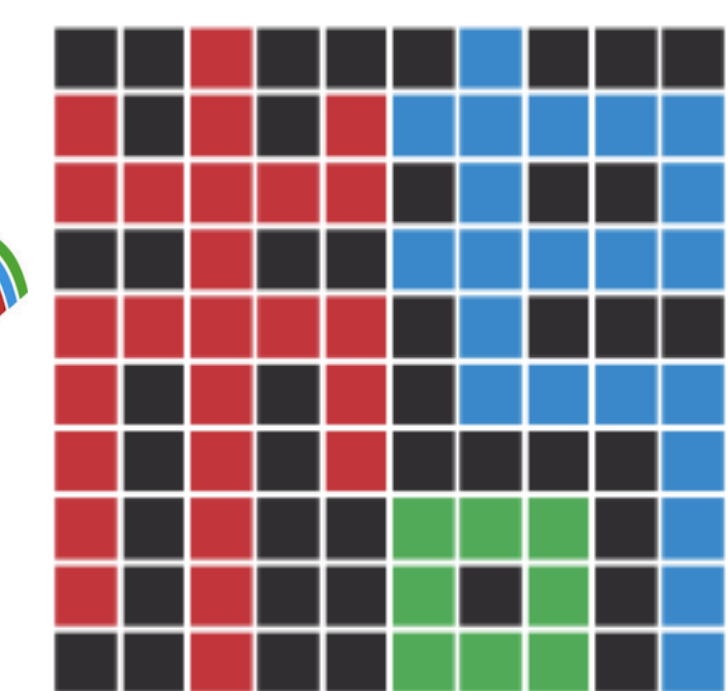
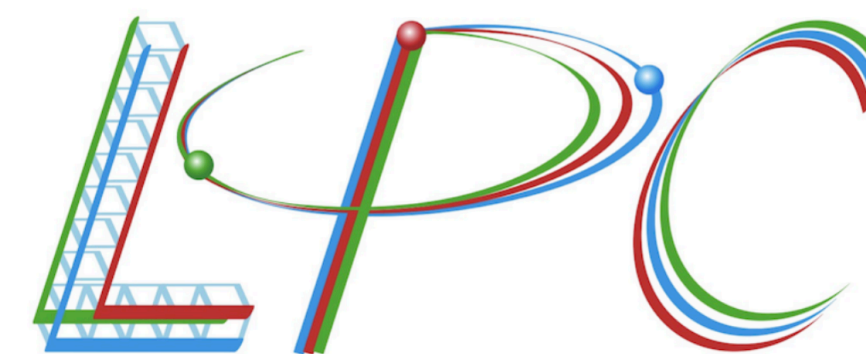




香港中文大學(深圳)

The Chinese University of Hong Kong, Shenzhen



CLQCD

**LaMET 2026**

# Progress in the Lattice of Jaffe- Manohar Spin Decomposition

2026.07.06 PM 4:00-4:30

Speaker: Dian-Jun Zhao

[Mainly based on arXiv: 2512.24315](https://arxiv.org/abs/2512.24315)

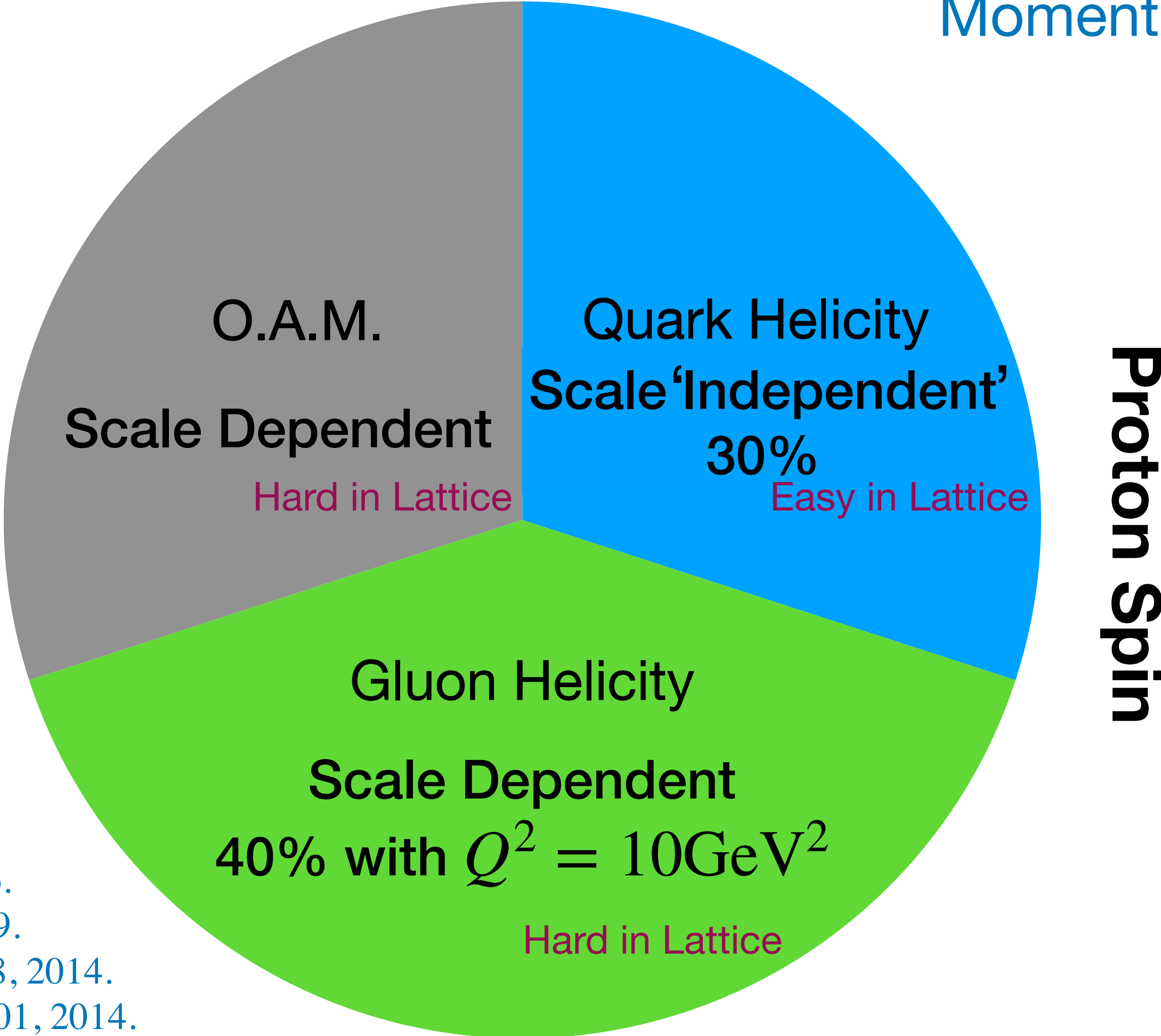
In Collaboration with LPC and CLQCD

# Introduction on spin structure

**Jaffe–Manohar spin decomposition:**  $\frac{1}{2} = L_q^{\text{JM}} + L_g^{\text{JM}} + \frac{1}{2} \Delta\Sigma + \Delta G$

R. L. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990).

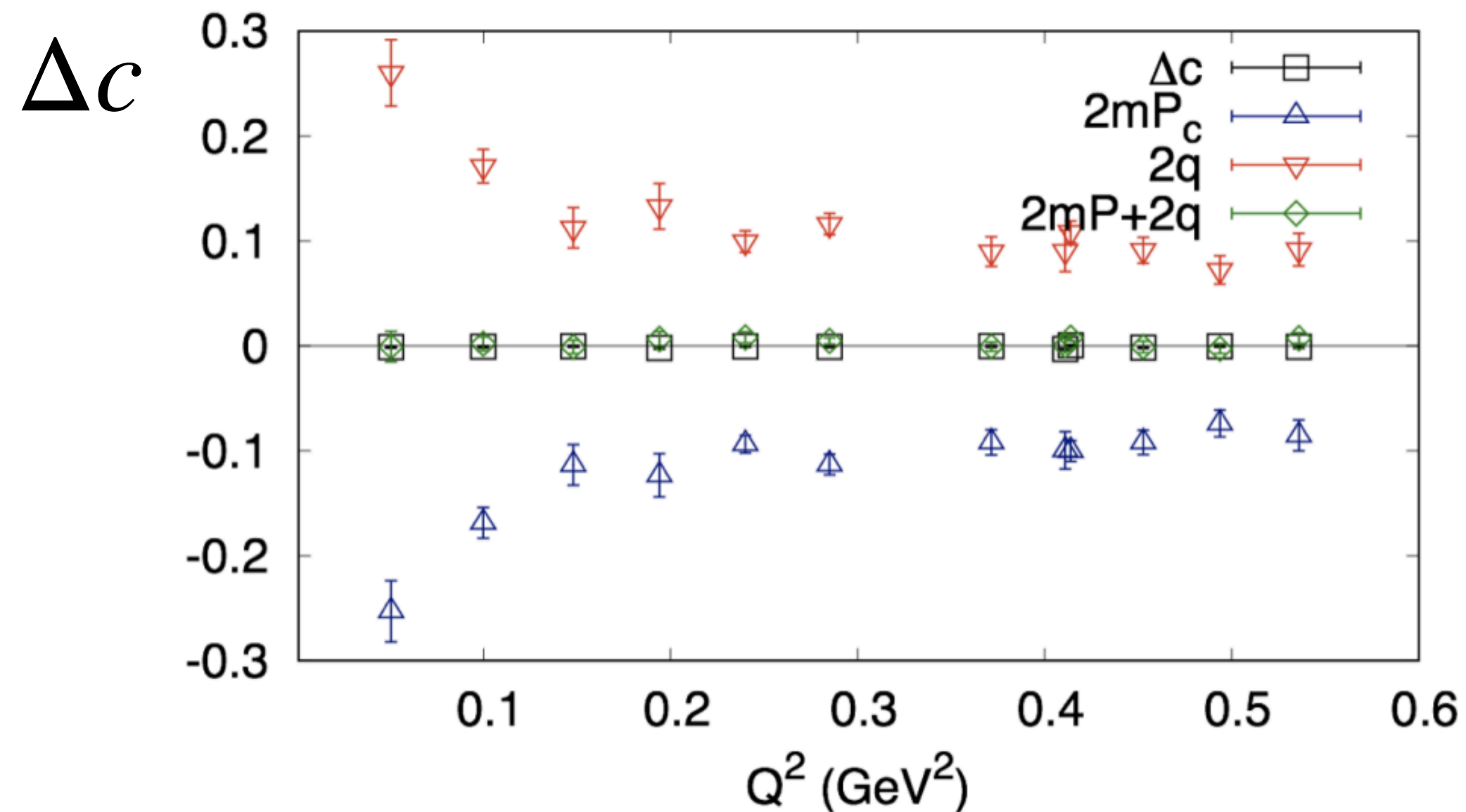
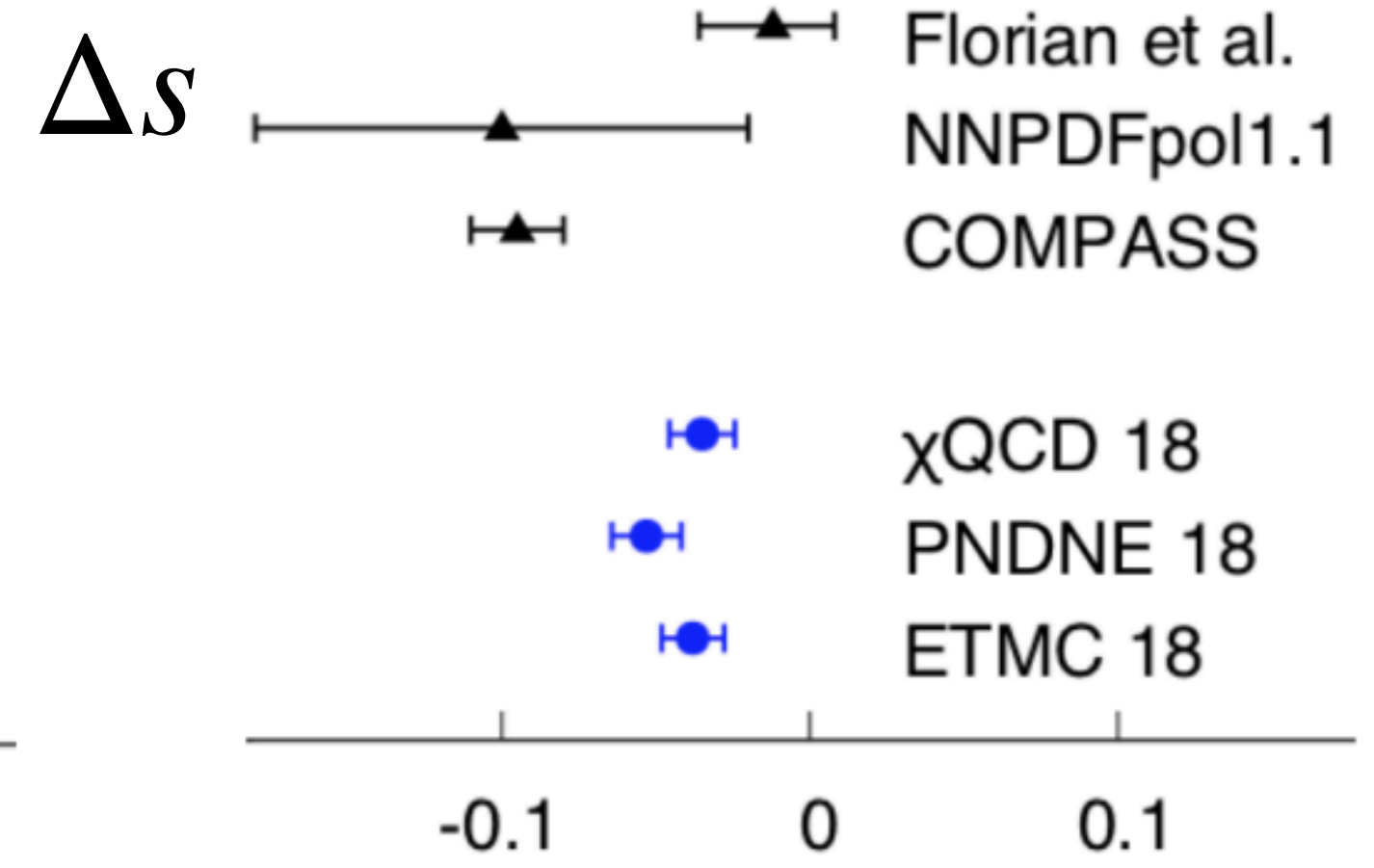
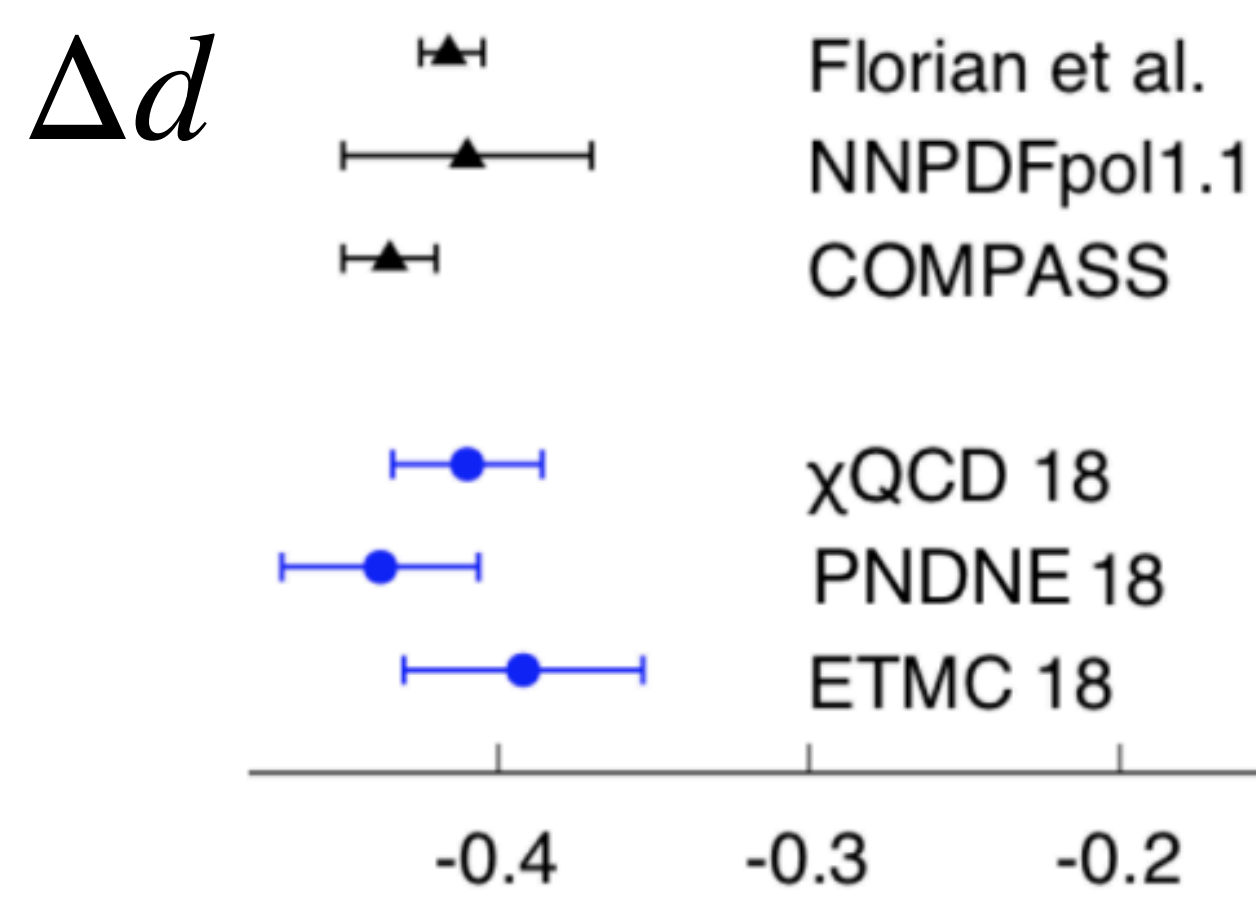
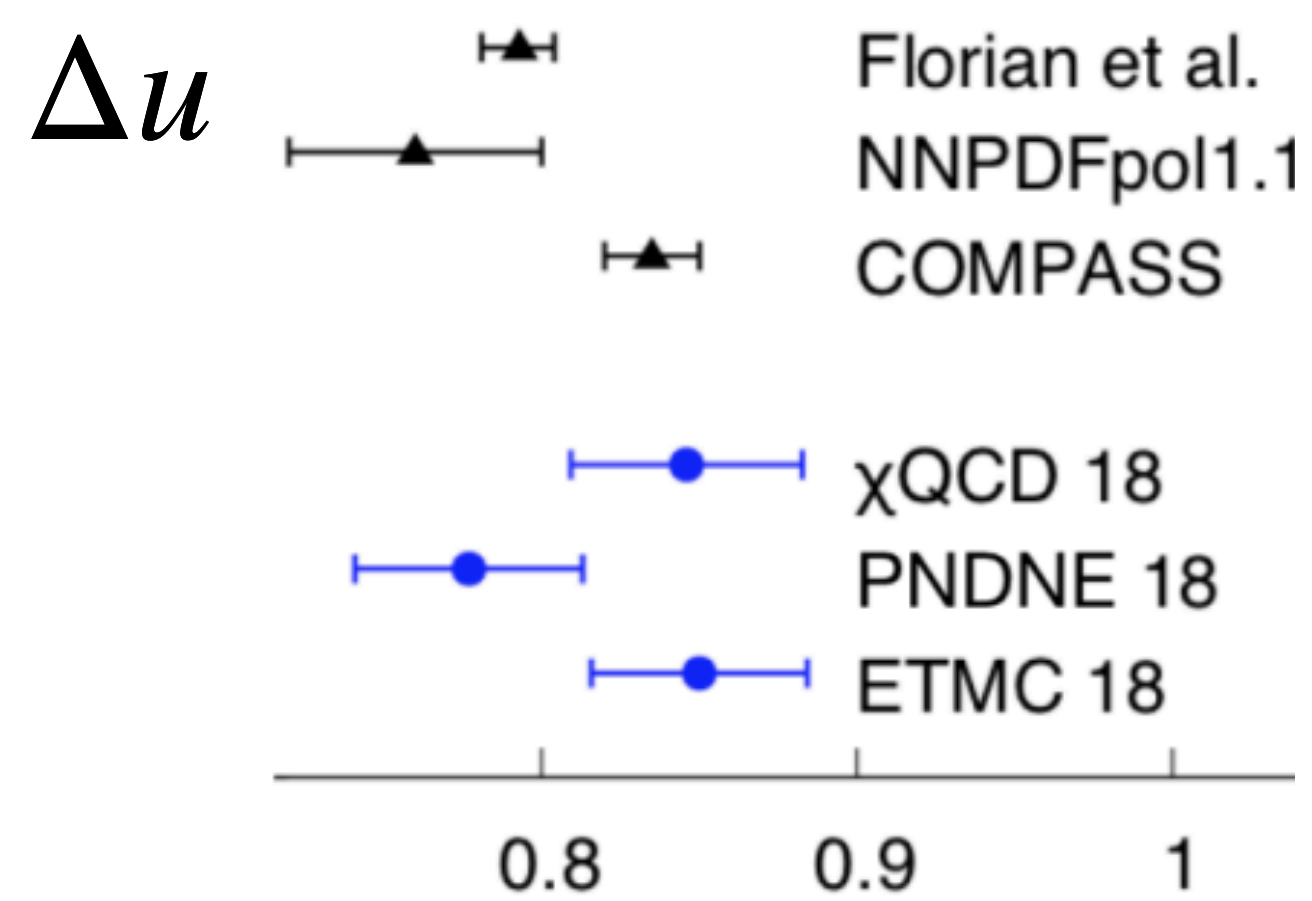
Moment of polarized PDF



C. Adolph et al. Phys. Lett., B753:18–28, 2016.  
 D. Florian et al. Phys. Rev. D, 80:034030, 2009.  
 E. R. Nocera, et al. Nucl. Phys., B887:276–308, 2014.  
 D. Florian, et al. Phys. Rev. Lett., 113(1):012001, 2014.

# Lattice interpretation of quark helicity

$$\Delta q = \langle PS | \bar{q} \gamma_5 \vec{\gamma} \cdot \vec{S} q | PS \rangle \quad \int d^3x (\bar{q} \gamma_\mu \gamma_5 q)(x) = 2m_f \int d^3x \vec{x} P(x) - 2i \int d^3x \vec{x} q(x)$$



$$\frac{1}{2} \Delta \Sigma = \frac{1}{2} \sum \Delta q = 0.155(25)$$

J. Liang, et al. Phys. Rev. D, 98(7):074505, 2018.

H.-W. Lin, et al. Phys. Rev. D, 98:094512, 2018.

C. Alexandrou, et al. Phys. Rev. Lett., 119(14):142002, 2017.

M. Gong, et al. Phys. Rev., D95(11):114509, 2017.

# Lattice interpretation of gluon helicity

$$\Delta G = \int dx \Delta g(x) = \int dx \frac{i}{2xP^+} \int \frac{d\varepsilon^-}{2\pi} e^{-ix\varepsilon^-P^+} \langle \text{PS} | F_a^{+\mu}(\varepsilon^-) \mathcal{L}_{ab}(\varepsilon^-, 0) \tilde{F}_{b,\mu}^+(0) | \text{PS} \rangle$$

Integrating LCPDF

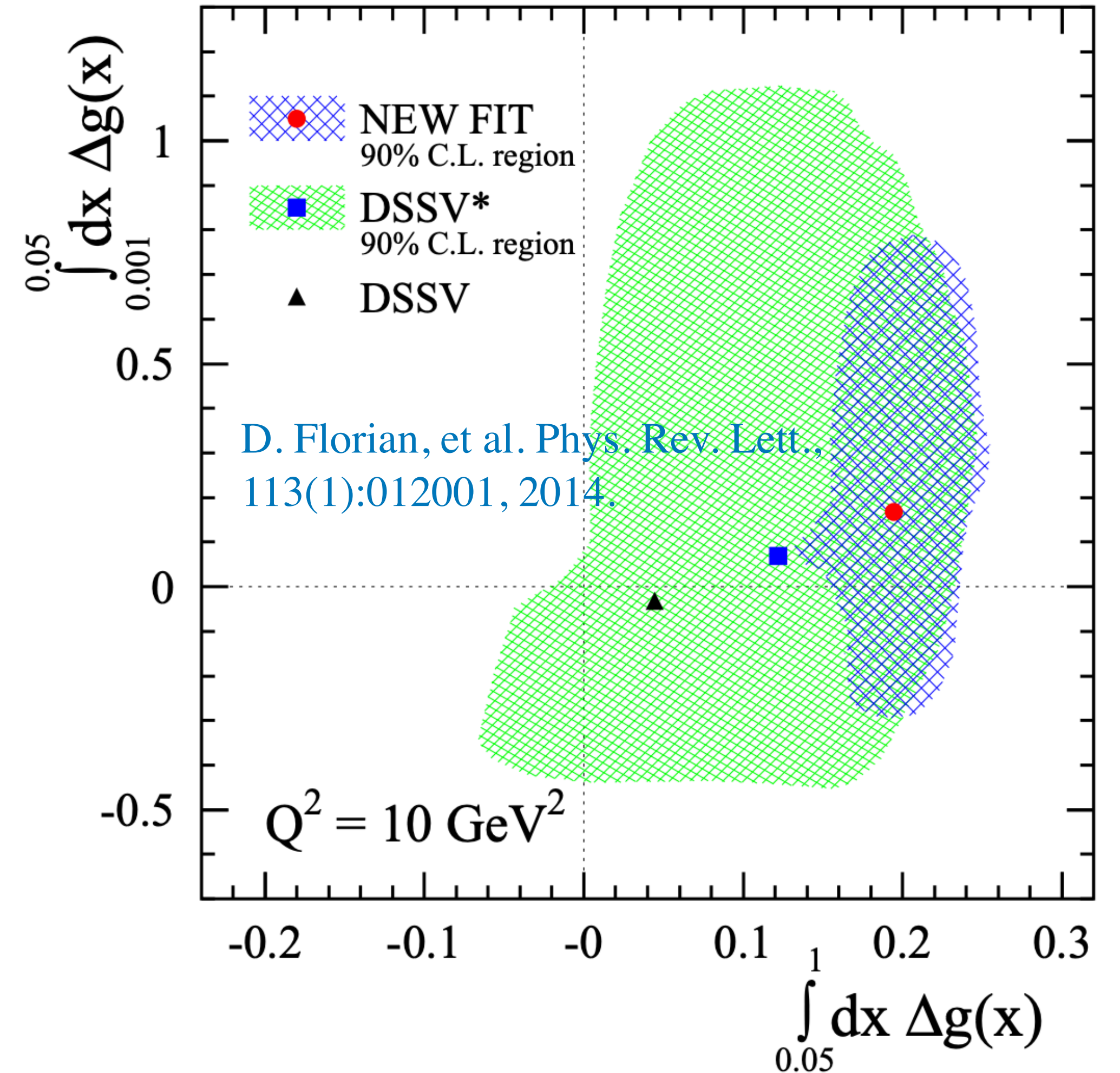
$\Delta G$  is difficult to calculate in LQCD because light-cone gauge (L.G.)  $\not\leftrightarrow$  Euclidean space!

**Gauge invariant non-local  $\longrightarrow$  Coulomb local**

↑

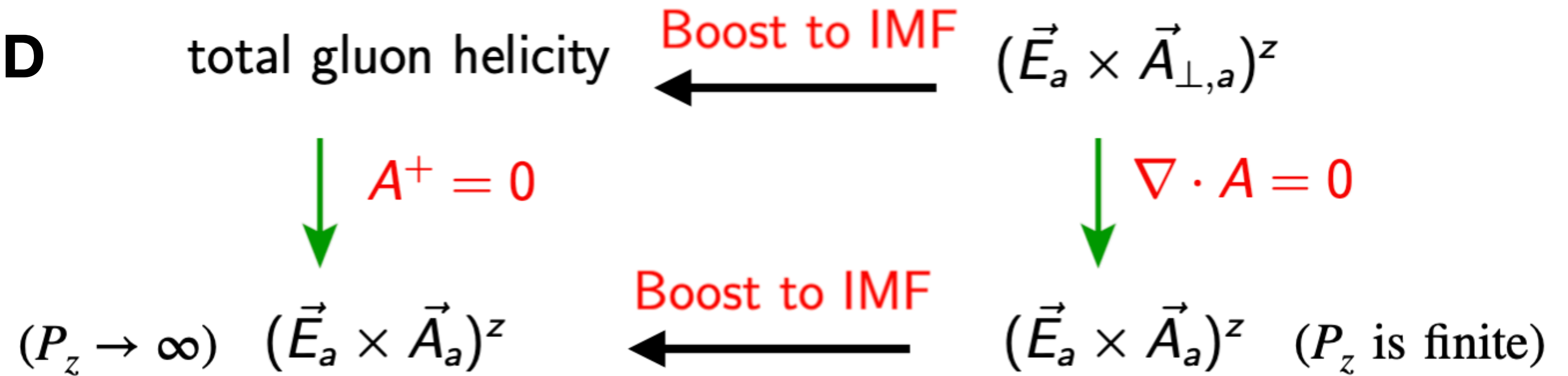
**LaMET** suggests that Coulomb gauge fixing (C.G.) condition become to L.G. when nucleon to IMF, then  $\Delta G \propto \langle \vec{E} \times \vec{A} \rangle_{\text{C.G.}} = S_G |_{\text{IMF}}$  with matching for their difference in UV behavior.

X.-D. Ji et al. [10.1103/PhysRevLett.111.112002](https://arxiv.org/abs/10.1103/PhysRevLett.111.112002)

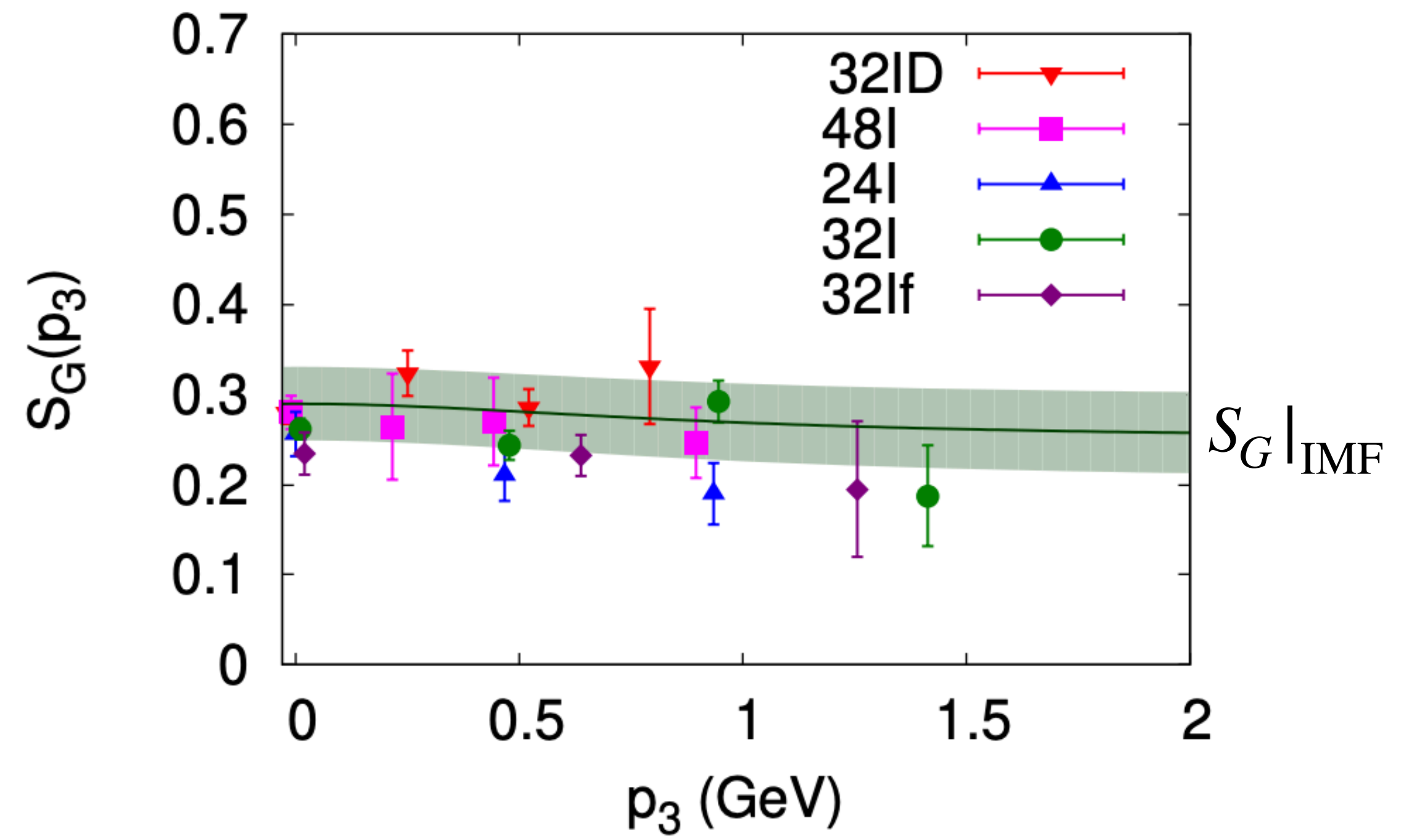
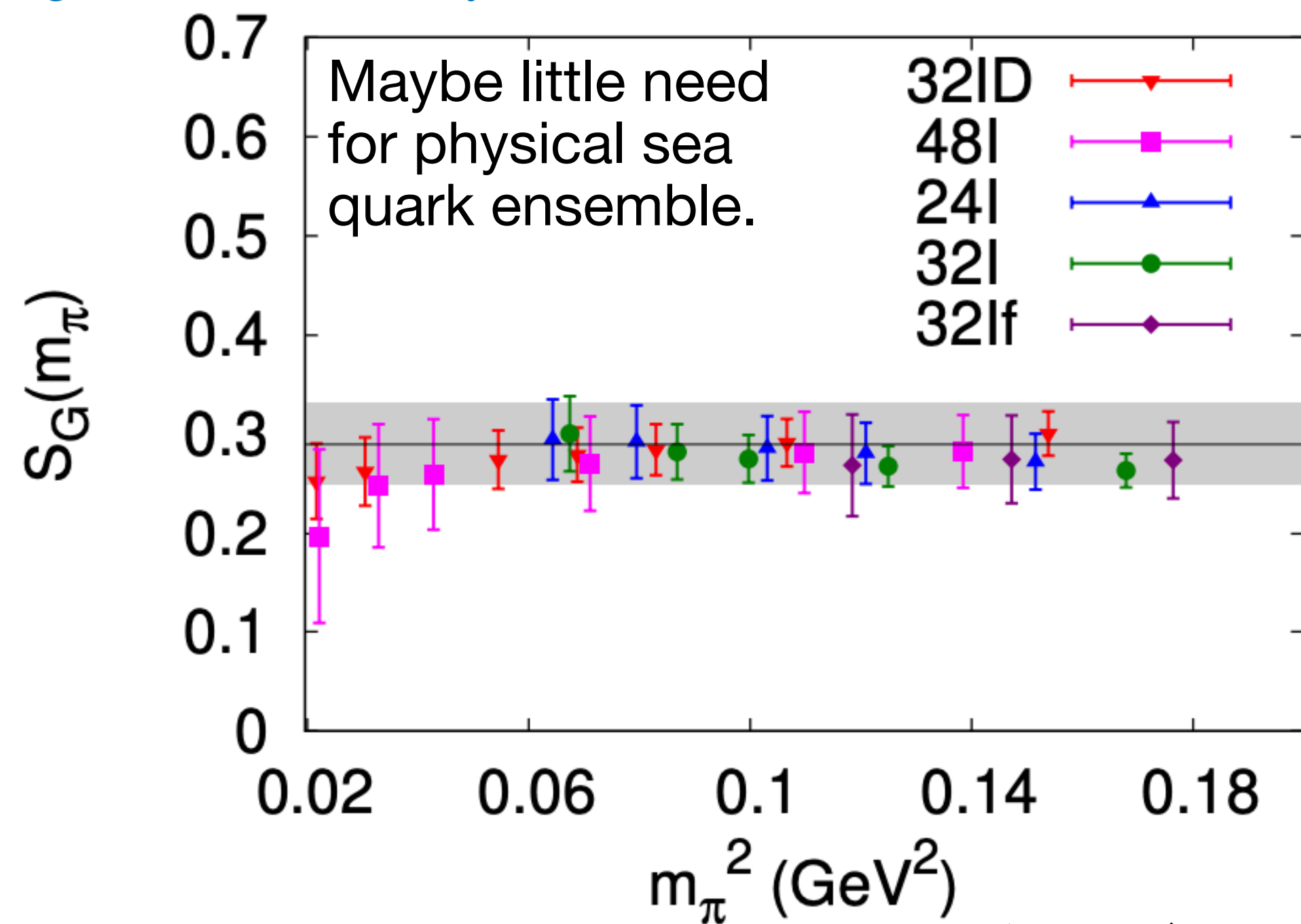


# Lattice interpretation of gluon helicity

## Gluon helicity in lattice QCD



Y.-B. Yang et al., [10.1103/PhysRevLett.118.102001](https://arxiv.org/abs/10.1103/PhysRevLett.118.102001)



$$\Delta G \propto \langle \vec{E} \times \vec{A} \rangle_{\text{C.G.}} = S_G|_{\text{IMF}} = 0.251(47)^{\text{sts.}}(16)^{\text{sym.}}$$

# Lattice interpretation of gluon helicity

Potential problems with  $\Delta G$  extraction using  $\langle \vec{E} \times \vec{A} \rangle_{\text{C.G.}}$

Y.-B. Yang et al., [10.1103/PhysRevLett.118.102001](https://arxiv.org/abs/10.1103/PhysRevLett.118.102001)

1. Large momentum external state is not large enough;
2. Should be non-perturbative renormalization + perturbative matching;

**Gluon helicity  $\Delta G$  from topological current** Z.-Y. Pang et al., [10.1007/JHEP07\(2024\)222](https://arxiv.org/abs/10.1007/JHEP07(2024)222)

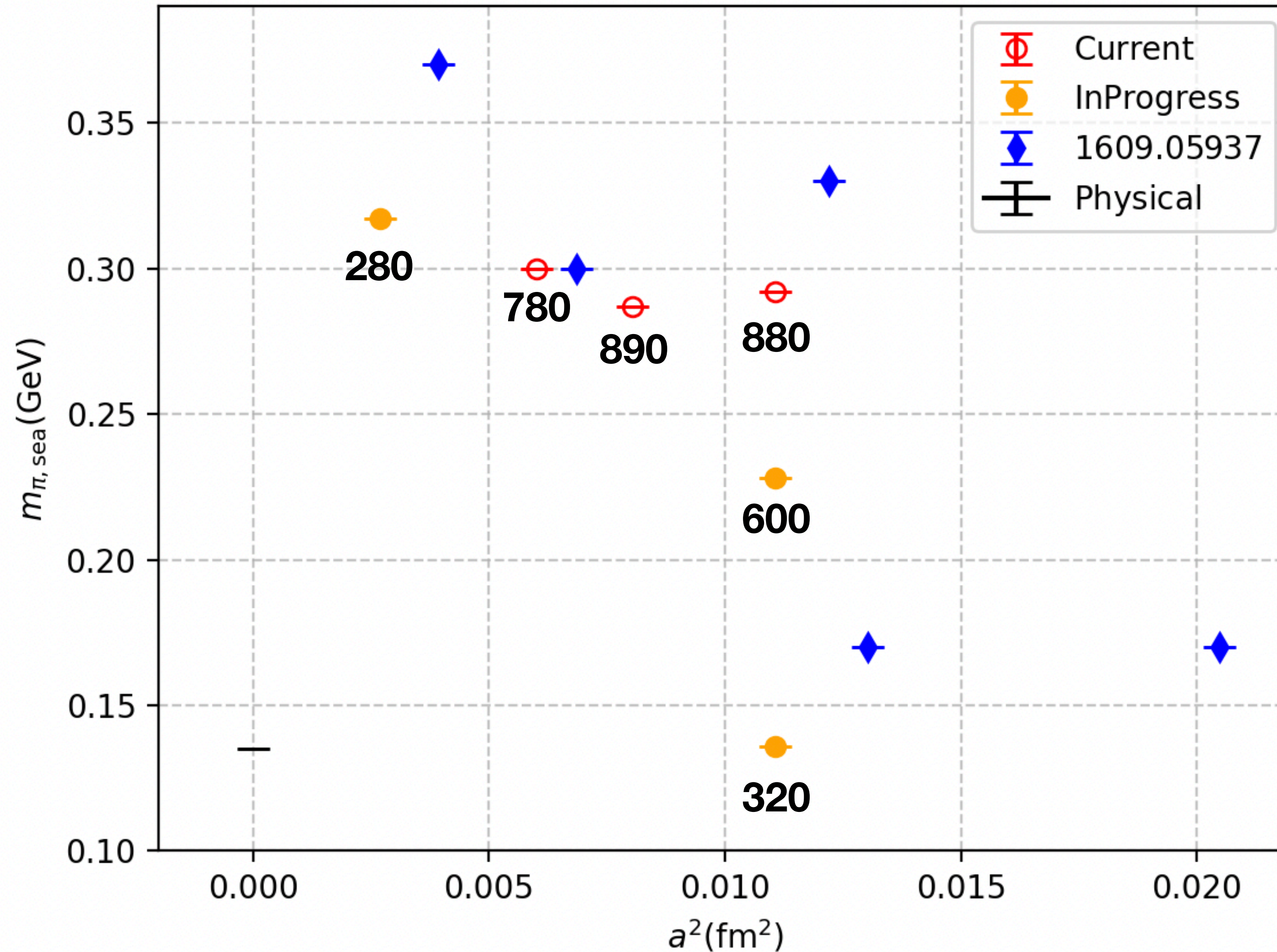
We propose a scheme that relates  $\Delta G$  to **local topological current**  $K_{\text{C.G.}}^\mu$ .  
This scheme solves all of these problems and doesn't require lattice perturbative theory.

$$\text{Topological Current } K^\mu(x) = \epsilon^{\mu\nu\rho\sigma} \text{Tr}[A_\nu F_{\rho\sigma} - 2ig_s A_\nu A_\rho A_\sigma / 3](x)$$

$$\text{Target Three-PT } \langle \text{PS}_{\text{Proj.i}} | K^{t/i} | \text{PS}_{\text{Proj.i}} \rangle_{\text{C.G.}} \propto S^{t/i} \Delta G + \text{h.t.}$$

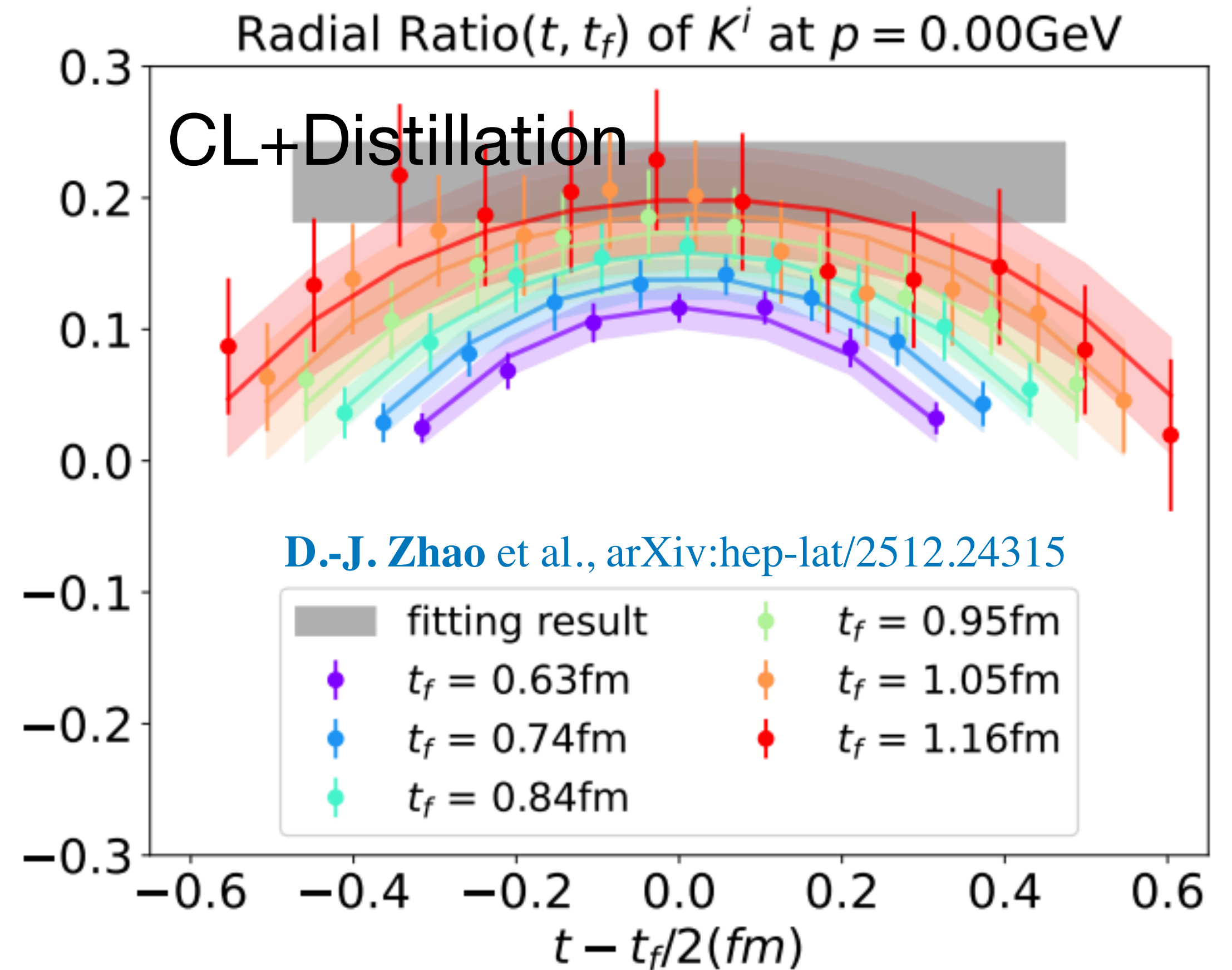
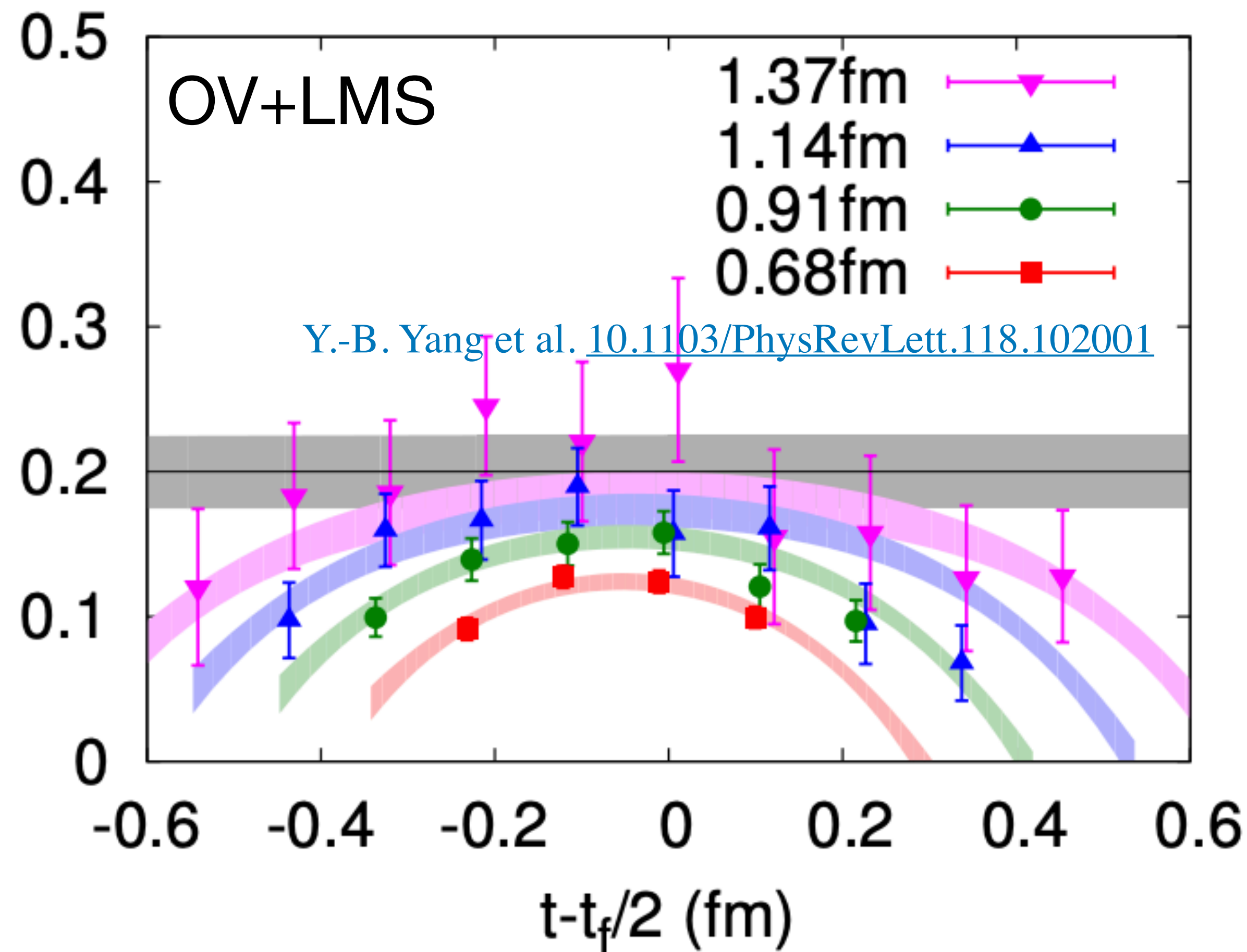
# Lattice Setup

Z.-C. Hu et al. [10.1103/PhysRevD.109.054507](https://arxiv.org/abs/10.1103/PhysRevD.109.054507).  
Y.-B. Yang et al., [10.1103/PhysRevLett.118.102001](https://arxiv.org/abs/10.1103/PhysRevLett.118.102001)



# BME with optimization smear methods

## Distillation for small $p_z$



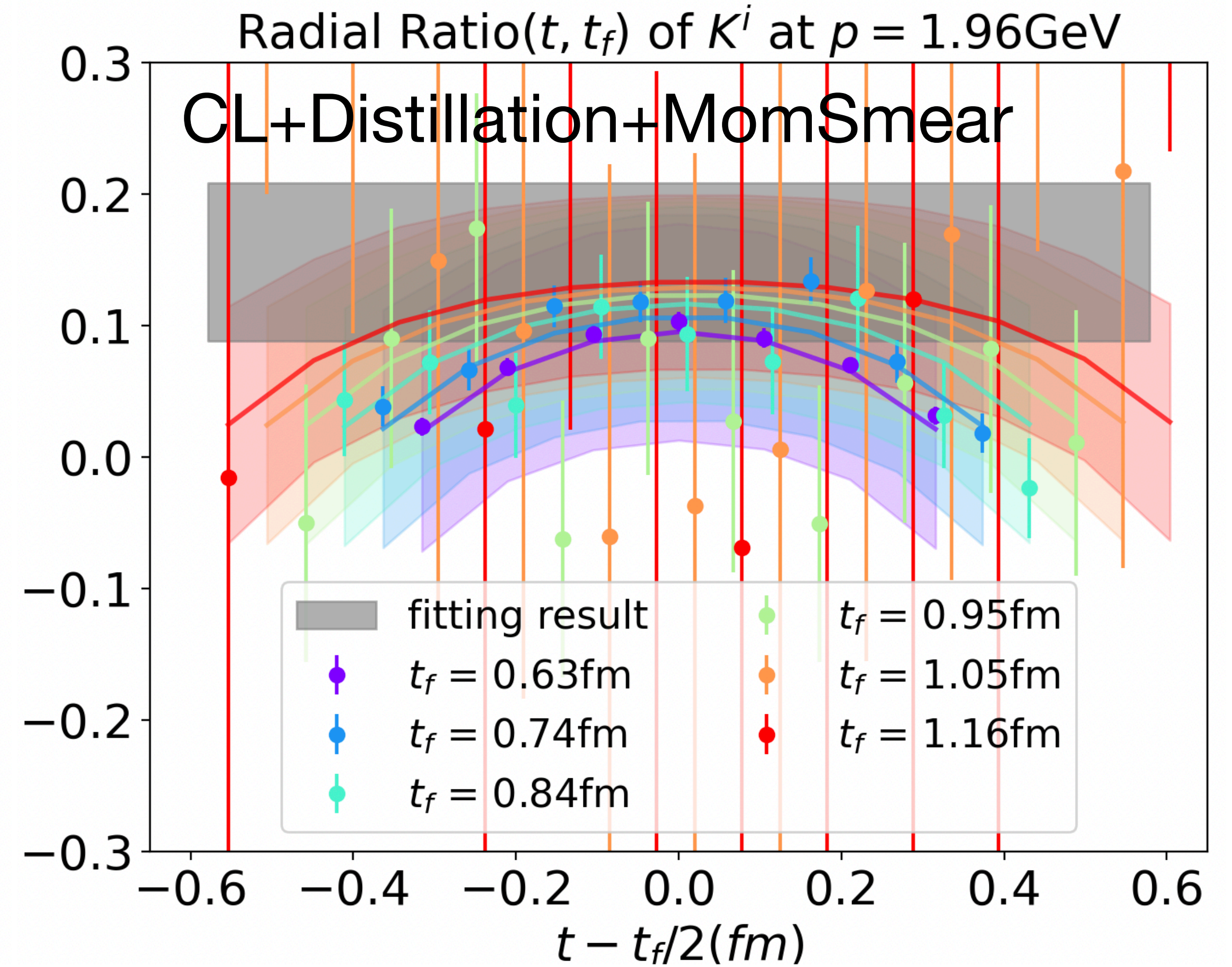
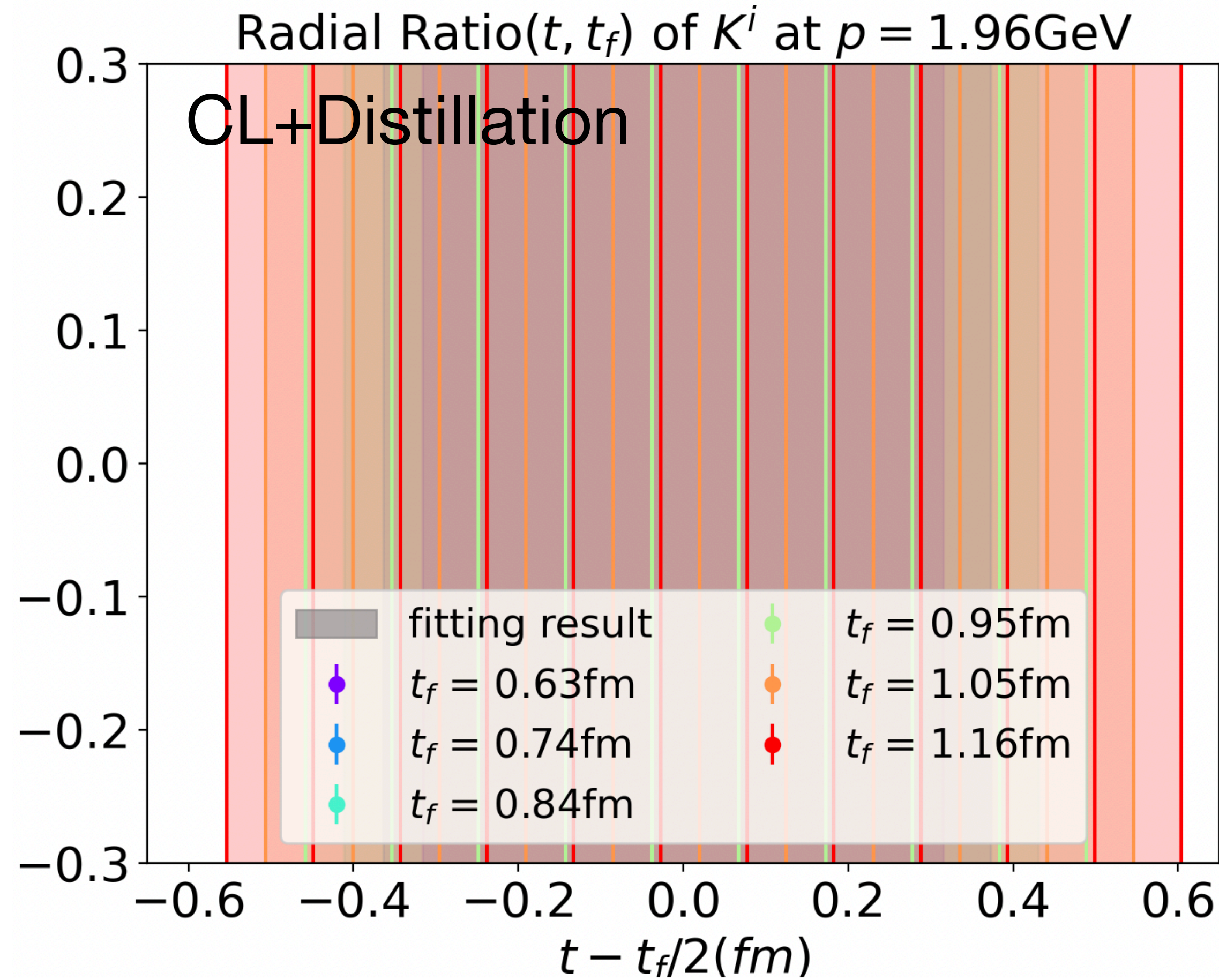
**Fitting Form:**  $R(p; t_f, t_i) = \langle \{K^\mu, E \times A\} \rangle_N^B(p) + c_1 e^{-\Delta E(t_f - t_i)} + c_2 e^{-\Delta E t_i}$

# BME with optimization smear methods

D.-J. Zhao et al., arXiv:hep-lat/2512.24315

C. Egerer et al., *Phys.Rev.D* 103 (2021) 3, 034502

## Distillation + Momentum Smear for large $p_z$

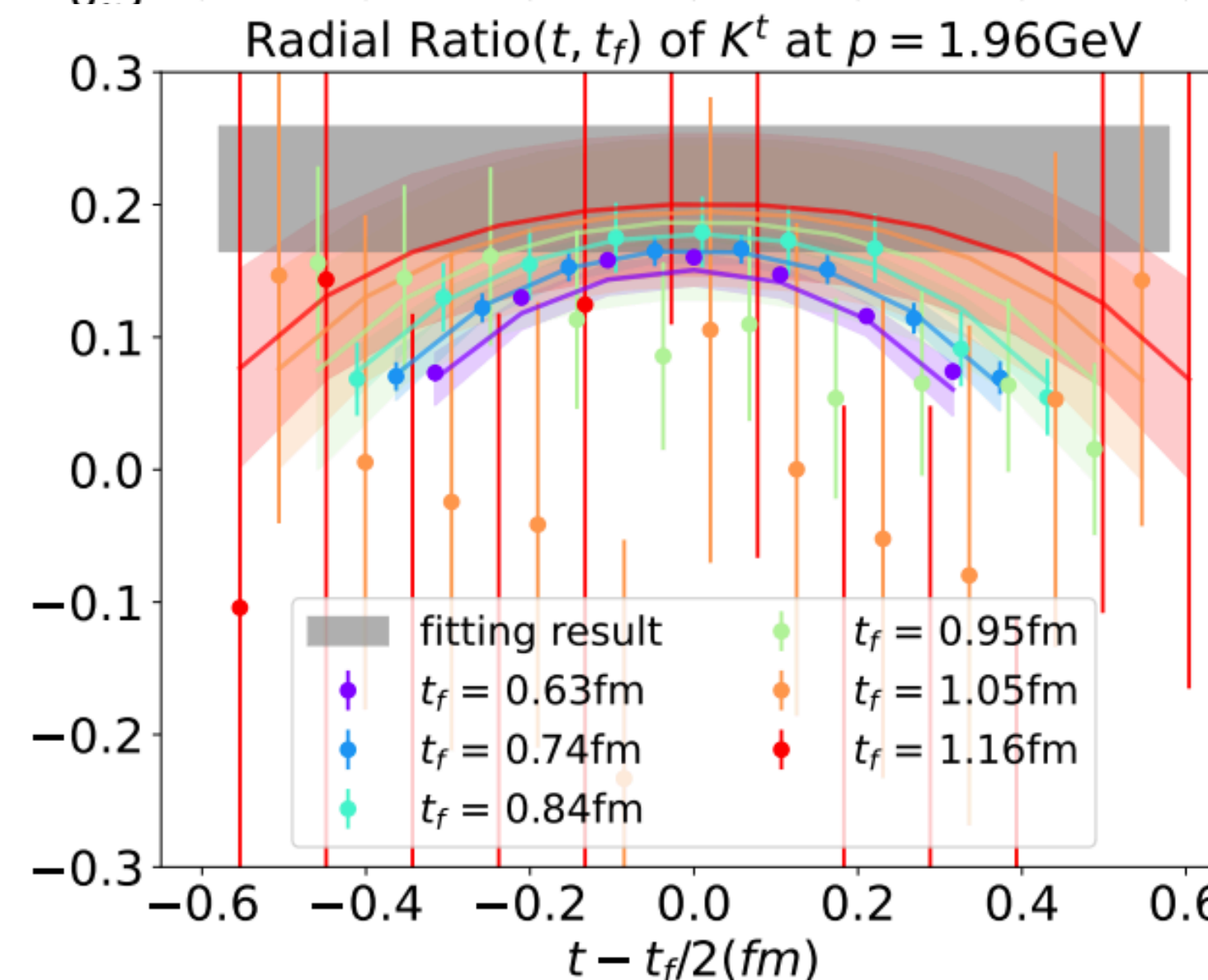
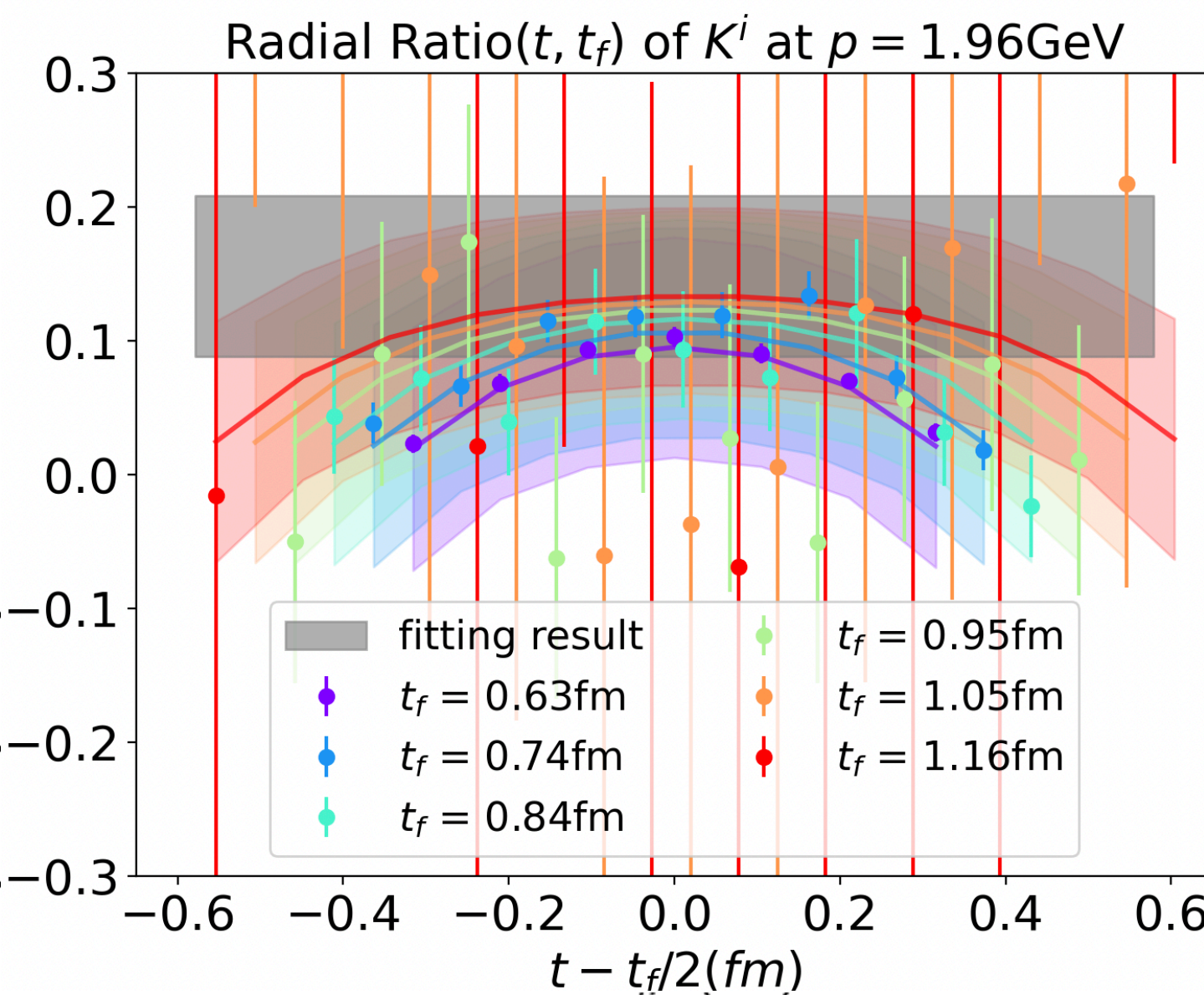
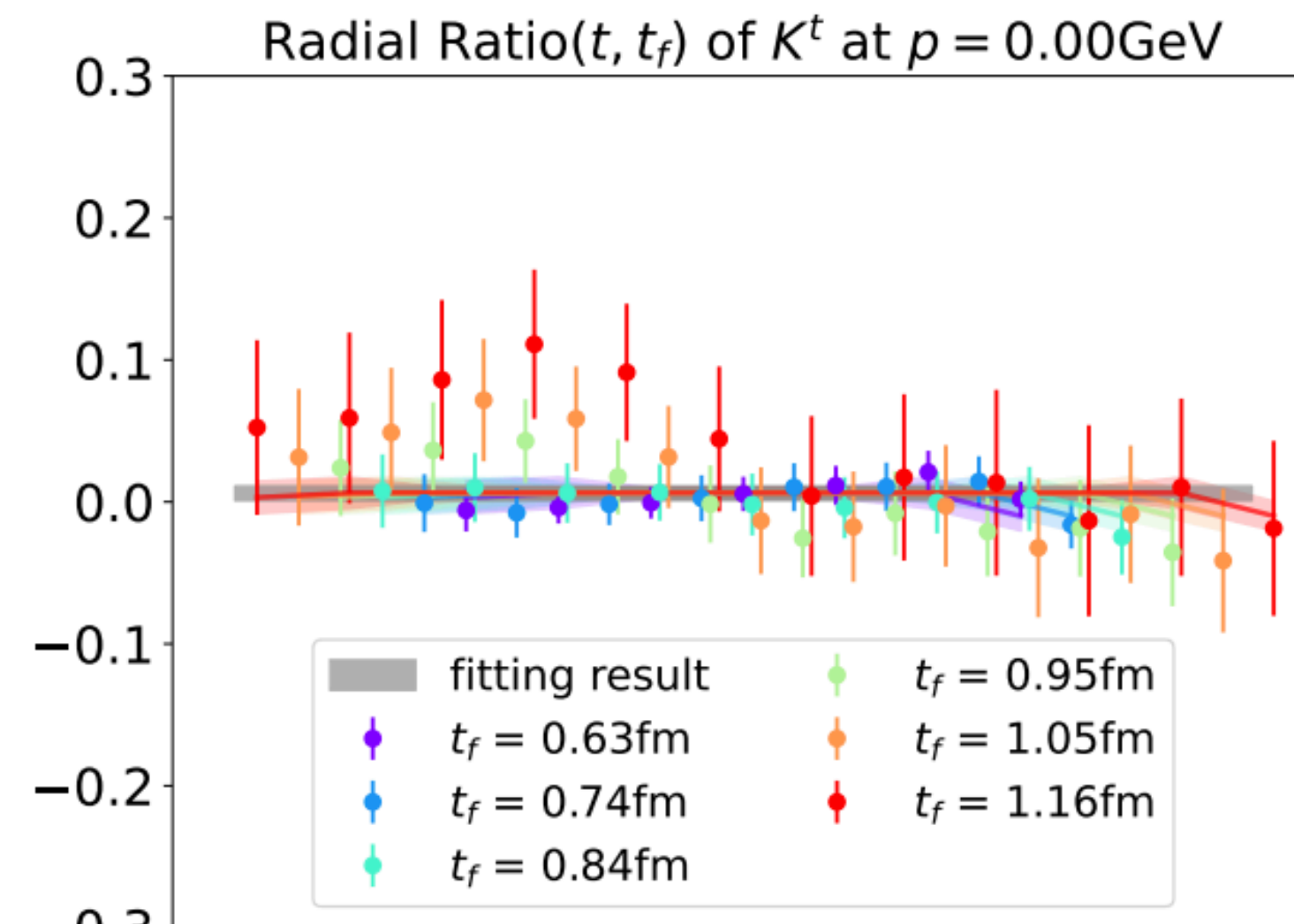
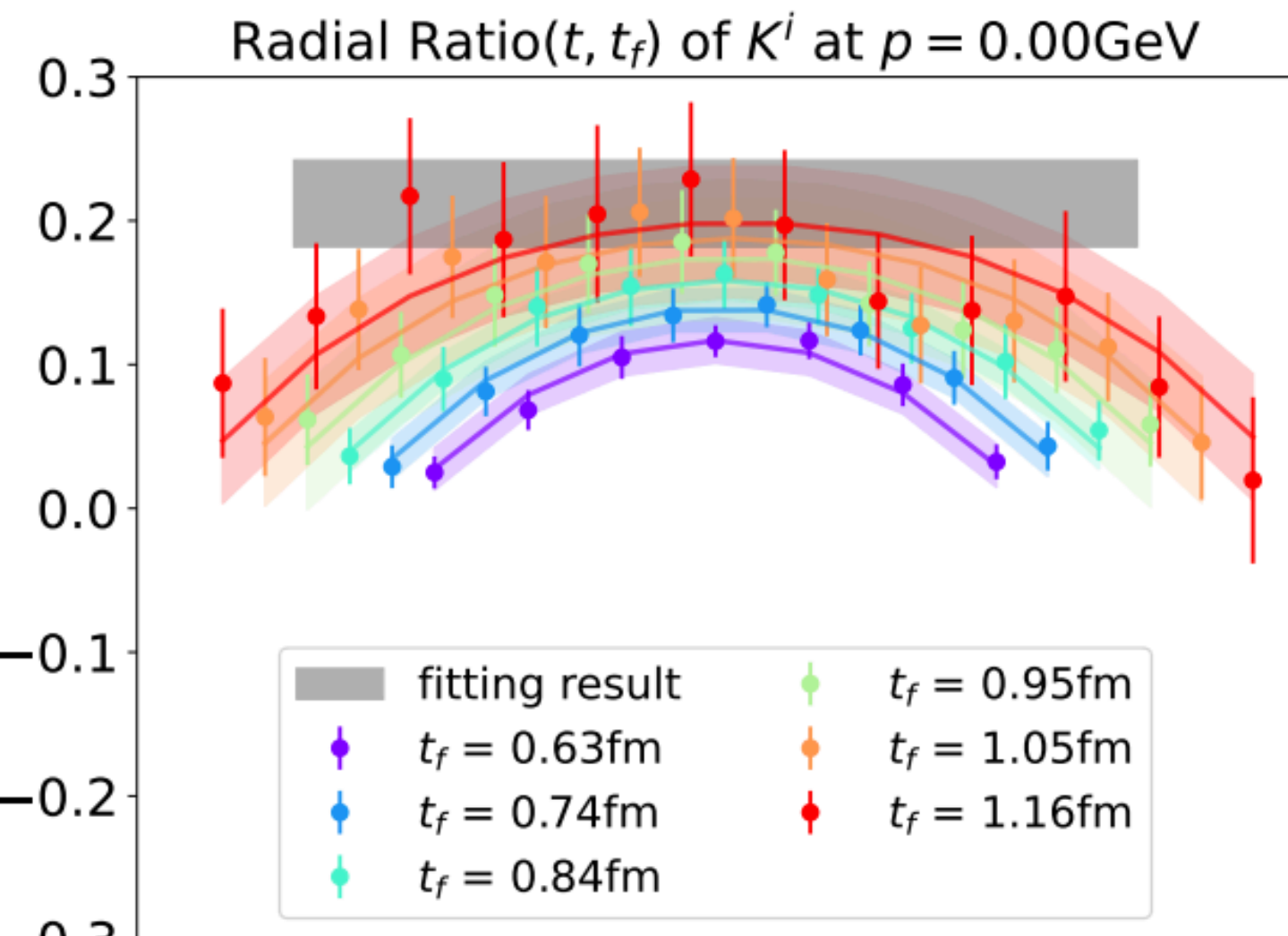


**Fitting Form:**  $R(p; t_f, t_i) = \langle \{K^\mu, E \times A\} \rangle_N^B(p) + c_1 e^{-\Delta E(t_f - t_i)} + c_2 e^{-\Delta E t_i}$

# BME Performance under different momentum

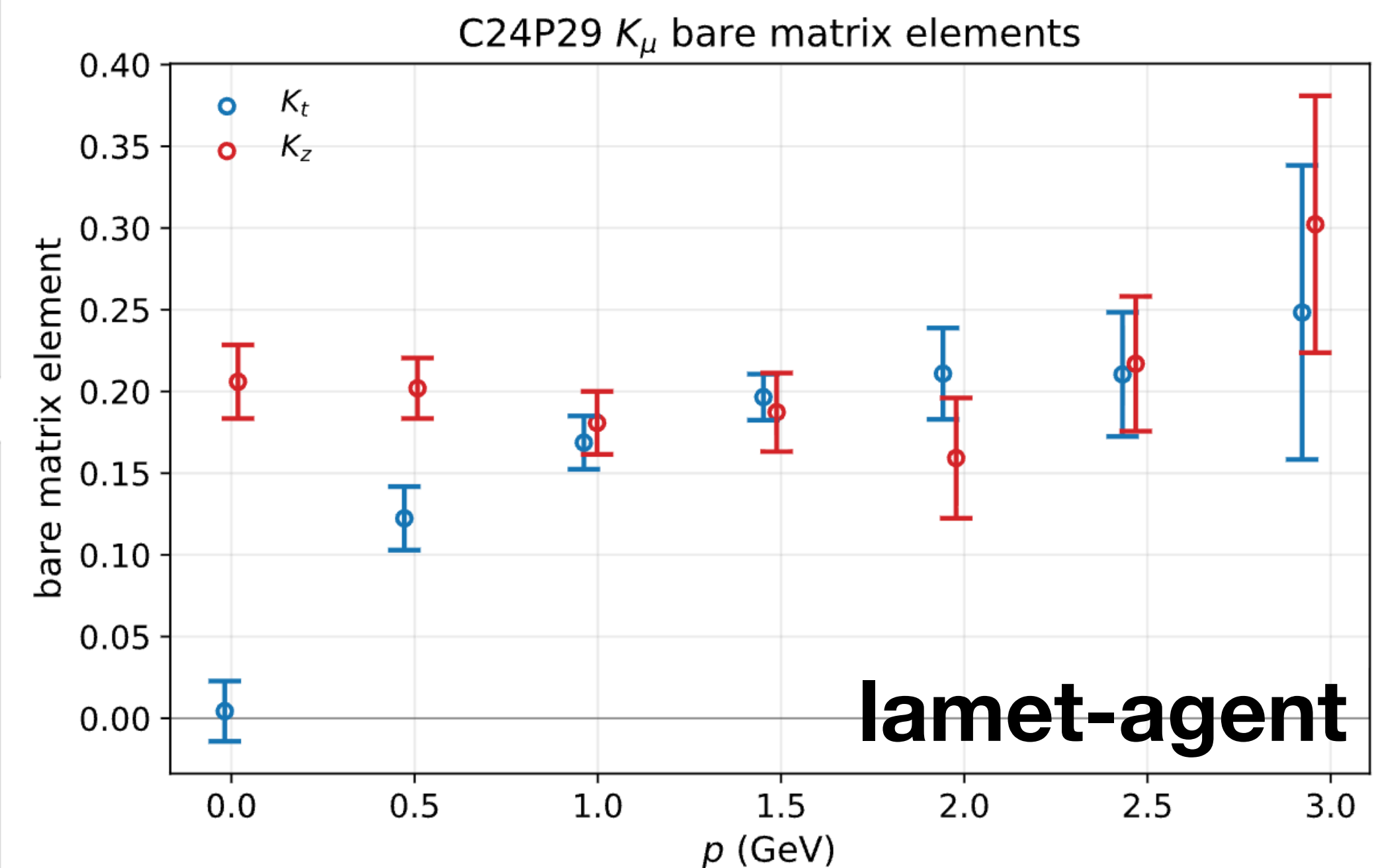
$a \approx 0.1\text{fm}$

D.-J. Zhao et al., arXiv:hep-lat/2512.24315



J. He, X. Jiang, F. Yao and D.-J. Zhao, in preparation

**LaMET-Agent** provided consistent results within the error range and supported BMA.



Refer to **Jinchen's** talk on **PM 2:00-2:30**  
**8th July** for other details on LaMET-Agent.

# Renormalization of topological current $K^\mu$

**RI/MOM** is non-perturbative renormalization scheme on LQCD.

Z.-Y. Pang et al. [10.1007/JHEP07\(2024\)222](https://arxiv.org/abs/10.1007/JHEP07(2024)222)

D.-J. Zhao et al., [arXiv:hep-lat/2512.24315](https://arxiv.org/abs/hep-lat/2512.24315)

## Lattice to RI/MOM

$$\langle PS | \widehat{\{K^\rho, J^{5\rho}\}} | PS \rangle^{\text{tree.}} = Z_{\{1,2\}1}^{\text{RI}} \langle PS | K^\rho | PS \rangle^{\text{lat.}} + Z_{\{1,2\}2}^{\text{RI}} \langle PS | J^{5\rho} | PS \rangle^{\text{lat.}}$$

Coupled

$$\langle K^\mu \rangle_N^{\text{RI}} = Z_{11}^{\text{RI}} \left( \langle K^\mu \rangle_N^{\text{B.}} - \frac{K^{\text{lat.,q}}}{J_5^{\text{lat.,q}}} \langle J^{5\rho} \rangle_N^{\text{B.}} \right)$$

## RG Running

$$R_{ij}^{3\text{loop}}(\mu^{\text{RI}}, \mu = e^{-\frac{5}{6}} \mu^{\text{RI}}) \rightarrow R_{ij}^{3\text{loop}}(\mu^{\text{RI}}, \mu = \sqrt{10} \text{GeV})$$

S. J. Brodsky, et al., Phys. Rev. D 28, 228 (1983).

**Improve the first few terms' convergence of perturbative series.**

## RI/MOM to $\overline{\text{MS}}$

$$\langle K^\mu \rangle_N^{\overline{\text{MS}}} = R_{11}^{3\text{loop}} \langle K^\mu \rangle_N^{\text{RI}} + R_{12}^{3\text{loop}} \langle J^{5\rho} \rangle_N^{\text{RI}} \approx \bar{Z}_{11}^{\text{lat}} \langle K^\mu \rangle_N^{\text{B.}}$$

$$\langle J^{5\rho} \rangle_N^{\overline{\text{MS}}} \approx \bar{Z}_{22}^{\text{lat}} \langle J^{5\rho} \rangle_N^{\text{B.}} \quad \text{where } \bar{Z}_{11}^{\text{lat}} \equiv R_{11}^{3\text{loop}} \frac{Z_{11}^{\text{RI}}}{\mathbf{2}}$$

# CDER and $a^2p^2$ poly-subtraction of $\bar{Z}_{11}^{\text{lat}}$

$a \approx 0.1 \text{ fm}$

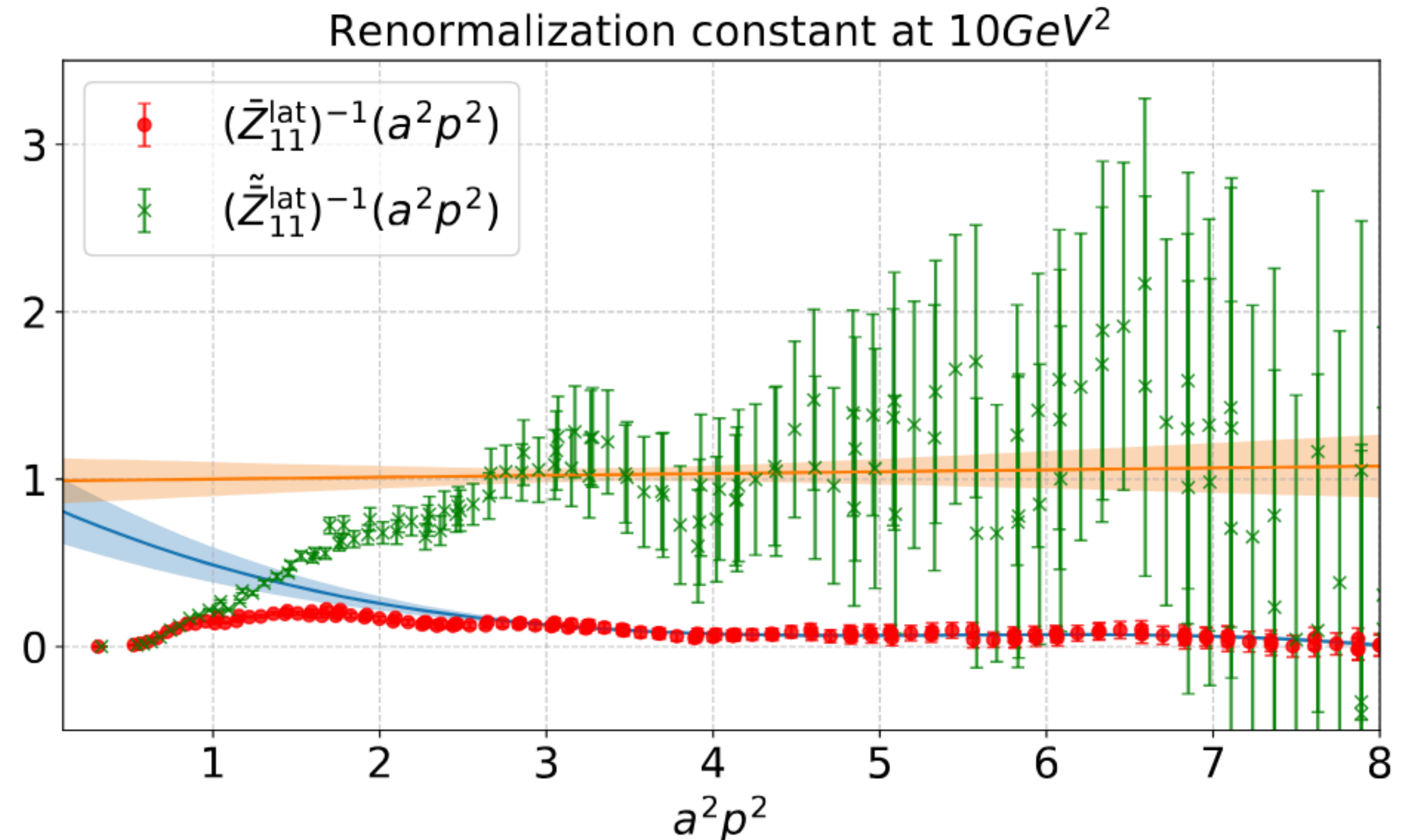
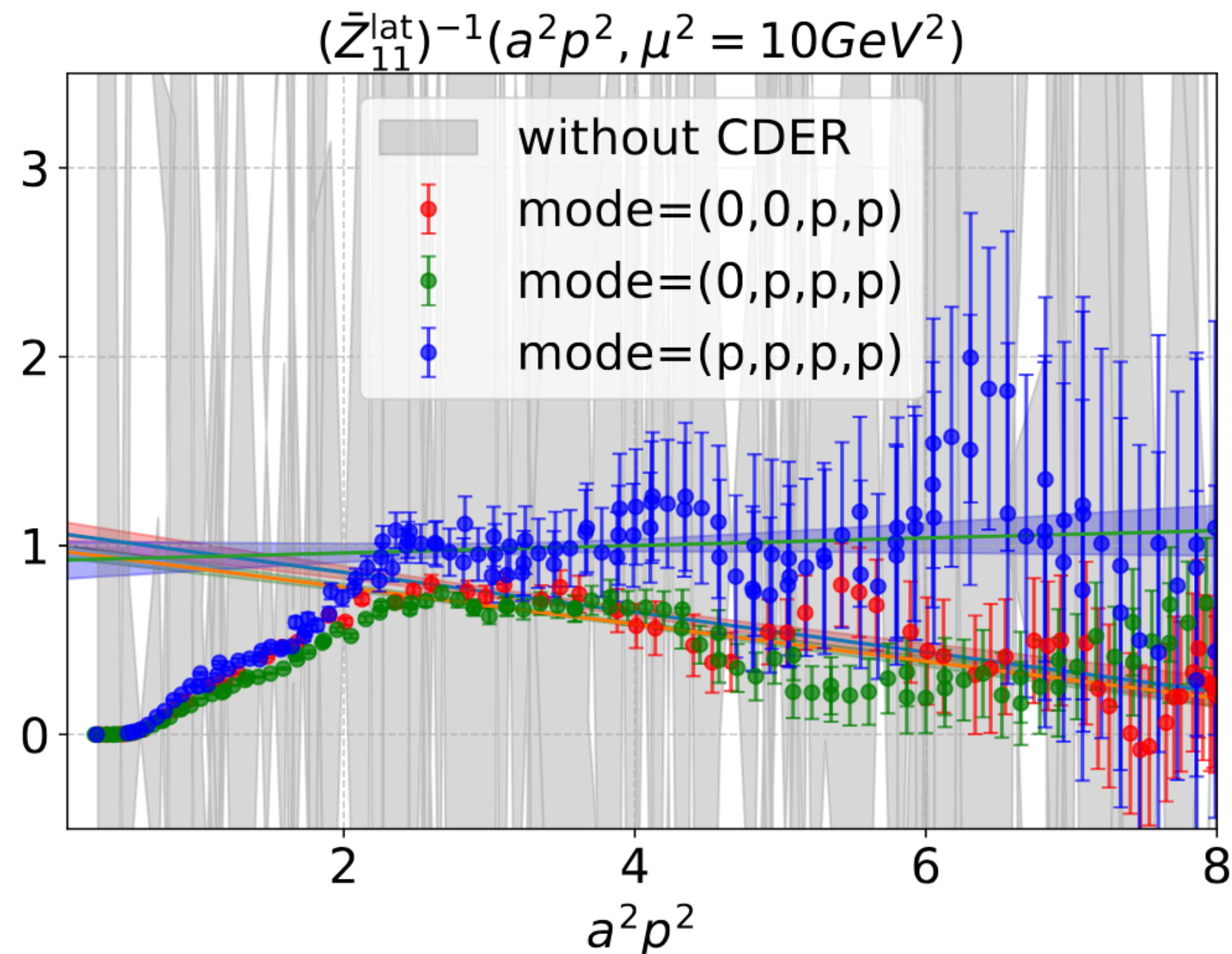
$$Z_{11}^{\text{RI}}(\mu^{\text{RI}}) = \frac{Z_g(\mu^{\text{RI}})}{\text{Tr}[S_g^{-1}(p_1)G_K(p_1,p_2)S_g^{-1}(p_2)(K^{\text{tree,g}})^{-1}]}$$

$Z_{11}^{\text{RI}}$  contributed by gluon Green functions is difficult to see signal, so we use **CDER** scheme to enhance SNR.

Y.-B. Yang et al., [10.1103/PhysRevD.98.074506](https://arxiv.org/abs/10.1103/PhysRevD.98.074506)

Y.-B. Yang, et al., Phys. Rev. Lett. 121, 212001

$$\tilde{Z}_{11}^{\text{lat}}(\mu^2, a^2p^2) = \bar{Z}_{11}^{\text{lat}}(\mu^2, a^2p^2) f(a^2p^2 \rightarrow 0) / f(a^2p^2) \quad \text{where } f(a^2p^2) = \frac{\langle \text{Tr}[A_\mu^{n-\text{HYP}}(p)A_\mu^{n-\text{HYP}}(-p)] \rangle}{\langle \text{Tr}[A_u(p)A_u(-p)] \rangle}$$

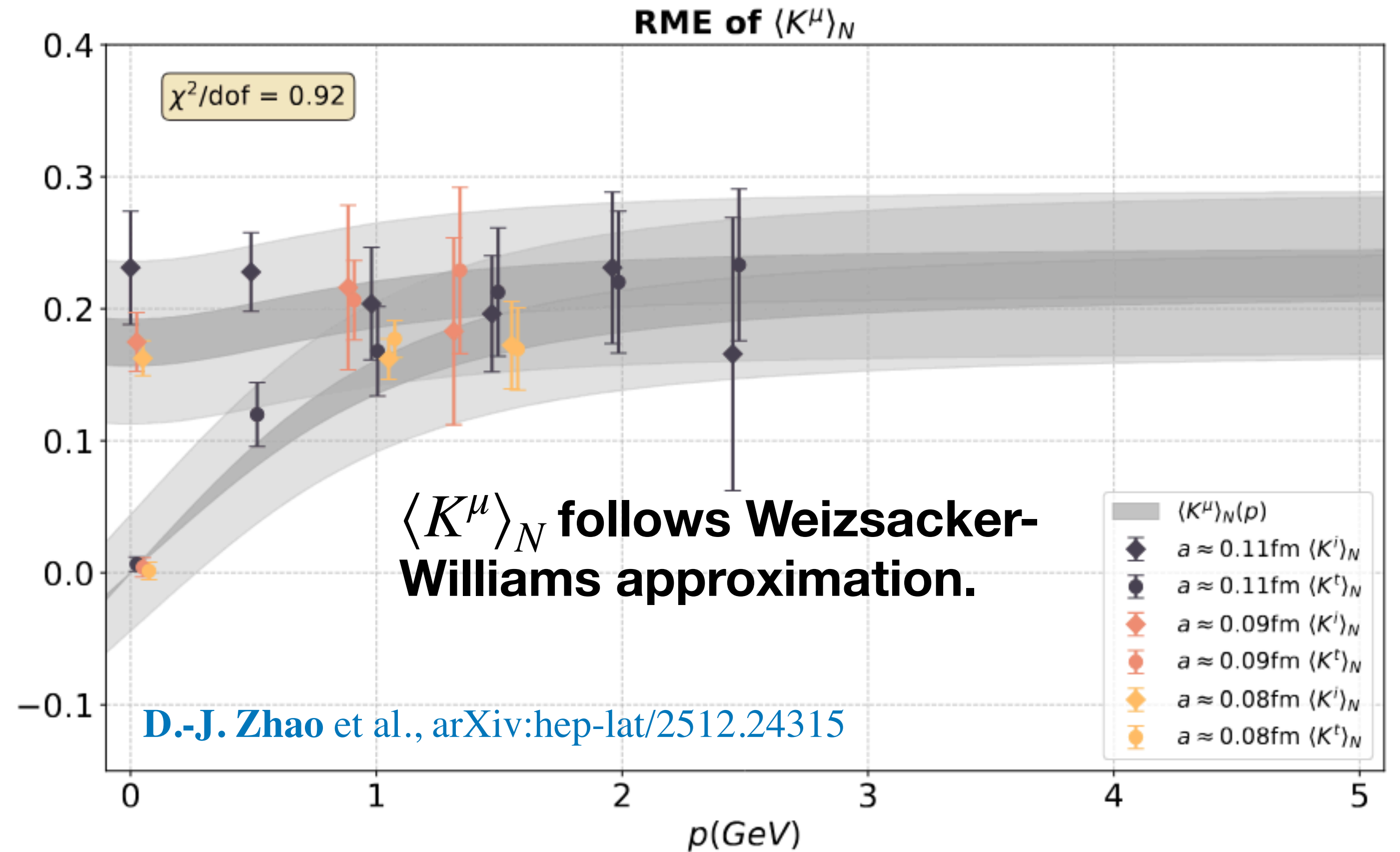


# Continuous limit extrapolation

$$\langle K^\mu \rangle_N^{\overline{\text{MS}}} = \bar{Z}_{11}^{\text{lat.}} \langle K^\mu \rangle_N^{\text{B.}} = \frac{S^\mu}{E} \left( \Delta G + \frac{c_{\text{h.t.}}}{E^2} \right) + c_a a^2$$

## Systematic error sources:

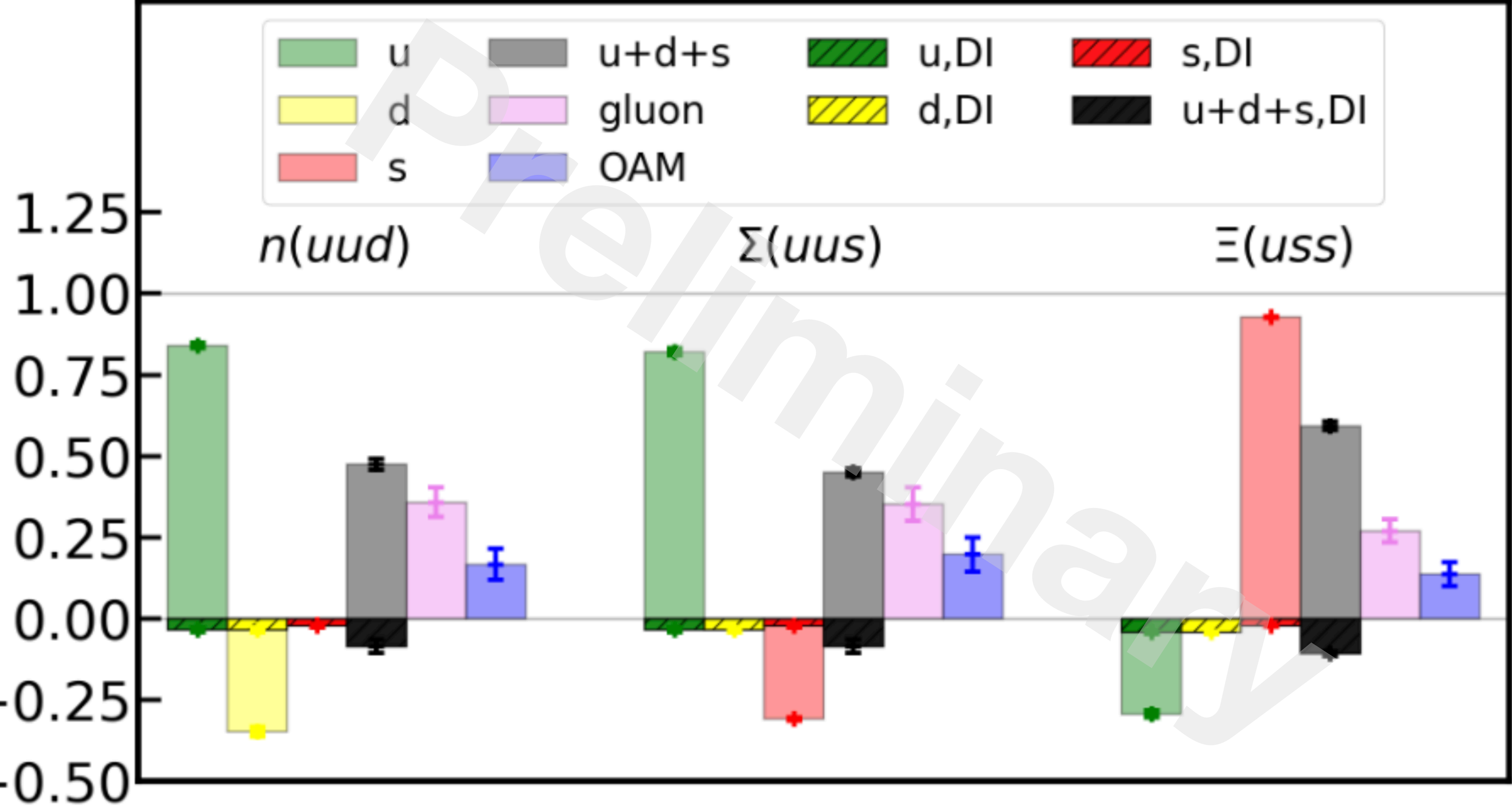
1. Zero-momentum external state data points
2. Finite lattice spacing
3. Choice of the UV window in the RI/MOM scale
4. Perturbative inputs



$$\Delta G(\mu^2 = 10\text{GeV}^2) = 0.231(17)^{\text{sta.}}(44)^{\text{sym.}}$$

# Preliminary exploration of octet states

**C32P23**  $m_\pi = 227\text{MeV}$   $a = 0.105\text{fm}$



**If the quark masses which make up hadrons increases, then gluon helicity will be squeezed.**

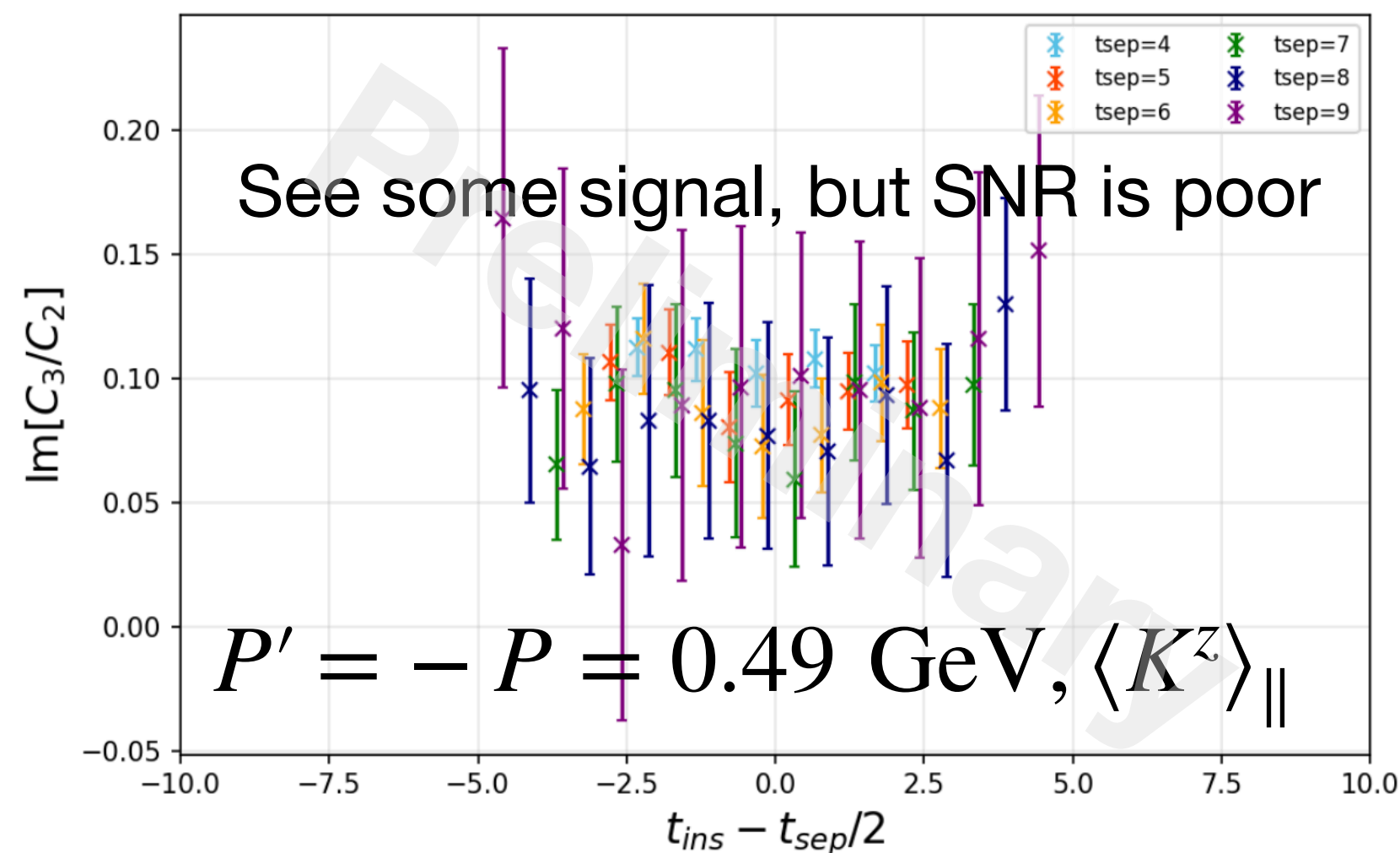
# Two subtopics under J-M spin decomposition

## Form factor of $K^\mu$

In the Breit Frame with z-polarization

$$\langle K^{t/x/y} \rangle_{\parallel} = 0 \quad \langle K^z \rangle_{\parallel} = 2M\tilde{G}_A(t) + \frac{t}{2M}\tilde{G}_P(t)$$

$$\langle K^{t/x/y} \rangle_{\perp} = 0 \quad \langle K^z \rangle_{\perp} = 2\sqrt{M^2 - t/4}\tilde{G}_A(t)$$



Spin distribution  $\tilde{G}_A$  and topological behavior  $\tilde{G}_P$  may be further studied after obtaining better SNR.

## Total OAM without LaMET matching

$$\begin{aligned} \frac{1}{2} &= \frac{1}{2}\Delta\Sigma + \Delta G + L_q^{\text{JM}} + L_g^{\text{JM}} \\ &= \bar{\psi}\gamma_z\gamma_5\psi + (E \times A)_z + L_q + L_g \\ &= \bar{\psi}\gamma_z\gamma_5\psi + K_z \left[ - (A_t B_z + A_t A_x A_y) + L_q + L_g \right] \end{aligned}$$

Those two items will only disappear after boosting to IMF and taking the light-cone.

We haven't found any  $L_{q/g}$  that doesn't require LaMET matching yet, but we can use this algebraic method to calculate total OAM that doesn't require LaMET matching.

# Summary and Outlook



## Summary

1. **Distillation + Momentum smear** (for B.M.E.) and **CDER** (for Renorm.) scheme.
2. After non-perturbative renormalization,  $\Delta G(\mu^2 = 10\text{GeV}^2) = 0.231(17)^{\text{sta.}}(44)^{\text{sym.}}$ , which accounts for 46(9) % of proton spin.

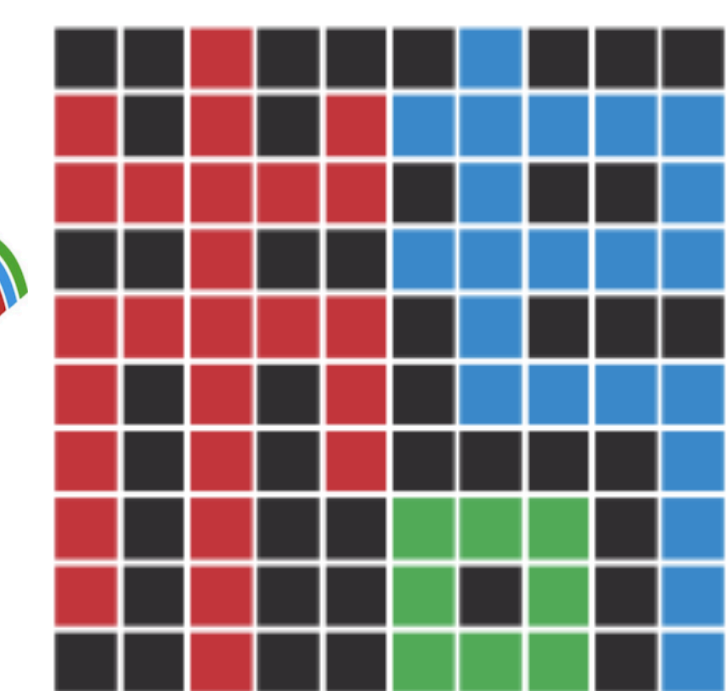
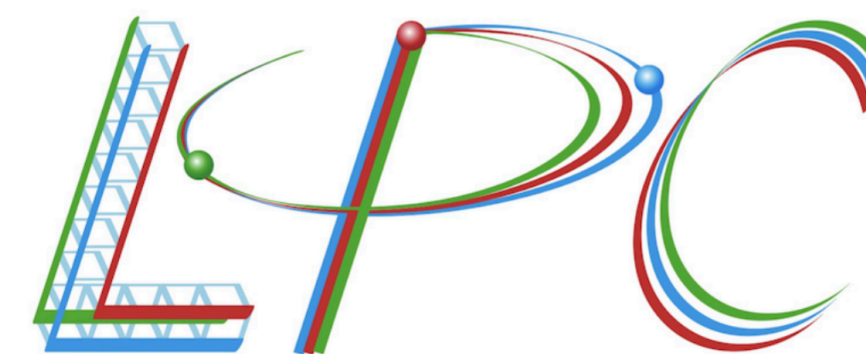
## Outlook

1. Combine  $\frac{1}{2}\Delta\Sigma$  using Blending to provide octet spin decomposition.
2. Refining the research of gluon helicity through the form factor of  $K^\mu$ .
3. Calculate total OAM without LaMET matching and aim to prove that J-M spin decomposition adds up to 1/2 within the error range.



香港中文大學(深圳)

The Chinese University of Hong Kong, Shenzhen



CLQCD

**LaMET 2026**

# Progress in the Lattice of Jaffe- Manohar Spin Decomposition

2026.07.06 PM 4:00-4:30

Speaker: Dian-Jun Zhao

[Mainly based on arXiv: 2512.24315](https://arxiv.org/abs/2512.24315)

In Collaboration with LPC and CLQCD