resistive Random Access Memory (RRAM)



A memristive chip with cross-point devices and 16×16 passive cross-bar array.



Endurance: up to 10^6 write/erase cycles
Switching time: < 70 ns

List of companies conducting active research on the development and creation of memristive devices:

Hynix, IBM, IMEC, Fujitsu, Samsung, SMIC, Sharp, TSMC, NEC, Panasonic, Macromix, Crossbar Inc., Qimonda, Ovonyx, Intel, KnowM, 4DS Memories Ltd., Global Foundaries, SanDisk, Nanya, NEC, Rambus, ST Microelectronics, Winbond, Adesto Technologies Corporation, HRL Laboratories LLC, Elpida.

Mathematical models of memristive devices

Dynamical

Chua 1976 - 2018
Strukov 2008
Ielmini 2011

First order differential equation. 1D.

- Mathematical formulation of generalized switching process.
- Does not involve fluctuations

Microstructural

Kim 2013
Kim 2014
Marchewka 2016

Large number of equations and all material properties in details. 3D.

- Less practical. Does not catch the general features
- Provide only qualitative fit to experimental data.

Thermo dynamical

Karpov 2017

FPE and Boltzmann distribution in equilibrium. 1D

- Naturally involves the fluctuations
- Need to calculate the thermodynamic potential (e.g. free energy)

Stochastic

Medeiros 2011
Stotland 2012
Patterson 2013
Naous 2016
Jiang 2017

First order Langevin equation. 1D, 3D.Corresponds to FPE

- Mathematical formulation of generalized switching process.
- Involve fluctuations

Dynamical models

For the ideal memristor the state-dependent Ohm's Law and its associated state equation are given by:

$$U(t) = R(q)I(t), \quad I(t) = \frac{dq}{dt}$$

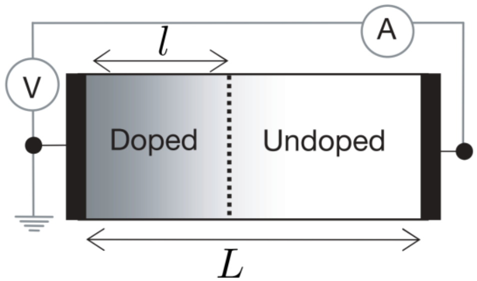
Chua et al., IEEE Trans. Circuit Theory, 1971

From the physical point of view the following monotonic exponential dependence of the resistance on charge is appropriate

$$R(q) = R_{ON} + \frac{\Delta R}{e^{-(q+q_1)/q_0} + 1}, \qquad \Delta R = R_{OFF} - R_{ON} \text{ and } R_{ON} \ll R_{OFF}$$

Strukov et al., Nature, 2008

$$U(t) = \left(R_{ON} l(t) + R_{OFF}\left(1 - l(t)\right)\right) I(t), \qquad \frac{dl(t)}{dt} = \frac{\mu_V R_{ON}}{L^2} I(t),$$

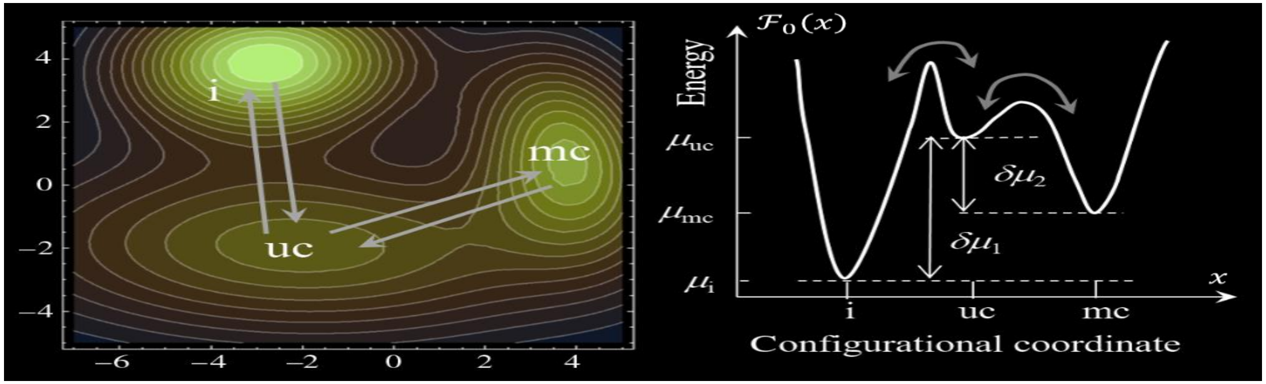


where L is the full size of the memristor with two states, $l(t)$ is the normalized size of the doped region, μ_V is the average ion mobility.

Switchings of memristor as phase transition of the first order

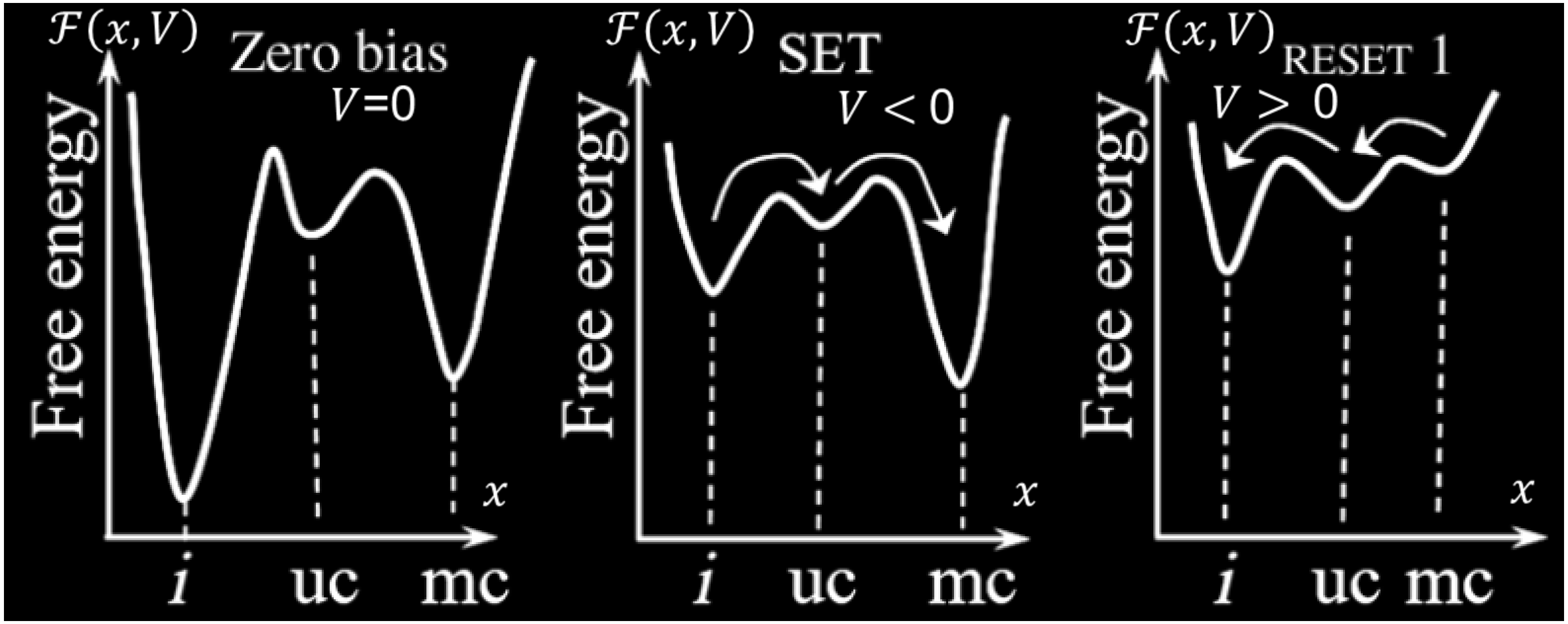
V.G. Karpov et al, Phys. Rev. A, V.8, 024028, 2017
Variable x is the parameter of order
External parameters: temperature, voltage
Thermodynamic potential: free energy

Free energy profile

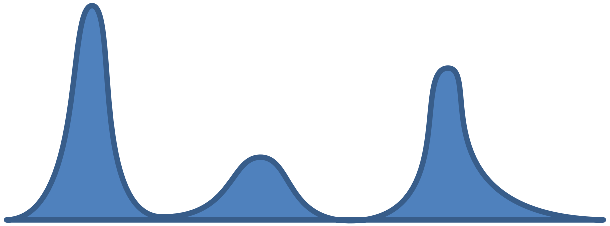


Three metastable states:
i – insulating state
uc – unstable conductive state
mc – metastable conductive state

Dependence on external parameter



Steady-state distributions



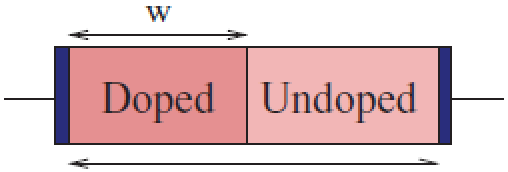
Stochastic models

PHYSICAL REVIEW E **85**, 011116 (2012)

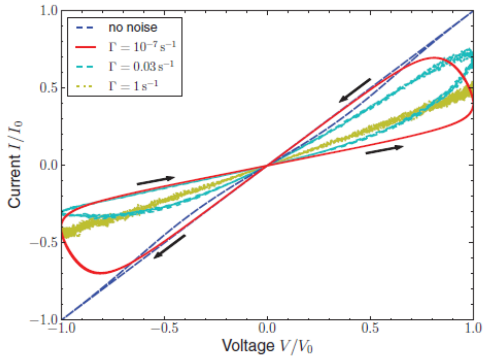
Stochastic memory: Memory enhancement due to noise

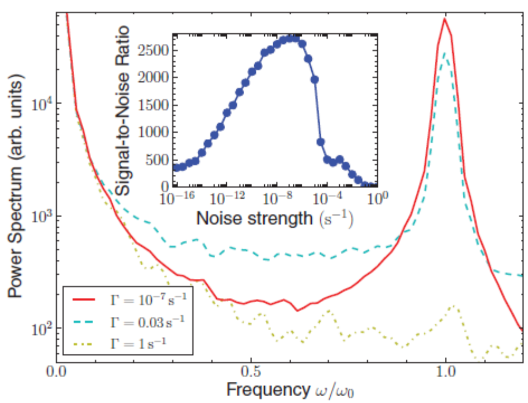
Alexander Stotland and Massimiliano Di Ventra

$$\{y(t)\}_\xi = \{g(x,u,t)\}_\xi u(t),$$
$$\dot{x} = f(x,u,t) + H(x,u,t)\xi(t),$$



$$\dot{g} = \frac{\partial g}{\partial x} f(x,u) + \frac{\partial g}{\partial u}\dot{u} + \frac{\partial g}{\partial x}\xi(t) + \frac{1}{2}\Gamma\frac{\partial^2 g}{\partial x^2},$$



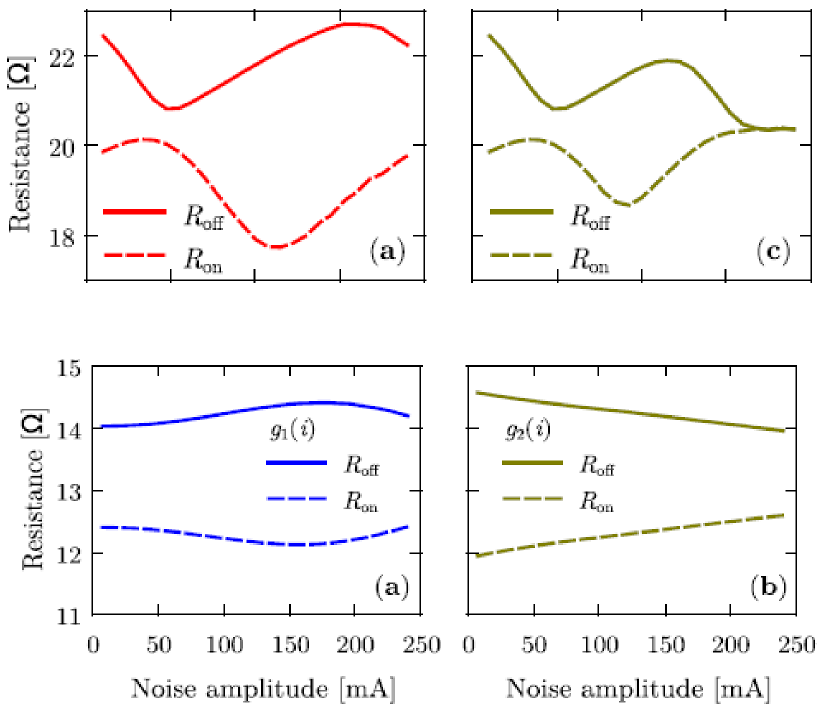


If vacancy diffusion is the main mechanism of memory in TiO2 thin films, the opening of the hysteresis should be strongly dependent on the external temperature, at least for small enough frequencies. This can be checked by lowering the frequency of the applied voltage below the optimal one for a wide hysteresis loop, tuning the temperature and observing the change in the shape of the hysteresis.

APPLIED PHYSICS LETTERS **103**, 074102 (2013)

On the beneficial role of noise in resistive switching

G. A. Patterson,[1,a)] P. I. Fierens,[2] and D. F. Grosz[2]



$$v(t) = R(x,i)\,i(t)\,,$$

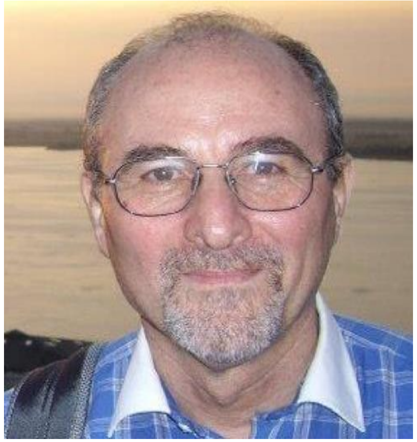
$$\frac{dx}{dt} = f(x,i)\,,$$

Local convexity for positive (negative) current leads to an enhancement of the contrast between high and low resistive states when noise is added.

StoLab

Laboratory of Stochastic Multistable Systems

Established in 2018 under Megagrant from Government of Russian Federation No 14.Y26.31.0021 “Comprehensive research of fluctuation phenomena in multistable systems for development of new generations of memristor-based electronic devices and neuromorphic technologies of artificial intelligence”
Project Terms 2018 – 2020
Funding: RUR 100,000,000



Prof. Bernardo Spagnolo University of Palermo (,Italy) Principal Investigator and Head of StoLab

Main research objectives of the project

•Investigation of the influence of external and internal noises on the behavior of multistable systems. Investigation and analysis of phenomena with constructive role of noise in multistable systems.
•Experimental study of behavior of memristive nanostructures based on oxide materials under the influence of external and internal noises. Development of an adequate physical macro-model of memristor taking into account the influence of external and internal noises and its connection with the physico-chemical micro-model of phenomena responsible for resistive switching
•Study of microscopic nature and impact of flicker and high frequency noise in memristive nanostructures.
•Experimental demonstration of new possibilities for stability increasing, behavioral prediction and parameter control of memristive devices in the prototypes of electronic devices and neuromorphic systems of next generation.

StoLab structure

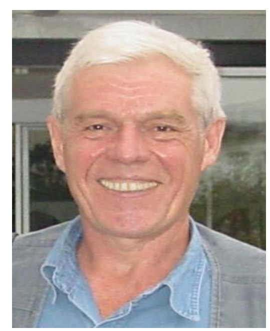
Head of laboratory
Prof. B. Spagnolo

- Sector of modern methods of stochastic analysis
- Sector of technology of memristive materials
- Sector of the microscopic probe investigations
- Sector for physics of noise
- Sector of neuromorphic technologies

Prof. A. Dubkov

Dr. A. Mikhaylov

Dr. O. Gorshkov



Prof. A. Yakimov

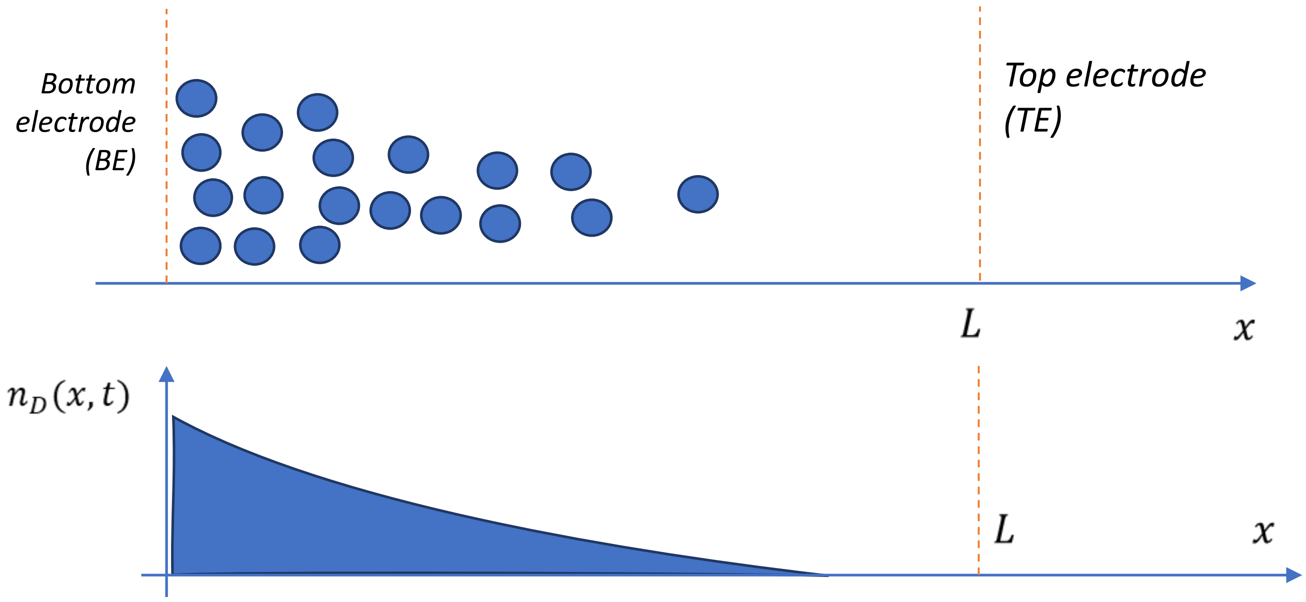
Dr. V. Demin

Theoretical investigations

Physical model

$$\frac{\partial}{\partial t} n_D(x,t) = \frac{\partial}{\partial x}\left[n_D(x,t)\frac{\mu dU(x)}{dx}\right] + D\frac{\partial^2}{\partial x^2} n_D(x,t)$$

$$n_D(0) = N_1,\ \ n_D(L) = N_2 \quad \rightarrow n_D(L)\text{ - reflecting boundary?}$$



Relaxation time

- ***How to reduce switching time?***
- ***How the values of diffusion coefficient D and external field $\mathbb{E}$ influence on the switching time?***
- **How to maximize the difference of R between LRS and HRS?**
- We obtain non-stationary solution $n_D(x, t)$ of the FPE. To do it we find a function of concentration as a sum of two terms:

$$n_D(x,t) = n_{st}(x) + n_{nst}(x,t),$$

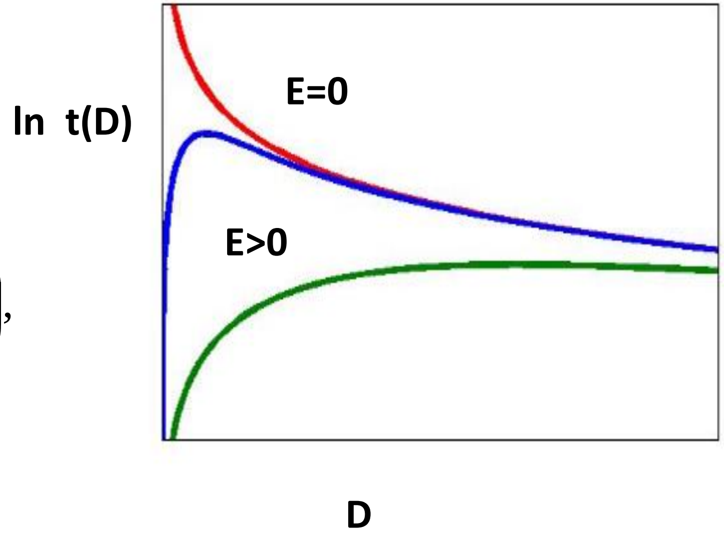
Using this approach we obtain

$$n_{nst}(x,t) = \exp\left(\frac{\mu \mathrm{E} x}{D}\right) \sum_{n=1}^{\infty} M(n) \exp[C(n)\, t] \cdot Sin\left(\frac{\pi n x}{L}\right),$$

where $C(n)$ are:

$$C(n) = \frac{D}{8}\left(-\frac{4(\pi n)^2}{L^2} - \left(\frac{2\mu \mathrm{E}}{D}\right)^2\right)$$

Then the relaxation time corresponds to the slowest exponent.



Model of ideal memristor with white and colored Gaussian noise

$$U(t) = R(q)I(t), \quad I(t) = \frac{dq}{dt}$$

The ideal memristor can be defined by an equivalent algebraic function

$$w(t) = \int_0^t U(t)dt = \int_0^{q(t)} R(q)dq = \Phi(q(t)).$$

Firstly, we consider **the voltage U(t) as stationary Gaussian noise** with non-zero mean U0 and the correlation function K(τ).

According to the Central Limit Theorem, the process w(t) is again a process with a Gaussian distribution or a Wiener process and it describes the free particle diffusion.

$$P_w(y,t) = \frac{1}{\sqrt{4\pi D(t)}} \exp\left\{-\frac{(y - U_0 t)^2}{4D(t)}\right\} \qquad D(t) = \int_0^t (t-\tau)\, K(\tau) d\tau.$$

We apply the theorem of the probability theory to calculate the PDF of the charge flowing through a memristor

$$P_q(z,t) = \frac{\Phi'(z)}{\sqrt{4\pi D(t)}} \exp\left\{-\frac{\left[\Phi(z) - U_0 t\right]^2}{4D(t)}\right\}$$

Also we can obtain the PDF of the resistance using the same technology

$$P_R(r,t) = \frac{r}{\sqrt{4\pi D(t)}} \sum_k \frac{1}{|\Phi''(q_k(r))|} \times \exp\left\{-\frac{\left[\Phi(q_k(r)) - U_0 t\right]^2}{4D(t)}\right\}$$

where $R = \Phi'(q)$

Due to the fact that the charge is a non-Gaussian random process, it is not possible to find the PDF of the current. However, we can find all the moments of the charge on the memristor and as a consequence the moments of the current setting a specific dependence R(q) and using the well-known relation.

We consider the following monotonic exponential dependence of the resistance on charge

$$R(q) = R_{ON} + \frac{\Delta R}{e^{-(q+q_1)/q_0} + 1}, \qquad \Delta R = R_{OFF} - R_{ON} \text{ and } R_{ON} \ll R_{OFF}$$

The exact expression for the PDF of the resistance has the following form

$$P_R(r,t) = \frac{q_o \Delta R}{\sqrt{4\pi D(t)}} \frac{r}{(R_{OFF} - r)(r - R_{ON})} \times \exp\left\{ -\frac{[\Phi(q(r)) - U_0 t]^2}{4D(t)} \right\},$$

where

$$\Phi(q(r)) = -q_1 R_{ON} - q_0 \Delta R \ln\left(e^{q_1/q_0} + 1\right) + q_0 R_{OFF} \ln\left(\frac{\Delta R}{R_{OFF} - r}\right) - q_0 R_{ON} \ln\left(\frac{\Delta R}{r - R_{ON}}\right)$$

$$K(\tau) = 2D\delta(\tau), \qquad D(t) = Dt$$

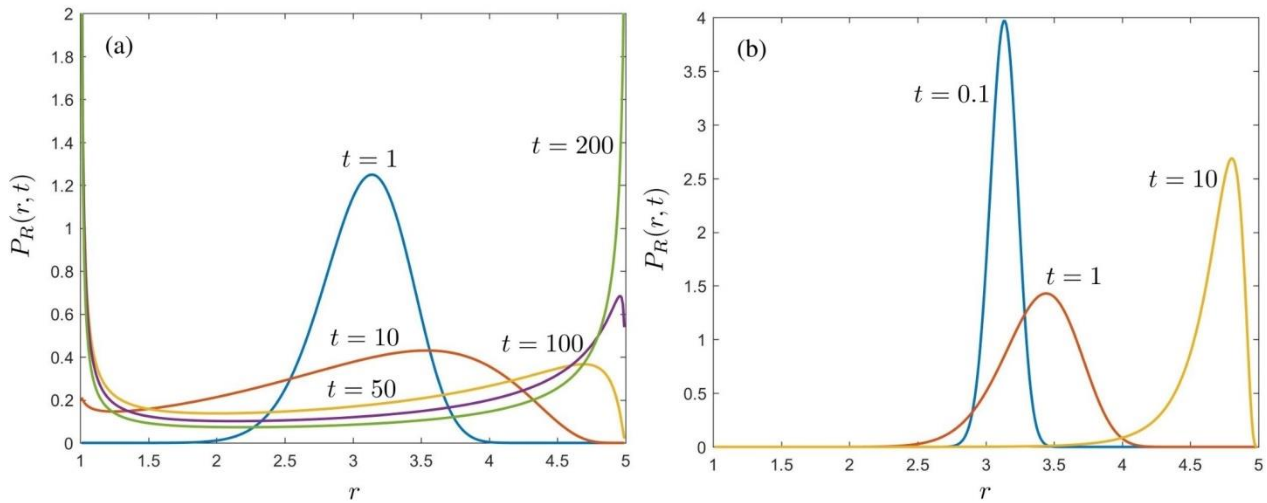
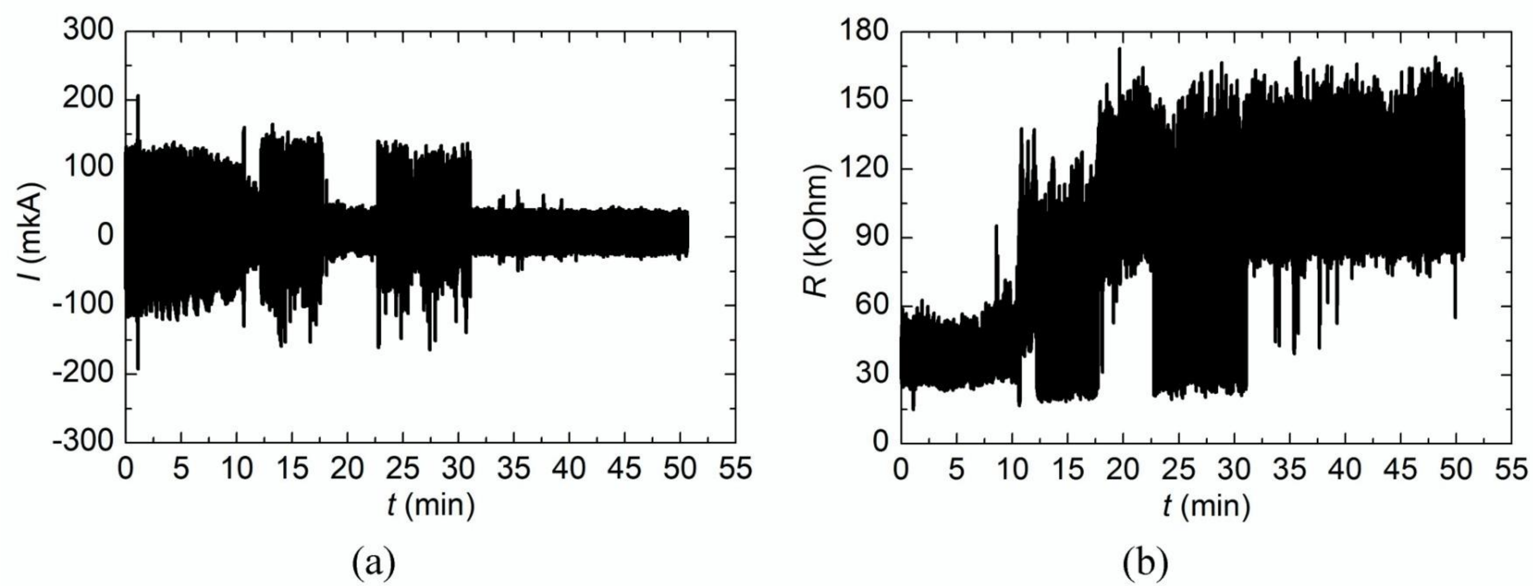


Fig.2. PDF of resistance in the case of colored Gaussian noise $U(t)$ for different time moments (a) $U_0 = 0$, (b) $U_0 = 5$.

The parameters are $q_0 = 1$, $q_1 = 0.1$, $R_{ON} = 1$, $R_{OFF} = 5$, $\sigma^2 = 1$ $\tau_c = 1$.

Experimental results

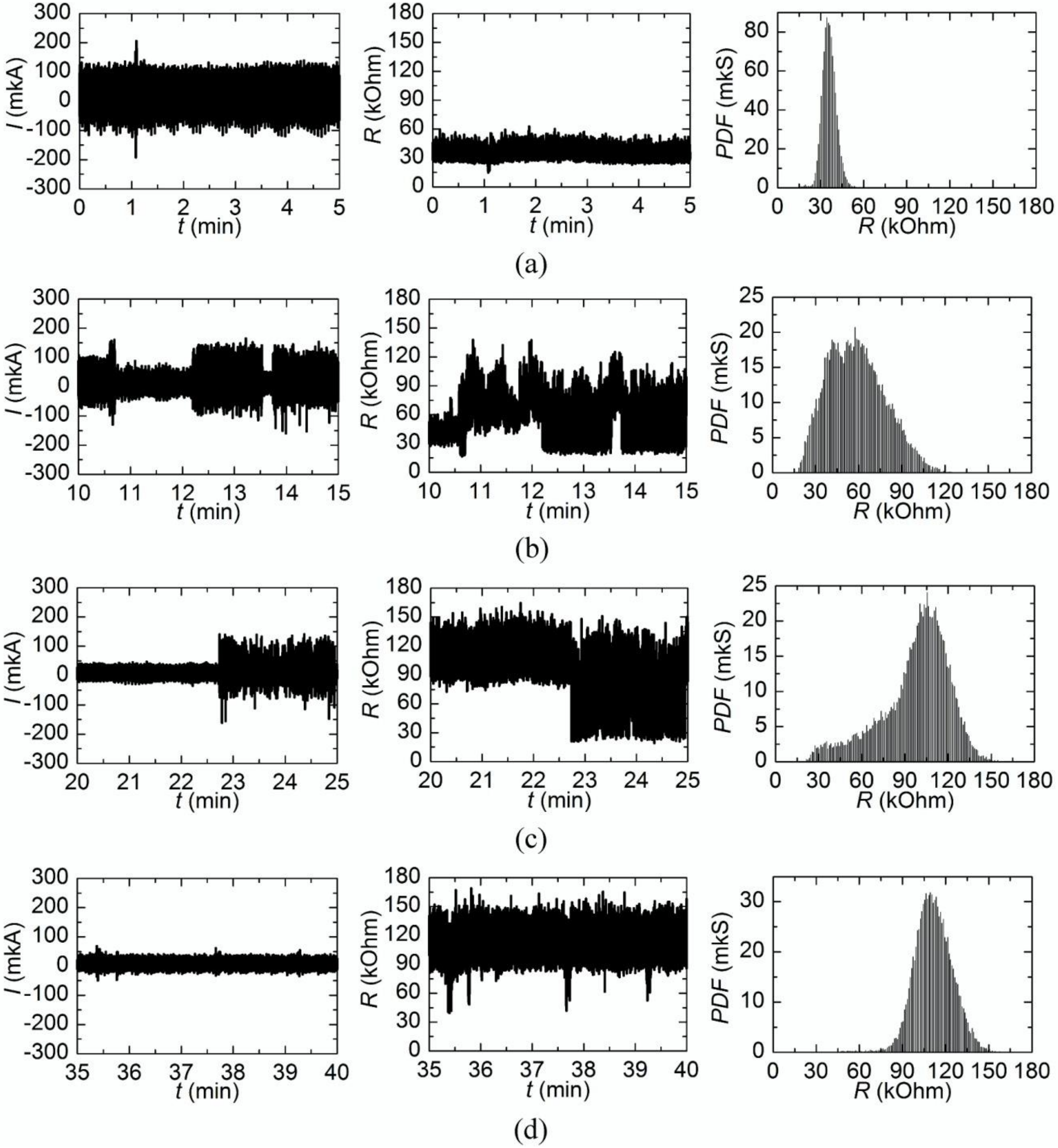


Long-time-scale waveforms *I(t)* (a) and *R(t)* (b) for the memristor based on a $ZrO_2(Y)/Ta_2O_5$ stack under Gaussian white noise signal with $V_{offset} = 0.35$ V; $\sigma_V = 0.5$ V.

$$R(t) = \frac{\sigma_V}{\sigma_I(t)}$$, where σ_V – standard deviation of input signal

$$\sigma_I(t) = \sqrt{\frac{\sum_{i=1}^{M}(I_i - \overline{I})^2}{M(M-1)}}$$ – standard deviation of current calculated for a short time period T_a ;

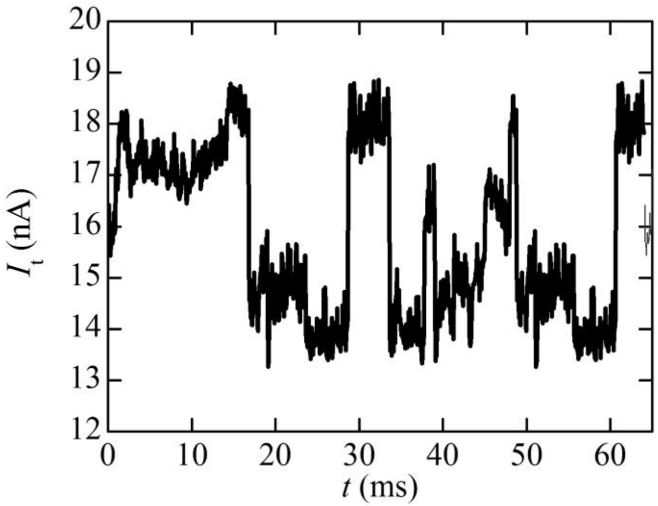
$M = 100$ – the number of points in the time series of the current I measured the period T_a ; $\overline{I}$ – the average current over time period T_a



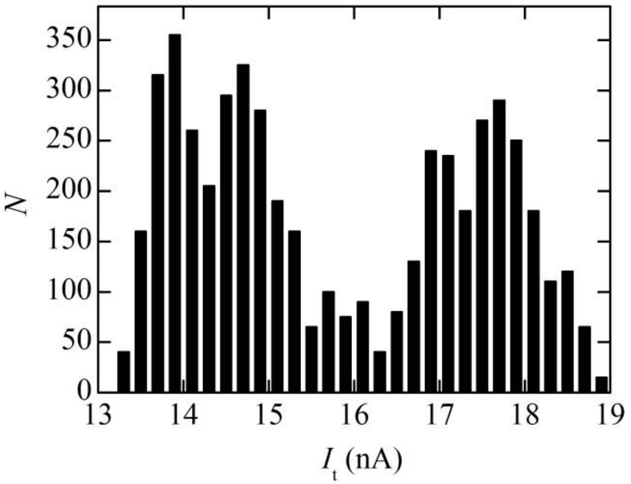
Example waveforms *I(t)* (left) and *R(t)* (middle) recorded in different time windows during the experiment on studying the response of the memristor to the white Gaussian noise signal (V_{offset} = 0.35 V; σ_V = 0.5 V): 0-5 min (a), 10-15 min (b), 20-25 min (c) and 35-40 min (d); corresponding distribution histograms of R (right).

RTS in Virtual Memristor

Low Resistant State (LRS) V_g = -2 V (No artificial noise signal)

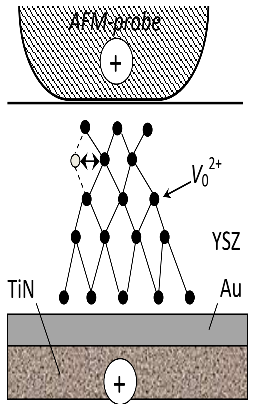


Waveform



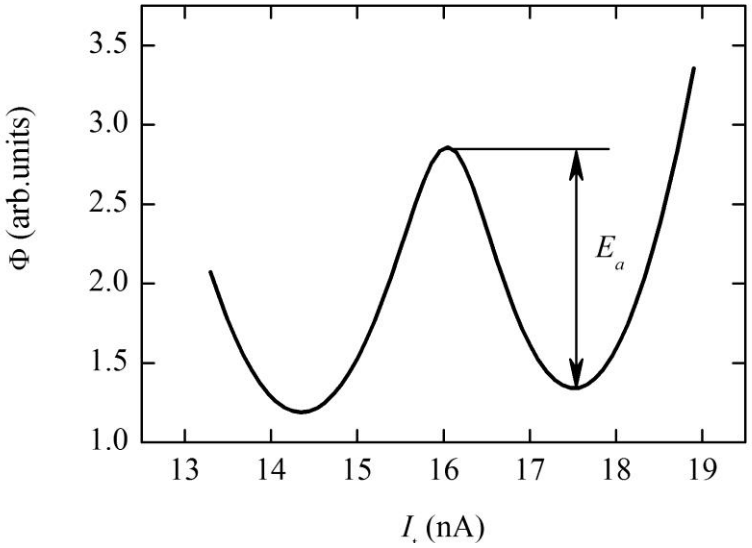
histogram

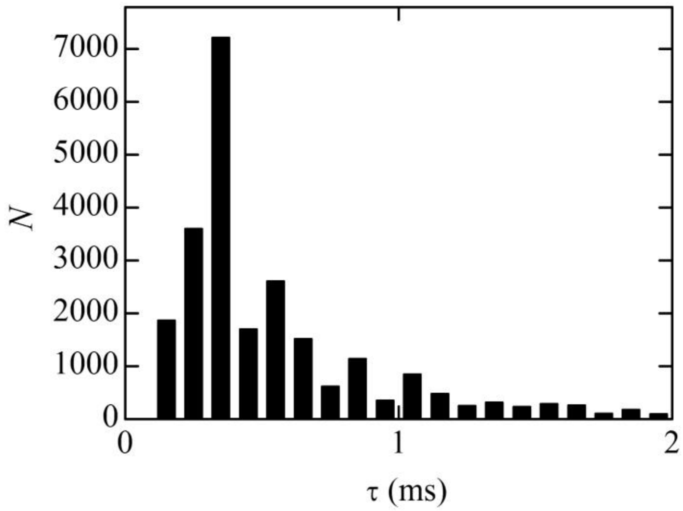
Mechanism of the Random Telegraph Signal Generation



- Filaments consist of a small (countable) number of the oxygen vacancies (V_Os)
- The mechanism of the electron transport in the filaments is hopping conducttivity via the V_Os
- Each jump of the O^{2-} ion onto an adjacent V_O results in a measurable change in the total conductivity of the filament
- The jumps of the O^{2-} ions via the V_Os are random in time

RTS time analysis



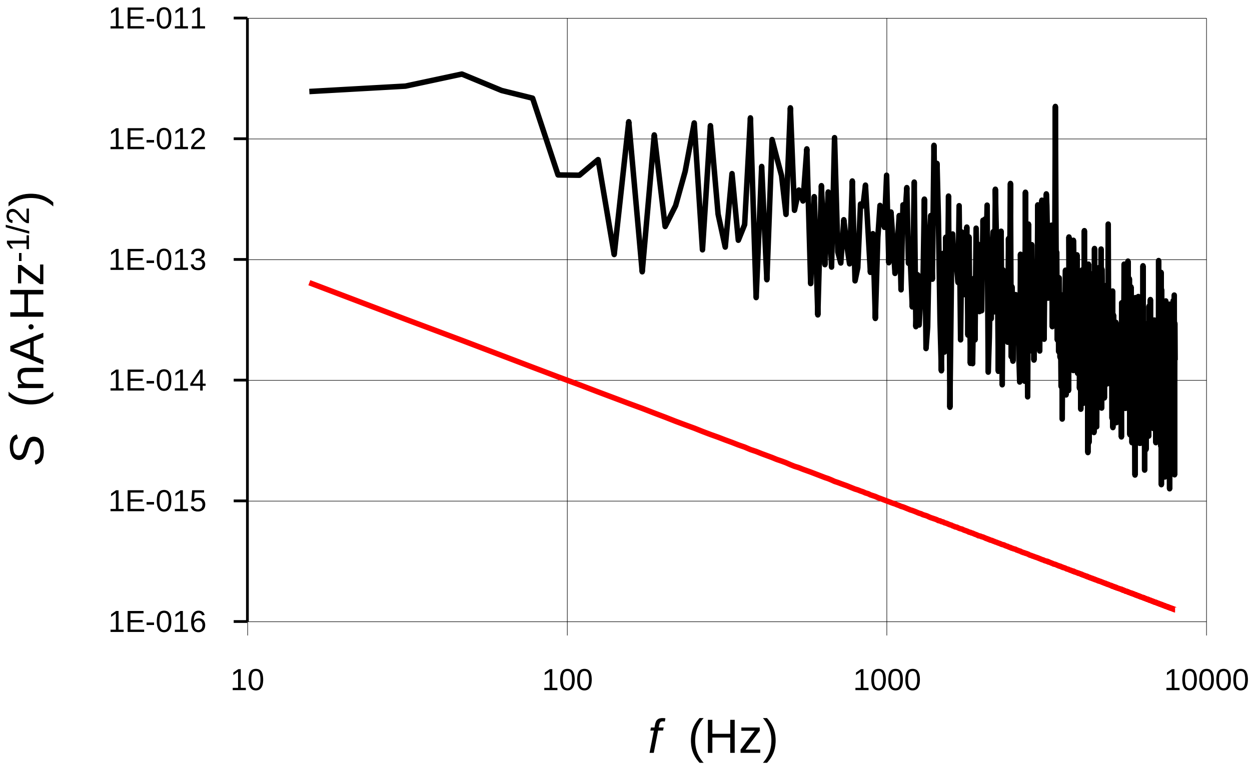


Potential profile calculated from the tip current distribution

E_a = 0.53–0.56 eV at 300 K (from ion migration polarization studies) [Tikhov S. et al. Adv. Condens. Matter Phys. 2018, 2028491]

Noise Spectrum Analysis

Flicker noise in virtual memristors due to oxygen ion diffusion



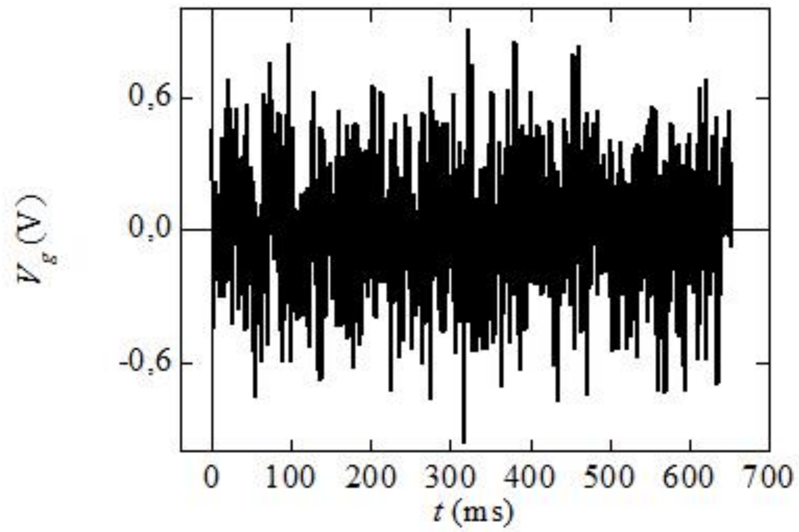
f = 0.4 – 8 kHz

E_a = 0.52–0.68 eV
300 K

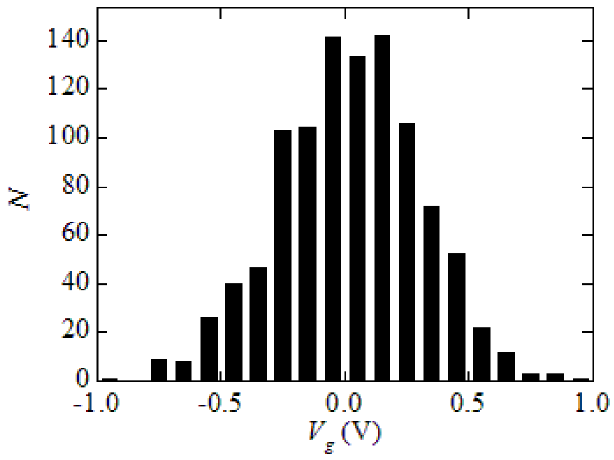
Yakimov A et al. Appl. Phys. Lett. 114, 253506 (2019)

Response of Virtual Memristor to Noise Signal

Artificial Noise Signal
Gaussian White Noise
Synthesized by **ADSView** Noise Generator software

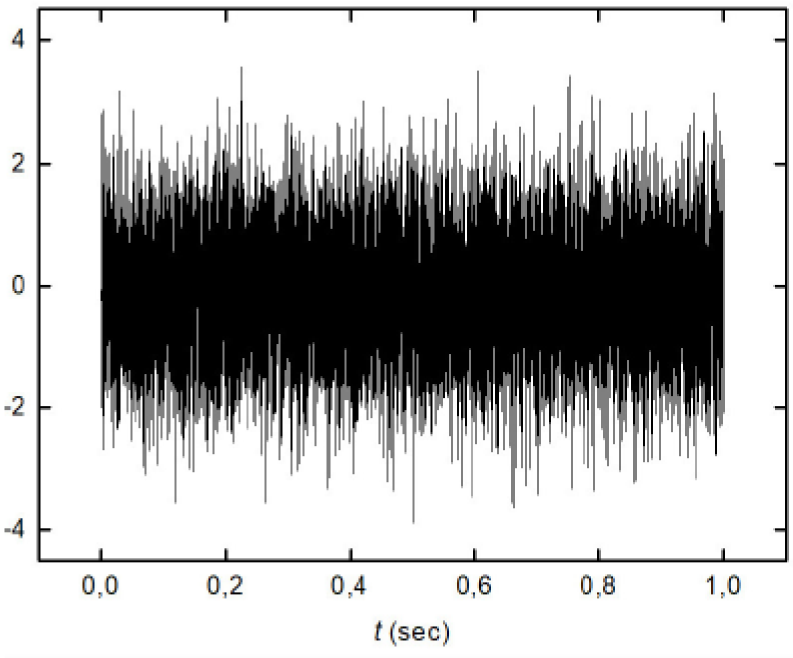


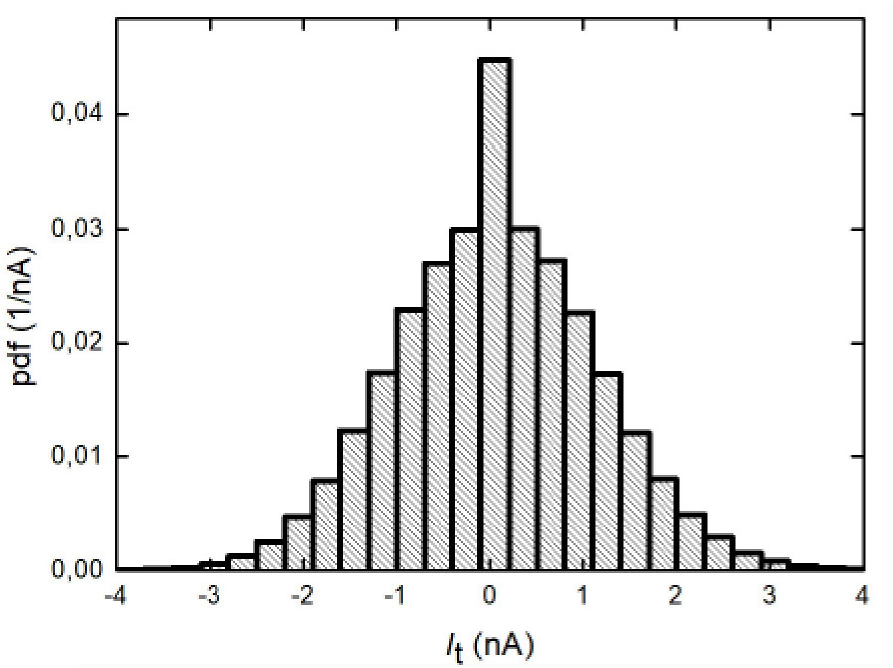
Waveform



Histogram

Background probe current
No input noise signal

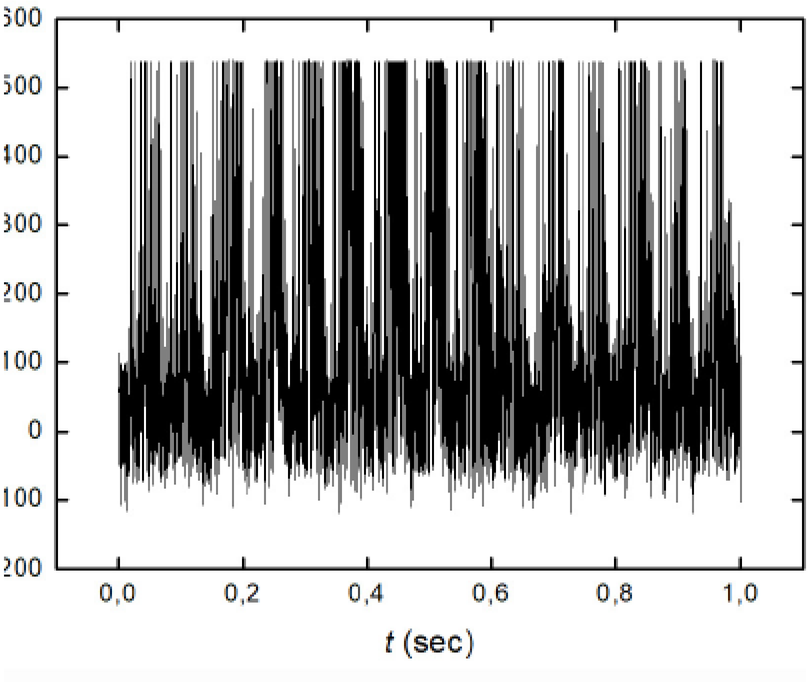


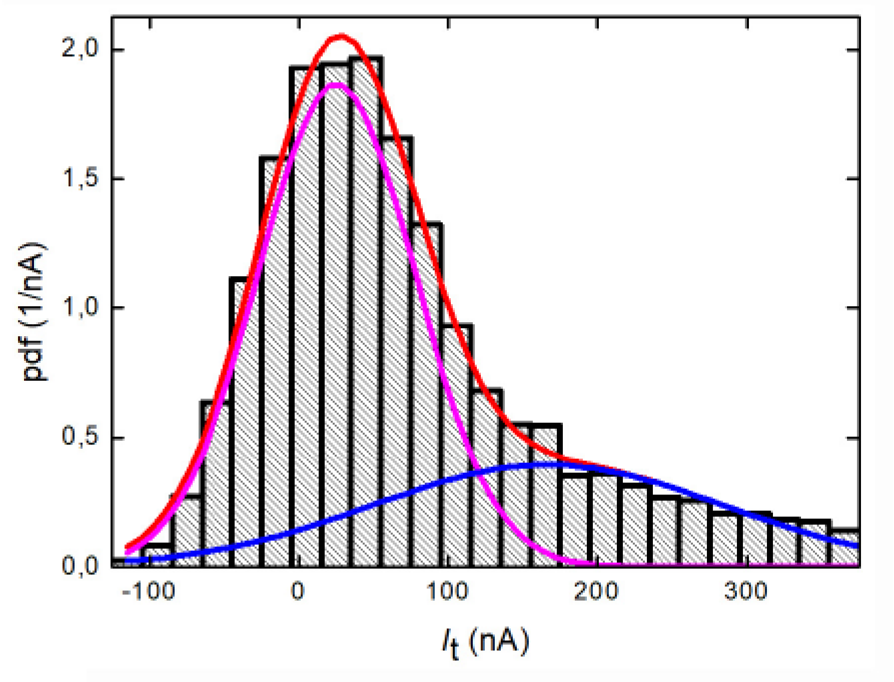


Current Waveform

Current Distribution

Distribution of the probe current film





Current Waveform

Probe Current Distribution

Some remarks

1. RTS has been observed in the noise generated in the virtual memristor in LRS that was attributed to the jumps of individual O^{2-} ions via the oxygen vacancies inside the filament. This observation manifests the intrinsic stochastic nature of the resistive phenomenon.
2. The evolution of the current response of the virtual memristor to external artificial Gaussian white noise signal has been studied. Although at high enough noise magnitude and appropriate offset, the virtual memristor response manifested a RTS pattern, in the long run it fell into either LRS or HRS that manifest the degradation of the virtual memristor.

Conclusions

1. A chaotic switching of the memristor between two or three metastable resistance states in the random telegraph signal mode has been observed.
2.The potential profiles extracted from the waveforms of the current response of the memristor to the white Gaussian noise signal manifested from 2 to 3 local minima corresponding to the number of levels of the memristor resistance switching observed in the experiment.
3. The potential profiles of the memristors were found to depend on the magnitude of the input noise signal that points to the multiplicative character of the impact of the noise on the memristors, i.e. the electrical properties of the memristors are affected by the noise signal itself.
4. The evolution of the potential profile of the memristor under the white Gaussian noise signal has been studied. The potential profile of the memristor was found to change abruptly that points to the sudden changes of the memristor properties under the impact of the noise.
5. The effect of noise on the electrical properties of the memristor was attributed to the burning out of the active filaments and the wakeup of the potential ones.
6. The results of the present study indicate the memristors to be more complex dynamic system than in a system with a simple two-well fixed potential. To work out more adequate theoretical description of the real memristors on the basis of the statistical physics approach, further development of the theory as well as further experiments on its verification are necessary.

Many questions

1. How do we take into account the thermal noise and flicker noise in our models of memristors?

2. Which is the response of memristor on a stochastic input?

3. Should we construct different stochastic models for different types of memristors?

4. Which statistical characteristics do we have to investigate: current, voltage, resistance,…