Fundamental limitations of the step quantum heat engine

Marcin Łobejko, Paweł Mazurek, Michał Horodecki

Intitute of Theoretical Physics and Astrophysics, University of Gdańsk, Gdańsk, Poland

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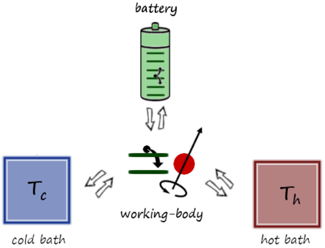
Step Quantum Heat Engine

$H_t = H_0 + V_t$

$H_0 = H_S + H_H + H_C + H_B$

$V_t = \sum_{k=1}^{N} [\lambda_H^{(k)}(t)\ V_{SH}^{(k)} + \lambda_C^{(k)}(t)\ V_{SC}^{(k)} + \lambda_B^{(k)}(t) V_{SB}^{(k)}]$

$$\rho \to U \rho U^\dagger$$



*Skrzypczyk *et al.*, Nat. Commun. 5, 4185
Alhambra *et al.* PRX 6, 041017

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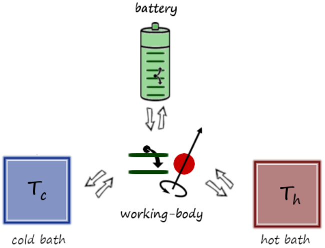

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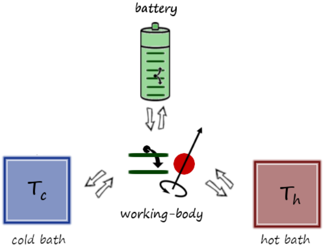

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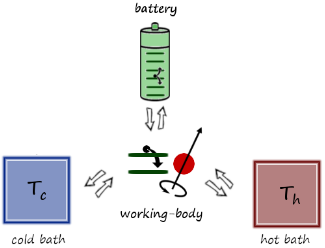

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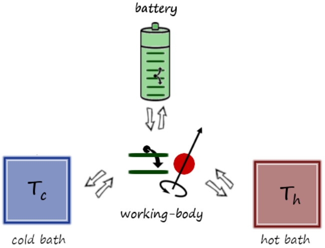

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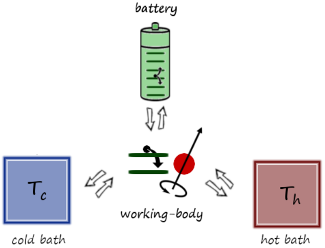

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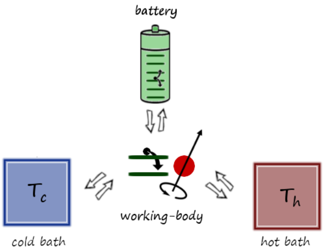

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   $\tau_A = \frac{1}{Z_A} e^{-\beta_A H_A}$
5. Cyclic working body (qubit)
   $\mathrm{Tr}_{H,C,B}[\rho] = \mathrm{Tr}_{H,C,B}[U \rho U^\dagger]$

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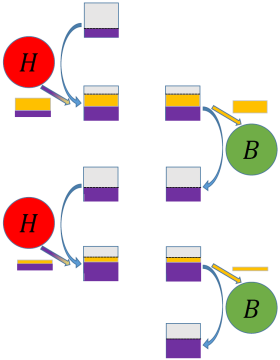
Single heat bath - work extraction

Heat $Q = \mathrm{Tr}[H_H(\rho - U\rho U^\dagger)]$

Work $W = \mathrm{Tr}[H_B(U\rho U^\dagger - \rho)]$



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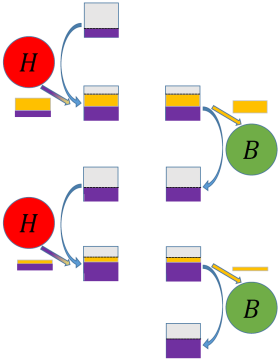

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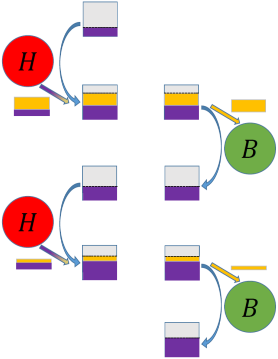

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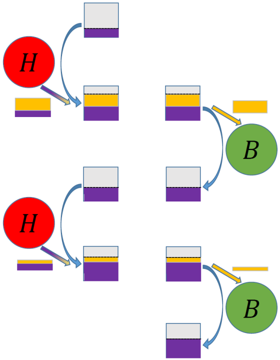

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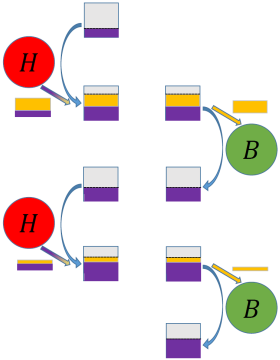

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- Ergotropy extraction U_H

  $\Delta R_S \leq 2(\omega - \epsilon)e^{-\beta_H \omega} - \omega$

  $\Delta P_S \geq (\omega - \epsilon)(1 - e^{-\beta_H \omega})$

  (thermal operations†)

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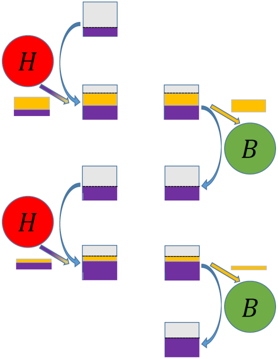

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- Clausius inequality

  $Q < T\Delta S_S$ irreversibility!

  and Second Law, $W < -\Delta F_S$

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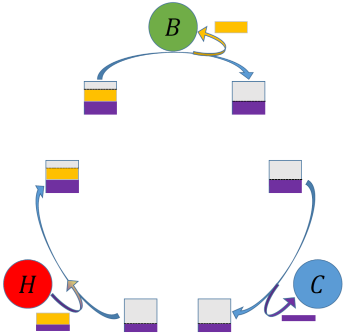
Two heat baths - heat engine



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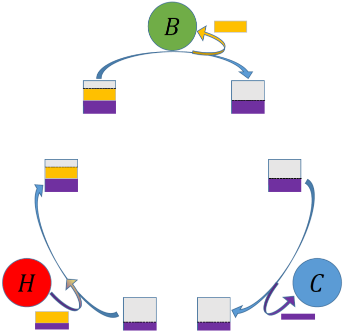

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- *Battery* is used to ergotropy storage.
- *Cold bath* is used to releasing passive energy.

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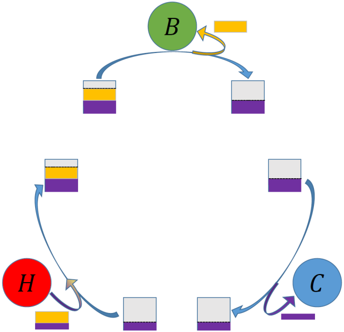

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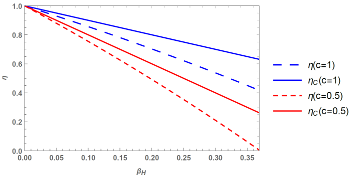
Maximal efficiency:

$$\eta = \frac{2 - e^{-\beta_C \omega} - e^{\beta_H \omega}}{1 - e^{-\beta_C \omega}} < \eta_C$$

Power:

$$P = \omega \left[\frac{2 e^{-\beta_H \omega}}{1 + e^{-(\beta_C + \beta_H)\omega}} - 1 \right]$$

Figures



Efficiency vs Carnot efficiency

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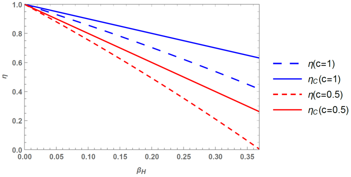

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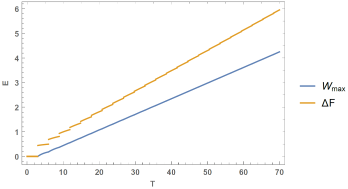

Work vs Free energy

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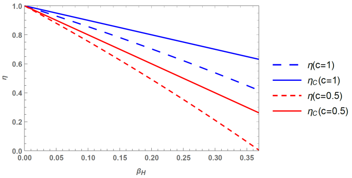

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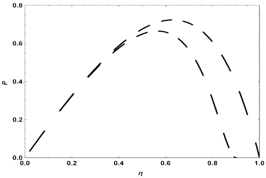

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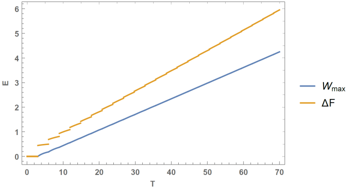

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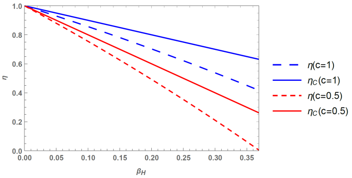

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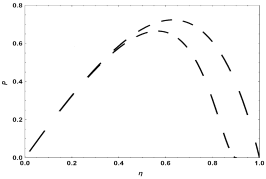

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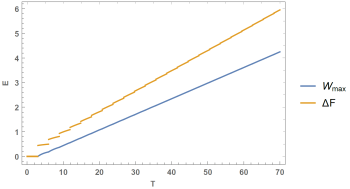

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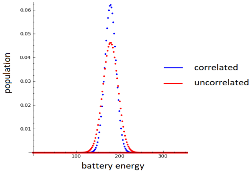

Fluctuations vs Correlations

Summary

- Charging battery is limited by the ergotropy.
- Extraction ergotropy from the heat bath is possible, but unavoidably accompanied with passive energy flow.
- Step operations are fundamentally irreversible (worse than Carnot).
- Correlations between working body and battery can reduce fluctuations of work.

Thanks for your attention!
(paper soon on arXiv)