

Theoretical and experimental determination of a boundary condition at a thin membrane for diffusion

We present a method of deriving a boundary condition for diffusion at a thin membrane from experimental data. Within this method the Laplace transform of a boundary condition is assumed to be in the form

$$\hat{C}_2(0^+, p) = \hat{\Phi}(p)\hat{C}_1(0^-, p),$$

where $\hat{\Phi}(p)$ is a function to be determined. Next, we find the Laplace transform of some theoretical function containing Φ , which is a relatively easy to measure experimentally. Then, this function is also determined by means of a numerical calculation of the Laplace transform of the experimental data obtained for normal diffusion of ethanol in water in a system with a nephrophan membrane. Finally, comparing both Laplace transforms mentioned above, we find the function Φ . The derived boundary condition at a membrane contains a term with a Riemann-Liouville fractional time derivative

$$\alpha C_2(0^+, t) + \beta \frac{\partial^{1/2}}{\partial t^{1/2}} C_2(0^+, t) = C_1(0^-, t).$$

Such a form of the boundary condition shows that particles transfer through a thin membrane is a “long-memory process.” The presented method is an example that an important part of the mathematical model of physical processes may be derived directly from experimental data.

This work was partially supported by the Polish National Science Centre under Grant No. 2014/13/D/ST2/03608.

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