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Generalized Poisson-Kac processes in statistical physics, thermodynamics and transport

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Langevin equations driven by vector-valued Wiener noise represent the prototypical model of evolution equations for a physical system driven by a deterministic velocity field in the presence of superimposed stochastic fluctuations. The statistical nature of a Wiener process can be regarded as the natural legacy of a large number ansatz, in which the effects of many unknown and uncorrelated perturbations justifies the Gaussian nature for the increments of the stochastic forcing. Analogously, in dealing with stochastic field equations (stochastic partial differential equations), $\partial\phi(\mathbf{x},t)/\partial t = \mathcal{N}[\phi(\mathbf{x},t)] + a(\phi(\mathbf{x},t))\, f_s(\mathbf{x},t)$, the most common assumption for the stochastic spatio-temporal forcing $f_s(\mathbf{x},t)$ is its delta-correlated nature in space and time ("derivative of a Wiener process").

Notwithstanding the analytical advantages, the assumption of stochastic perturbation of Wiener nature entails some intrinsic shortcomings. The most striking one is the unbounded speed of propagation of stochastic perturbations that, at a microscopic level, is one-to-one with the fractal nature (almost nowhere differentiability) of the graph of a generic realization of a Wiener process. The resolution of the infinite propagation velocity problem has been proposed by C. Cattaneo in the form of a hyperbolic diffusion equation, now bearing his name. In 1974 M. Kac provided a simple stochastic model, for which the associated probability density function is a solution of the Cattaneo equation. In point of fact, it is well known that the Cattaneo model in spatial dimension higher than one does not admit any stochastic interpretation and that the solutions of the Cattaneo model do not preserve positivity.

In order to overcome this problem and to provide a stochastic background to the extended thermodynamic theories of irreversible phenomena, the original Kac model has been recently extended and generalized in any spatial dimension via the concept of Generalized Poisson-Kac (GPK) processes. In this presentation, after a brief review of GPK theory we discuss some new results and applications in statistical physics.

Specifically:

(i) Motivated by the title of the present conference "On the Uniformity of Laws of Nature", it is addressed how Poisson-Kac and GPK processes permit to resolve the "singularities" in the solutions of classical parabolic transport equations. This is not only related to the resolution of the paradox of infinite propagation velocity,

but involves also the description of boundary-layer dynamics and the group properties of the associated Markov operator.

(ii) The latter issue is closely related to the intrinsic “spinorial” statistical description of GPK processes, that naturally emerges from the relativistic description of stochastic kinematics.

(iii) It is addressed how the application of GPK fluctuations in stochastic partial differential equation ensures the preservation of positivity of the field variable (if required by physical principles, for instance whenever $\phi(\mathbf{x}, t)$ represents a concentration) and avoids the occurrence of diverging correlation function, problem that arises even in the simplest (linear) stochastic partial differential equations in the presence of delta-correlated noise fields. The most striking example is the Edwards-Wilkinson model in spatial dimensions higher than one.

(iv) Finally, the application of GPK is addressed in connection with the modeling of systems of interacting particles.

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