# High precision nucleon matrix elements based on the unbiased estimate of all-to-all fermion propagators


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CLQCD


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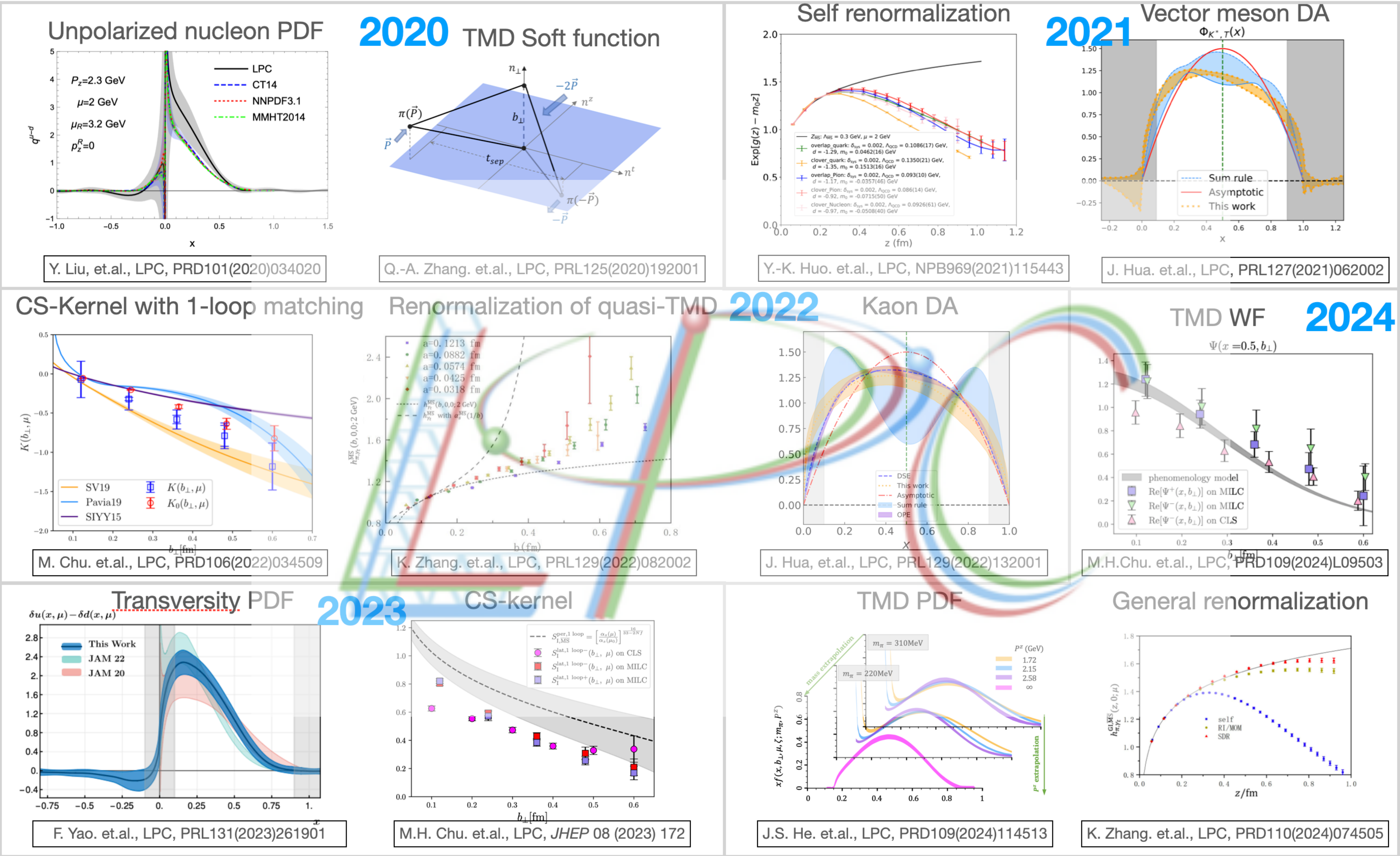
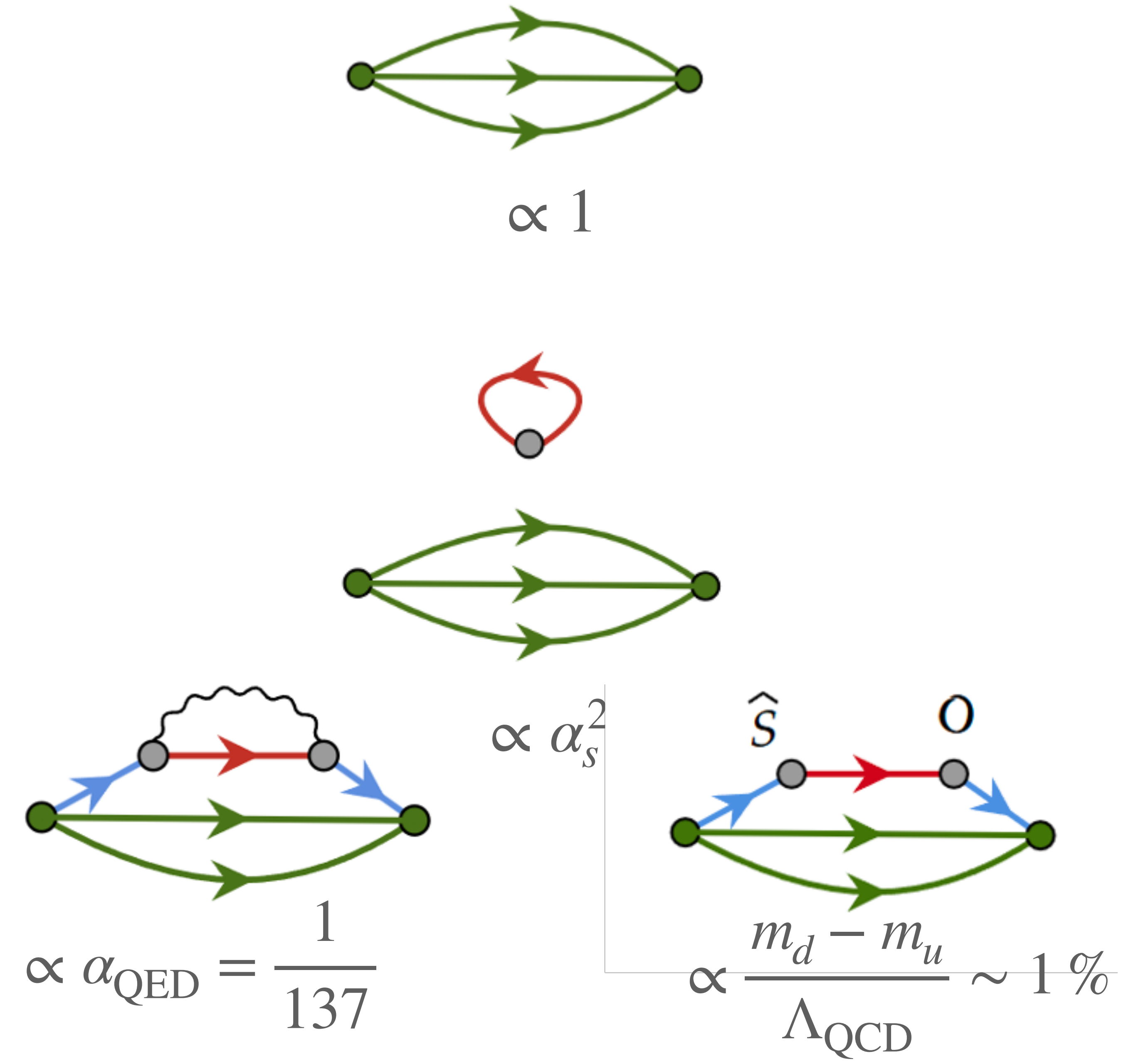
# Background

## High precision hadron structure



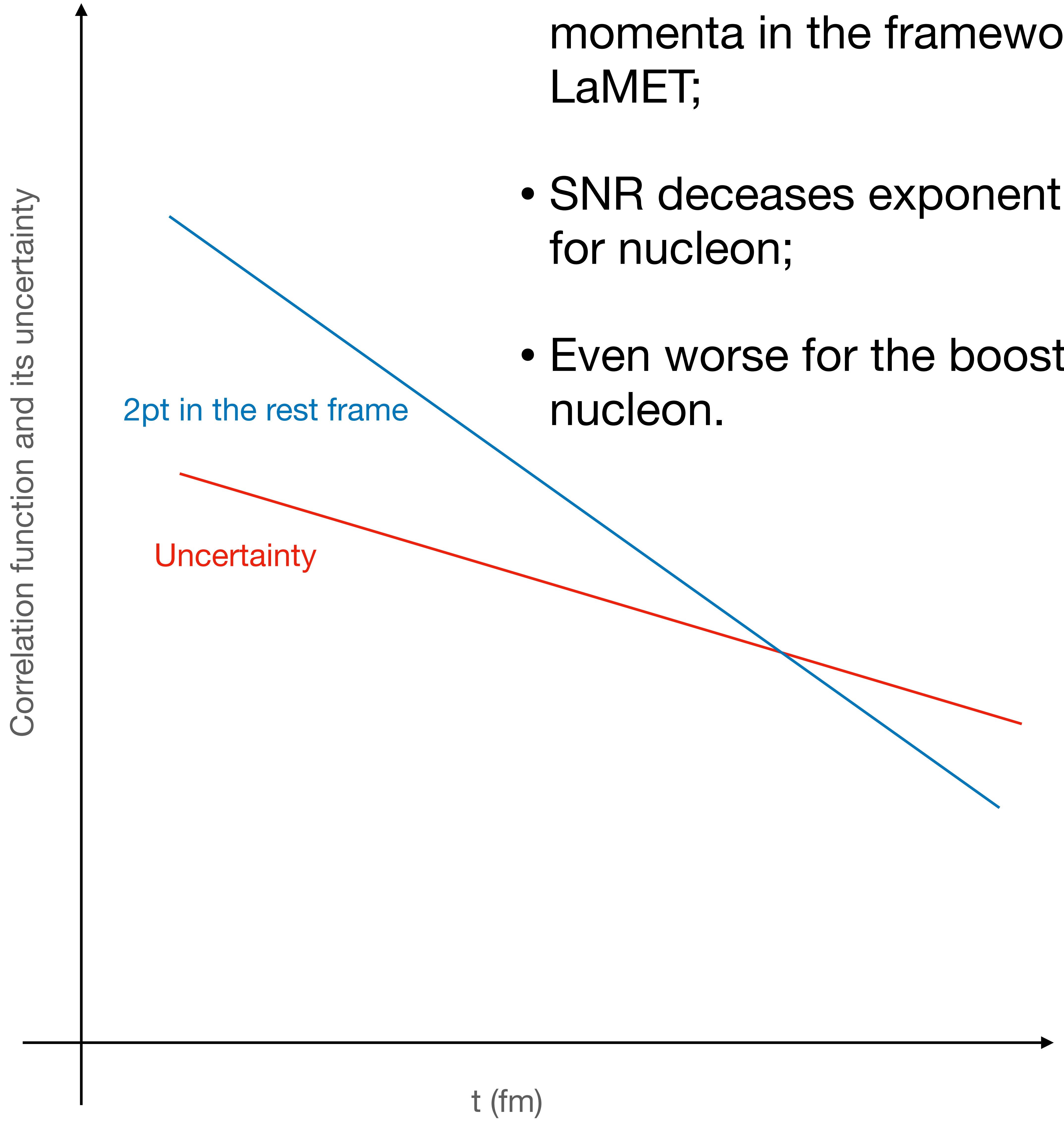




- Lattice QCD calculations of hadron structure have reached the precision frontier, driven by a variety of recent theoretical innovations;
- Technical improvement from lattice become more important to satisfy the increasing need, plus the control on kinds of sub-leading corrections.

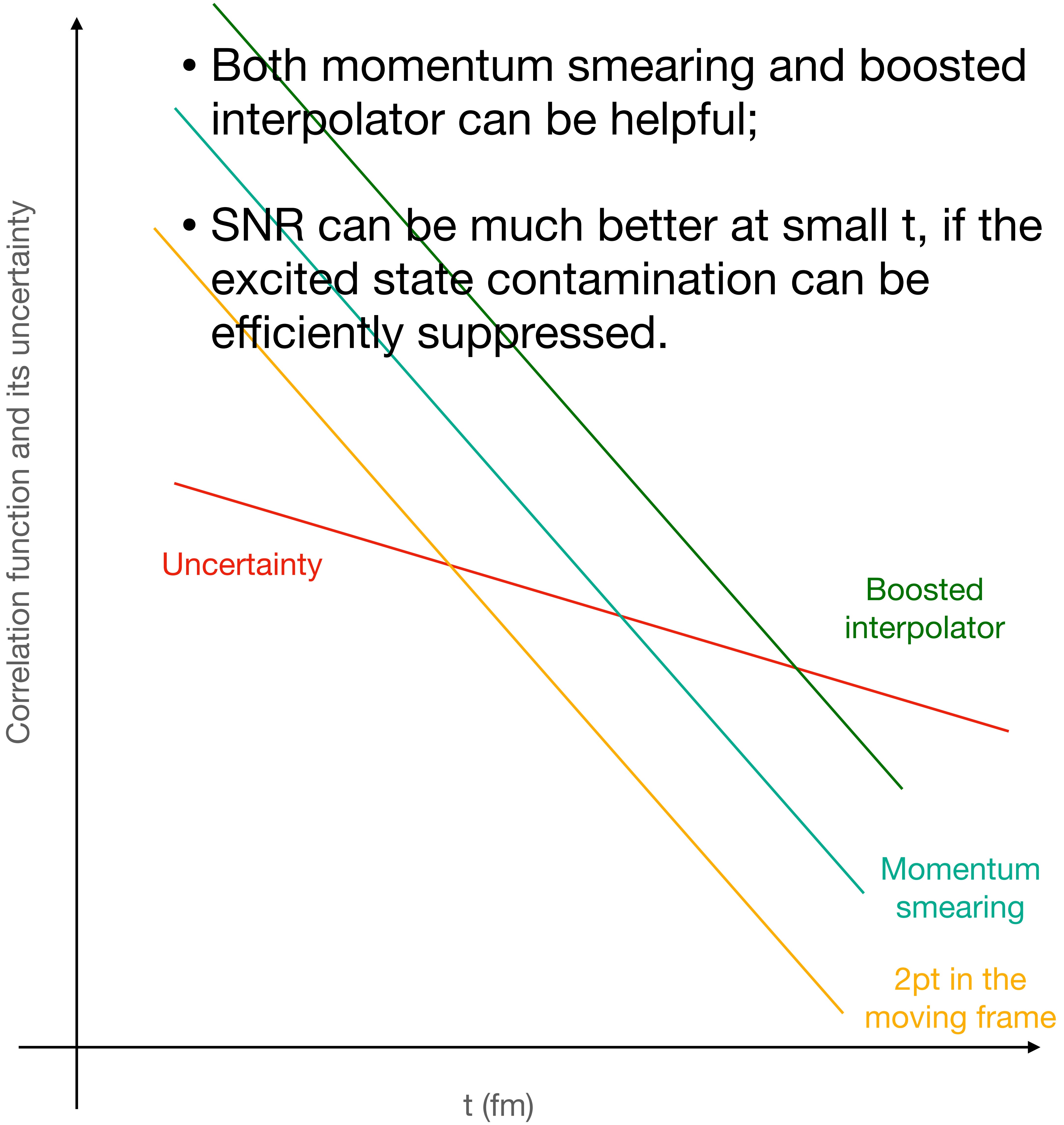
# Background

High precision LaMET calculations

- Hadrons should be boosted to large momenta in the framework of LaMET;
- SNR deceases exponentially on t for nucleon;
- Even worse for the boosted nucleon.

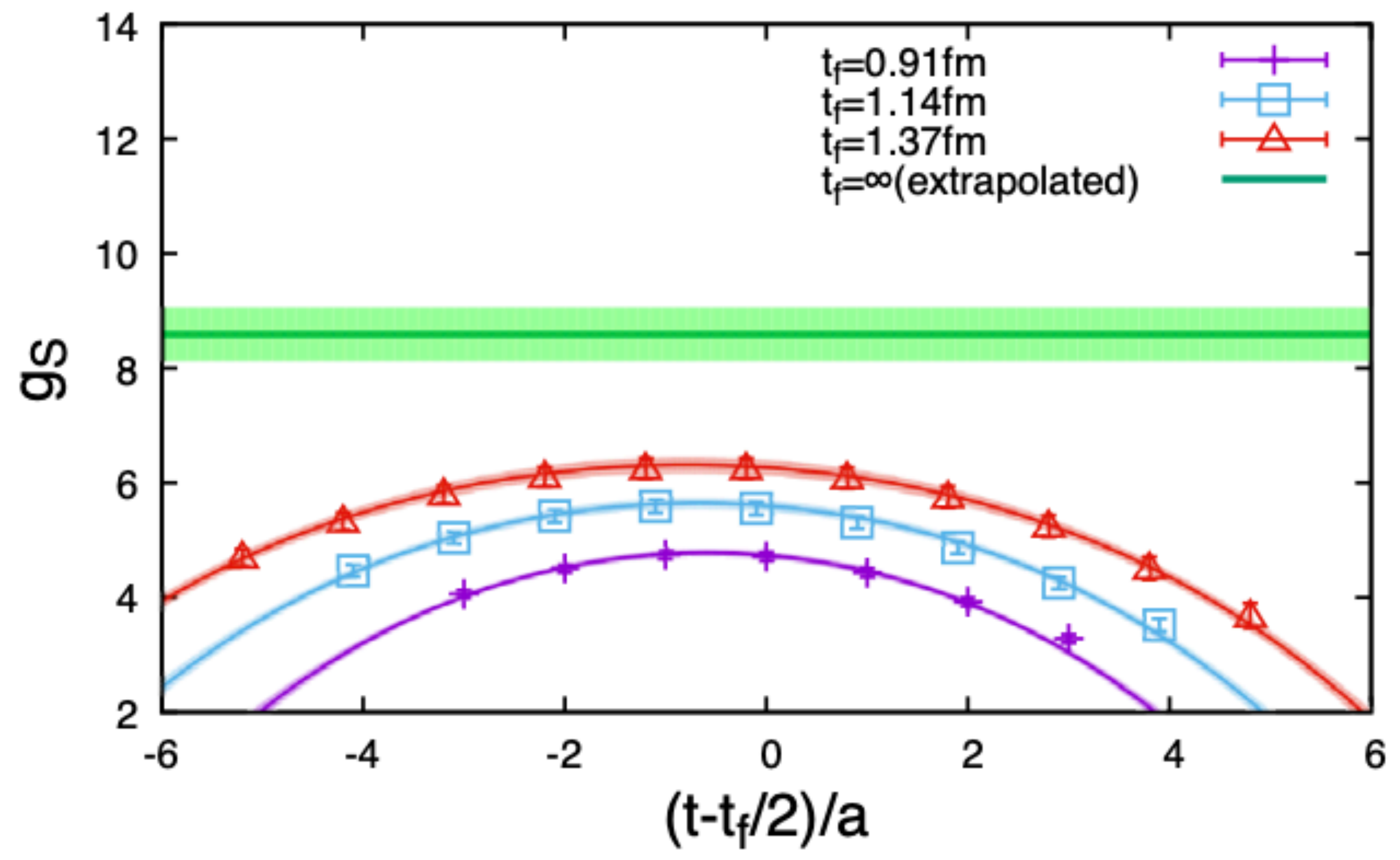



- Both momentum smearing and boosted interpolator can be helpful;
- SNR can be much better at small t, if the excited state contamination can be efficiently suppressed.



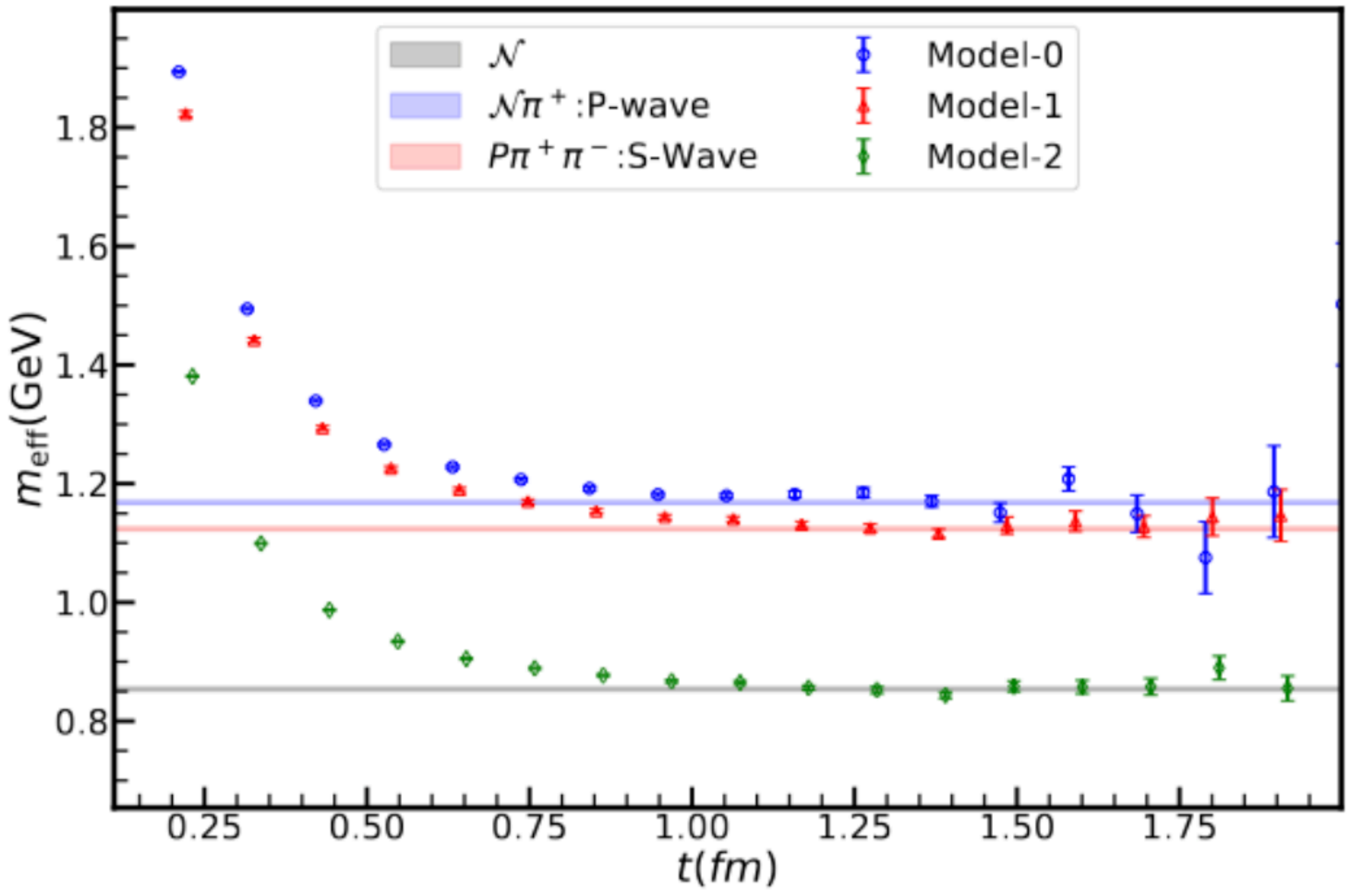
# Background






Excited state contaminations can be sizable and then make the extracted ground state matrix element to be much more noisier than the ratio at finite source-current-sink separations;

But there are so many excited states in the spectrum of the nucleon, it can be extremely challenging to eliminate them one by one.



# Blending method



- The "blending" method projects the identity matrix into exact low momentum modes plus stochastic samples of high momentum modes:

$$\hat{I} = \sum_{i=1}^{\dim \mathscr{L}} |V_i\rangle\langle V_i| \approx \sum_{k=1}^{N_e+N_{\rm st}} \Omega_k^{(1)} |\phi_k\rangle\langle\phi_k|,$$ where

$$\{\phi_i\}_{i=1}^{N_e+N_{\rm st}} \equiv \{v_{\lambda_1}, \ldots, v_{\lambda_{N_e}}, \eta_1, \ldots, \eta_{N_{\rm st}}\}.$$

- The quark bilinear local current without momentum transfer, $\mathcal{O} = \int d^3x \bar{q}(x)\Gamma q(x)$, can be projected onto the blending space:

$$\mathcal{O}_i \equiv \Omega_i^{(1)} \int d^3z \langle \phi_i(z)|\Gamma|\phi_i(z)\rangle = \Omega_i^{(1)}\Gamma.$$

$$\Omega_k^{(1)} = \begin{cases} 1 & \text{for } k \le N_e, \\ \frac{\dim \mathscr{L}_2}{N_{\rm st}} & \text{for } k > N_e. \end{cases}$$

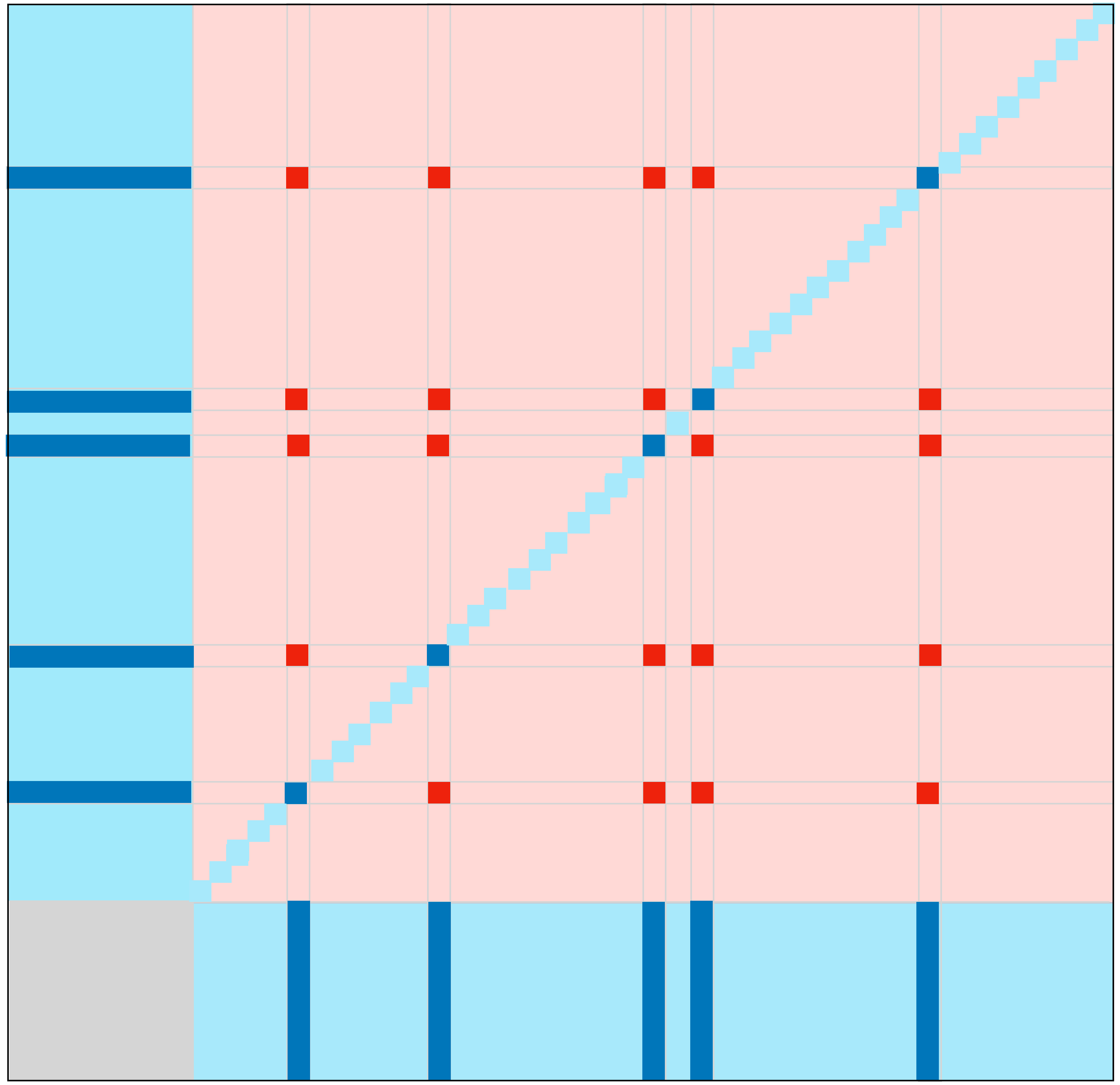
Z.C.Hu, J.H. Wang et. al., arXiv:2505.01719

# Blending method

## More general projected operators

- The quark bilinear local current with momentum transfer, $\mathcal{O} = \int d^3x e^{\mathrm{i}x\cdot p}\bar{q}(x)\Gamma q(x)$, can also be projected onto the blending space:
$$\mathcal{O}_{ij} \equiv \Omega^{(2)}_{ij}\Gamma \int d^3x \langle \phi_i(x) | e^{ip\cdot x} | \phi_j(x)\rangle.$$

- The quark bilinear non-local current without momentum transfer,
$$\mathcal{O} = \int d^3x \bar{q}(x) U(x, x+z)\Gamma q(x+z),$$ can also be projected onto the blending space:
$$\mathcal{O}_{ij} \equiv \Omega^{(2)}_{ij}\Gamma \int d^3x \langle \phi_i(x) | U(x, x+z) | \phi_j(x+z)\rangle.$$



$$\Omega^{(2)}_{ij} = \begin{cases} 1 & \text{for } i, j \le N_e, \\ \Pi^{1}_{i=0}\frac{V - N_e - i}{N_{st} - i} & \text{for } i, j > N_e,\ i \ne j \\ \frac{V - N_e}{N_{st}} & \text{for the other cases,} \end{cases}$$

Z.C.Hu, J.H. Wang et. al., arXiv:2505.01719

# Blending method

## Basic idea

- Then the correlation function can be projected to the subspace of those modes:

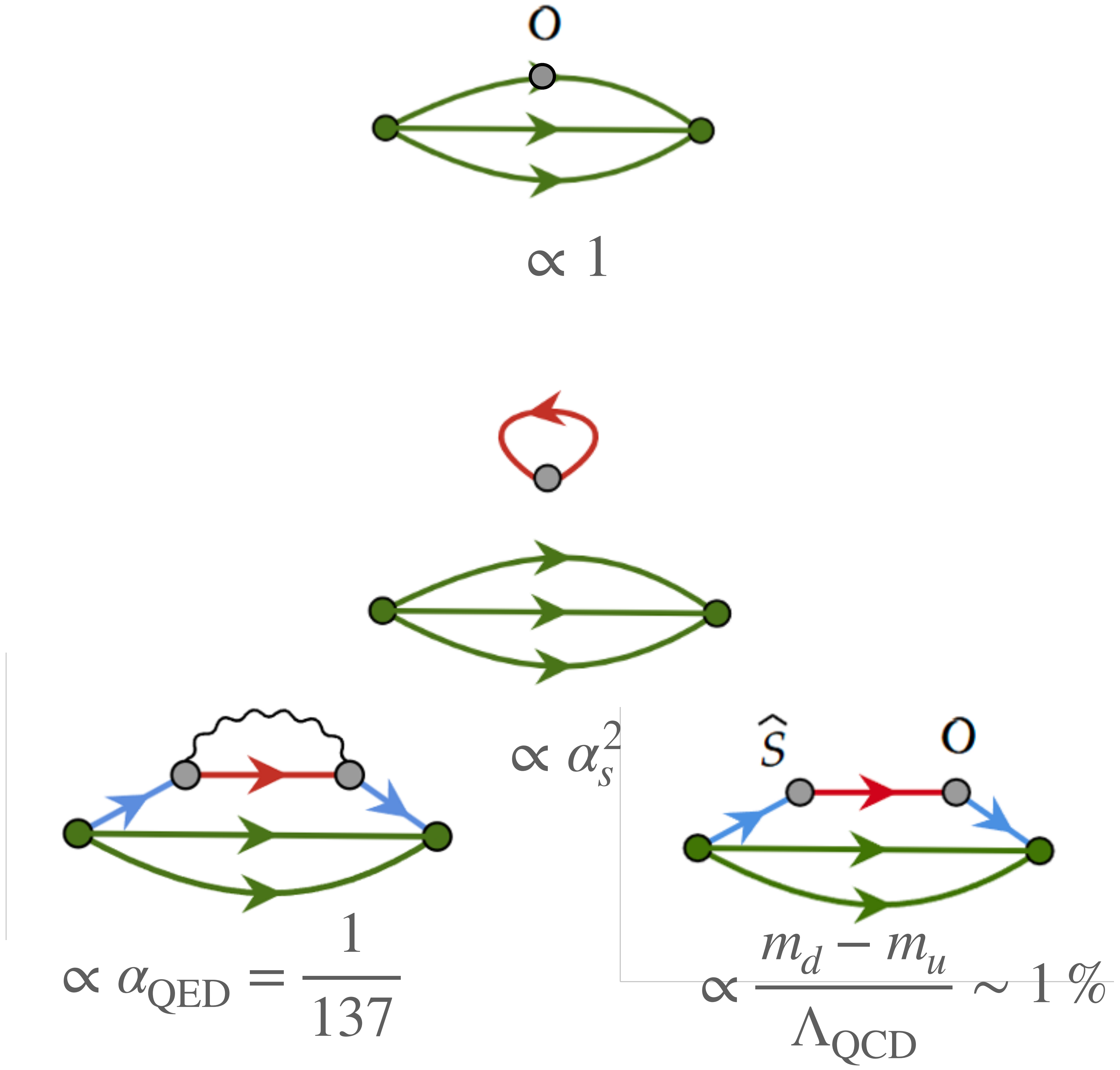
$$\sum_{x,y,z,w} \langle \ldots S(x,y)\mathcal{O}(y,z)S(z,w)\ldots \rangle = \sum_{i,j,k,l} \langle \ldots S_{ij}\mathcal{O}_{ik}S_{kl}\ldots \rangle + \mathcal{O}(\frac{1}{N_{\rm st}})$$

with the projected propagators

$$S_{ij} \equiv \int \mathrm{d}^3z\mathrm{d}^3w \langle \phi_i(z) | S(z,w) | \phi_j(w) \rangle$$

and also the projected operators $\mathcal{O}_{ij}$;

- Thus the hadron matrix element with kinds of high order diagrams (disconnected quark diagram, QED correction, iso-spin breaking effect et al.) can be extracted through the blending method.

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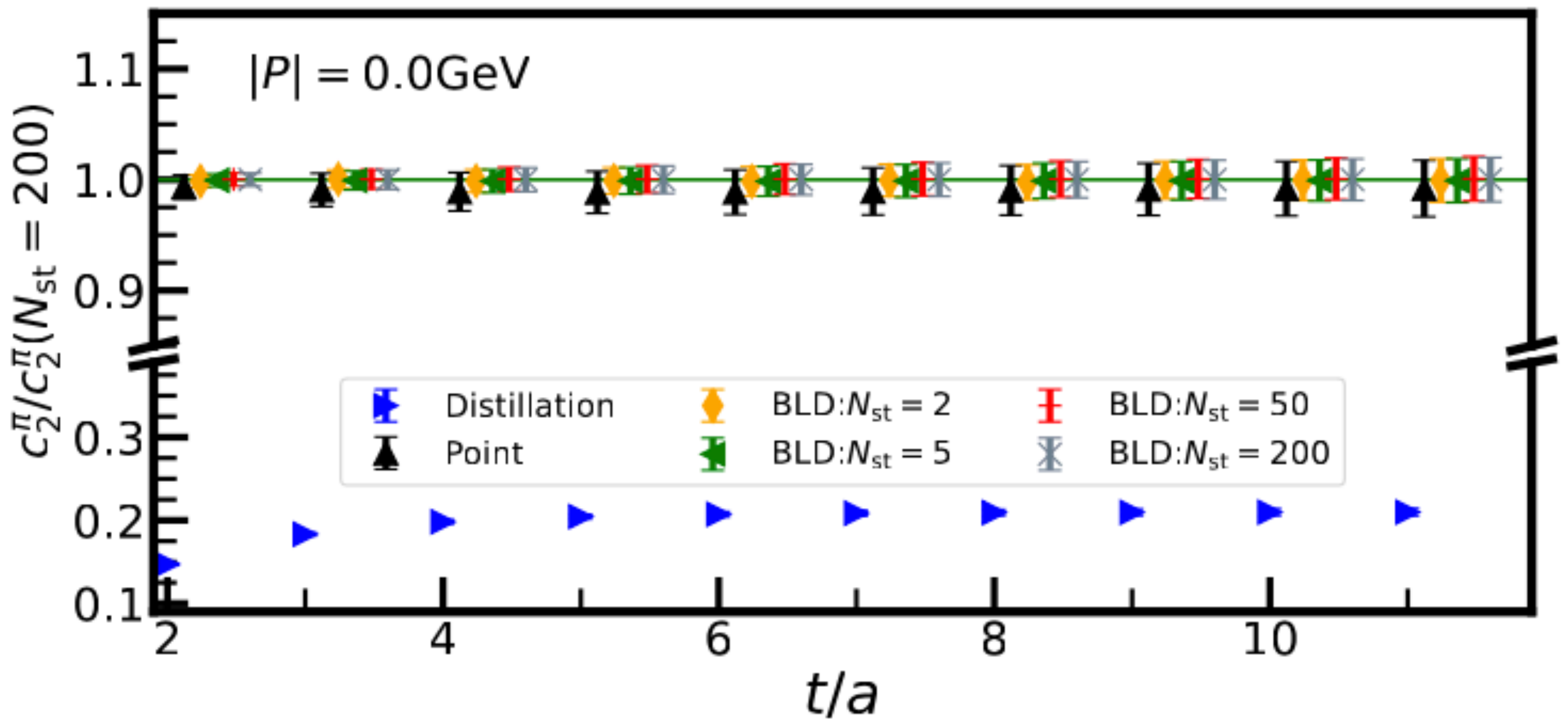
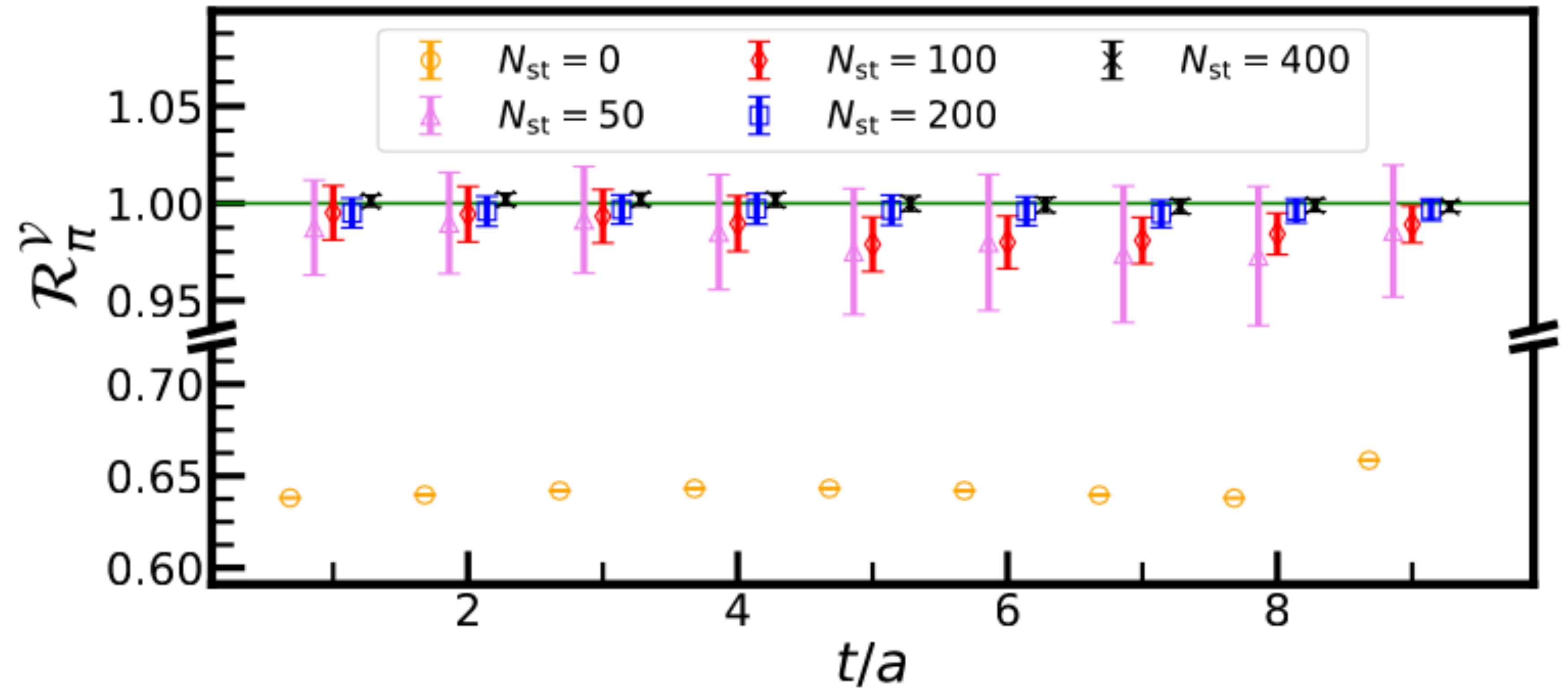
# Blending method



$C_2^{\mathrm{BLD}}(t)/C_2^{\mathrm{pt}}(t) = 1$

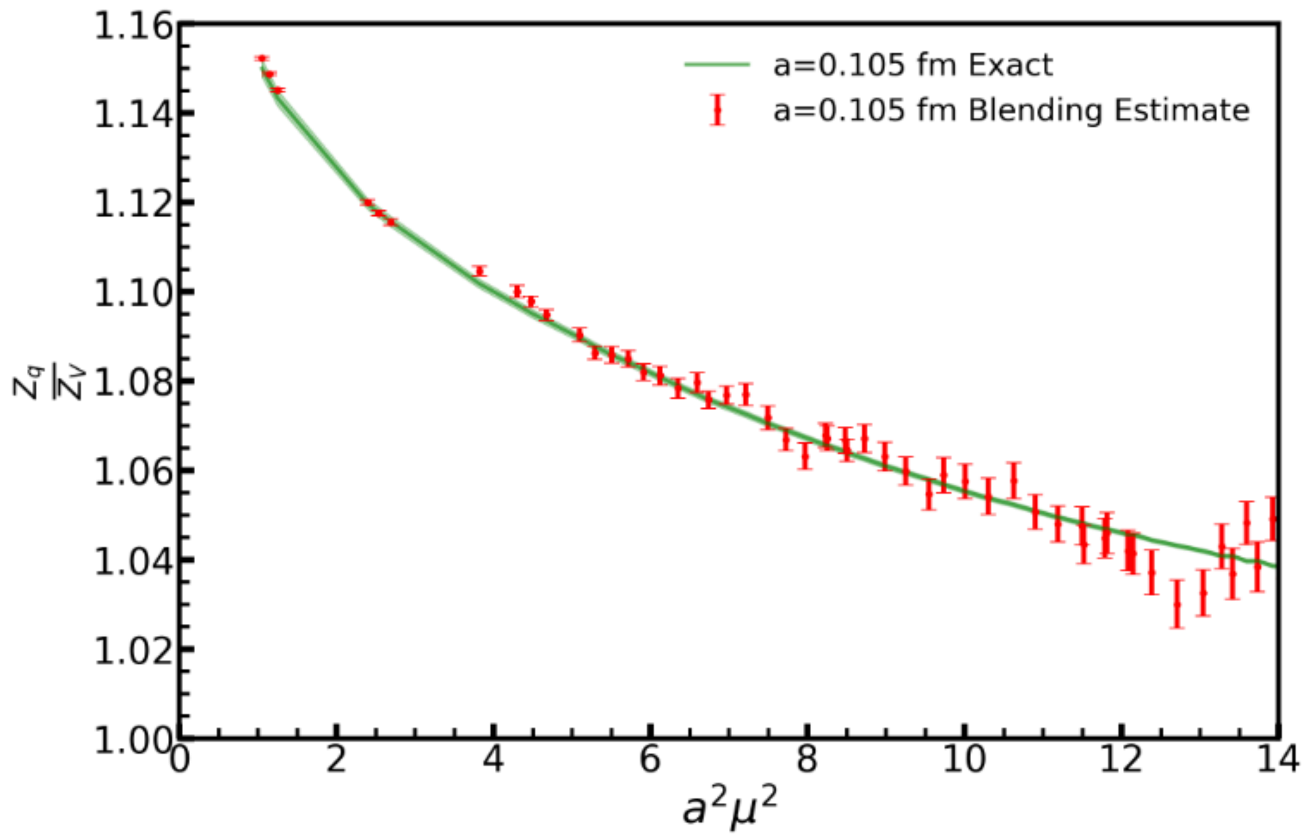



$\langle \mathscr{V}_4^{\mathrm{conserved}} \rangle_\pi = 1$

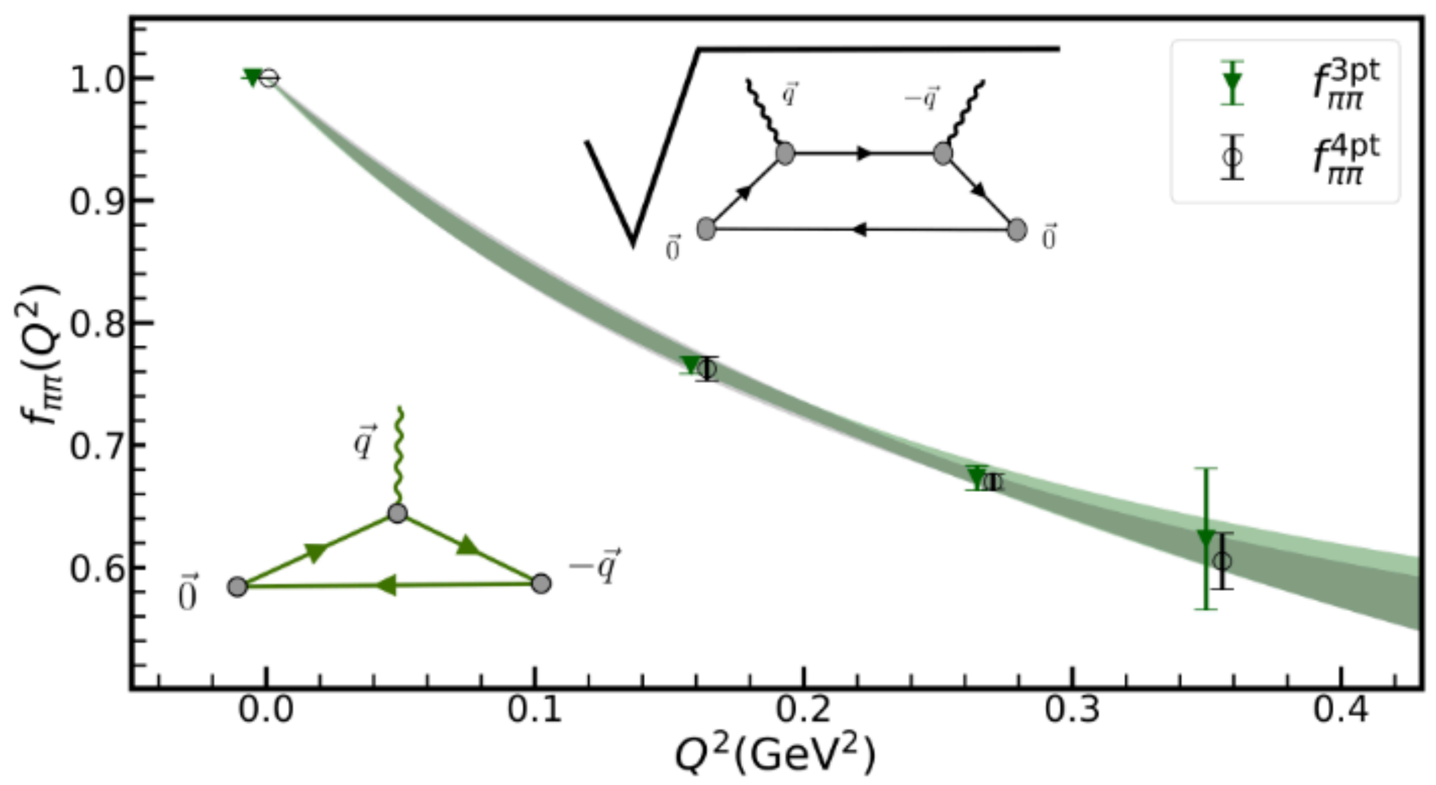



- All-to-all propagator can be approximated unbiasedly with $\mathcal{O}(1\%)$ inversions;
- The high momentum mode turns out to be important in the full correlation functions and can not be ignored.
- Unbiasedness has been verified using two-, three-, and four-point functions.

$Z_q^{\mathrm{BLD}}(a^2\mu^2)/Z_q^{\mathrm{vol}}(a^2\mu^2) = 1$




$f_{\pi\pi}^{\mathrm{3pt,BLD}}(Q^2) = f_{\pi\pi}^{\mathrm{4pt,BLD}}(Q^2)$



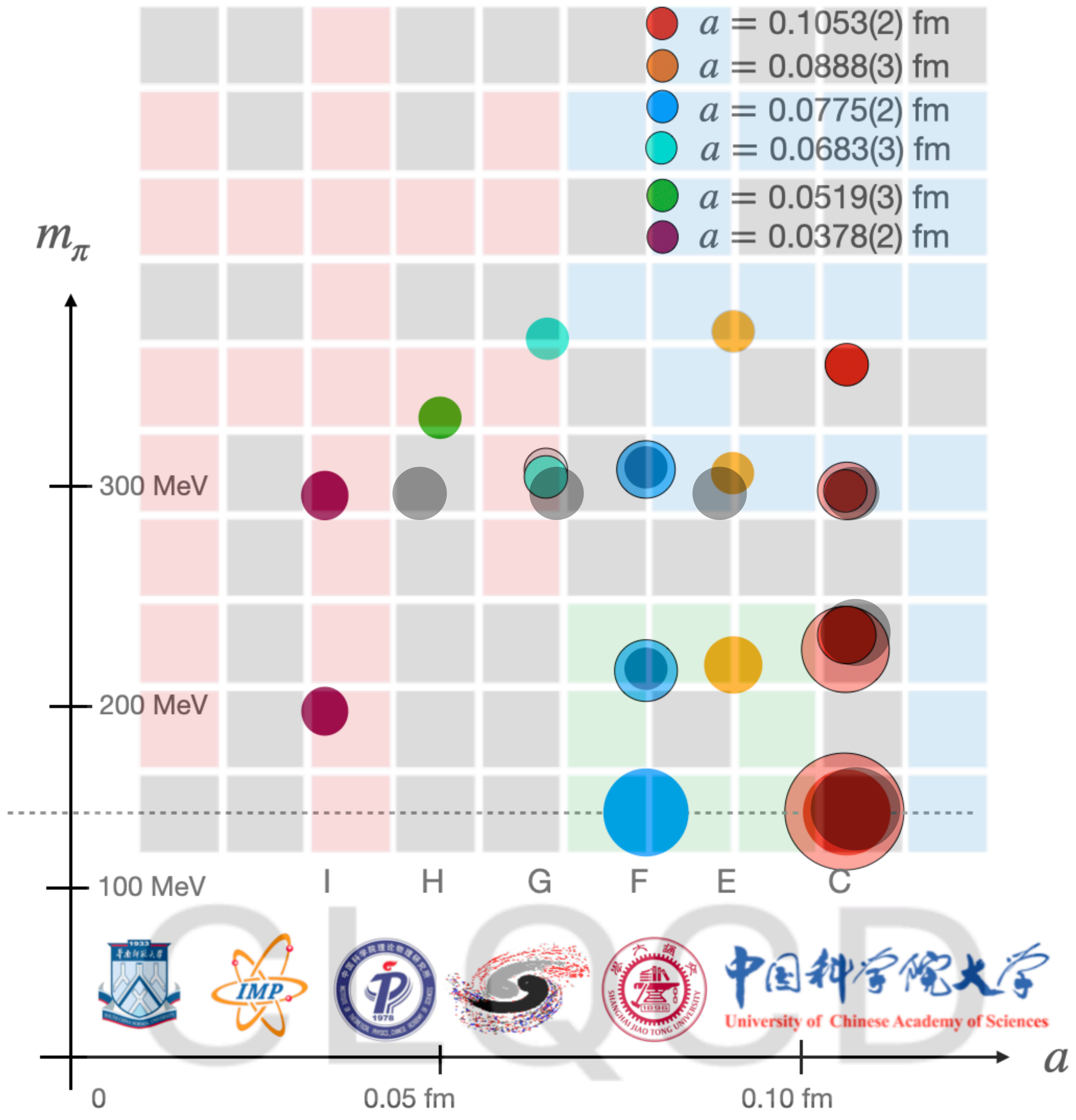

Z.C.Hu, J.H. Wang et. al., arXiv:2505.01719

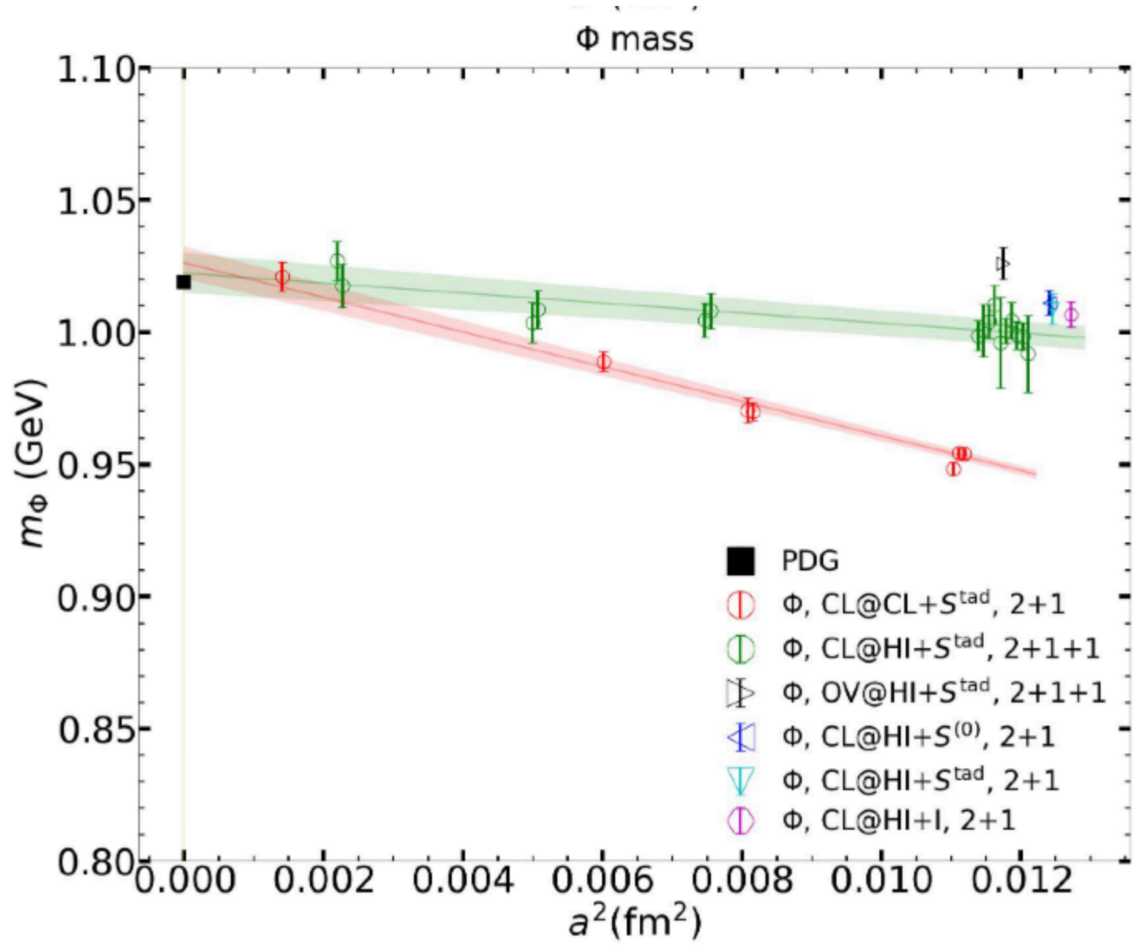
# CLQCD ensembles



| | Country/ Region | Smallest lattice spacing | No. of physical point ensembles | Largest spacial size | No. of fermion discretization |
|---|---|---|---|---|---|
| MILC | US | 0.03 fm | 5 | 5.8 fm | 1 |
| RBC | US | 0.06 fm | 3 | 5.5 fm | 1 |
| BMW | EN | 0.05 fm | 15 | 10 fm | 2 |
| CLS | EN | 0.04 fm | 2 | 5.5 fm | 1 |
| ETM | EN | 0.05 fm | 5 | 6.3 fm | 1 |
| PACS | JP | 0.06 fm | 3 | 10 fm | 1 |
| **CLQCD** | **CN** | **0.04 fm** | **4** | **6.7 fm** | **2** |

- First ensemble set from China which can control most of the systematic uncertainties；
- Unique advantage on finite volume studies.

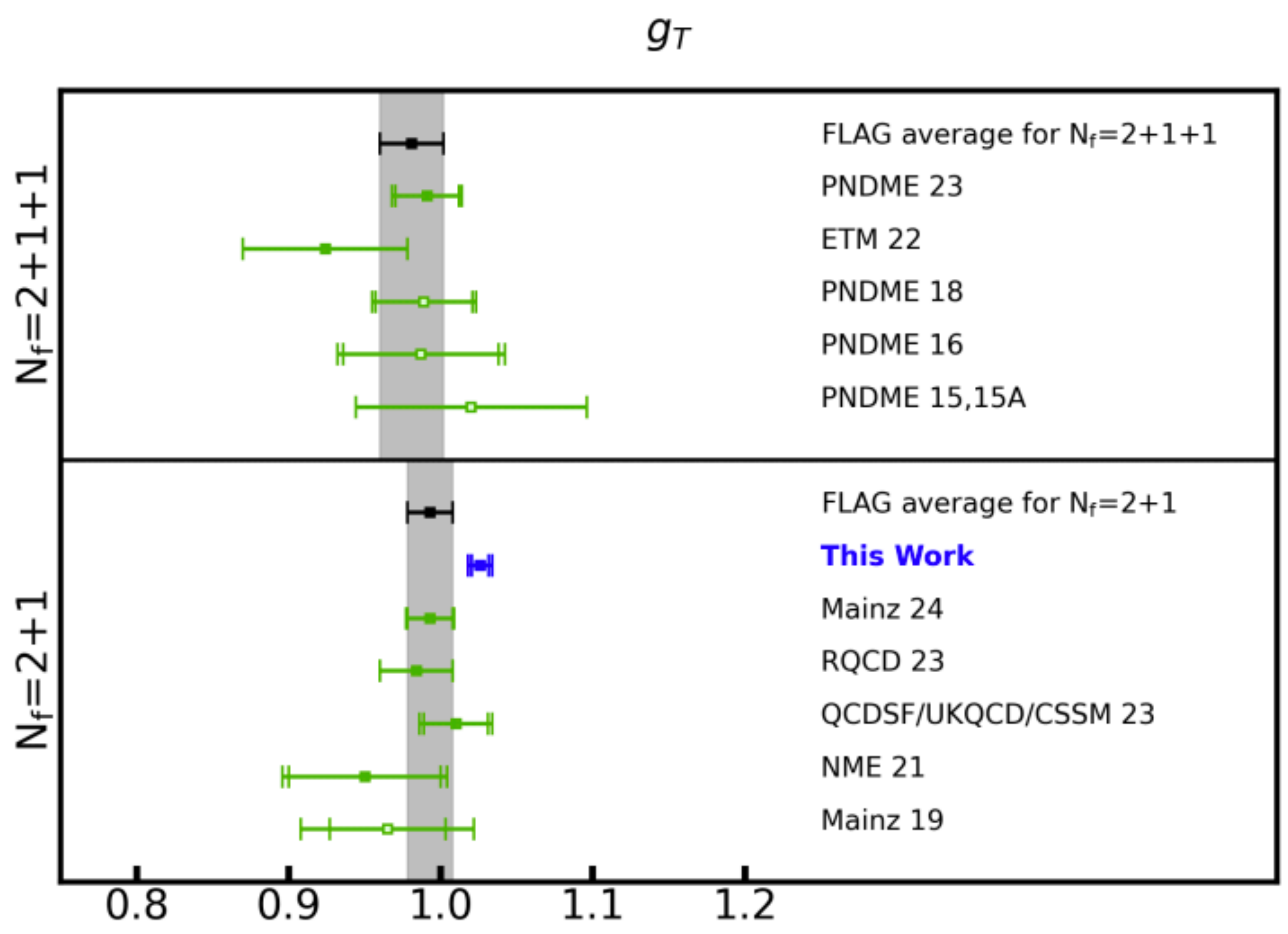


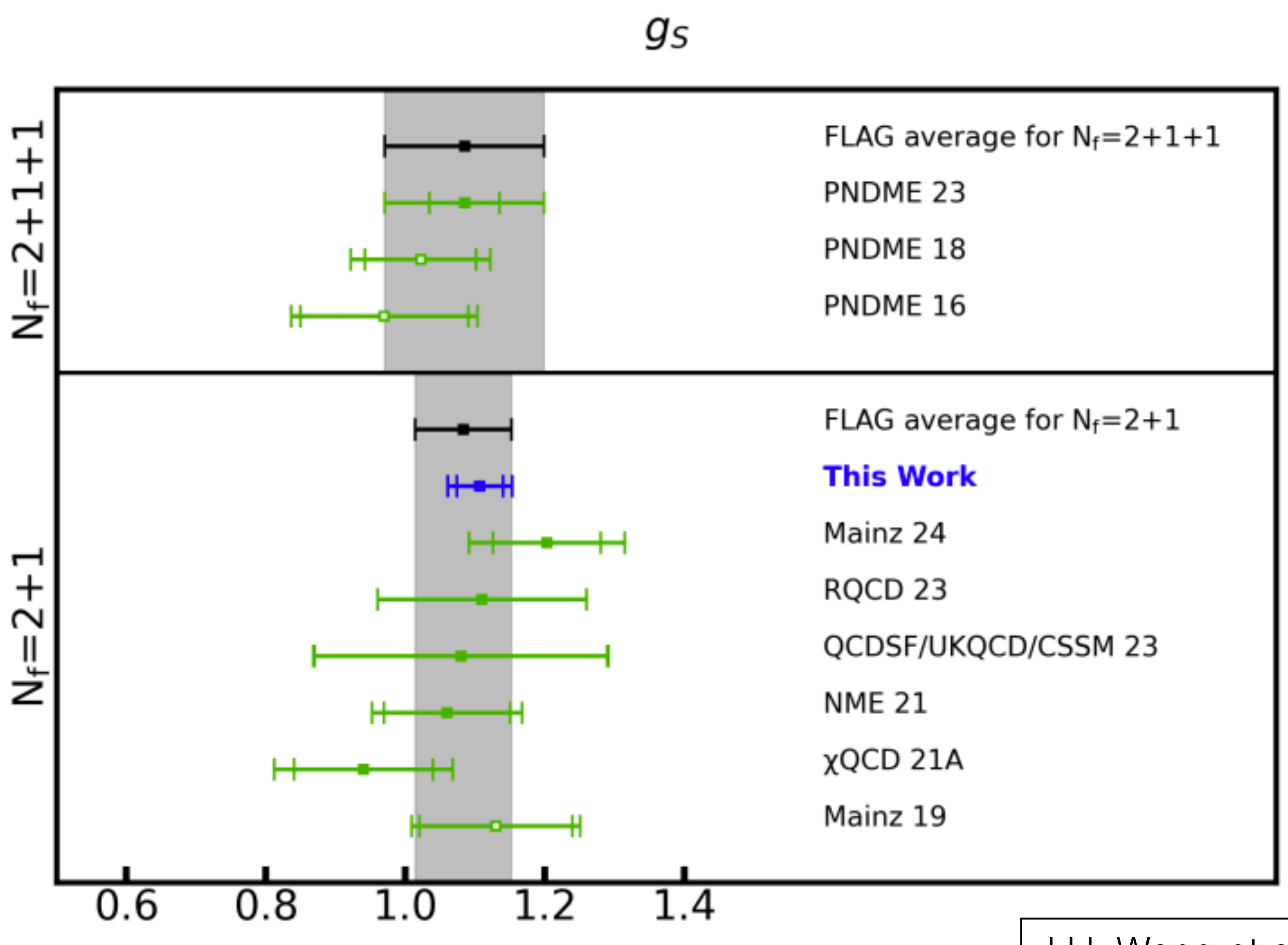




- New ensembles $(\mathrm{HI} + \mathrm{S}^{\mathrm{tad}})$ with 2+1+1 flavor HISQ fermion can provide proper estimate of the charm sea effects;
- Compared to the current 2+1 flavor Clover fermion ensembles $(\mathrm{CL}^{\mathrm{stout}} + \mathrm{S}^{\mathrm{tad}})$, the discretization errors are also suppressed in kinds of the cases.

# CLQCD ensembles

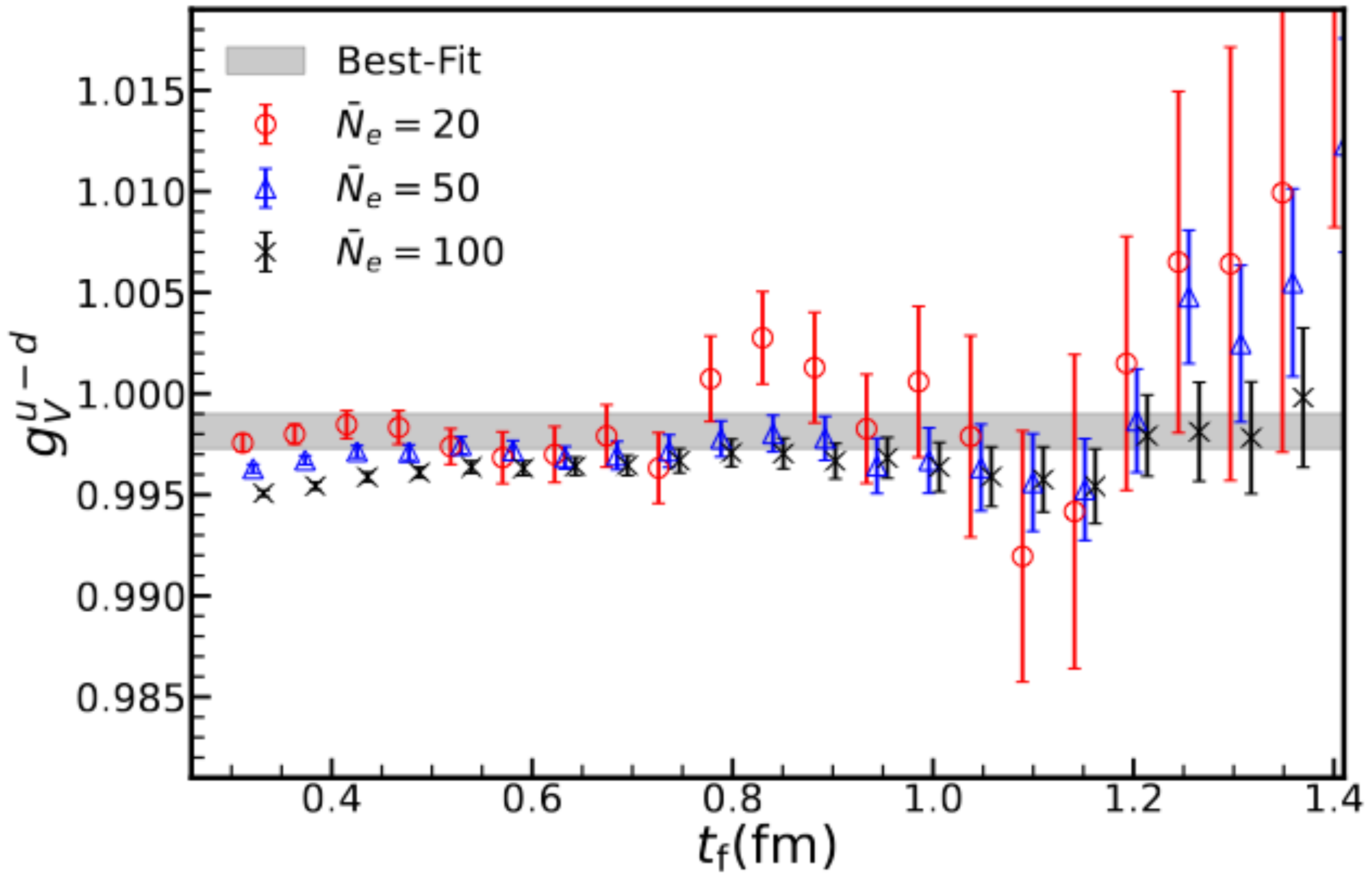


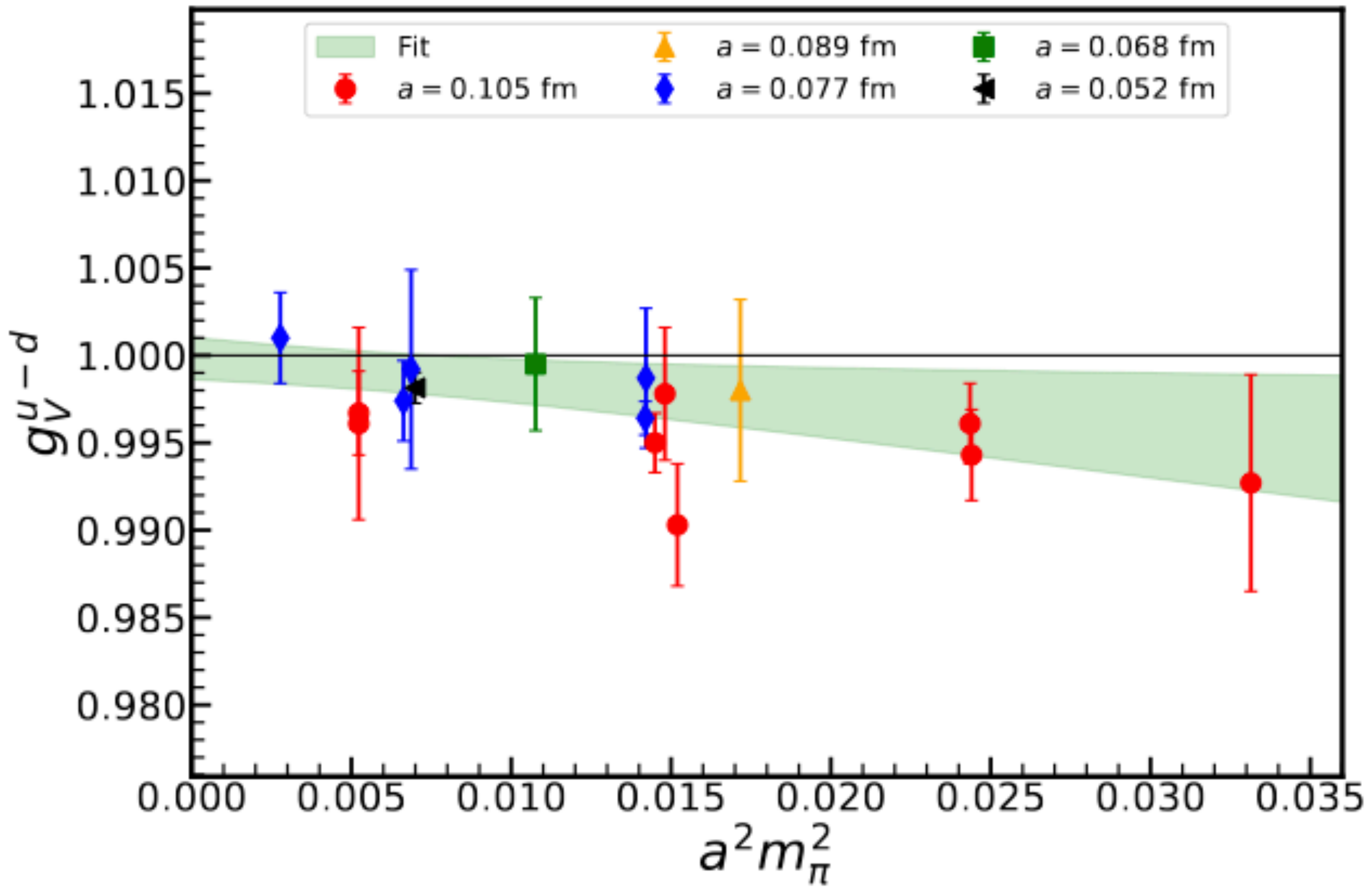




J.H. Wang et al. [CLQCD], arXiv:2511.02326

# Verification on $g_V^{u-d}$




| | $a$(fm) | $n_L^3 \times n_T$ | $m_\pi$(MeV) | $m_\pi L$ | $m_{\eta_s}$(MeV) | $n_{\rm cfg}$ | interval | $\tau$ | $N_{\rm e}$ | $N_{\rm st}$ | $g_V^{2-\rm state}$ | $g_S^{2-\rm state}$ | $g_S^{3-\rm state}$ | $g_T^{2-\rm state}$ | $g_T^{3-\rm state}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| C24P34 | 0.1053 | 24×64 | 341.1(1.8) | 4.38 | 748.61(75) | 48 | 200 | 1.000 | 50 | 250 | 0.9927(62) | 1.036(18) | 1.038(23) | 1.0182(58) | 1.0167(67) |
| C24P29 | | 24×72 | 292.7(1.2) | 3.75 | 657.83(64) | 190 | 200 | 0.707 | 50 | 50 | 0.9943(26) | 1.027(29) | 1.027(37) | 0.9933(37) | 0.9904(45) |
| C32P29 | | 32×64 | 292.4(1.1) | 5.01 | 658.80(43) | 196 | 200 | 0.700 | 60 | 60 | 0.9961(23) | 1.126(13) | 1.091(13) | 1.0099(20) | 1.0007(55) |
| C24P23 | | 24×64 | 229.5(3.0) | 2.93 | 645.67(99 | 169 | 20 | 1.000 | 50 | 50 | 0.9903(35) | 0.880(24) | 0.942(33) | 0.9661(40) | 0.9670(39) |
| C32P23 | | 32×64 | 228.0(1.2) | 3.91 | 643.93(45) | 333 | 50 | 0.700 | 60 | 60 | 0.9978(38) | 1.058(29) | 1.039(19) | 0.9873(37) | 0.9796(38) |
| C48P23 | | 48×96 | 225.6(0.9) | 5.79 | 644.08(62) | 54 | 50 | 0.700 | 100 | 200 | 0.9950(17) | 1.115(19) | 1.107(19) | 1.0008(36) | 0.9881(59) |
| C48P14 | | 48×96 | 135.5(1.6) | 3.47 | 706.55(39) | 56 | 100 | 1.000 | 150 | 200 | 0.9961(55) | 1.029(30) | 1.050(34) | 0.9777(69) | 0.9743(81) |
| C64P14 | | 64×128 | 134.5(1.6) | 4.63 | 706.55(39) | 38 | 20 | 1.000 | 150 | 50 | 0.9967(24) | 1.099(25) | 1.111(27) | 0.9866(85) | 0.9885(75) |
| E32P29 | 0.08973 | 32×64 | 286.7(1.8) | 4.19 | 701.37(92) | 99 | 20 | 1.000 | 50 | 50 | 0.9980(52) | 1.046(30) | 1.012(29) | 1.0121(45) | 1.0103(64) |
| F32P30 | 0.07753 | 32×96 | 303.2(1.3) | 3.56 | 675.98(97) | 91 | 200 | 0.500 | 100 | 200 | 0.9964(10) | 0.954(26) | 1.004(40) | 1.0054(28) | 1.0059(37) |
| F48P30 | | 48×96 | 303.4(0.9) | 5.72 | 674.76(58) | 40 | 100 | 0.500 | 100 | 200 | 0.9987(40) | 1.124(16) | 1.091(17) | 1.0276(24) | 1.0217(35) |
| F32P21 | | 32×64 | 210.9(2.2) | 2.67 | 658.79(94) | 369 | 50 | 0.500 | 50 | 50 | 0.9992(57) | 0.904(34) | 0.910(42) | 0.9683(56) | 0.9685(38) |
| F48P21 | | 48×96 | 207.2(1.1) | 3.91 | 661.94(64) | 150 | 20 | 0.500 | 100 | 60 | 0.9974(23) | 1.084(26) | 1.049(26) | 0.9951(26) | 0.9897(33) |
| F64P13 | | 64×128 | 134.1(1.5) | 3.37 | 681.48(59) | 46 | 20 | 1.000 | 140 | 60 | 1.0010(26) | 0.990(23) | 0.997(29) | 0.9927(64) | 0.9982(39) |
| G36P29 | 0.06887 | 36×108 | 297.2(0.9) | 3.73 | 693.05(46) | 43 | 40 | 1.000 | 60 | 140 | 0.9995(38) | 0.970(14) | 1.005(17) | 1.0142(61) | 1.0120(61) |
| H48P32 | 0.05199 | 48×144 | 317.2(0.9) | 4.00 | 691.88(65) | 46 | 50 | 1.000 | 100 | 200 | 0.9981(09) | 0.919(19) | 0.982(36) | 1.0243(47) | 1.0260(53) |
| $g_{V/S/T}^{\rm physical}$ | | | | | | | | | | | 0.9998(12) | 1.098(27) | 1.106(31) | 1.0236(52) | 1.0264(53) |

- Apply the mass-dependent $Z_V \equiv 1/\langle \pi^+ | \bar{u}\gamma_t u | \pi^+ \rangle$ with unitary pion mass to eliminate the $\mathcal{O}(m_q a)$ correction on the quark bilinear operator of the clover fermion;

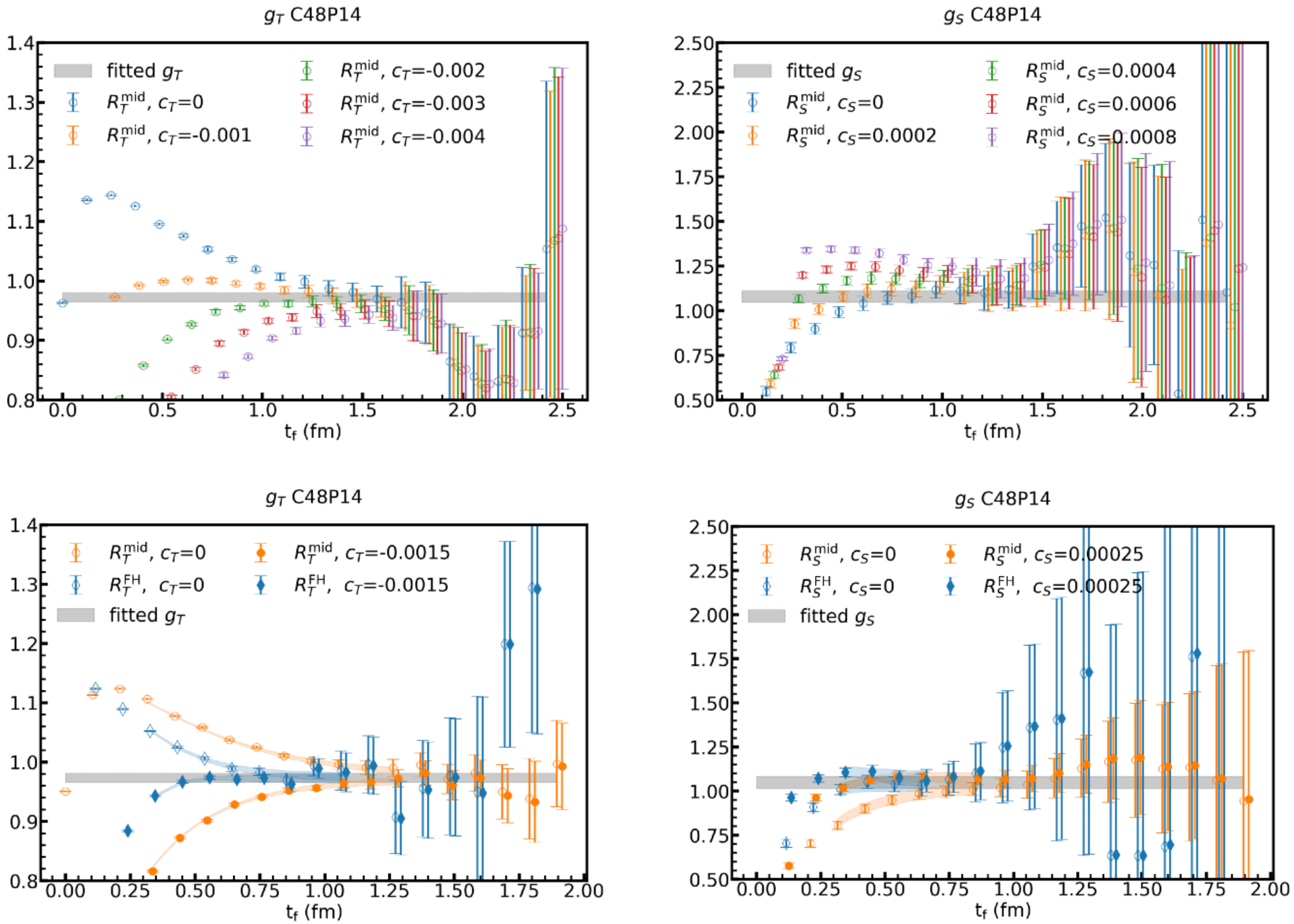
- $g_V(a = 0.052 \text{ fm}, m_\pi = 0.32 \text{ GeV}) = 0.9981(9)$ with 46 configurations;

- The joint fit with the ansatz $g_V^R(m_\pi, a) = g_V^\chi(1 + c_1 m_\pi^2 a^2 + c_2 a^2)$ gives $g_V^\chi = 0.9998(12)$.

J.H. Wang et al. [CLQCD], arXiv:2511.02326

# $g_{S,T}^{u-d}$

# Current-involved interpolator basis

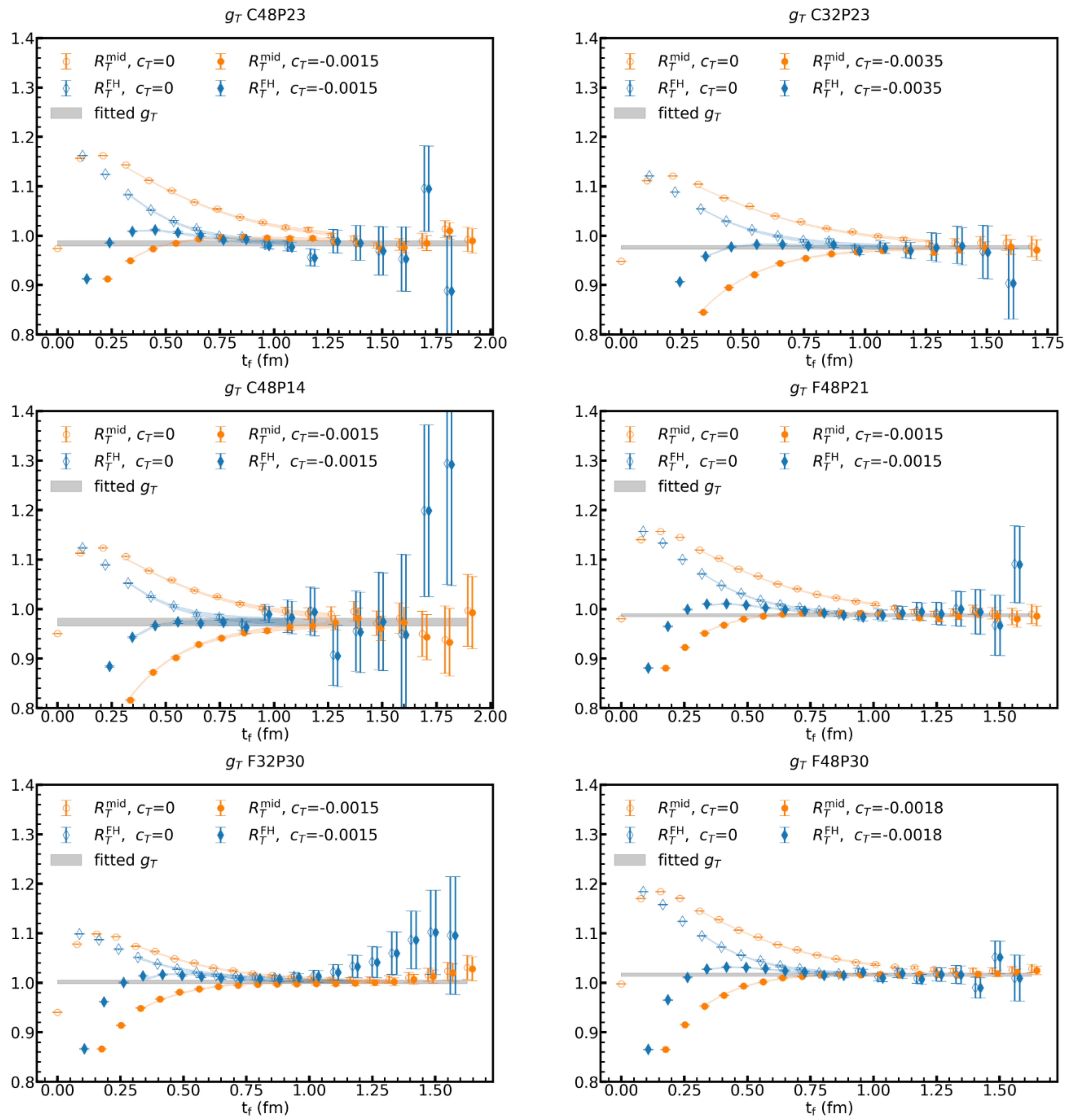



- Excite state contamination can be very sensitive to the mixing coefficient when we use the $\mathcal{N} + c_{S,T}\mathcal{N}_{S,T}$ interpolator with $\mathcal{N}_X = \mathcal{N}\sum_x X(x)$;
- Both the mid-point ratio $R(t_f/2, t_f)$ and Feynman-Hellman summed differential ratio $R_{\rm FH}(t_f)$ can take advantage of the tuning on the mixing coefficient.
- 3-state joint fit with $c_{S,T} = 0$ and $c_{S,T}^{\rm opt} \neq 0$ is applied to avoid the dependence on the optimal tuning of $c_{S,T}^{\rm opt}$.

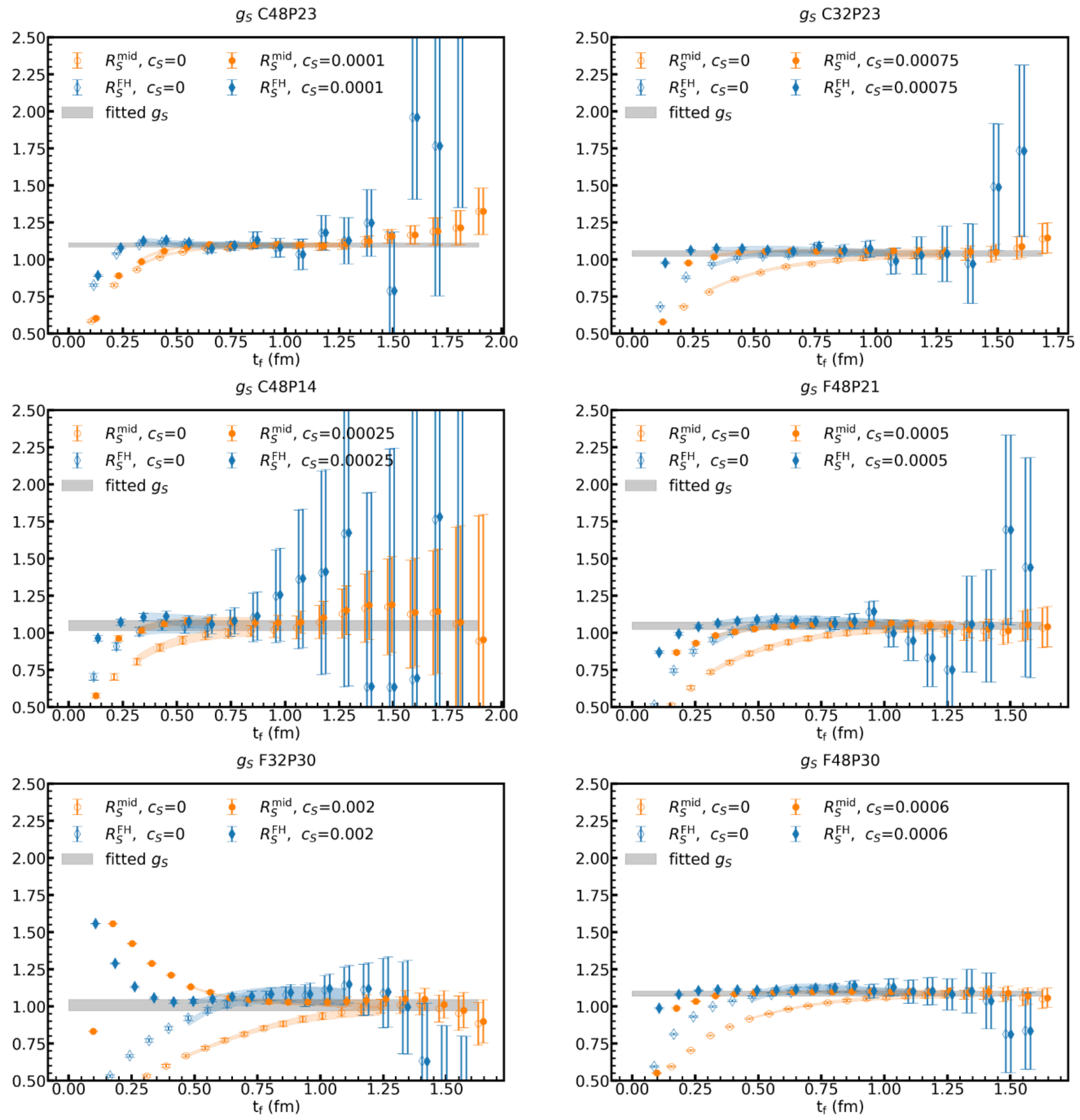
J.H. Wang et al. [CLQCD], arXiv:2511.02326

# $g_{S,T}^{u-d}$

# Current-involved interpolator basis




The current involved (enhanced) interpolator $\mathcal{N}_X = \mathcal{N} \sum_x X(x)$ is always important regardless the pion mass, lattice spacing, volume, and the current involved.

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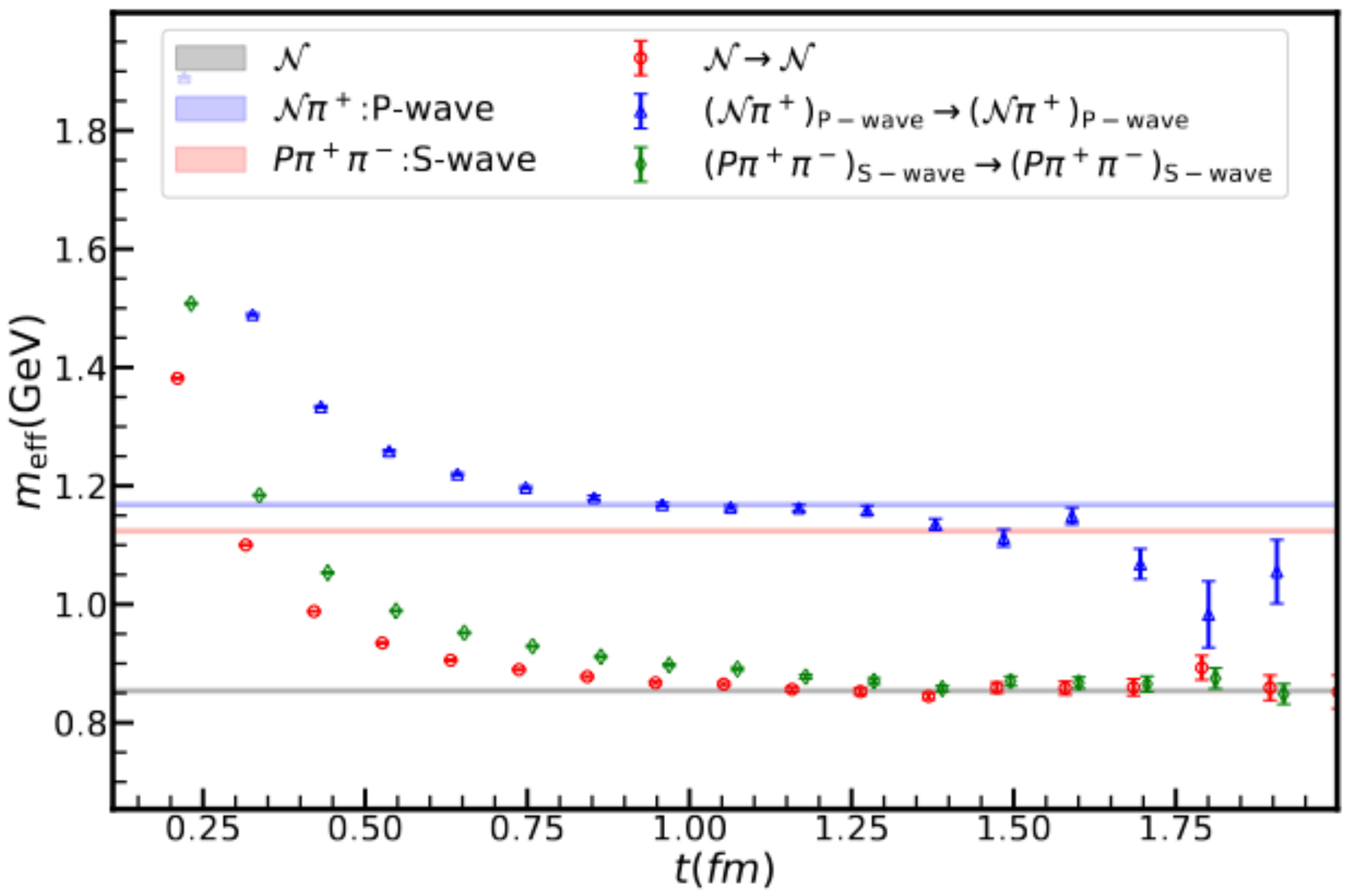
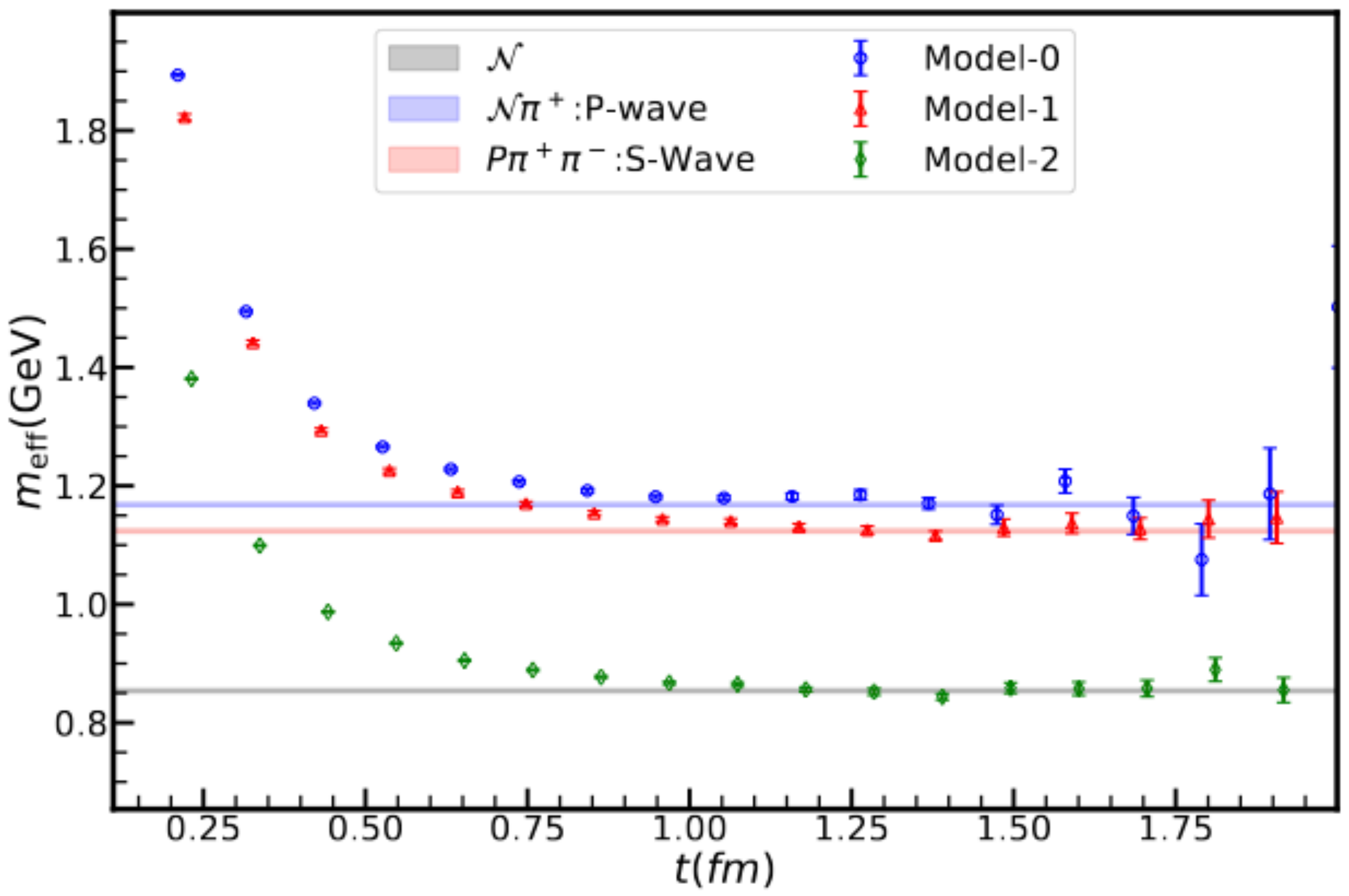
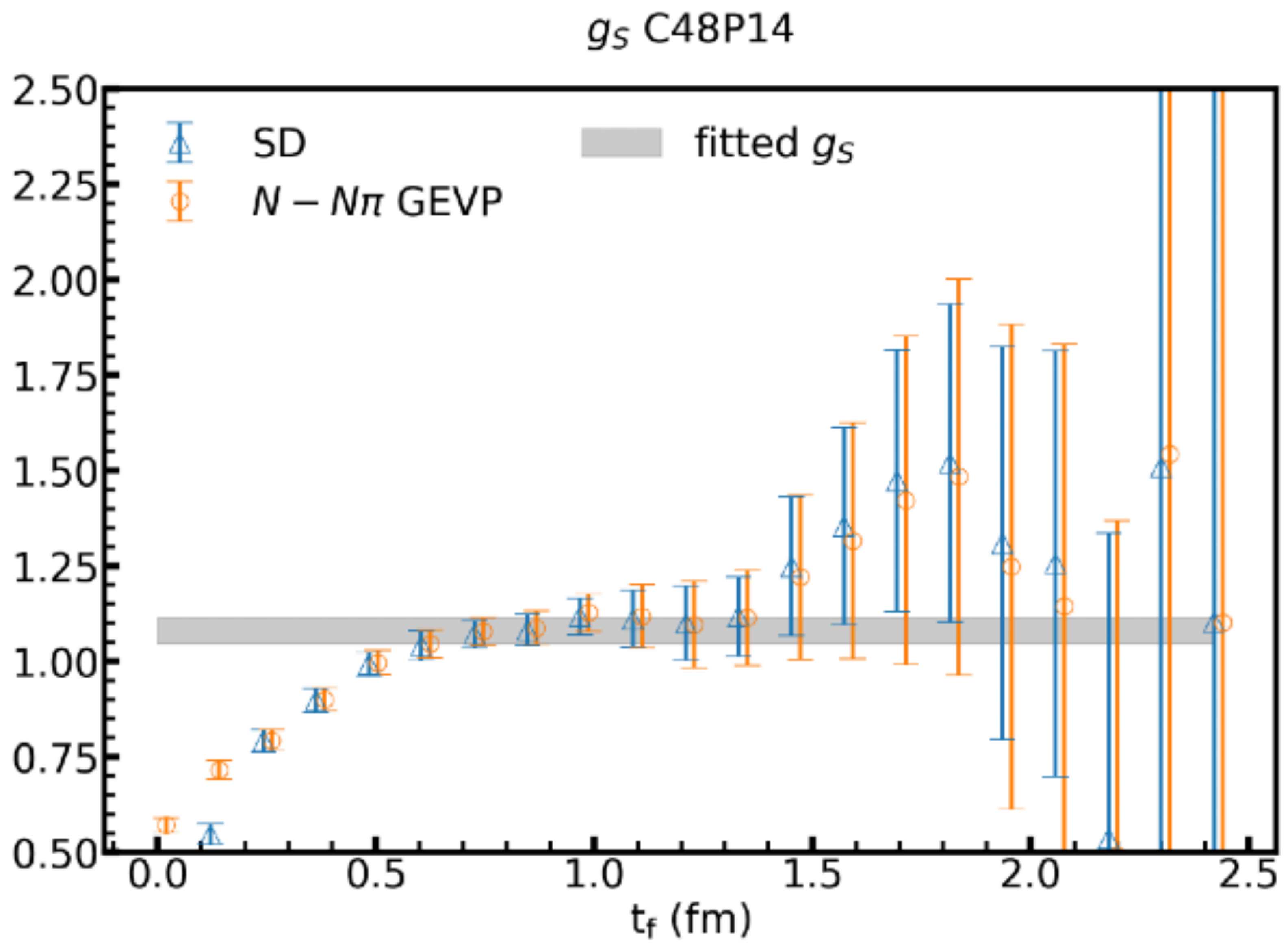
# Influence from $N\pi$




- The blending method also allows us to investigate the GEVP with the $N\pi$ and $N\pi\pi$ states;

- But the impact from the $N\pi$ turns out to be small for $g_s^{u-d}$, compared with the $Na_0$ state even though the previous one has lower energy.

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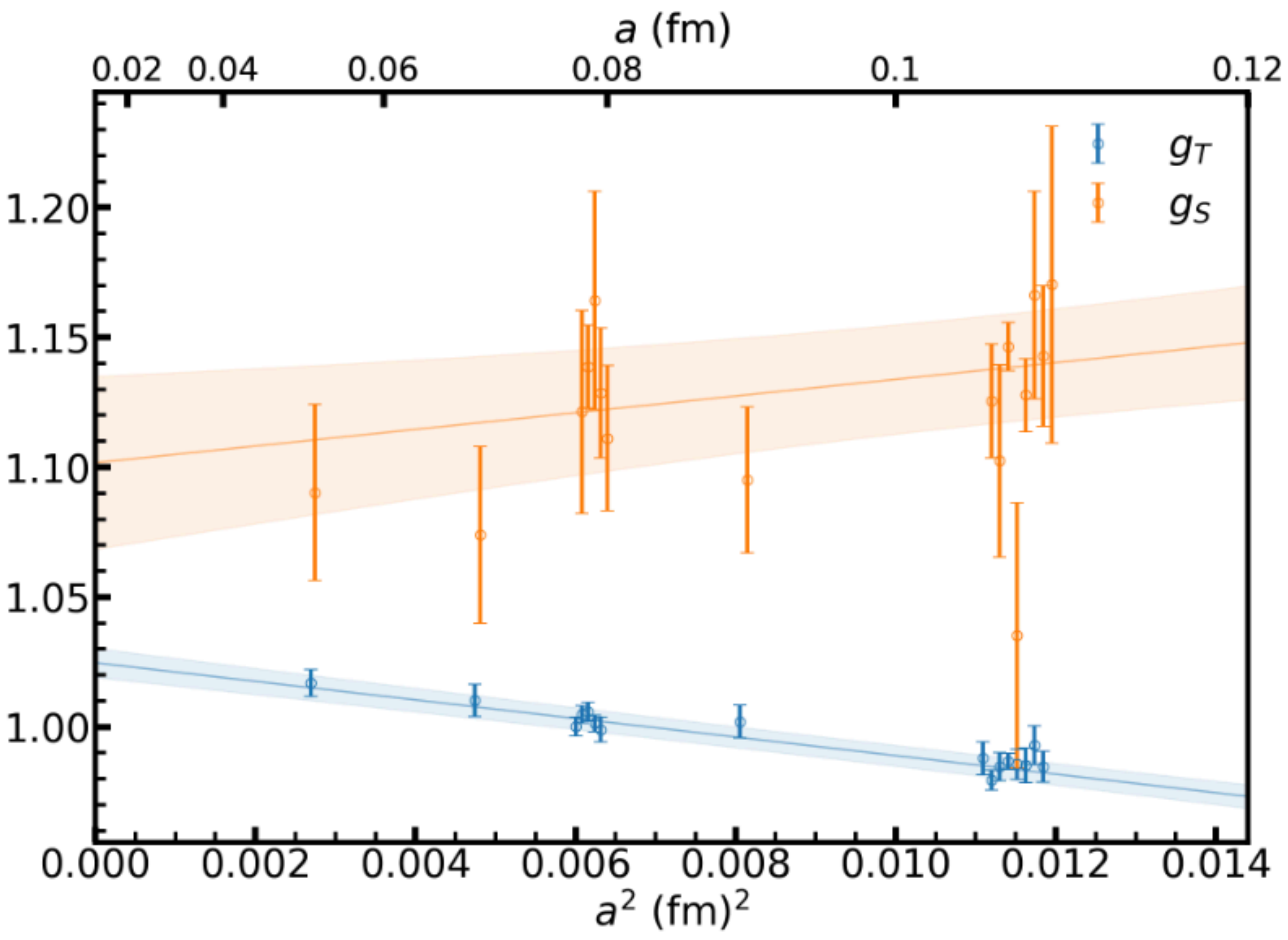
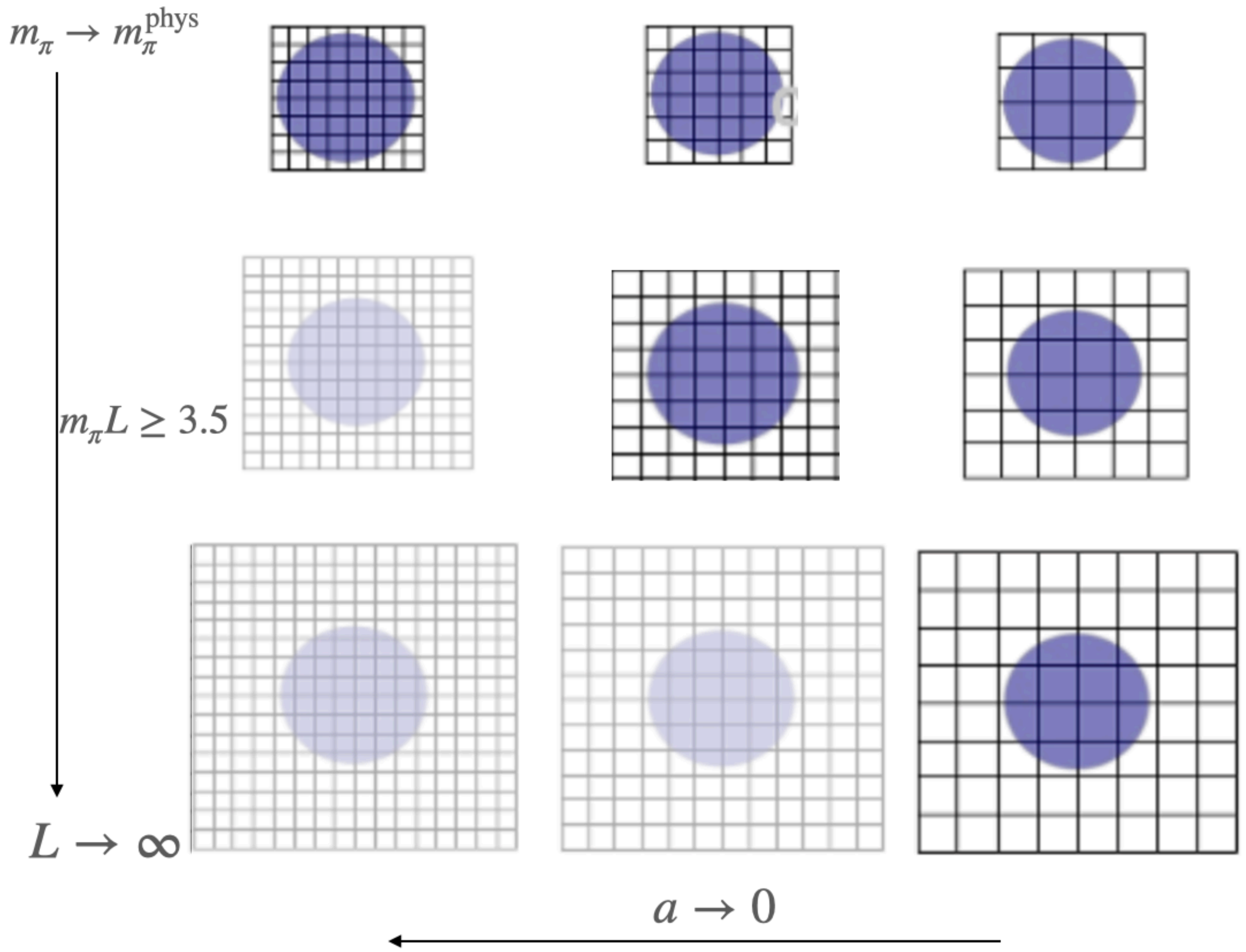
# Systematic uncertainties

## Continuum extrapolation



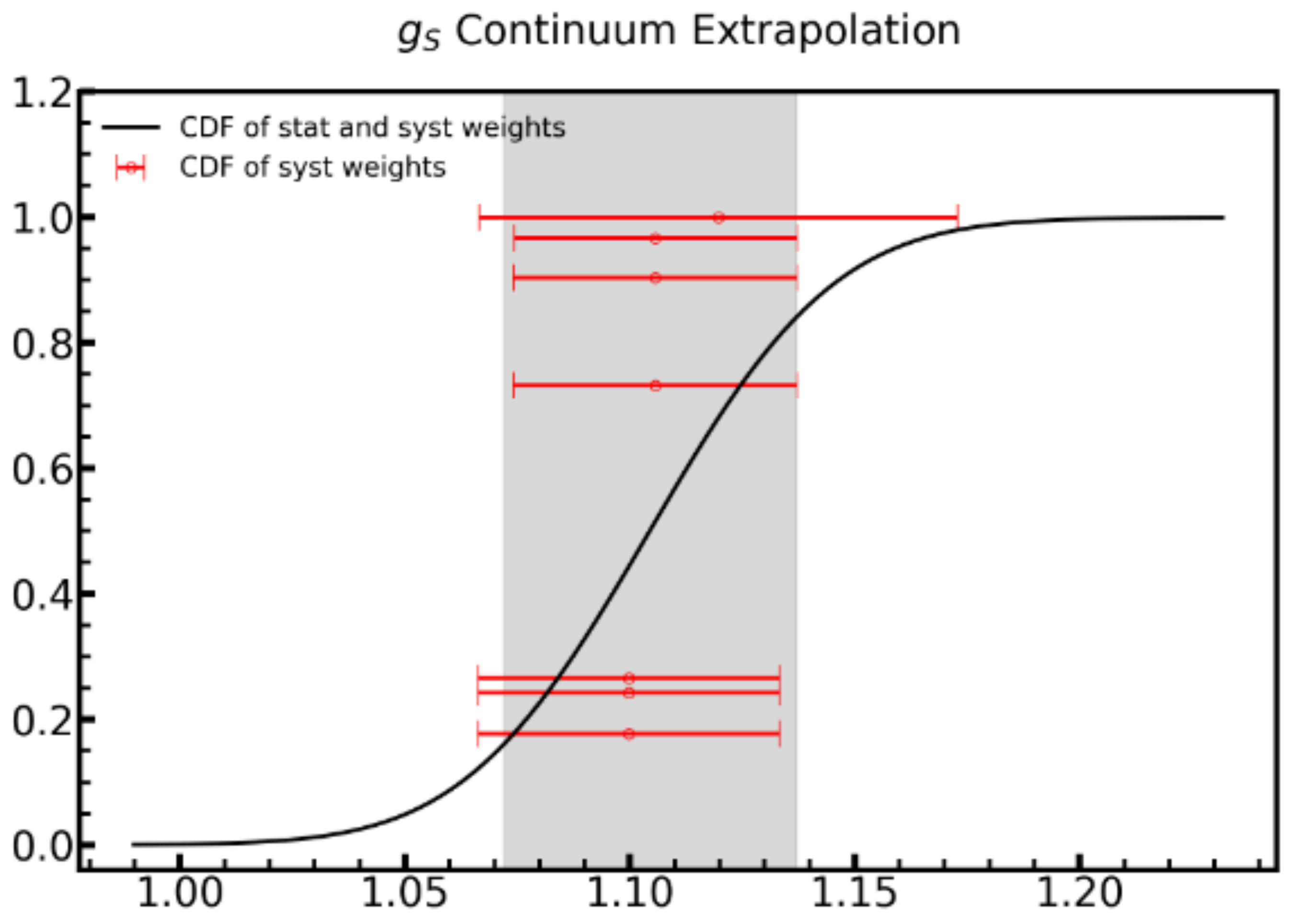

| Ensemble | L | T | a(fm) | $m_\pi$ |
|---|---|---|---|---|
| H48P32 | 48 | 96 | 0.052 | 320 |
| I64P29 | 64 | 128 | 0.038 | 290 |

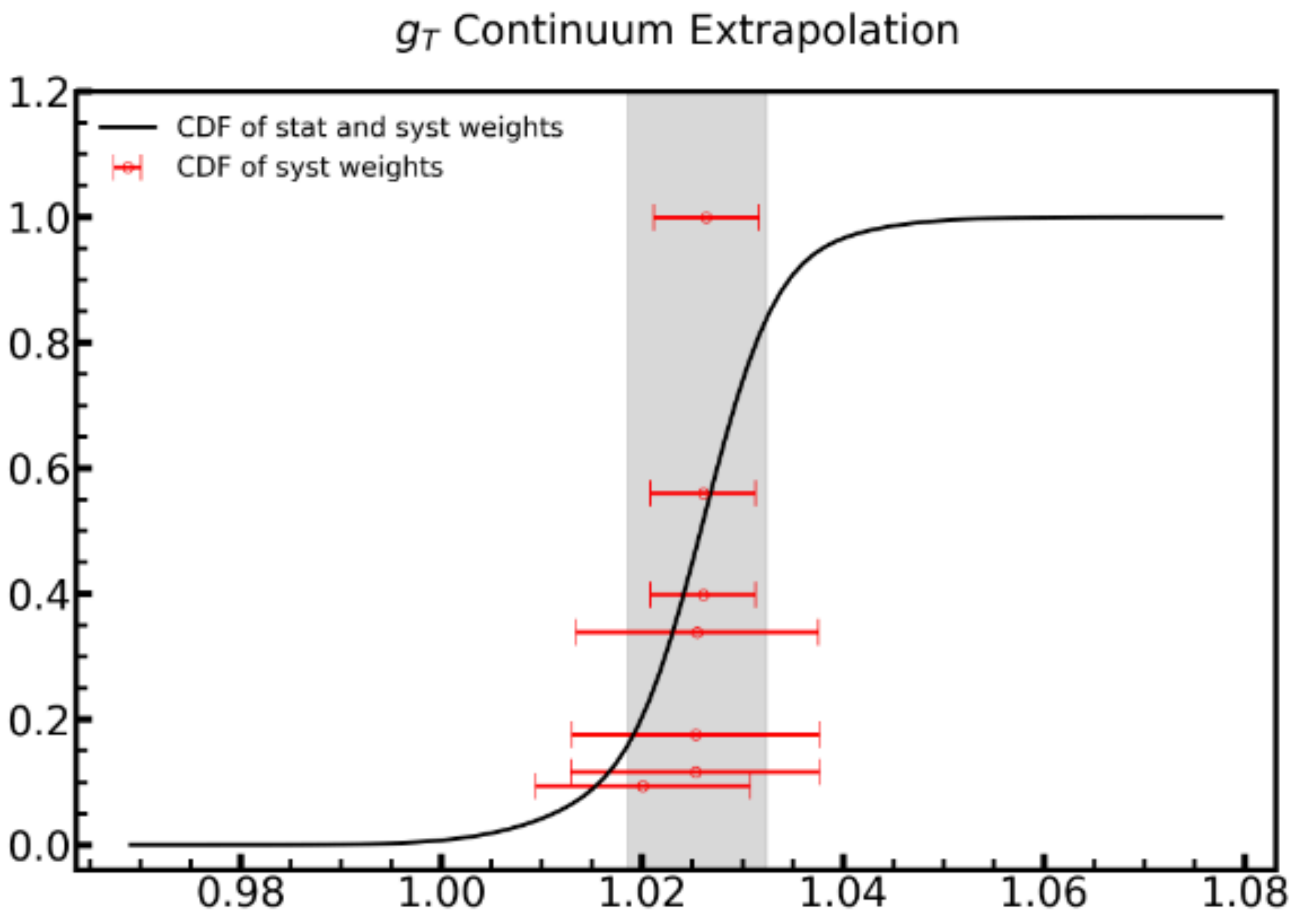
- The cost at $a \sim 0.05$ fm is ~16x of that at $a \sim 0.10$ fm with the same $V$;
- Different discretized fermion and gauge action can only have consistent prediction after the continuum extrapolation.
- Current FLAG **"green star"** grade requires 3 different lattice spacing $a$ with two of them smaller than 0.1 fm, and $a^2_{\rm max}/a^2_{\rm min} \geq 2$.
- **Such a requirement can be satisfied efficiently using the ensembles at relatively heavy $m_\pi \simeq 300$ MeV.**

J.H. Wang et al. [CLQCD], arXiv:2511.02326

# Systematic uncertainties

## Continuum extrapolation




- Simple $a^2$ form for the $\mathcal{O}(a)$-improved clover fermion action, $c_a^{(2)}a^2$;
- $a^2 + a^3$ form for higher order correction, $c_a^{(2)}a^2 + c_a^{(3)}a^3$;
- $a^2 + a^3 + a^4$ form for even higher order correction, $c_a^{(2)}a^2 + c_a^{(3)}a^3 + c_a^{(4)}a^4$;
- Simple $a^2$ form with the data at $a < 0.1$ fm only;
- $a^2$ form with additional $a\log(u_0)$ term for the residual $a\alpha_s$ effect, $c_a^{(1)}a\log(u_0) + c_a^{(2)}a^2$;
- $a\log(u_0) + a^2 + a^3$ form for higher order correction, $c_a^{(1)}a\log(u_0) + c_a^{(2)}a^2 + c_a^{(3)}a^3$
- Most conservative $a\log(u_0) + a^2 + a^3 + a^4$ form, $c_a^{(1)}a\log(u_0) + c_a^{(2)}a^2 + c_a^{(3)}a^3 + c_a^{(4)}a^4$

- The systematic uncertainty from the continuum extrapolation is estimated based on the CDF of different fit ansatz with the AIC weight

$$\omega_i = \frac{\exp\left[-\frac{1}{2}(\chi_i^2 + 2n_{i,\text{par}} - n_{i,\text{data}})\right]}{\sum_j \exp\left[-\frac{1}{2}(\chi_j^2 + 2n_{j,\text{par}} - n_{j,\text{data}})\right]};$$

- The $a\log(u_0) \propto a\alpha_s$ term is introduced in some of the ansatz to estimate the $\mathcal{O}(a\alpha_s)$ effect in the quark bilinear operator, after the $\mathcal{O}(a)$ is removed by the mass-dependent vector current normalization.

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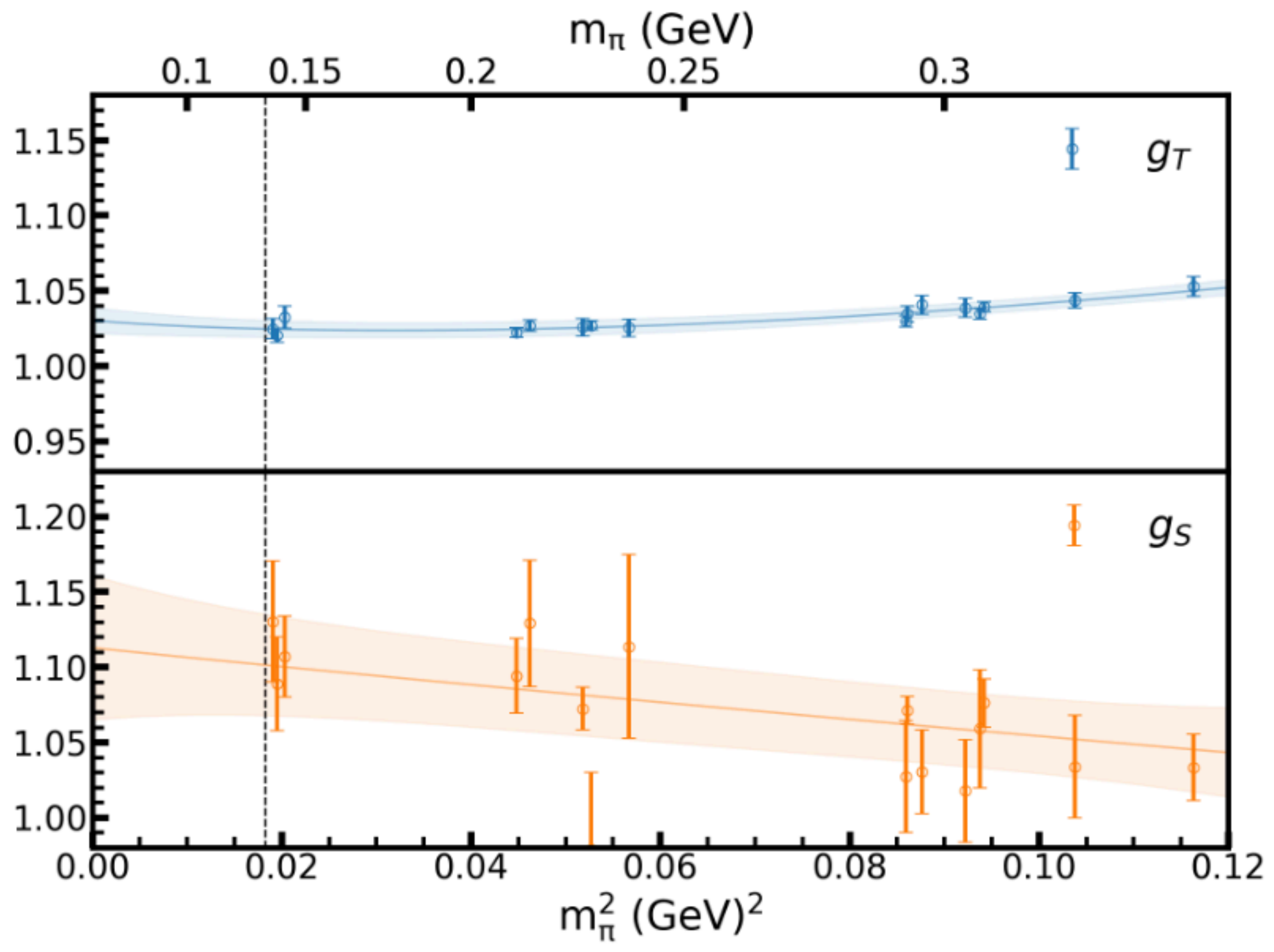
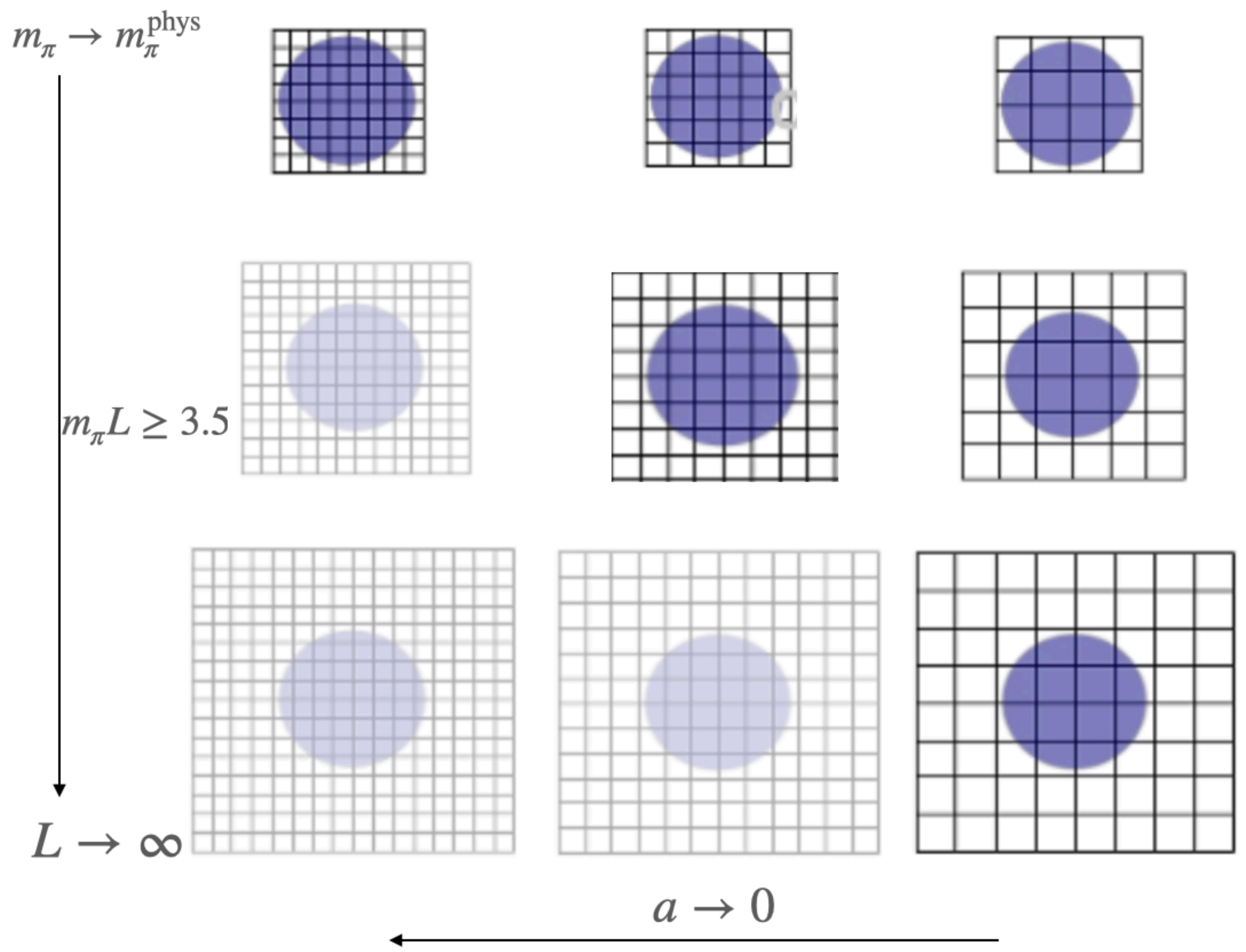
# Systematic uncertainties





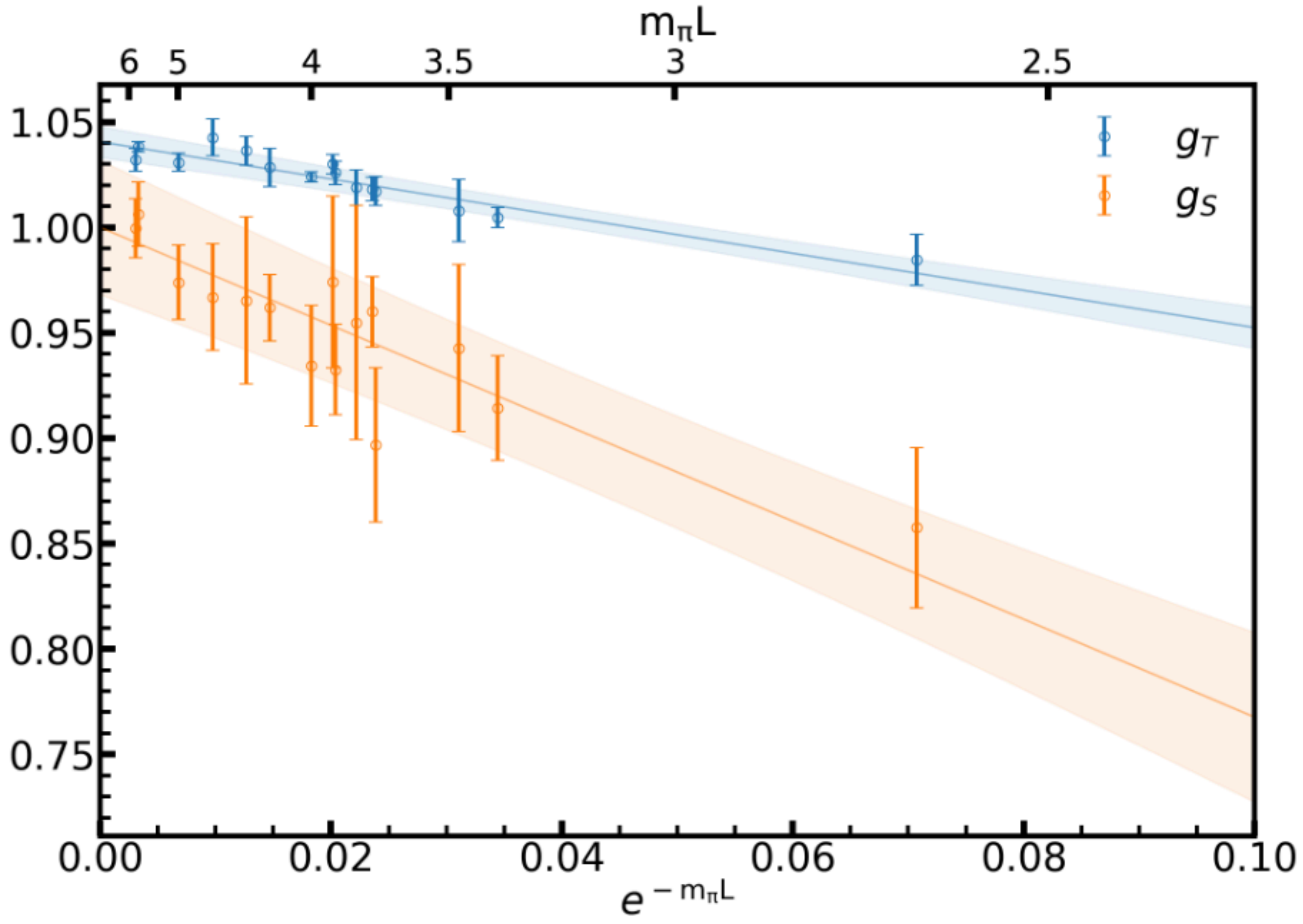

| Ensemble | L | T | a(fm) | $m_\pi$ |
|---|---|---|---|---|
| C48P14 | 48 | 96 | 0.105 | 135 |
| C64P14 | 64 | 96 | 0.105 | 135 |
| F64P13 | 64 | 128 | 0.077 | 134 |

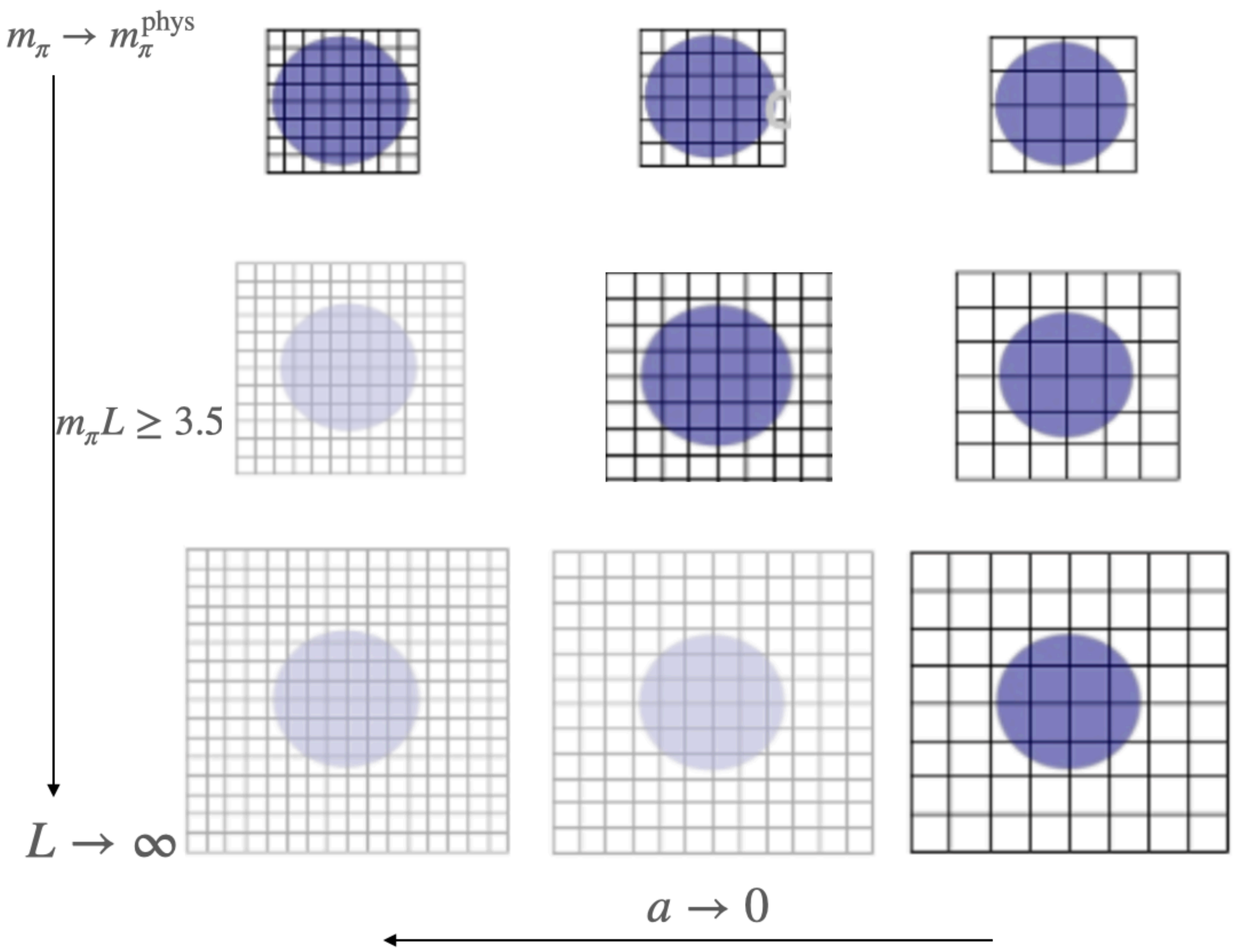
- The cost at $m_\pi \simeq 135$ MeV is ~4x of that at $m_\pi \simeq 310$ MeV, and requires additional 4x statistics to reach similar precision;
- Current FLAG **"green star"** grade requires 3 different $m_\pi$ with $m_{\pi,\rm min} < 200$ MeV in the chiral extrapolation, or $m_{\pi,\ \rm case1} = 135 \pm 10$ MeV and $m_{\pi,\ \rm case2} < 200$ MeV.
- **Such a requirement can be satisfied efficiently using the ensembles at the coarsest lattice spacing.**

J.H. Wang et al. [CLQCD], arXiv:2511.02326

# Systematic uncertainties

## Infinite volume extrapolation



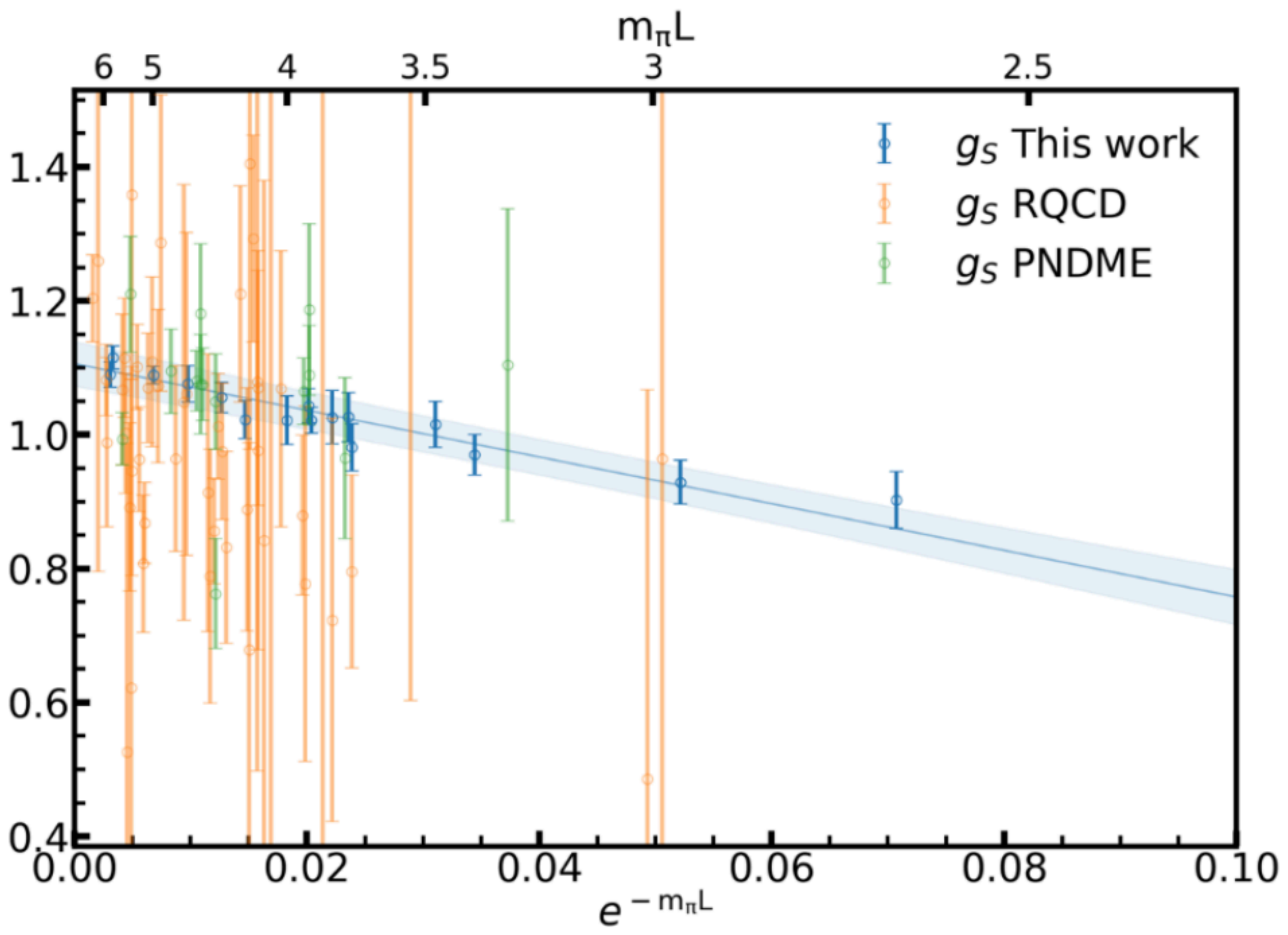

| Ensemble | L | T | a(fm) | |
|---|---|---|---|---|
| C48P14/C64P14 | 48/64 | 96 | 0.105 | 135 |
| C24P23/C32P23/C48P23 | 24/32/48 | 64/96 | 0.105 | 227 |
| C24P29/C32P29/C48P29 | 24/32/48 | 64/72 | 0.105 | 290 |
| F32P21/F48P21 | 32/48 | 96 | 0.077 | 210 |
| F32P30/F48P30 | 32/48 | 64/96 | 0.077 | 300 |

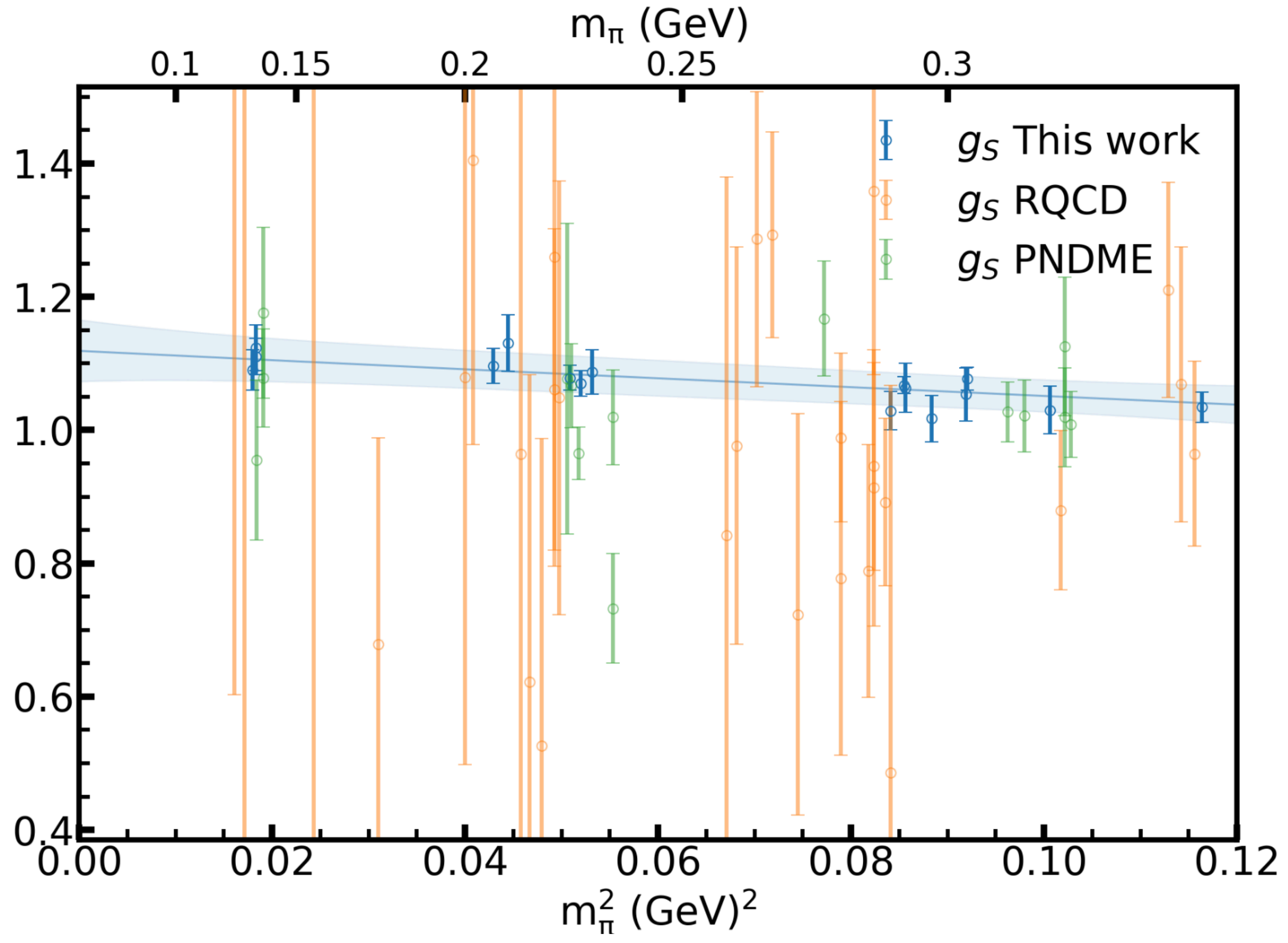
- The cost at $L \sim 5$ fm is 8x of that at $L \sim 2.5$ fm, and $\chi$PT suggests an $e^{-m_\pi L}$ behavior with $m_\pi L \geq 3$;
- Current FLAG **"green star"** grade requires $m_\pi L \sim 3.2$ with $m_\pi \sim$ 135 MeV, or at least three $L$ in the infinite volume extrapolation.
- **Such a requirement can be satisfied efficiently using the ensembles at the coarsest lattice spacing and or heavier $m_\pi$;**

J.H. Wang et al. [CLQCD], arXiv:2511.02326

# Systematic uncertainties

## Infinite volume extrapolation




J.H. Wang et al. [CLQCD], arXiv:2511.02326

| Ensemble | L | T | a(fm) | |
|---|---|---|---|---|
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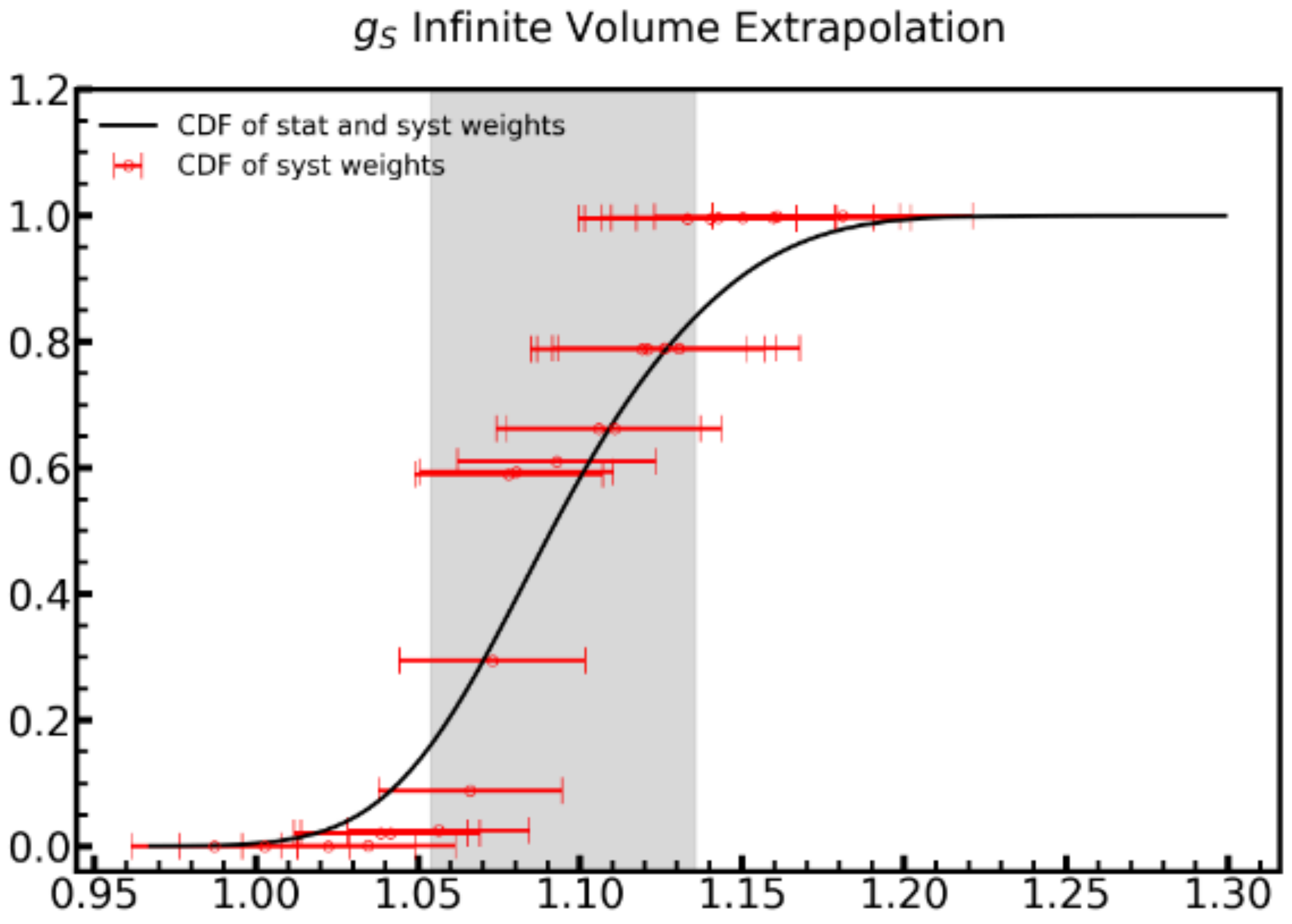
| F32P30/F48P30 | 32/48 | 64/96 | 0.077 | 300 |

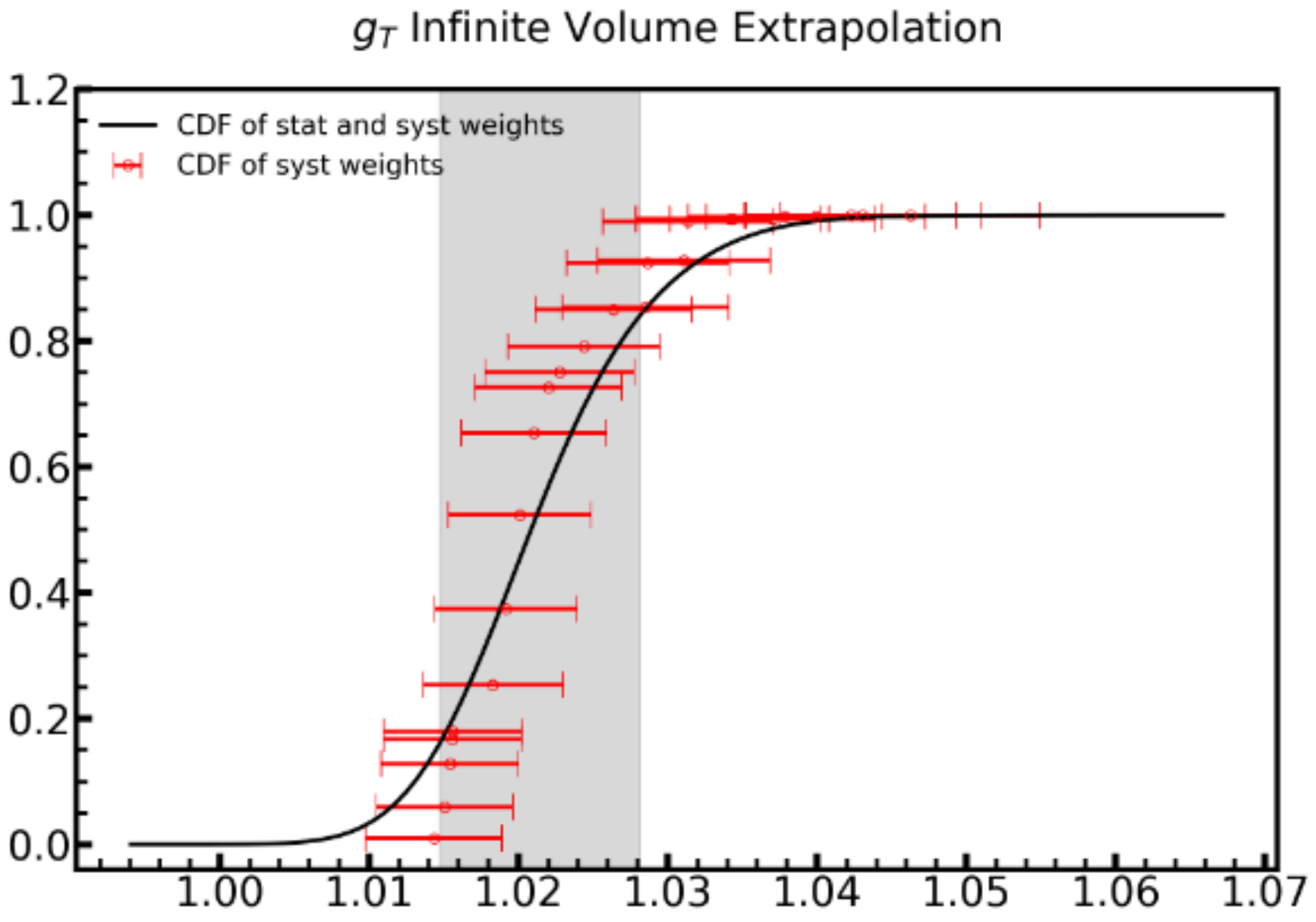
- Best parameter combinations to constraint the finite volume effects at different lattice spacing and light quark masses;

- Much better signal at physical pion mass compared to all the previous results.

# Systematic uncertainties

## Infinite volume extrapolation




- We use a general finite volume effect (FVE) model $m_\pi^i (m_\pi L)^{j/2} e^{-m_\pi L}$ with $i, j \in [-2,2]$ in the AIC average to estimate the systematic uncertainty from FVE;
- The HB$\chi$PT ansatz corresponds to $i = 2, j = -1$ which has a suppressed weight in the average;
- Eventually we give $g_T^{\mathrm{QCD}} = 1.0264[77]_{\mathrm{tot}}(53)_{\mathrm{stat}}(13)_a(46)_{\mathrm{FV}}(01)_\chi(28)_{\mathrm{ex}}(04)_{\mathrm{re}}$ and $g_S^{\mathrm{QCD}} = 1.106[43]_{\mathrm{tot}}(31)_{\mathrm{stat}}(03)_a(28)_{\mathrm{FV}}(01)_\chi(08)_{\mathrm{ex}}(08)_{\mathrm{re}}$.

J.H. Wang et al. [CLQCD], arXiv:2511.02326

$g_{S,T}^{u-d}$

# Cost comparison

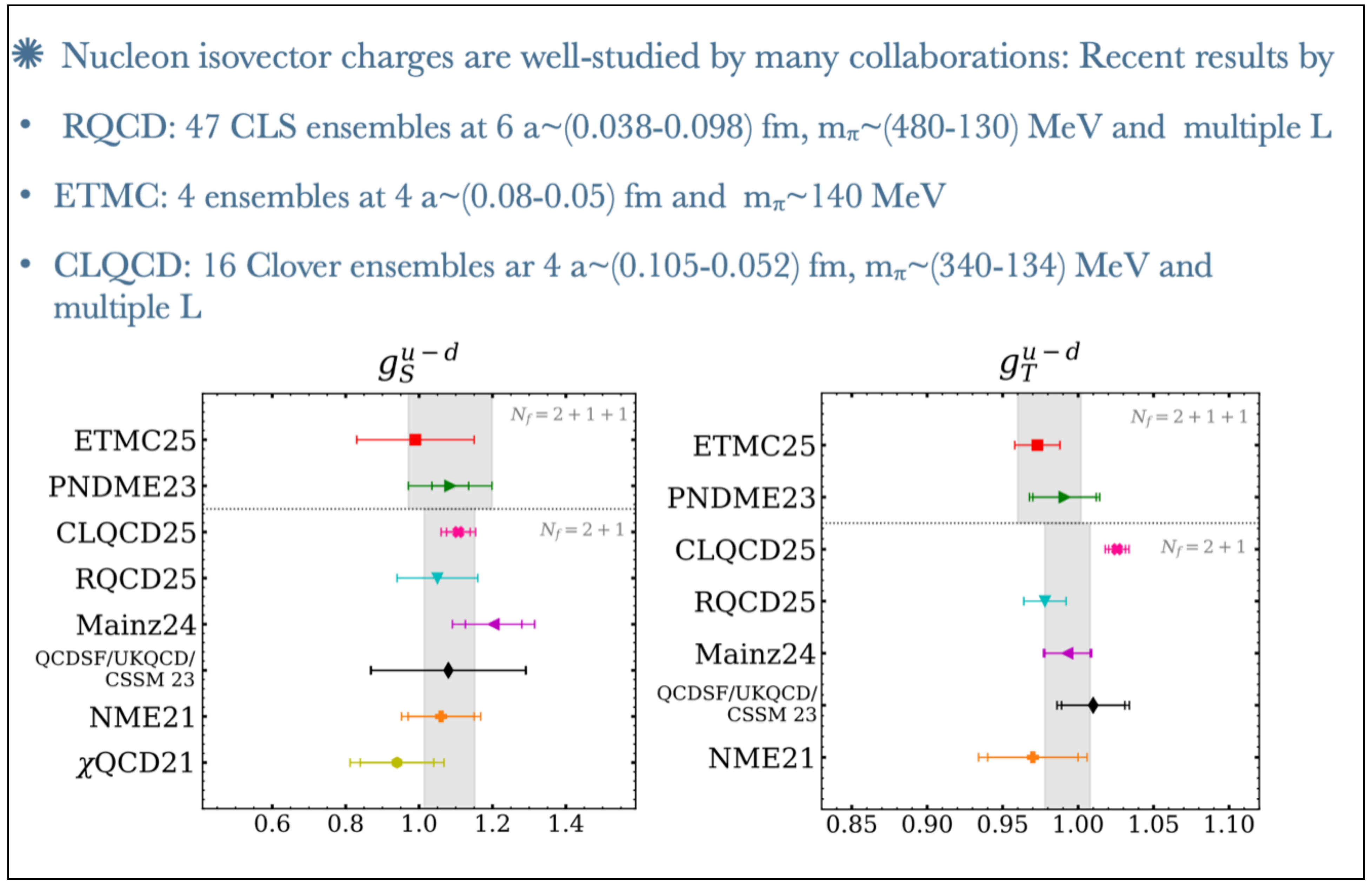
| Collaborations | Ensemble | L | T | a(fm) | $m_\pi$(MeV) | $n_{\text{cfg}}$ | $g_S$ | $g_T$ | Inversions | Inversion for our precision of $g_S$ | Inversion for our precision of $g_T$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ETMC(2020) | cB211.072.64 | 64 | 128 | 0.08 | 139 | 750 | 1.35(17) | 0.939(027) | 1.71M | 32M | 92M |
| RQCD(2023) | D452 | 64 | 128 | 0.076 | 156 | 1000 | -0.6(3.0) | 0.870(110) | 0.01M | 364M | 11M |
| PNDME(2023) | a09m130 | 64 | 96 | 0.09 | 138 | 1290 | 1.05(23) | 1.010(006) | 1.8M | 103M | 4.2M |
| **This Work** | F64P13 | 64 | 128 | 0.078 | 134 | 46 | 1.00(03) | 0.998(004) | 0.39M | 0.39M | 0.39M |

- Comparison based on the physical point ensembles with L=64 and a~0.08 fm;
- The blending method can reach similar precision with 6 to 877 times fewer inversion;
- Much more efficient usage of the configuration given limited statistics.
- Using the 2-state fit with the optimal interpolator can achieve even higher precision, compared to most conservative 3-state fit we used here.

J.H. Wang et al. [CLQCD], arXiv:2511.02326

# Related prediction

## Proton-neutron mass difference




"Hadron Structure in the Context of the EIC Program",
Constantia Alexandrou, Lattice 2025 Plenary talk

- Most precise prediction on $g_{S,T}^{u-d}$ so far;
- One can have more precise prediction on $m_n - m_p$ based on our prediction of $g_S^{u-d}$.

$$m_n - m_p = m_u\left(\frac{\partial m_n}{\partial m_u} - \frac{\partial m_p}{\partial m_u}\right) + m_d\left(\frac{\partial m_n}{\partial m_d} - \frac{\partial m_p}{\partial m_d}\right) + \delta^{\mathrm{QED}} m_p^{\mathrm{isoQCD}} = (m_d - m_u) g_S^{u-d} + \delta^{\mathrm{QED}} m_p^{\mathrm{isoQCD}}$$

$$= 1.51[0.28]_{\mathrm{tot}}(16)_{\mathrm{stat}}(23)_{\mathrm{sys}}\ \mathrm{MeV}$$ From BMWc, Science 347(2015)1452

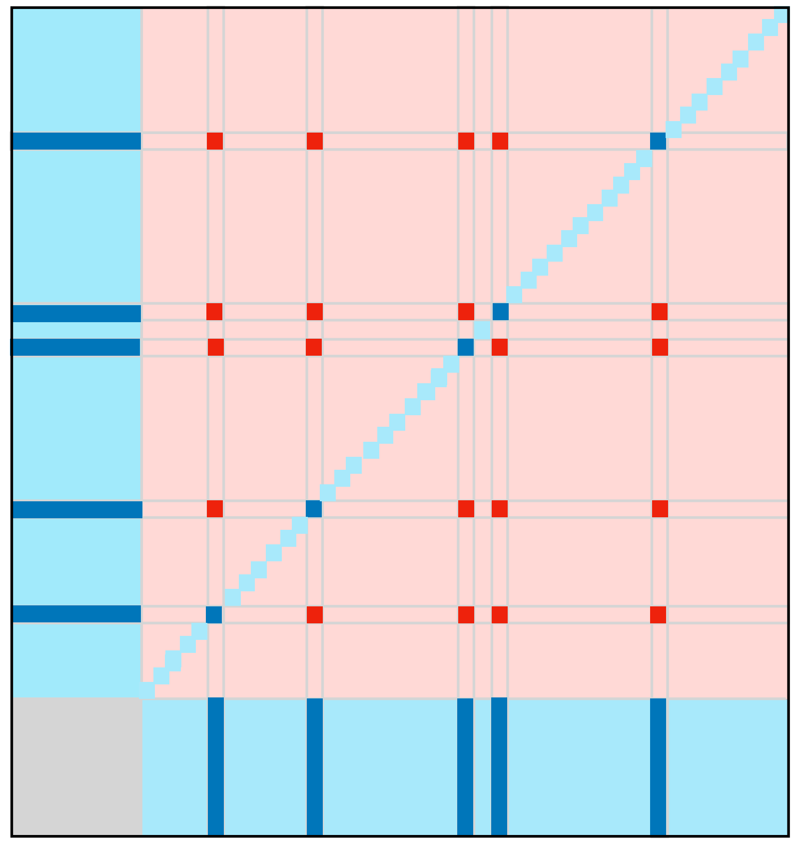
$$= 1.60[0.23]_{\mathrm{tot}}(11)_{g_S}(13)_{\mathrm{ISB}}(16)_{\mathrm{QED}}\ \mathrm{MeV}$$ From CLQCD, arXiv:2511.02326

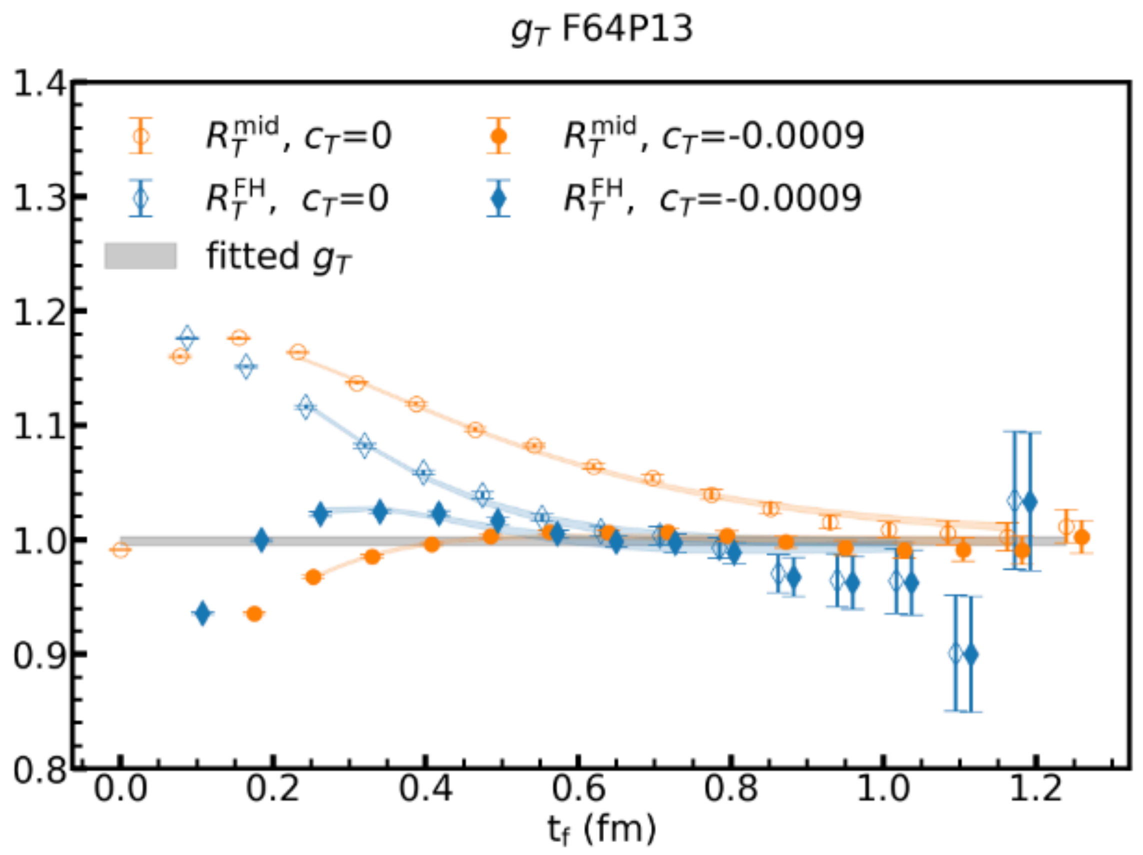
$$m_n^{\mathrm{exp}} - m_p^{\mathrm{exp}} = 1.293\ \mathrm{MeV}$$

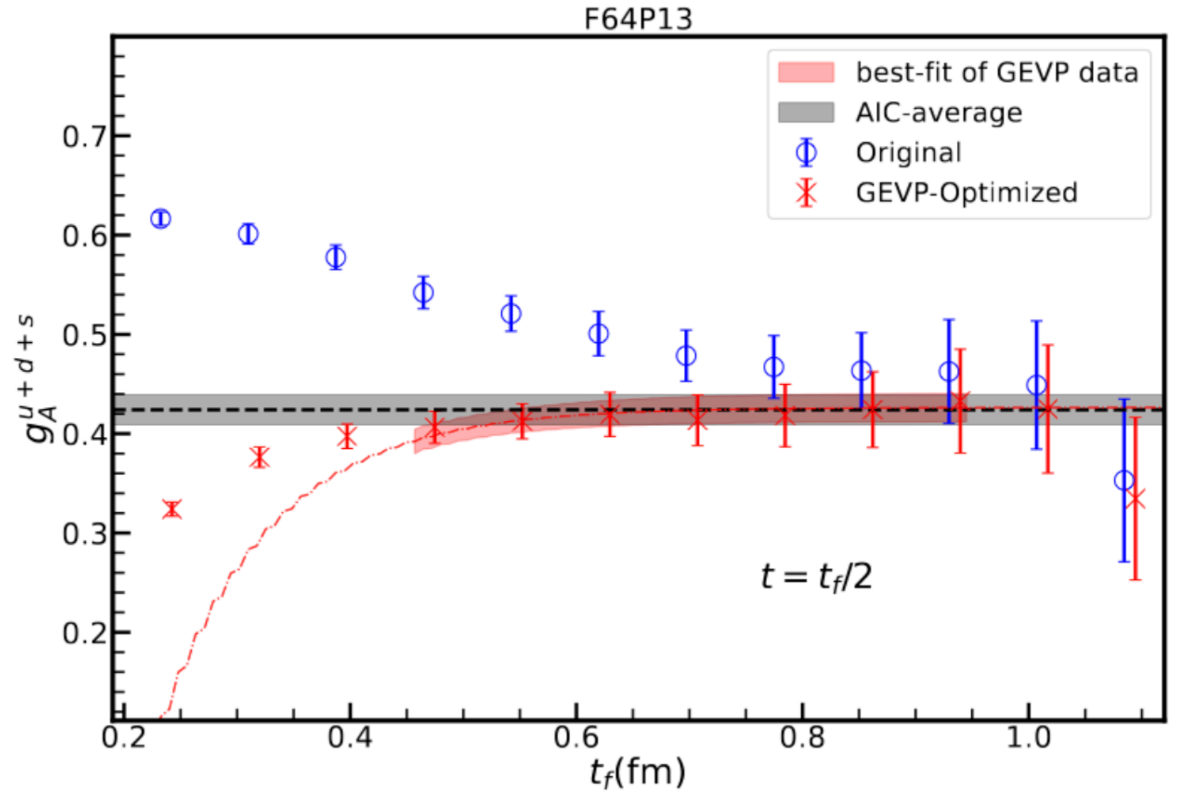
**ISB** : Flavor lattice average group, 2411.04268
**QED:** BMWc, Science 347(2015)1452

# Summary

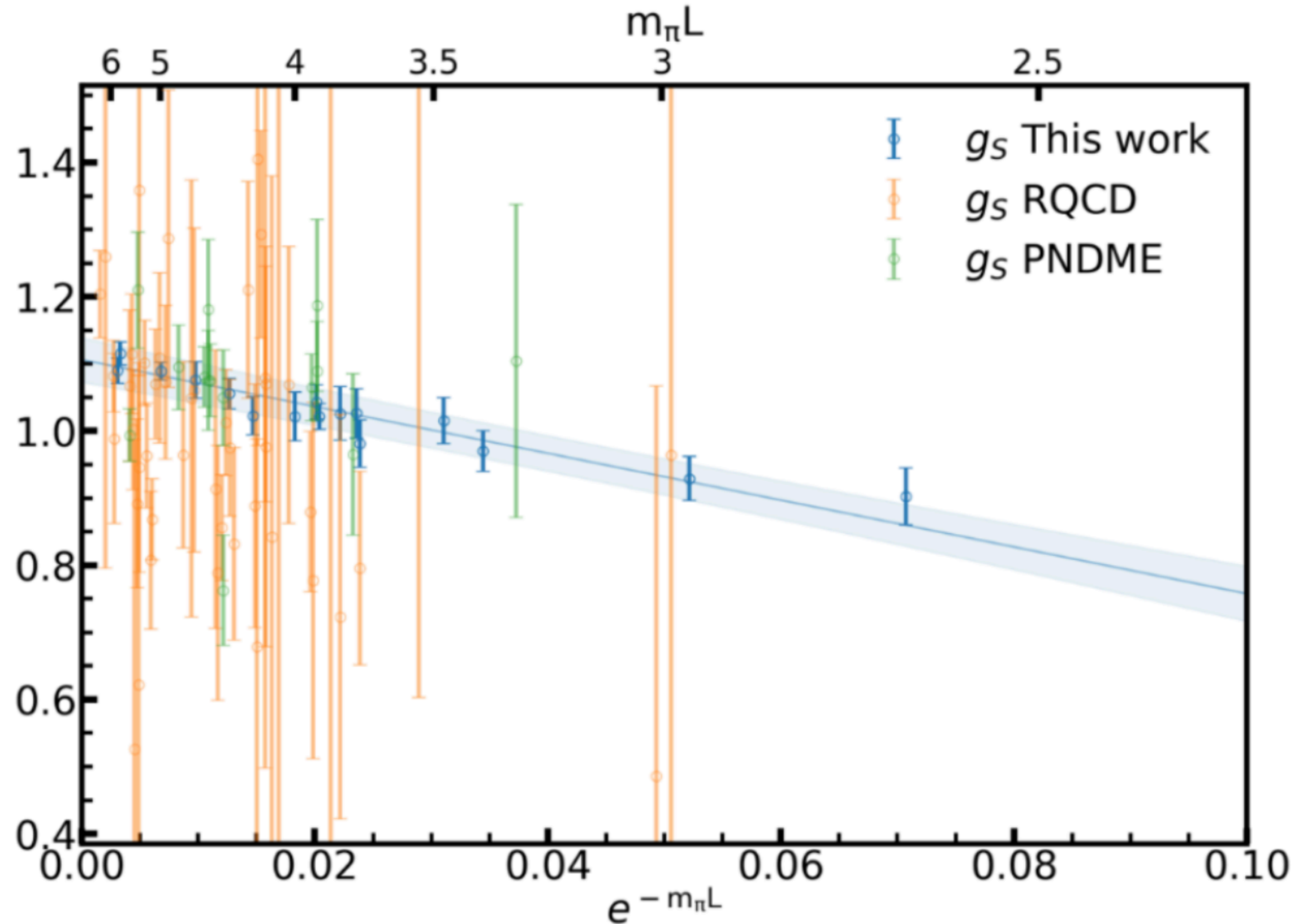
- The blending method provides an efficient choice to obtain the projected propagators which can be used for general purpose;






- The current-involved interpolator basis is very helpful to suppress the major excited state contamination, in all the cases we investigated.




- With diminishing statistical uncertainties, the finite volume effect constitutes a systematic error that merits more careful consideration.