Parton structure from Hamiltonian lattice gauge theory

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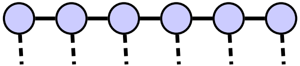


[arXiv:2504.07508]
[arXiv:2409.16996]

XIIIth Meeting on Lattice Parton Physics
from Large Momentum Effective Theory (LaMET 2026)
@ Cracow, Poland
9 July 2026

Collaborators

Mari Carmen Bañuls

C.-J. David Lin

Krzysztof Cichy

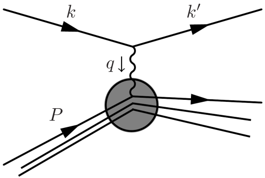
Outline

1. Motivation & Goal: Parton Distribution Functions
2. Method: Tensor Network States
3. Application: Schwinger Model
4. Results
5. Summary & Outlook

Hadron structure and light-cone matrix elements

- hadron structure functions:
  universal, non-perturbative inner structure of hadrons
- parton distribution functions (PDF):
  collinear momentum fraction ξ of partons in hadron

$$f_\Psi(\xi) = \int dz^- e^{-i\xi P^+ z^-} \langle P | \bar{\psi}(z^-)\gamma^+ W(z^- \leftarrow 0)\psi(0) | P \rangle$$



[Schwartz 2014]

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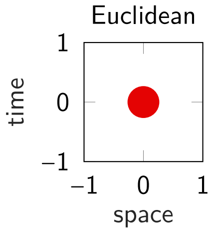

- integration along light-front direction z^-
- lattice QCD in euclidean space: light cone → point
- indirect methods (e.g. LaMET): very successful in certain regimes, but challenges in others (e.g. soft partons)

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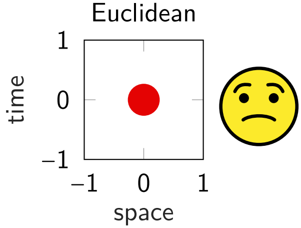
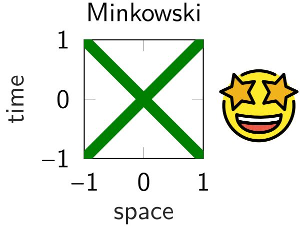
- Hamiltonian formalism: light cone in Minkowski space
- → use tensor network states/quantum devices





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Tensor Networks

- generic state scales exponentially

$$|\psi\rangle = \sum_{s_1, s_2, \dots, s_N} \Psi^{s_1 s_2 \dots s_N} |s_1\rangle \otimes |s_2\rangle \otimes \dots \otimes |s_N\rangle$$

Tensor Networks

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- tensor network state as ansatz
- 1d: matrix product state (MPS)

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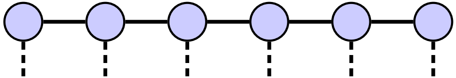
$$\Psi^{s_1 s_2 \ldots s_N} = \sum_{\{i_x\}} A^{1,s_1}_{i_1} \cdot A^{2,s_2}_{i_1,i_2} \cdot A^{3,s_3}_{i_2,i_3} \ldots A^{N,s_N}_{i_{N-1}}$$

$=$ 

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- truncation to bond dimenion D
- polynomial resource scaling

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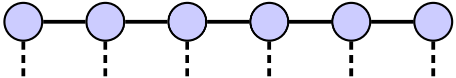
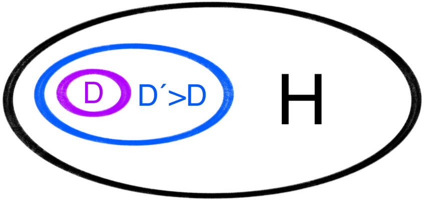
$=$ 



Tensor Networks

- generic state scales exponentially
- tensor network state as ansatz
- 1d: matrix product state (MPS)
- truncation to bond dimenion D
- polynomial resource scaling
- good approximation for ground states and low excited states [Hastings 2007]
- no sampling, fundamentally different systematics compared to Monte Carlo

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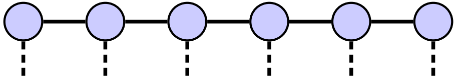
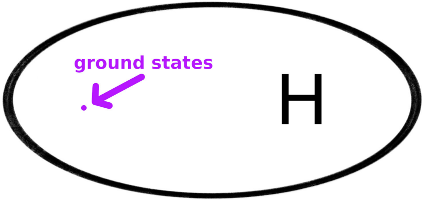
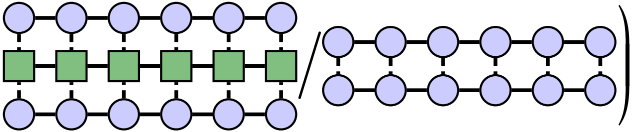




Efficient Tensor Network operations

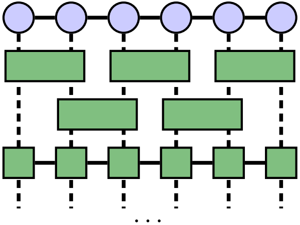
- Find groundstate and excited states

$$\min\left(E = \frac{\langle \Psi | \hat{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \right)$$

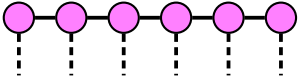


- Apply operators / time evolution

$$\hat{O}\,|\Psi\rangle =$$

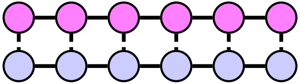


$$\longrightarrow |\Phi\rangle =$$



- Calculate overlap

$$\langle \Psi | \Phi \rangle =$$



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Schwinger model [Schwinger 1962; Hamer et al. 1997]

- quantum electrodynamics in 1+1 dimensions, $U(1)$ symmetry
- fermion couples to gauge boson → partons
- bound states → hadrons [Bañuls et al. 2013]
- scattering → PDF [Dai et al. 1995]

$$\mathcal{L} = \bar{\Psi}(i\not{\partial} - g\not{A} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_0\rho \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{H} = -i\bar{\Psi}\gamma^1(\partial_1 - igA_1)\Psi + m\bar{\Psi}\Psi + \frac{1}{2}E^2 + A_0\left(g\Psi^\dagger\Psi + \rho - \partial_1 E\right)$$

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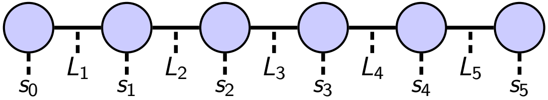
- on the lattice:



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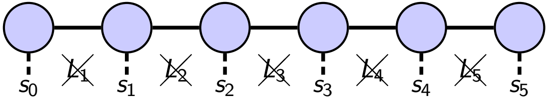

$$H = x\sum_{n=0}^{N-2}\left[\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right] + \frac{\mu}{2}\sum_{n=0}^{N-1}\left[1 + (-1)^n\sigma_n^z\right] + \sum_{n=0}^{N-2}\left[\frac{1}{2}\sum_{k=0}^{n}\left((-1)^k + \sigma_k^z + 2q_k\right)\right]^2$$

$$\left(x = \frac{1}{a^2g^2}, \mu = \frac{2m}{ag^2}\right)$$

Lightfront matrix elements

$$\left\langle P \,\middle|\, \bar{\Psi}(z^-)\gamma^+ W(z^- \leftarrow 0)\Psi(0) \,\middle|\, P \right\rangle$$
$$\rightarrow \langle P| \, \sigma^+(z^-) \, W_{z^- \leftarrow 0} \sigma^-(0) \, |P\rangle + \ldots$$

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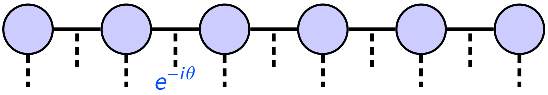
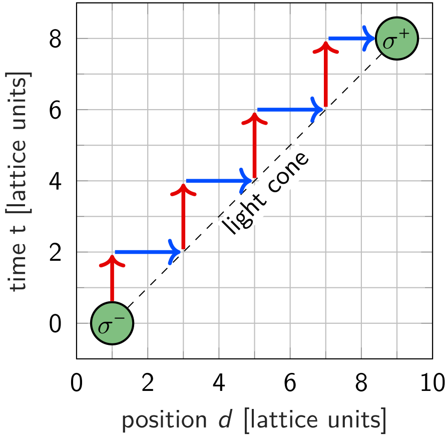
- lightcone
  $\rightarrow$ small time- and space-like steps
- time evolution:
  $e^{-i\tau H} \approx \left(e^{-i\delta\tau H_{eo}} e^{-i\delta\tau H_{oe}} e^{-i\delta\tau H_L}\right)^{N_\tau}$
- spatial evolution:
  change electric field along the path
  $\rightarrow$move static charges





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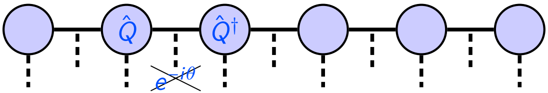

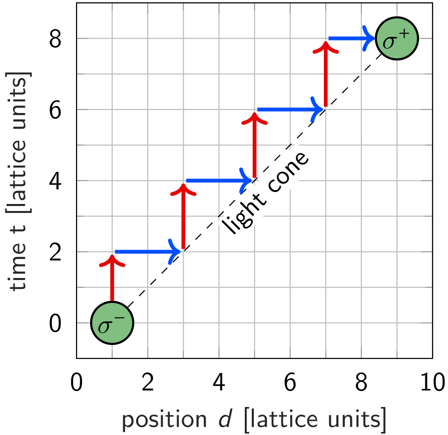

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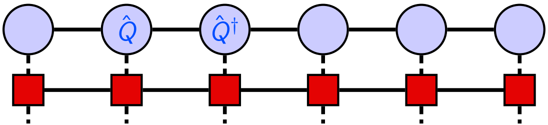

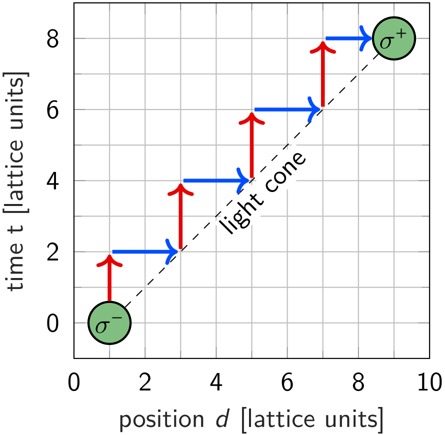

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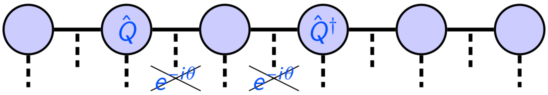

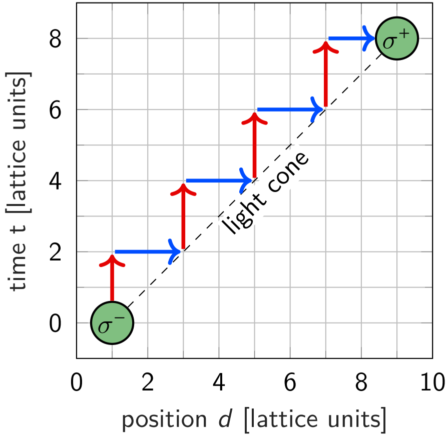

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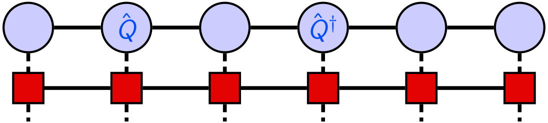

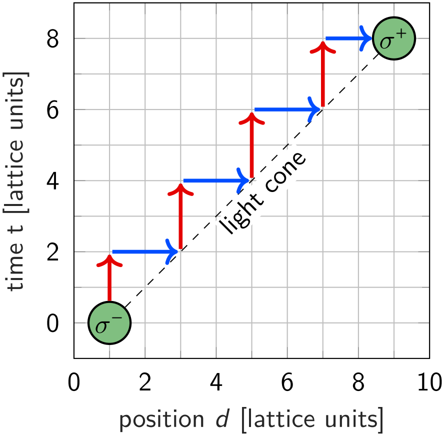

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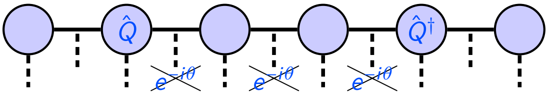

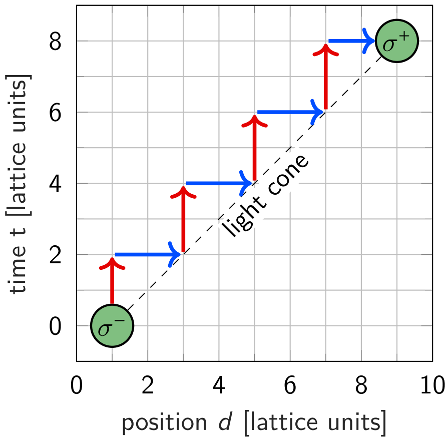

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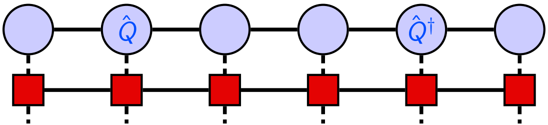

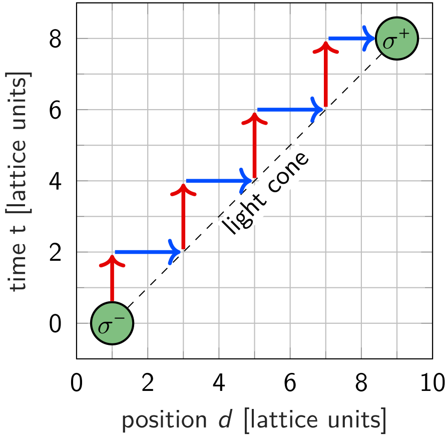

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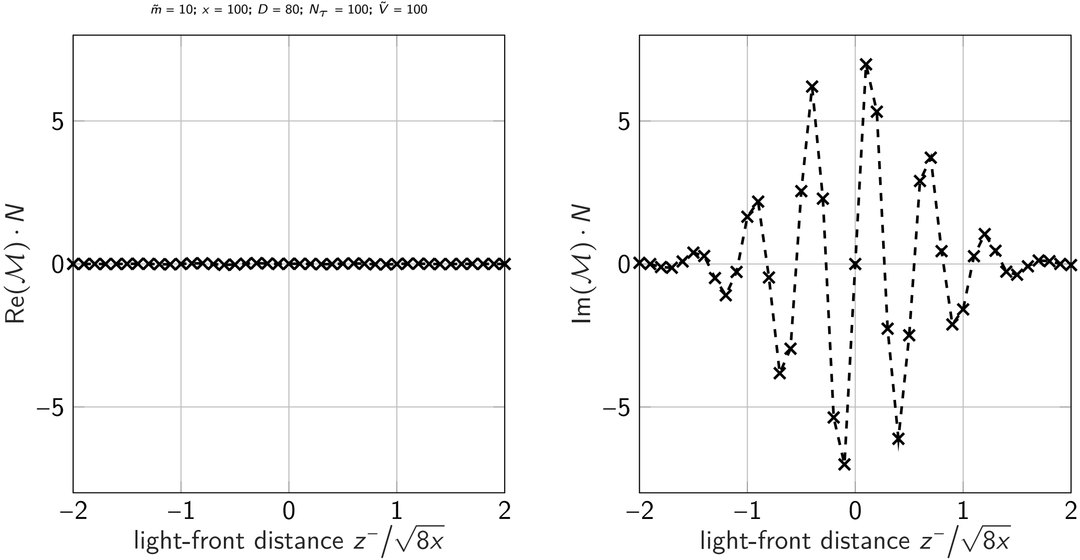
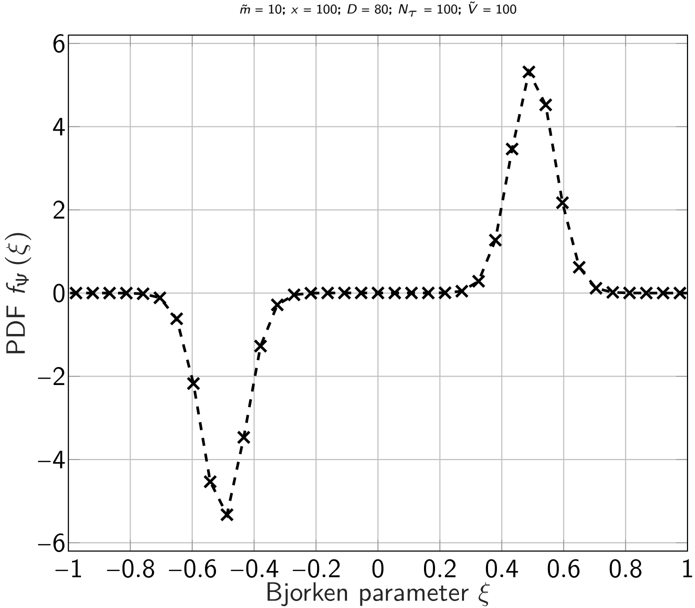


Results: PDF



observations:

- $\xi > 0$: $f_\psi \approx$ symmetric around $\xi = 0.5$
- antifermion PDF from negative ξ:

$$f_{\overline{\psi}}(\xi) = -f_\psi(-\xi)$$

- observed symmetry $\rightarrow f_{\overline{\psi}}(\xi) = f_\psi(\xi)$ $\Rightarrow$ meson ✓

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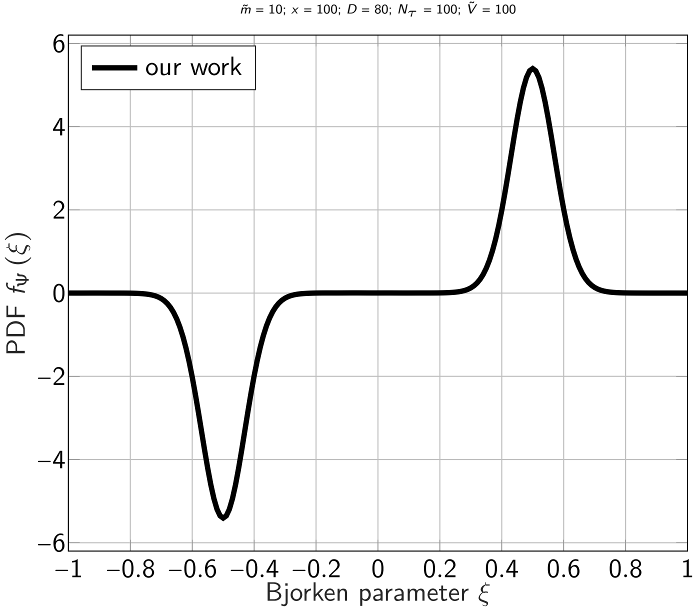

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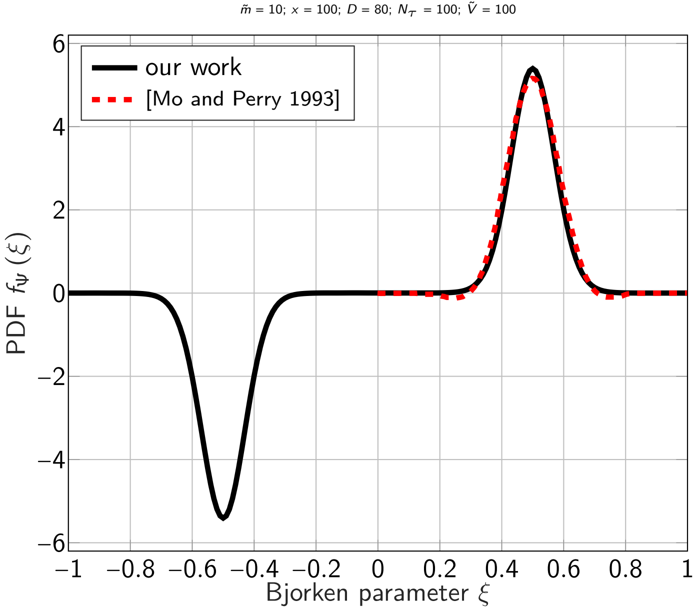

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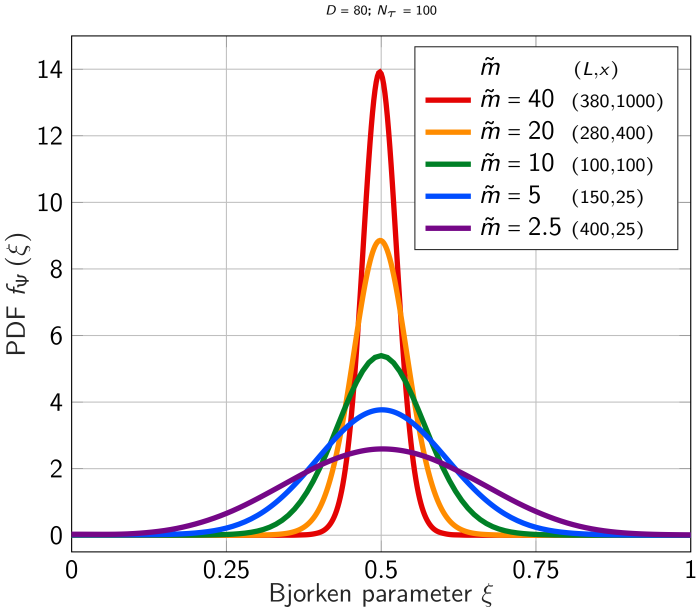

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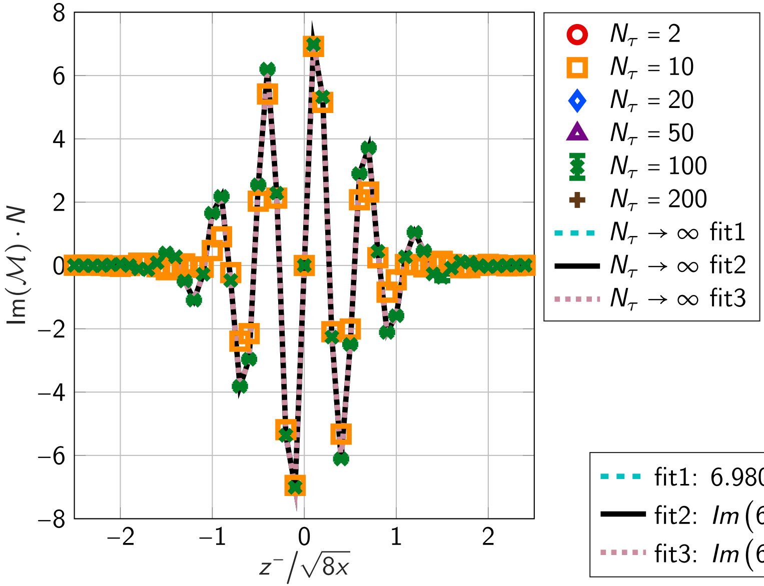
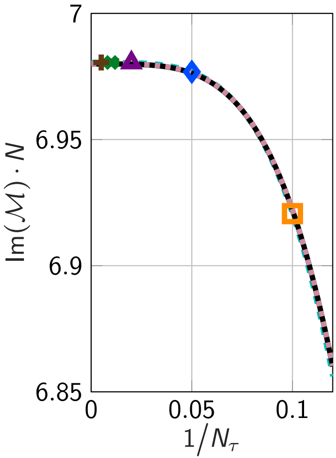
- peak broadens with decreasing fermion mass ✓

Results: N_τ-dependence

$\tilde{m} = 10;\ x = 100;\ \tilde{V} = 100;\ D = 80$



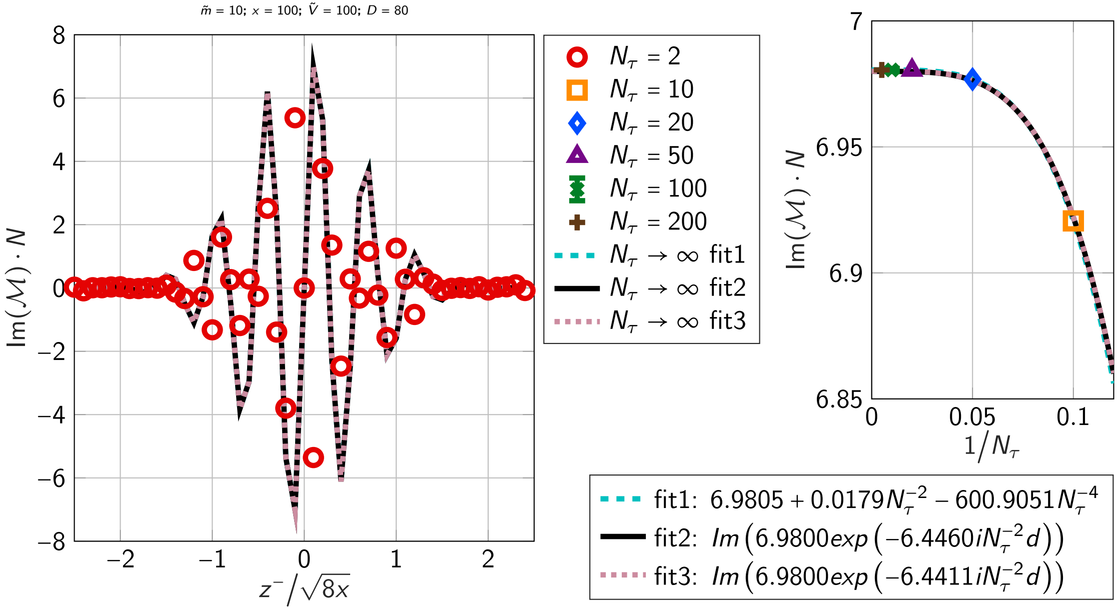


fit1: $6.9805 + 0.0179 N_\tau^{-2} - 600.9051 N_\tau^{-4}$

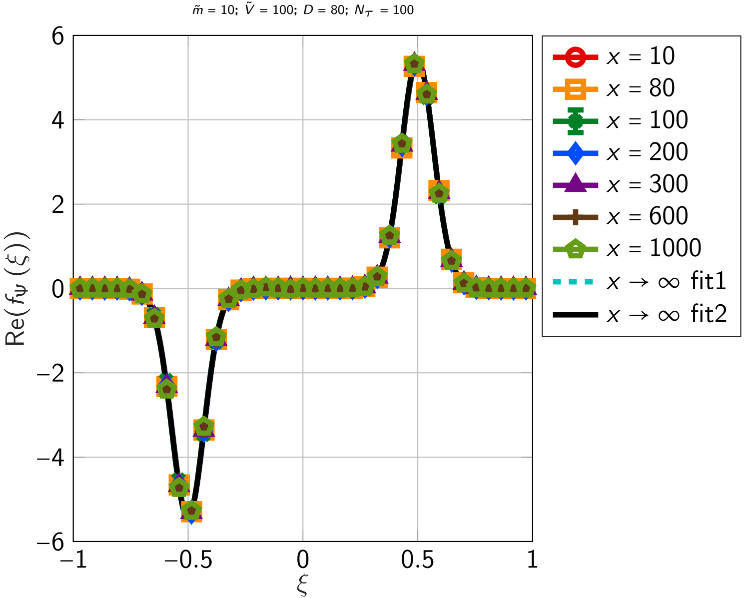
fit2: $Im\left(6.9800 exp\left(-6.4460 i N_\tau^{-2} d\right)\right)$

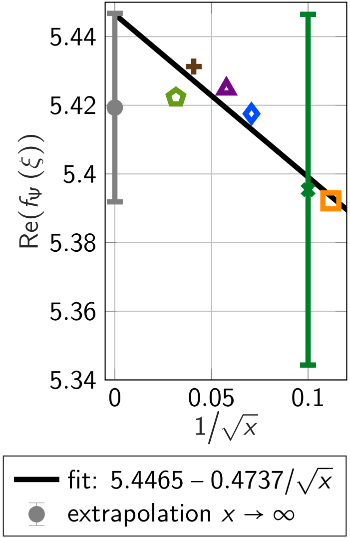
fit3: $Im\left(6.9800 exp\left(-6.4411 i N_\tau^{-2} d\right)\right)$

Results: N_τ-dependence – too small

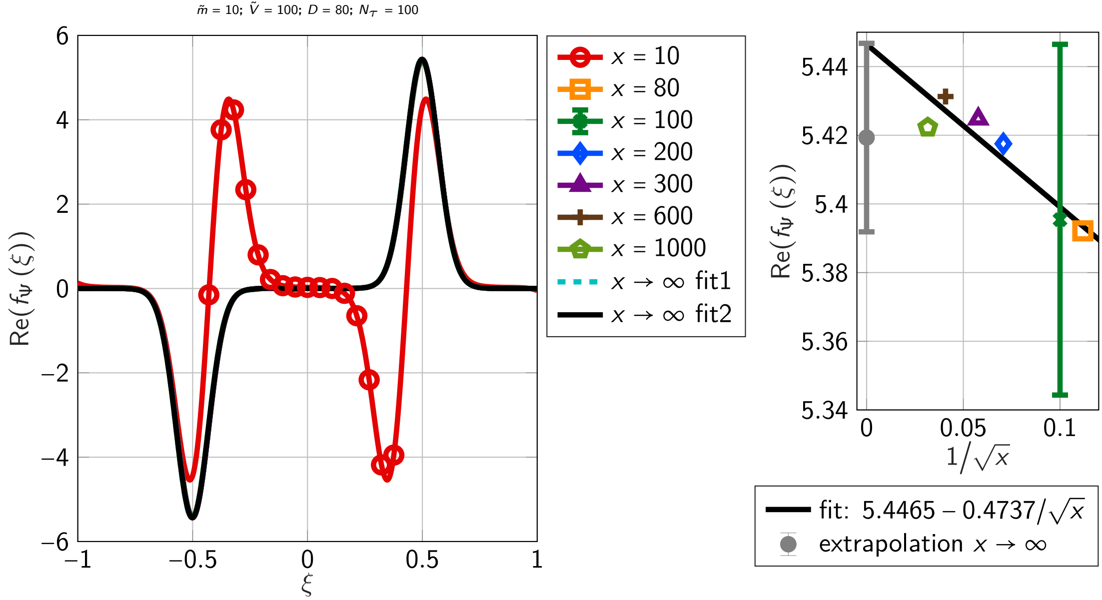


Results: x-dependence

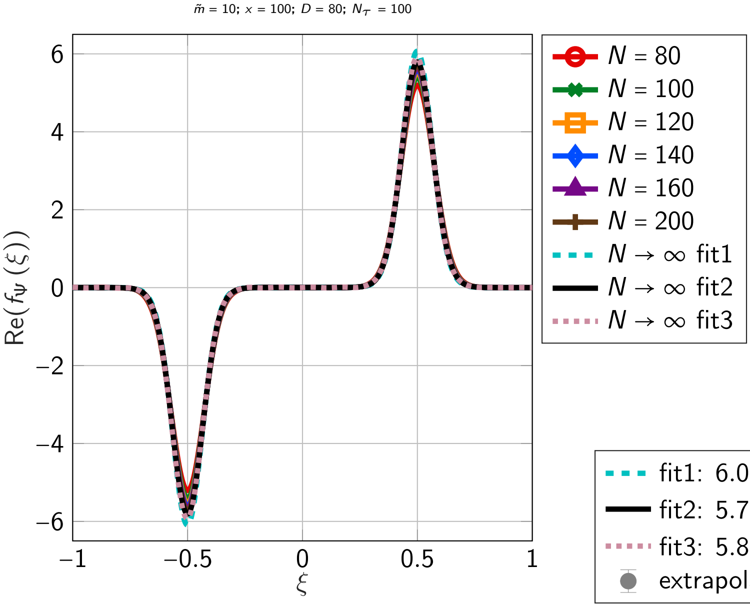


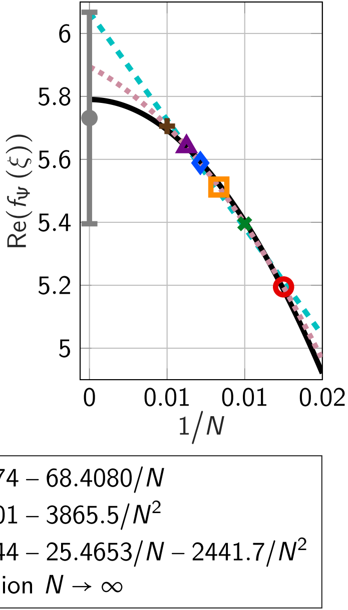


Results: x-dependence – too small

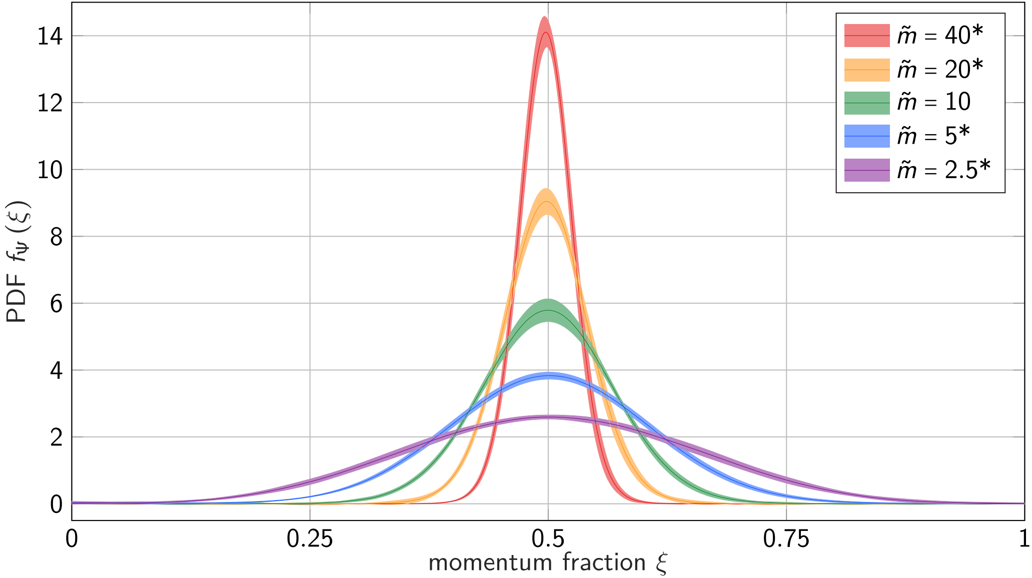


Results: N-dependence



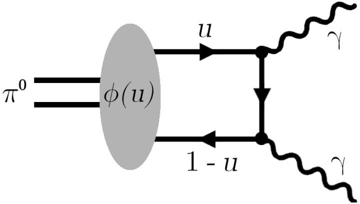


Results: PDF [*preliminary]



Lightcone Distribution Amplitude (LCDA) [preliminary]

LCDA: decay or hadronization

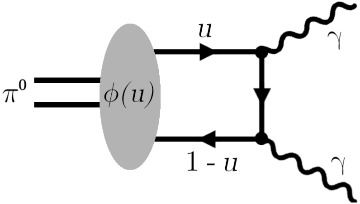


$$\pi^0 \to q(u)\bar{q}(1-u) \to \gamma\gamma$$

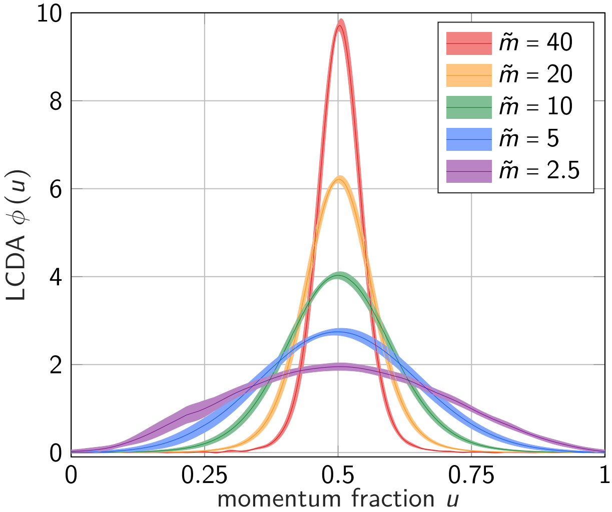
$$if\phi(u) = \int dz^- e^{iuP^+z^-} \langle 0 | \bar{\psi}(z^-)\gamma^+\gamma_5 W(z^- \leftarrow 0)\psi(0) | P \rangle$$

Lightcone Distribution Amplitude (LCDA) [preliminary]

LCDA: decay or hadronization



$$\pi^0 \to q(u)\bar{q}(1-u) \to \gamma\gamma$$



$$if\phi(u) = \int dz^- e^{iuP^+z^-} \langle 0 | \bar{\psi}(z^-)\gamma^+\gamma_5 W(z^- \leftarrow 0)\psi(0) | P \rangle$$

Outline

1 Motivation & Goal: Parton Distribution Functions

2 Method: Tensor Network States

3 Application: Schwinger Model

4 Results

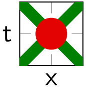
5 Summary & Outlook

Summary

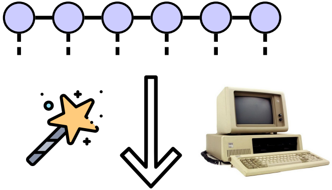
Summary:

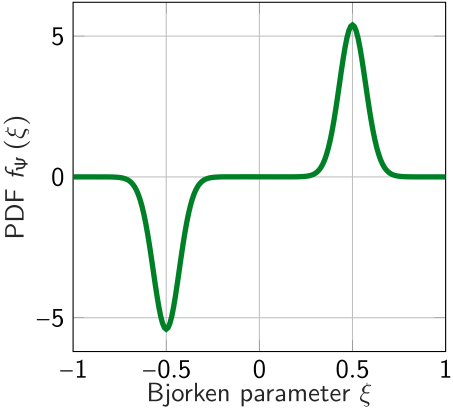
- PDF → universal structure of hadrons

- Euclidean space: lightcone → point



- ⇒ use tensor network states / quantum devices
- Schwinger model:
  fermion- and anti-fermion-PDF for the vector meson





Outlook:

- further lightcone observables
- same analysis for QCD ☺

[arXiv:2504.07508]
[arXiv:2409.16996]

1 M. C. Bañuls, K. Cichy, C.-J. D. Lin, and M. Schneider, "Parton distribution functions in the schwinger model from tensor network states," Phys. Rev. D **113**, L011502 (2026) doi:10.1103/lcyh-rxf3.

2 M. Schneider, M. C. Bañuls, K. Cichy, and C.-J. D. Lin, "Parton Distribution Functions in the Schwinger Model with Tensor Networks," in Proceedings of the 41st international symposium on lattice field theory — pos(lattice2024), Vol. 466 (2025), p. 024, doi:10.22323/1.466.0024.

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5 M. C. Bañuls, K. Cichy, J. I. Cirac, and K. Jansen, "The mass spectrum of the schwinger model with matrix product states," JHEP **2013**, 158 (2013) doi:10.1007/JHEP11(2013)158.

6 J. Dai, J. Hughes, and J. Liu, "Perturbative analysis of the massless schwinger model," Phys. Rev. D **51**, 5209–5215 (1995) doi:10.1103/PhysRevD.51.5209.

7 J. Schwinger, "Gauge invariance and mass. ii," Phys. Rev. **128**, 2425–2429 (1962) doi:10.1103/PhysRev.128.2425.

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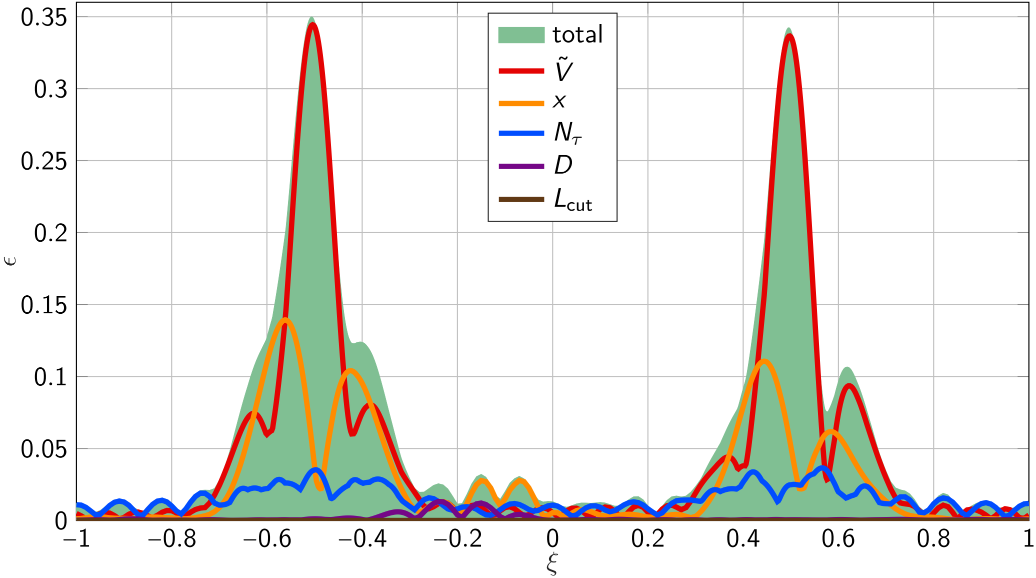
[9] Y. Mo and R. J. Perry, "Basis function calculations for the massive schwinger model in the light-front tamm-dancoff approximation," Journal of Computational Physics **108**, 159–174 (1993) doi:10.1006/jcph.1993.1171.

[10] Further image sources, EIC, www.computerhistory.org/timeline/1981, https://openmoji.org, www.flaticon.com/free-icons/search.

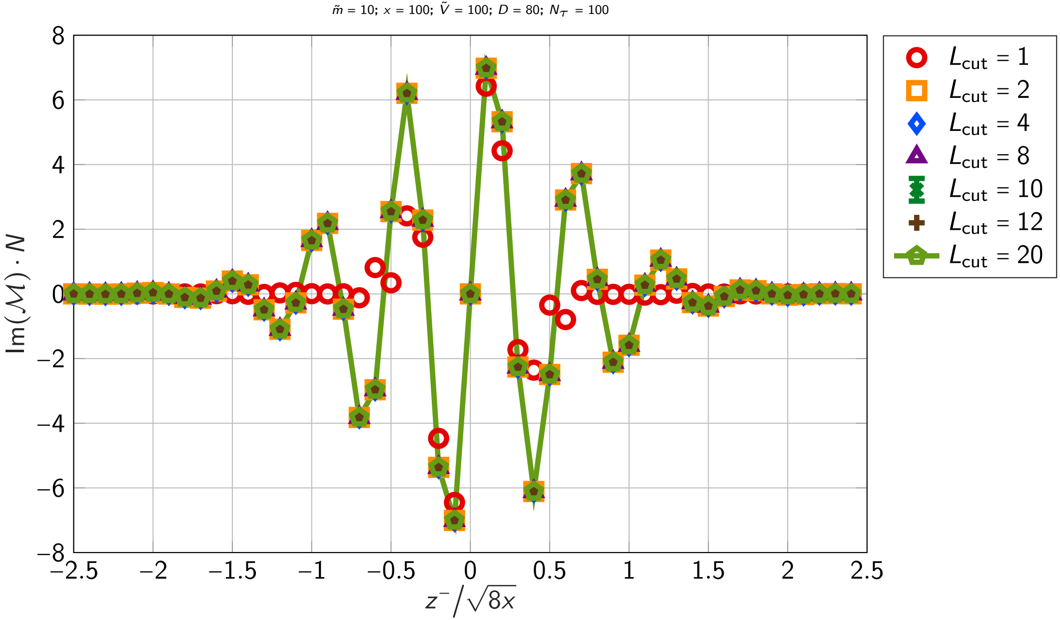
Outline

6 Backup

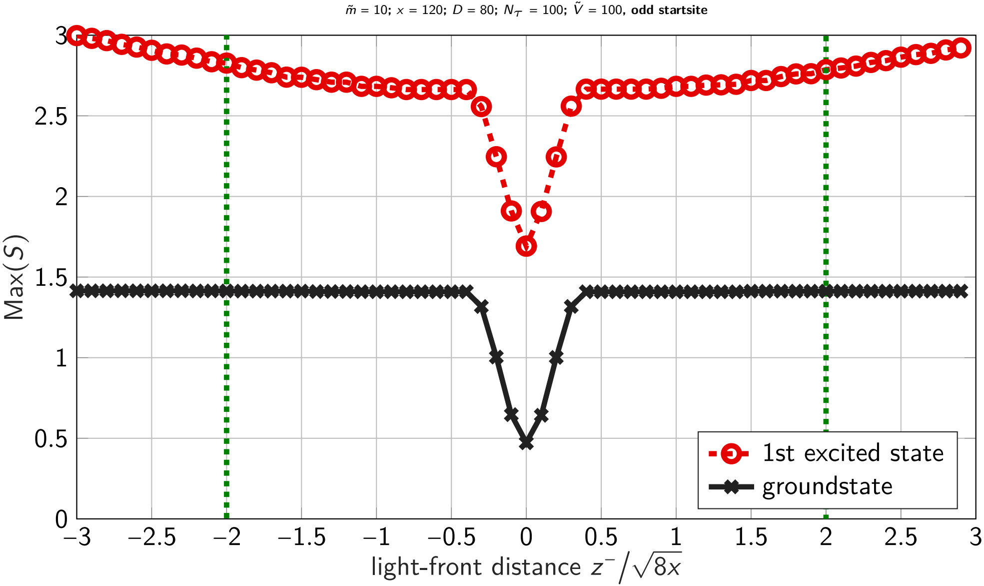
Contributions to error



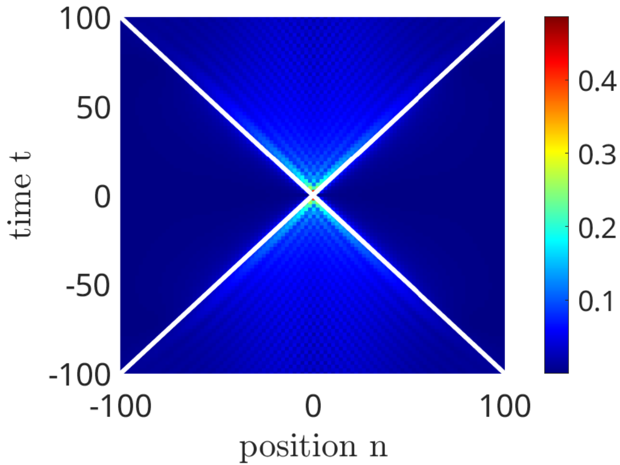
L_{cut}-dependence: truncation of electric field



Results: Entanglement entropy



Light-cone structure



$$\langle P|\, e^{iHt} \prod_{k<n} (i\sigma_k^z)\, \sigma_n^+ e^{-iH_0 t} \prod_{k'<0} (-i\sigma_{k'}^z)\, \sigma_0^- \,|P\rangle$$

- even-to-even matrix element
- calculated to each site at each timeslice
- static charge fixed at orign

Spin formulation of the Schwinger Model

$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - gA\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_0\rho$$

$$\mathcal{H} = -i\bar{\Psi}\gamma^1(\partial_1 - igA_1)\Psi + m\bar{\Psi}\Psi + \frac{1}{2}E^2 + A_0\left(g\Psi^\dagger\Psi + \rho - \partial_1 E\right)$$

Legendre transformation
$(E = F_{01})$

Spin formulation of the Schwinger Model

$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - gA\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_0\rho$$

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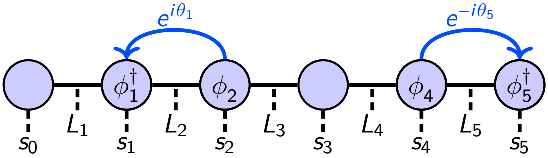
temporal Gauge: $A_0 \equiv 0$

Spin formulation of the Schwinger Model



$$\mathcal{L} = \bar{\Psi}(i\not{\partial} - g\not{A} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_0\rho$$

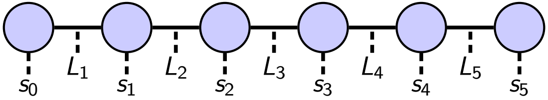
$$\mathcal{H} = -i\bar{\Psi}\gamma^1(\partial_1 - igA_1)\Psi + m\bar{\Psi}\Psi + \frac{1}{2}E^2 + A_0\left(g\Psi^\dagger\Psi + \rho - \partial_1 E\right)$$

$$H = -\frac{i}{2a}\sum_n\left(\phi_n^\dagger e^{i\theta_n}\phi_{n+1} - \phi_{n+1}^\dagger e^{-i\theta_n}\phi_n\right) + m\sum_n(-1)^n\phi_n^\dagger\phi_n + \frac{ag^2}{2}\sum_n L_n^2$$

staggered fermions
$(\theta = agA_1, gL = E)$

$$\phi_n \sim \begin{cases}\Psi_{\text{upper}}(x) & \text{if } n \text{ even}\\ \Psi_{\text{lower}}(x) & \text{if } n \text{ odd,}\end{cases}$$

Spin formulation of the Schwinger Model



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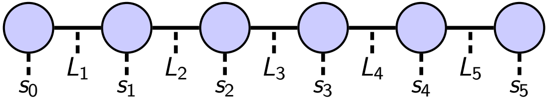
decoupling

$$\phi_n \to \prod_{k<n}\left(e^{-i\theta_k}\right)\phi_n$$

Spin formulation of the Schwinger Model



$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - gA\!\!\!/ - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_0\rho$$

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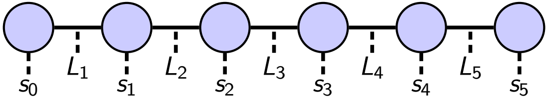
$$H = \frac{1}{2a}\sum_n(\sigma_n^+\sigma_{n+1}^- + \sigma_{n+1}^-\sigma_n^+) + \frac{m}{2}\sum_n[1 + (-1)^n\sigma_n^z] + \frac{ag^2}{2}\sum_n L_n^2$$

Jordan-Wigner transformation

$$\hat{\phi}_n = \prod_{k<n}(i\sigma_k^z)\,\sigma_n^-$$

Spin formulation of the Schwinger Model



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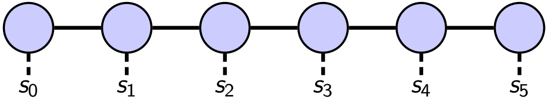
$$H = \frac{1}{2a}\sum_n(\sigma_n^+\sigma_{n+1}^- + \sigma_{n+1}^-\sigma_n^+) + \frac{m}{2}\sum_n[1 + (-1)^n\sigma_n^z] + \frac{ag^2}{2}\sum_n L_n^2$$

Gauss's law:
$(x = \frac{1}{a^2g^2}, \mu = \frac{2m}{ag^2})$

$$L_n - L_{n-1} = Q_n = \frac{1}{2}[(-1)^n + \sigma_n^z] + q_n$$

Spin formulation of the Schwinger Model



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$$\mathcal{H} = -i\bar{\Psi}\gamma^1(\partial_1 - igA_1)\Psi + m\bar{\Psi}\Psi + \frac{1}{2}E^2 \cancel{+A_0\left(g\Psi^\dagger\Psi + \rho - \partial_1 E\right)}$$

$$H = -\frac{i}{2a}\sum_n\left(\phi_n^\dagger \cancel{e^{i\theta_n}}\phi_{n+1} - \phi_{n+1}^\dagger \cancel{e^{-i\theta_n}}\phi_n\right) + m\sum_n(-1)^n\phi_n^\dagger\phi_n + \frac{ag^2}{2}\sum_n L_n^2$$

$$H = \frac{1}{2a}\sum_n(\sigma_n^+\sigma_{n+1}^- + \sigma_{n+1}^-\sigma_n^+) + \frac{m}{2}\sum_n[1 + (-1)^n\sigma_n^z] + \frac{ag^2}{2}\sum_n L_n^2$$

$$H = x\sum_{n=0}^{N-2}[\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+] + \frac{\mu}{2}\sum_{n=0}^{N-1}[1 + (-1)^n\sigma_n^z] + \sum_{n=0}^{N-2}\left[\sum_{k=0}^{n} Q_k\right]^2$$

Gauss's law:

$$\left(x = \frac{1}{a^2g^2}, \mu = \frac{2m}{ag^2}\right)$$

$$L_n - L_{n-1} = Q_n = \frac{1}{2}[(-1)^n + \sigma_n^z] + q_n$$

Time evolution with MPS

$$H = x\sum_{n=0}^{N-2}\left[\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right] + \frac{\mu}{2}\sum_{n=0}^{N-1}\left[1+(-1)^n\sigma_n^z\right] + \sum_{n=0}^{N-2}\left[\frac{1}{2}\sum_{k=0}^{n}\left((-1)^k+\sigma_k^z+2q_k\right)\right]^2$$

Suzuki-Trotter decomposition:

$$e^{-i\tau H} \approx \left(e^{-i\delta\tau H_{eo}} e^{-i\delta\tau H_{oe}} e^{-i\delta\tau H_L}\right)^{N_\tau}$$

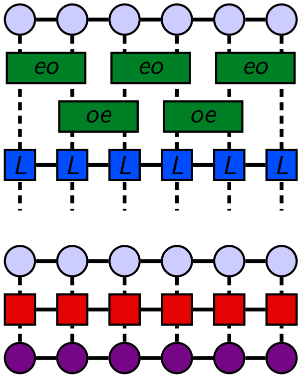
$$e^{-i\delta\tau H_{eo}} = \prod_{n=0,2,\ldots} e^{-i\delta\tau x\left[\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right]}$$

$$e^{-i\delta\tau H_{oe}} = \prod_{n=1,3,\ldots} e^{-i\delta\tau x\left[\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right]}$$

$e^{-i\delta\tau H_L}$: MPO with indices $L_n \in [-n, n]$

→ truncate to L_{cut}

Optimize to get new MPS



Lattice forumlation of PDF

PDF for lattice spin model:

$$f_\Psi(\xi) = \frac{NM}{8\pi x} \sum_{d=0,2,4,\dots,L} e^{-i\xi \frac{Md}{2x}} \mathcal{M}(d)$$

$$\mathcal{M}(d) = \mathcal{M}_{0,0}(d) - \mathcal{M}_{0,1}(d) - \mathcal{M}_{1,0}(d) + \mathcal{M}_{1,1}(d)$$

$$\mathcal{M}_{a,b}(d) = \langle h| \, e^{iHt_d} \prod_{k<d+a} \left(-i\sigma_k^z\right) \sigma_{d+a}^+ e^{-iH_{d-1}\delta t} \dots e^{-iH_3\delta t} e^{-iH_1\delta t} \prod_{k'<b} \left(i\sigma_{k'}^z\right) \sigma_b^- \, |h\rangle_c \, .$$