華南師範大學
SOUTH CHINA NORMAL UNIVERSITY

Lattice Parton
Collaboration

Recent Progress of LCDAs on Lattice with LaMET

Jun Hua South China Normal University

On behalf of Lattice Parton Collaboration

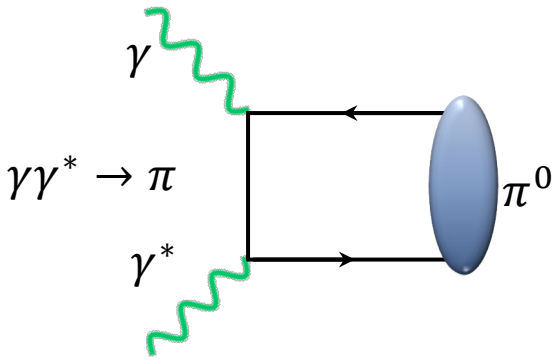
07/06 @ LaMET2026, Jagiellonian University

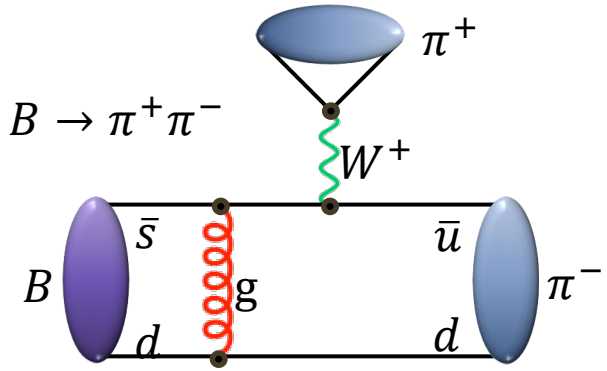
OUTLINE

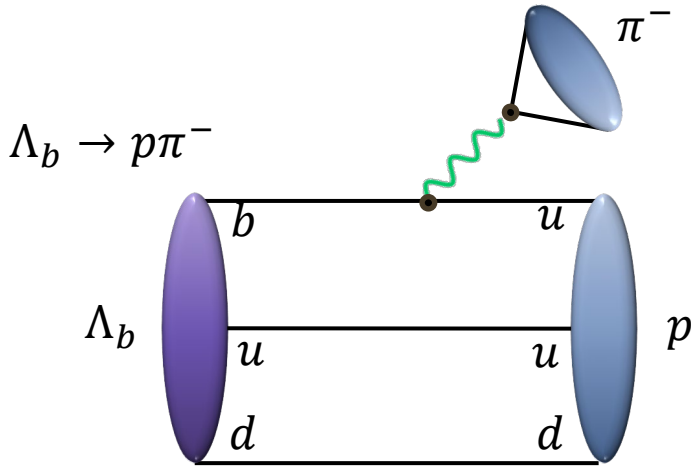
About LCDA

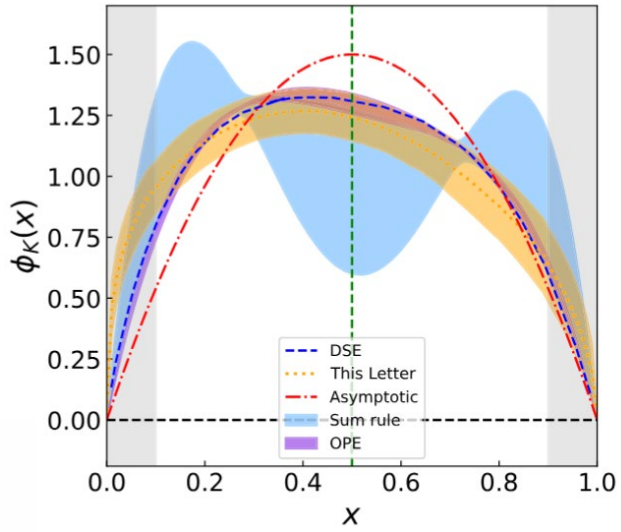
- Richer QCD dynamical information and most important input during describing

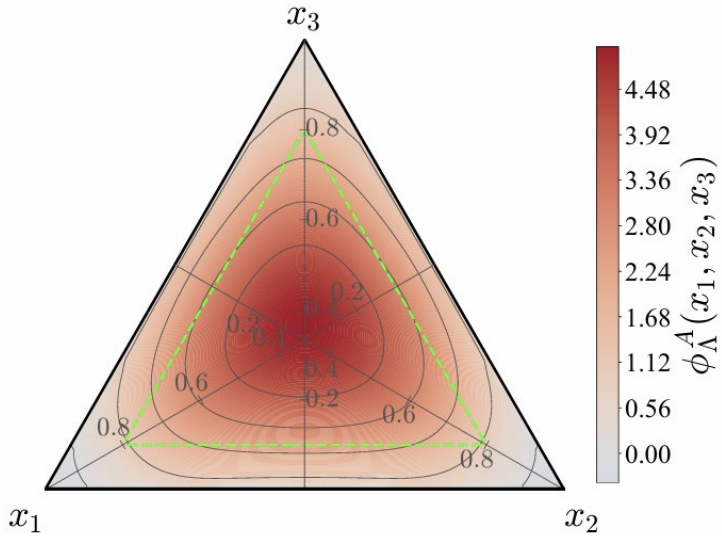
Exclusive scattering — LCDAs



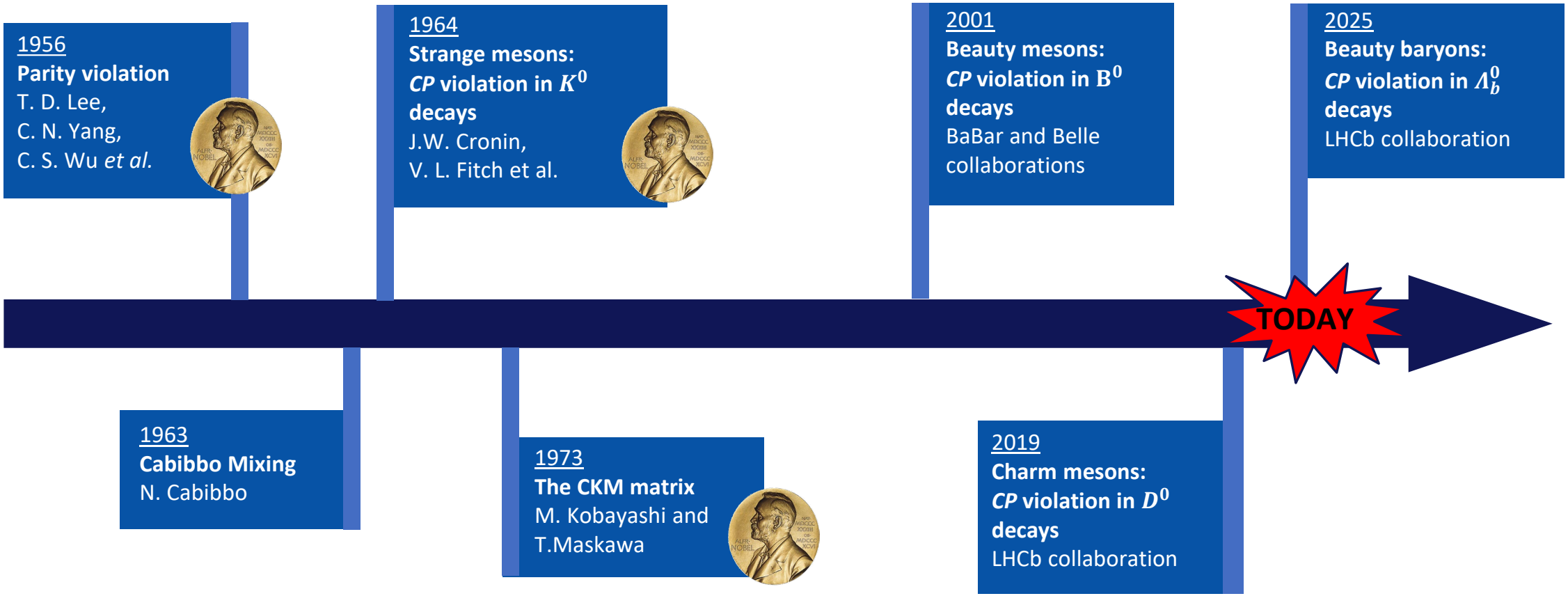




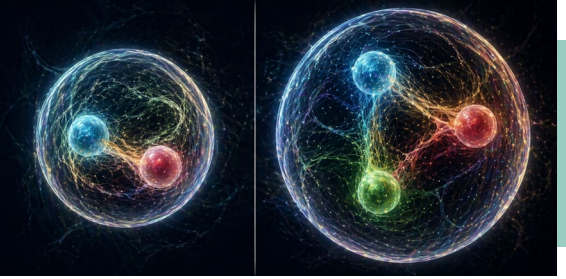




About LCDA



- **Among these, the LCDAs serve as a key theoretical cornerstone for investigating CP violation**



LCDAs of Meson & Baryon: dynamical progress

Results @ Indirect (moments / priors) reconstruction
Results @ Direct x-dependent distribution

Light Meson

Analytic

Sum Rule

NPB 201 492 (1982)
ZPC 44 157 (1989)
PRD 54 2182 (1996)
NPB 529 323 (1998)
JHEP 05 004 (2006)
…

Model

PRD 66 094016 (2002)
PRD 84 033001 (2011)
PRD 88 034013 (2013)
PRD 104 016021(2021)
…

Global Fits

PRD 62 116002 (2000)
PRD 67 074012 (2003)
PLB 578 91 (2004)
PRD 83 054020 (2011)
PRD 102 074022 (2020)
…

DSE

PRL 110 132001 (2013)
PRL 111 092001 (2013)
PRD 90 014011 (2014)
PPNP 120 103883 (2021)
…

OPE Moments

Lattice OPE

PRD 74 074501 (2006)
PRD 92 014504 (2015)
[RQCD]PLB 774 91 (2017)
[RQCD] JHEP 08 065 (2019)
…

Pseudo

PRD 95 056020 (2017)
PRD 100 116011 (2019)
arXiv:2401.06858
…

Current Current

EPJC 78 217 (2018)
PRD 98 094507 (2018)
…

Full Distribution

LaMET

PRD 95 094514 (2017)
PRD 102 094519 (2020)
[LPC] PRL 127 062002 (2021)
[LPC] PRL 129 132001 (2022)
[BNL/ANL]PRD 106 074505 (2022)
[BNL/ANL] JHEP 07 211 (2024)
…

Heavy Meson

Model

PRD 55 272 (1997)
PRD 69 034014 (2004)
JHEP 07 154 (2018)
…

HQ-LaMET

PRD 102 011502 (2020)
[LPC]PRD 111 L111503 (2025)
[LPC]PRD 111 034503 (2025)
[LPC]2604.25802, 2605.10946
…

Light Baryon

Sum Rule

NPB 279 785 (1987)
PRD 47 R3685 (1993)
…

Model

PRD 65 074011 (2002)
JHEP 11 191 (2013)
…

Lattice OPE

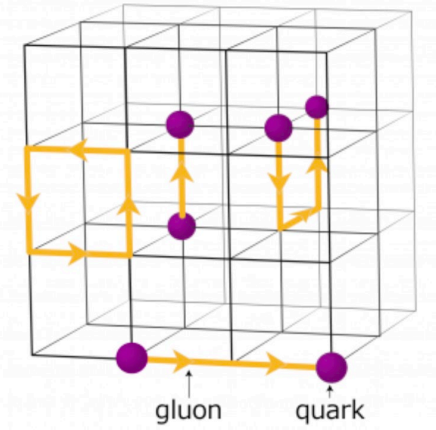
[QCDSF] PRL 101 112002 (2008)
[QCDSF] PRD79 034504 (2009)
[RQCD] EPJA 55 116 (2019)
[RQCD] PRD111 094517 (2025)
…

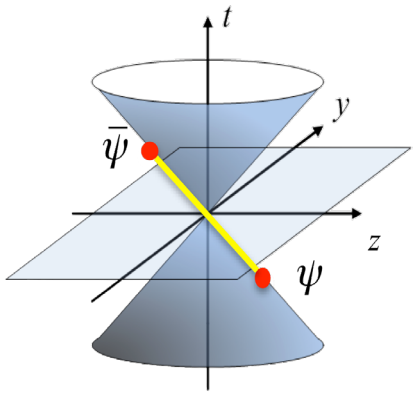
LaMET

[LPC] PRD 111 034510 (2025)
[LPC] PRD 112 114515 (2025)
…

LQCD for LCDAs OPE VS LaMET

LQCD is formulated as a Feynman path integral on a 4D Euclidean grid, with a Wick rotation from real time to imaginary time.





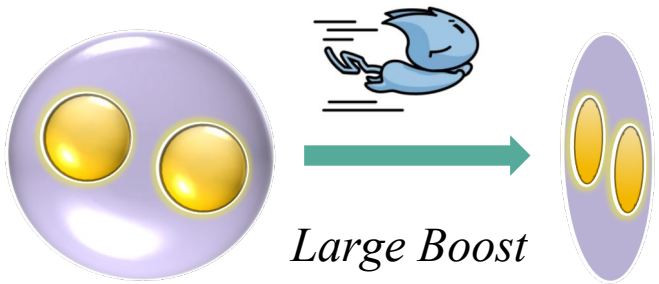
π LCDA: $\langle 0|\bar{q}_2(\xi^+)\gamma^+\gamma_5 U\left(0,\xi^-\right)q_2\left(\xi^-\right)|\pi\rangle$

- **OPE** ~ in local limit: → moments of DA

$$\langle 0|\bar{q}_2\left(\xi^+\right)\gamma^+\gamma_5 U\left(\xi^+,\xi^-\right)q_2\left(\xi^-\right)|\pi\rangle \overset{\xi\to 0}{\sim} C_m(z)^m\bar{q}_2(0)\left(\gamma^t+\gamma^z\right)\gamma_5(in_z\cdot\overleftrightarrow{D})^m q_1(0)$$

- **LaMET** ~ in large P^z limit: → shape of DA

$$\langle 0|\bar{q}_2\left(\xi^+\right)\gamma^+\gamma_5 U\left(\xi^+,\xi^-\right)q_2\left(\xi^-\right)|\pi\rangle \overset{P^z\to\infty}{\sim} \langle 0|\bar{q}_2\left(z_+\right)(\gamma^t+\gamma^z)\gamma_5 U\left(z_+,z_-\right)q_2\left(z_-\right)|\pi(P^z)\rangle$$



From Meson LCDA

Meson LCDAs by LaMET: (2021~ …)

- π, K LCDA $a \to 0$

  R.Zhang, H.W.Lin et.al. PRD 102,094519

- K^*, ϕ; π, K LCDA $a \to 0, \mathrm{m}_\pi \to 130\mathrm{MeV}$

  (LPC) PRL 127 062002(2021); PRL 129 132001(2022)

- Hybrid and Self renormalization

  (LPC) NPB 964 115311(2021); NPB 969 115443(2021)…

- RGR resummation & Renormalon resummation (LRR)

  Y.S.Su et.al. NPB 993 116282(2023)

- Threshold resummation

  Y.S.Su et.al. JHEP 03 045(2025)

- Two loop matching

  F.Yao et.al. arXiv:2504.09367; PRD 113 014505(2026)

- Heavy Meson DA (HQ-LaMET)

  (LPC) PRD L111503, 111 034503; 2604.25802,2605.10946

- Not physical π mass, RI/MOM scheme, λ truncation

- Physical π mass with $a \to 0$
- Hybrid scheme, Self renormazaliton

- Towards high precision
- Not yet done with lattice results

- A new frame work including 2-step matching
- High precision is challenge

From Meson LCDA

Lattice Advances: (2021 ~ …)

- Kinematically-enhance

  PRD 112 L051502(2025), arXiv: 2606.02447
- Precision check for meson LCDA

  … JHW et.al.
- Large λ Extrapolation

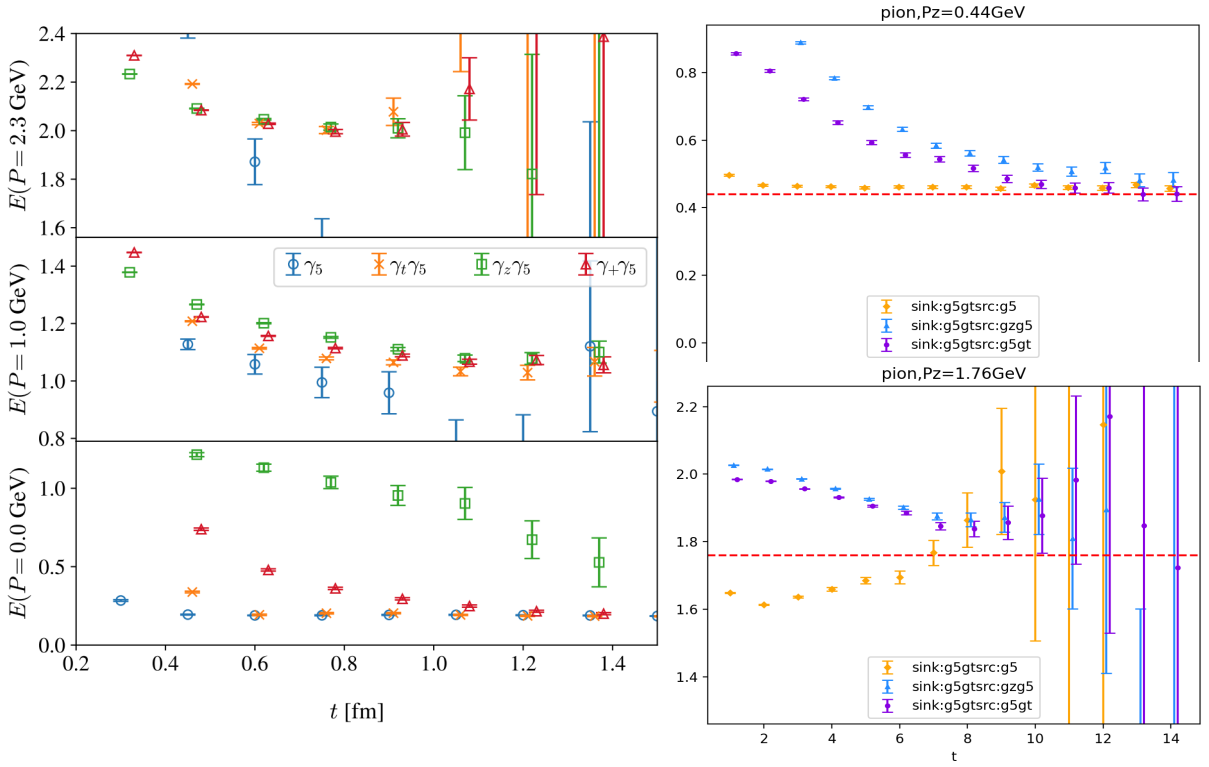
  PRD 113 014509(2026), arXiv: 2601.12189
- CLQCD ensembles & Pyquda

  CLQCD PRD 109(2024) , arXiv: 2411.08461…
- … Gradient flow

  JHEP 06 210 (2024), PRD 112 094504(2025)…

➢ Precision !

Signal Improve



Self renorm & Renormalon resum

JH et.al. NPB 993 116282(2023), XDJ et.al. JHEP 03 045(2025)

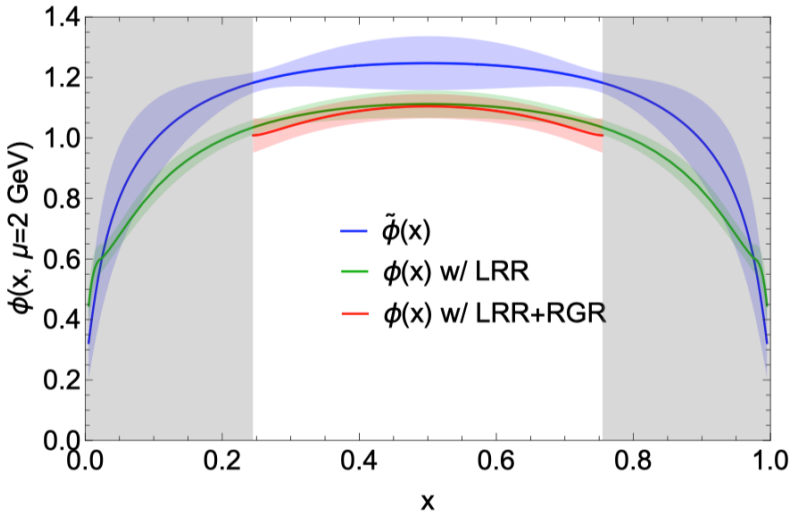
Linear divergence → Multi-a, extract linear-div → Match with perturbative $\overline{MS}$

$$Z(z,a)_R = \exp\left[\frac{kz}{a\ln\left[a\Lambda_{\rm QCD}\right]} + m_0 z + f(z)a + \frac{3C_F}{b_0}\ln\left[\frac{\ln\left[1/\left(a\Lambda_{\rm QCD}\right)\right]}{\ln\left[\mu/\Lambda_{\rm QCD}\right]}\right] + \ln\left[1+\frac{d}{\ln\left(a\Lambda_{\rm QCD}\right)}\right]\right]$$

- Extract from multi lattice spacings of zero momentum matrix
- Extract from matching with perturbative $\overline{MS}$ quasi

Renormalon ambiguity occurs

LRR+RGR



A LRR guarantee a more comprehensive self renormalization

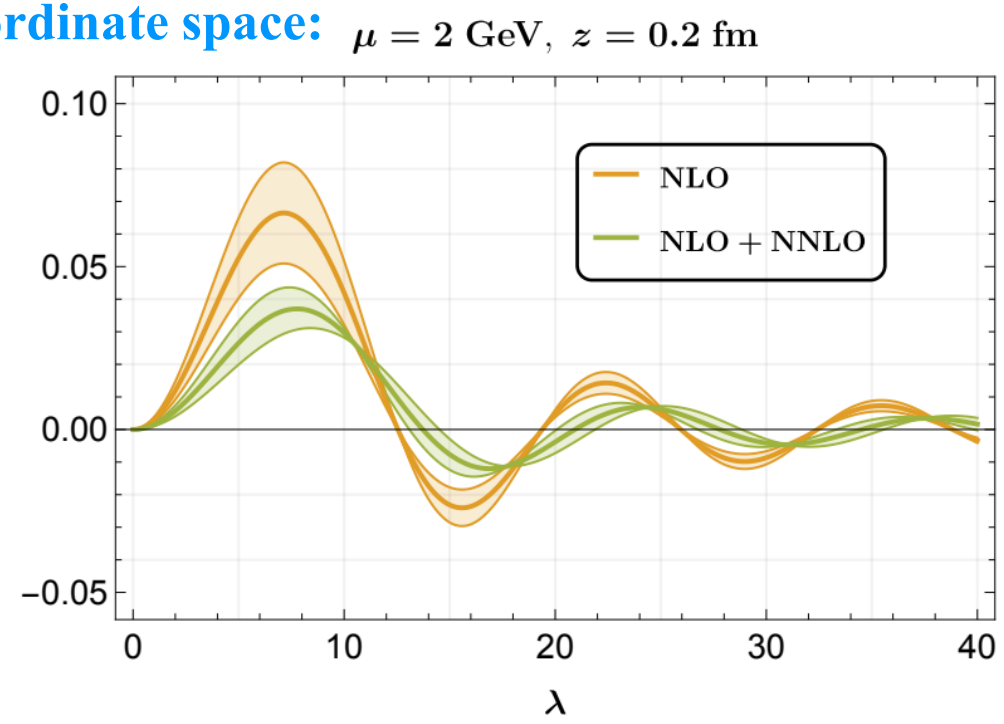
NNLO corrections:

YJ, FY & JHZ arXiv:2504.09367

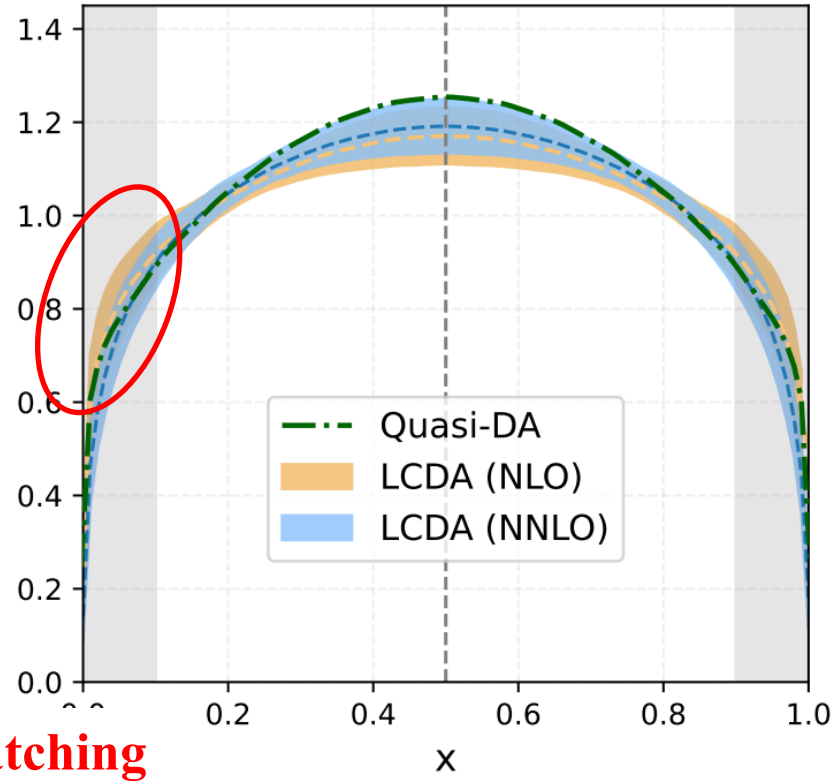
$$\phi_R(x, P^z) = \int dy C\left(x, y, \frac{\mu}{P^z}\right)\phi_R(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(P^z)^2}\right)$$

Matching kernel

Coordinate space:

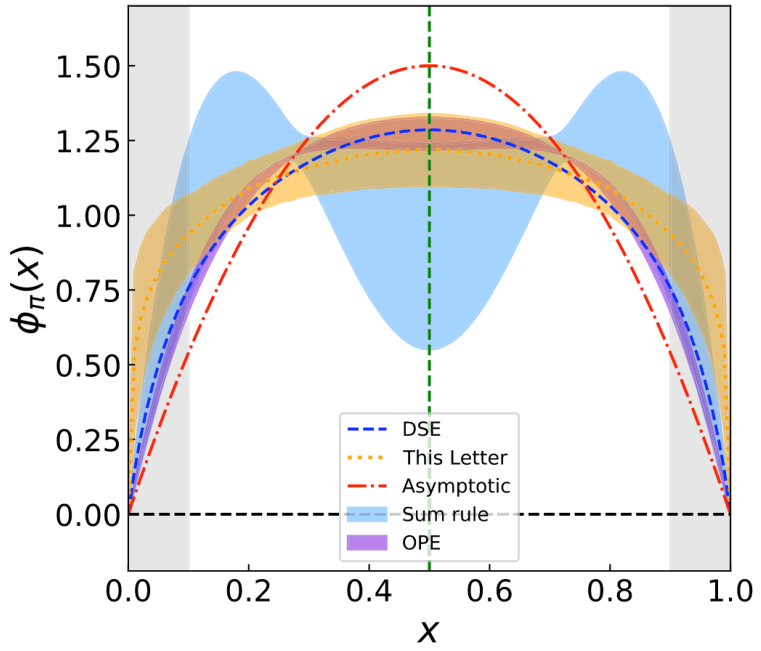


Momentum space:

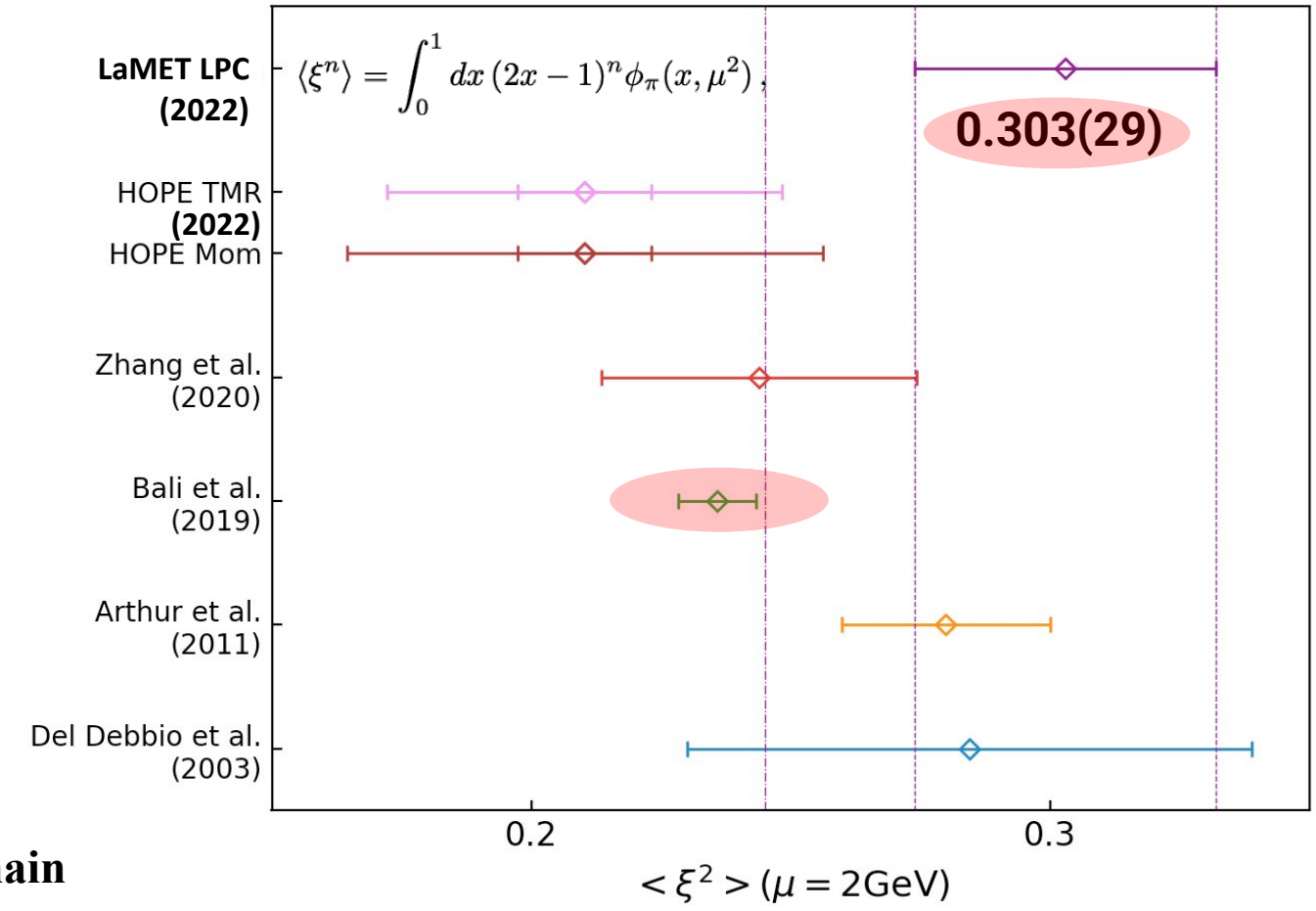


A high precision matching

Moments tension: Why higher precision for light meson LCDA



PRL 129,132001 (2022)



- **High precision under going in LaMET**
- **Moments tension between LaMET &OPE remain**

Benchmark moments & LaMET

(LPC) arXiv:26XXX

➢ Benchmark of Local Moments on CLS & MILC ensembles

	$a_\pi^{(2)}$	$a_K^{(1)}$	$a_K^{(2)}$	
RBC/UKQCD [22]	0.23(3)(6)	0.036(1)(2)	0.18(3)(6)	
RQCD [24]	0.116^{+19}_{-20}	0.0525^{+31}_{-33}	0.106^{+15}_{-16}	
LPC (2026)	0.187(15)	0.0481(23)	0.158(14)	
	$a_\rho^{(2)}$	$a_{K^*}^{(1)}$	$a_{K^*}^{(2)}$	$a_\phi^{(2)}$
RBC/UKQCD [22]	0.20(3)(6)	0.037(1)(2)	0.15(6)(6)	0.15(6)(3)
RQCD [25]	0.132(27)			
LPC (2026)	0.102(18)	0.0609(15)	0.085(17)	0.069(17)
	$a_\rho^{(2)T}$	$a_{K^*}^{(1)T}$	$a_{K^*}^{(2)T}$	$a_\phi^{(2)T}$
RQCD [25]	0.101(22)			
LPC (2026)	0.133(20)	0.0301(16)	0.087(19)	0.041(19)

➢ Benchmark of Local Moments and Quasi-DAs

Moments (slope) extracted from LCDAs calculated by LaMET

$a_\pi^{(2)}$	$a_K^{(1)}$	$a_K^{(2)}$	
✓	✓	✓	
$a_\rho^{(2)}$	$a_{K^*}^{(1)}$	$a_{K^*}^{(2)}$	$a_\phi^{(2)}$
✓	✗	✓	✓
$a_\rho^{(2)T}$	$a_{K^*}^{(1)T}$	$a_{K^*}^{(2)T}$	$a_\phi^{(2)T}$
✗	✓	✗	✗

See Ji-Hao's Talk

Light Baryon LCDA

Light Baryon LCDAs: (on lattice ~ …)

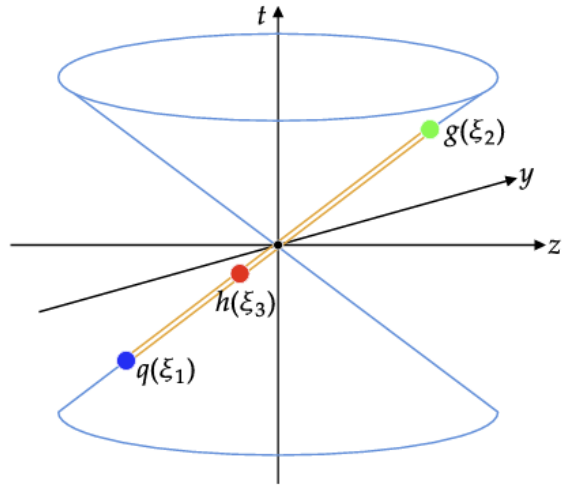
- Previous

  All results based on models …
- Lattice QCD with OPE

  QCDSF, 2008, 2009; RQCD, 2016, 2019(2025) ……
- Lattice QCD with LaMET

  LPC, 2024,2025; ……

  PRD 111 034510, PRD 112 114515

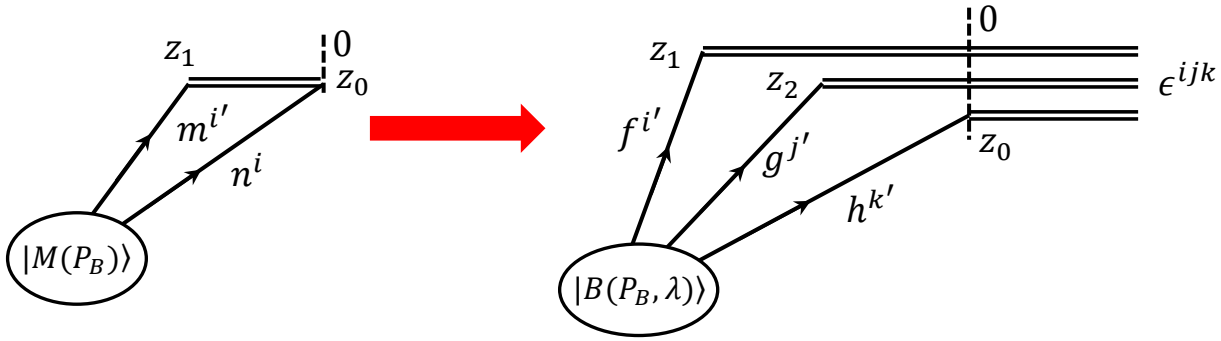


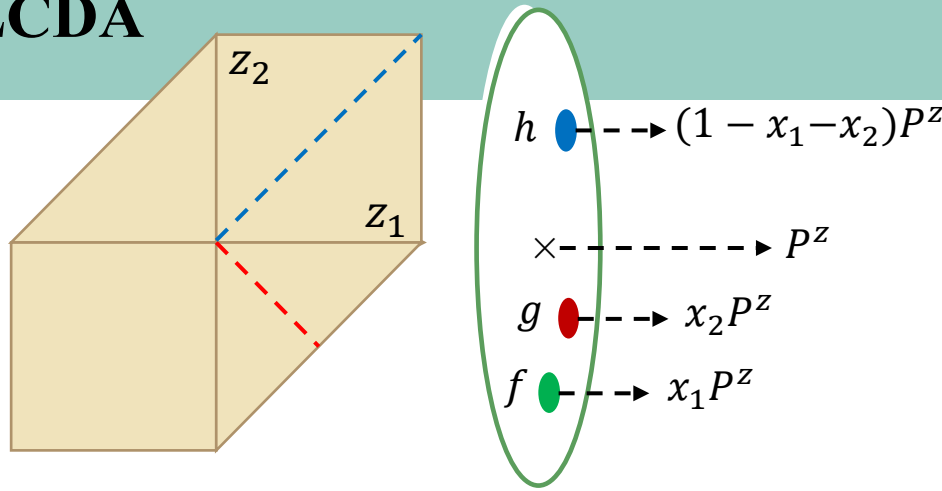
$$\langle 0|f_\alpha(z_1 n)g_\beta(z_2 n)h_\gamma(z_3 n)|B(P_B,\lambda)\rangle$$
$$= \frac{1}{4}f_V\left[(P_B C)_{\alpha\beta}(\gamma_5 u_B)_\gamma V^B(z_i n\cdot P_B) + (P_B\gamma_5 C)_{\alpha\beta}(u_B)_\gamma A^B(z_i n\cdot P_B)\right]$$
$$+ \frac{1}{4}f_T(i\sigma_{\mu\nu}P_B^\nu C)_{\alpha\beta}(\gamma^\mu\gamma_5 u_B)_\gamma T^B(z_i n\cdot P_B),$$

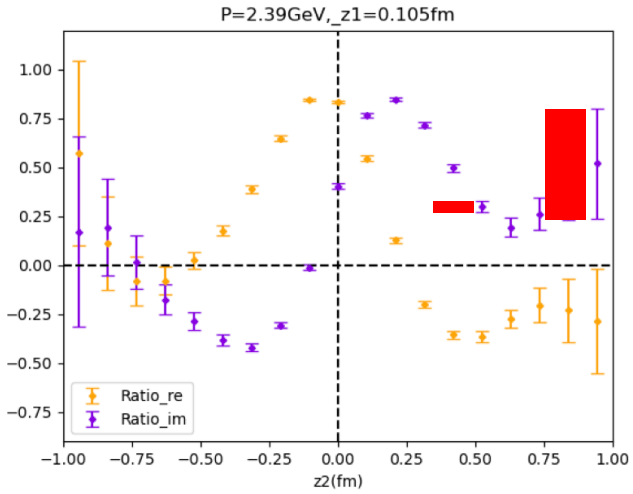
Octet	n	p	Λ
fgh	d d u	u u d	u d s

Light Baryon LCDA

Challenge from light meson LCDA to light baryon LCDA







- From effective Wilson length → **Signal to Noise Ratio**
  - Signal to Noise Ratio * 8 -> Statistics * 64
  - Totally 64*10*2
- **Dynamic enhance: SNR 200% improve (eff 400%) !**
- **Coding in Pyquda: Computing efficiency 800% !**

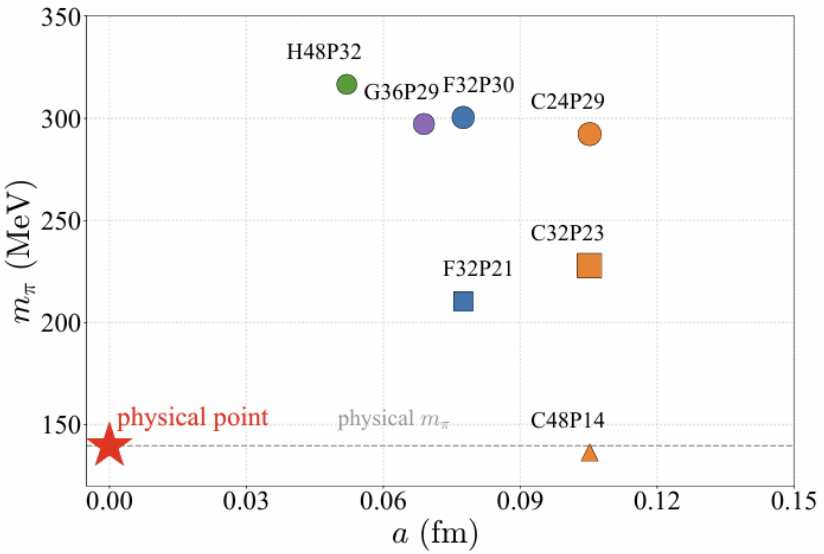
X.Y.Jiang arXiv:2411.08461

Light Baryon LCDA

CLQCD Ensembles —— fully exploiting available

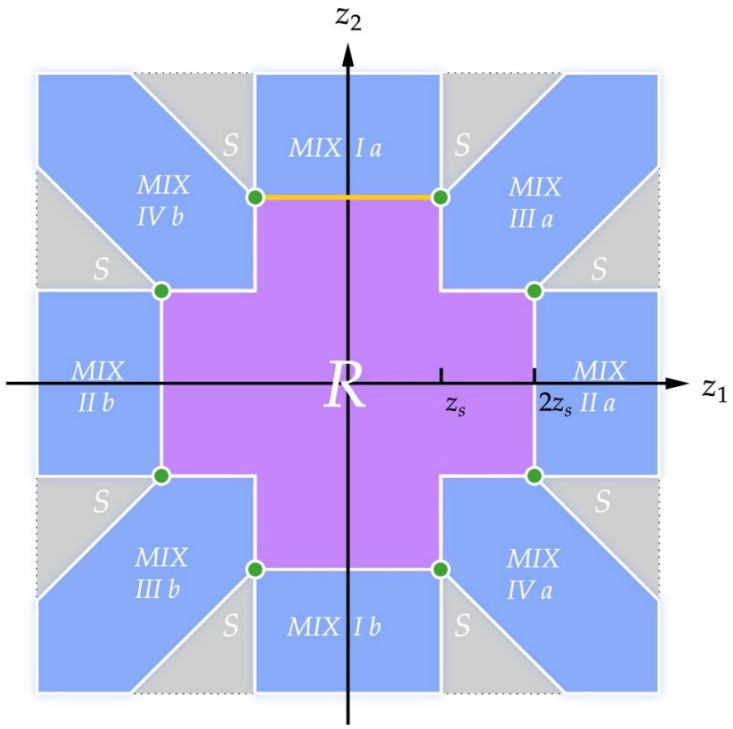
Ensemble	a (fm)	m_π (MeV)	P^z (GeV)	n_{stat}
C24P29	0.1052	292.3	1.96, 2.45, 2.94	864×36
C32P23	0.1052	227.9	1.84, 2.21, 2.57, 2.94	954×32
C48P14	0.1052	136.4	1.96, 2.45, 2.94	302×64
F32P30	0.0775	300.4	2.00, 2.49, 2.99	777×32
F32P21	0.0775	210.3	2.00, 2.49, 2.99	459×64
G36P29	0.0689	297.2	2.00, 2.50, 3.00	656×32
H48P32	0.0520	316.6	1.98, 2.48, 2.98	550×54

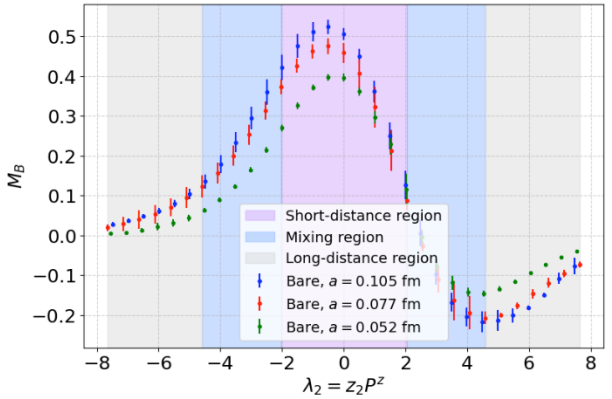
(CLQCD) PRD 109 054507 (2024)



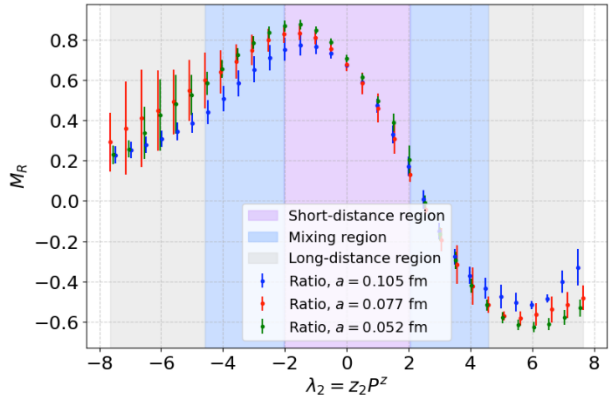
Light Baryon LCDA

Hybrid renormalization —— Done

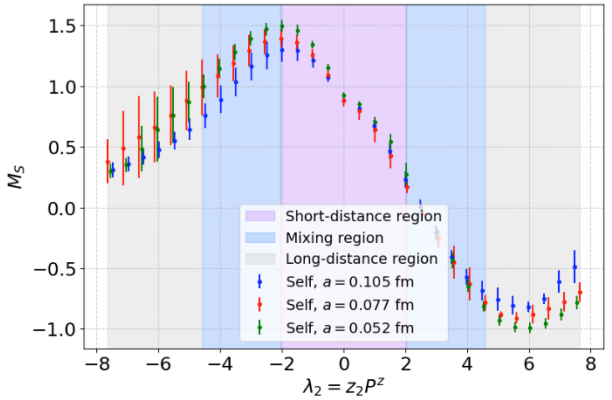




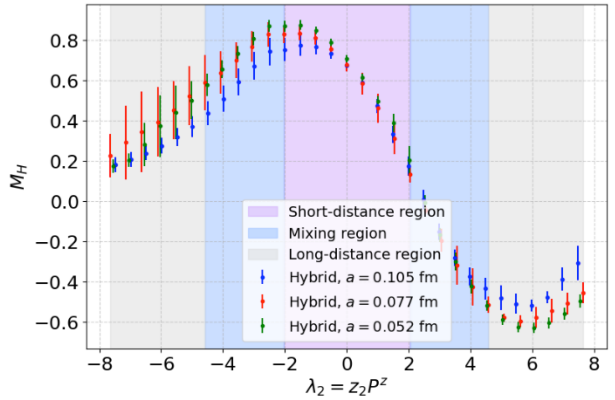
(a) Bare result of Λ at $P = 2.0$ GeV



(b) Ratio scheme result of Λ at $P = 2.0$ GeV



(c) Self renormalized result of Λ at $P = 2.0$ GeV



(d) Hybrid scheme result of Λ at $P = 2.0$ GeV

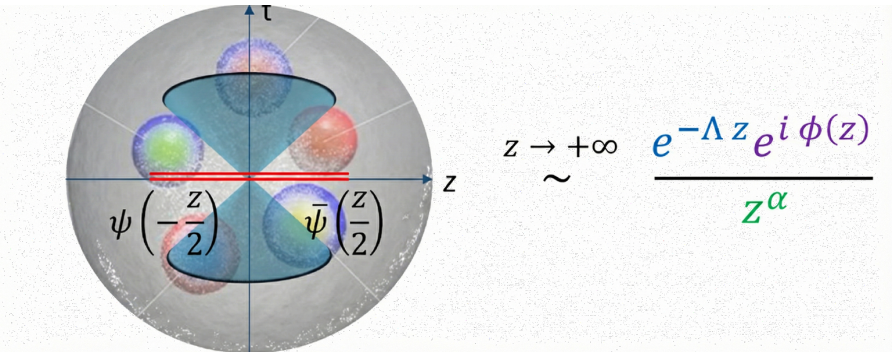
(LPC) PRD 112 114515 (2025)

Light Baryon LCDA

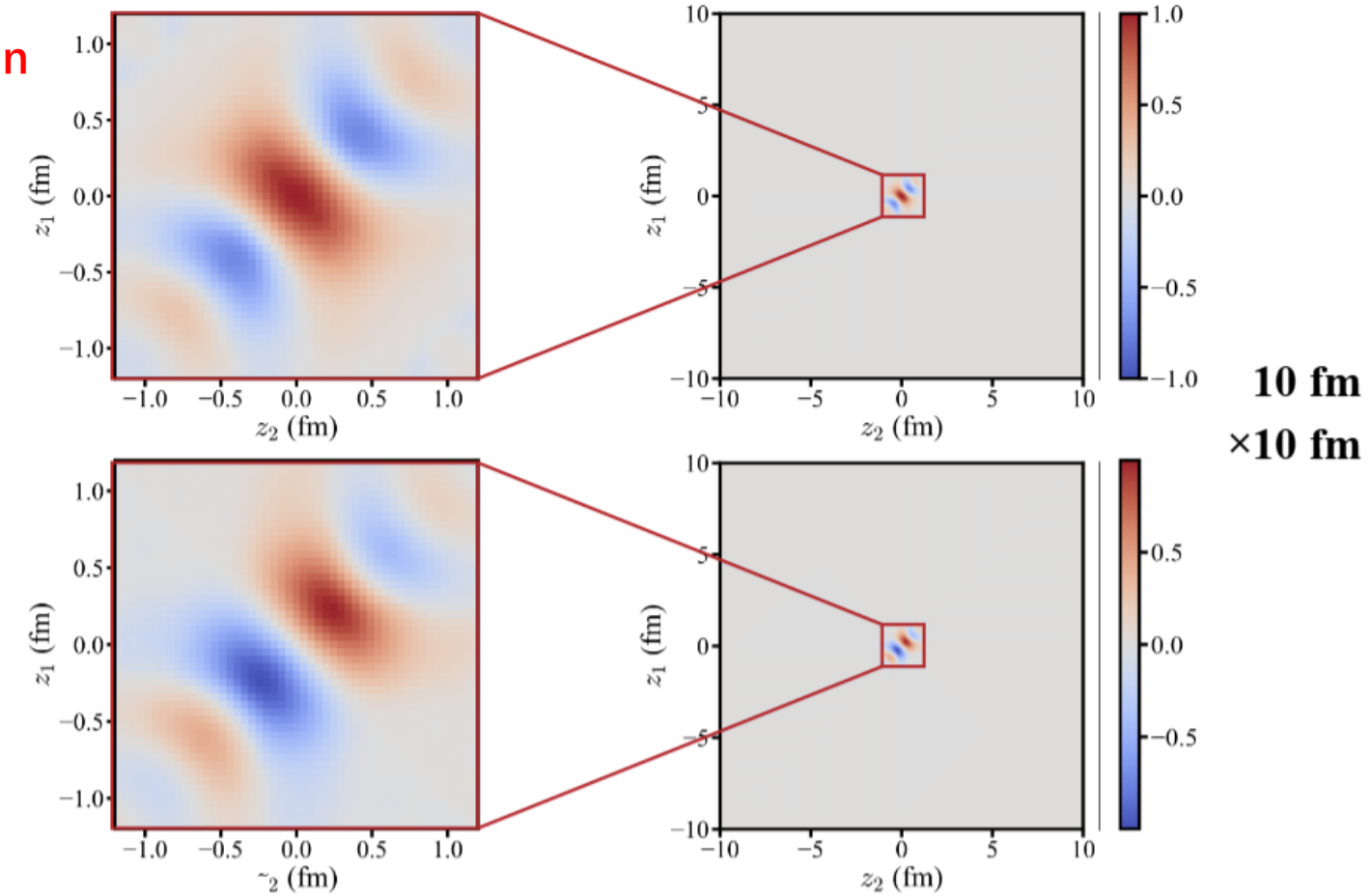
New λ-Extrapolation —— Done

Asymptotic Long-Distance Expansion of Euclidean Correlators in Lattice Parton Applications

X. Ji, Y. Liu, Y. Su, arXiv:2601.12189



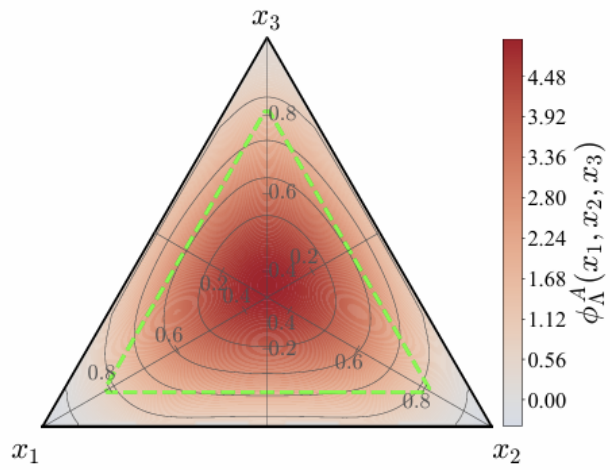
$$\underset{z\to+\infty}{\sim} \frac{e^{-\Lambda z} e^{i\phi(z)}}{z^{\alpha}}$$

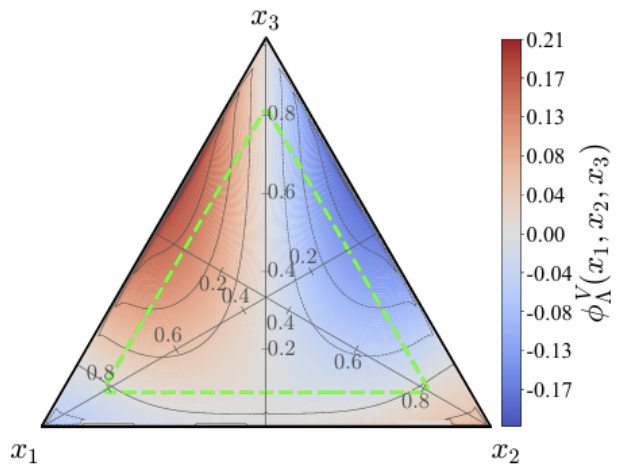


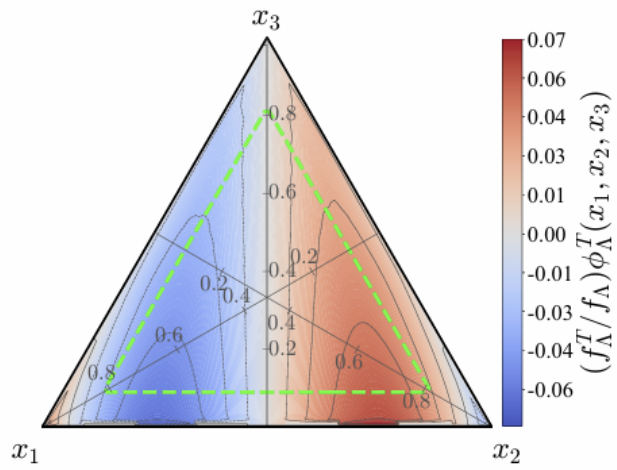
Light Baryon LCDA

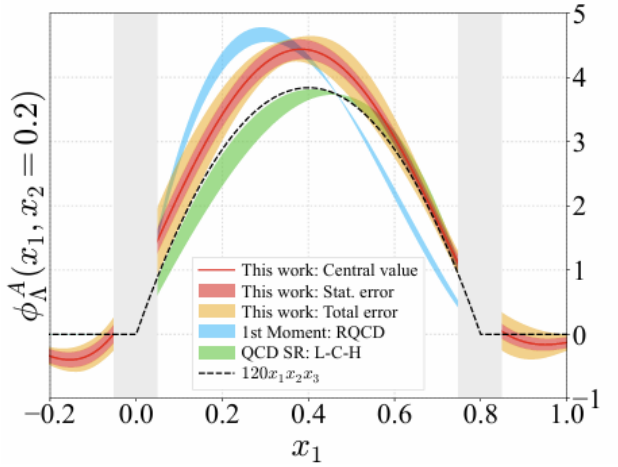
Numerical Results

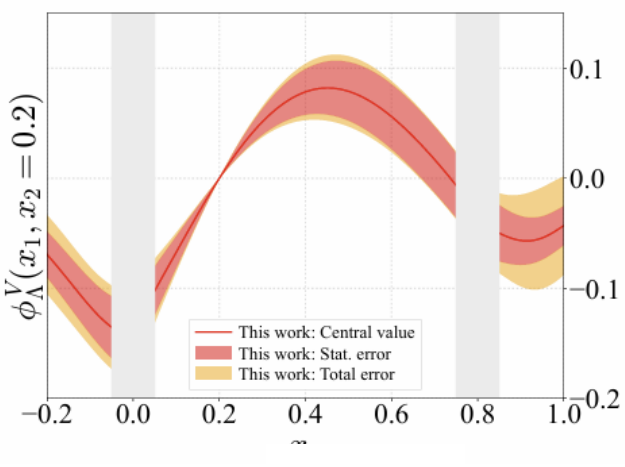
(LPC) arXiv:2606.xxxx, 2606.xxxx

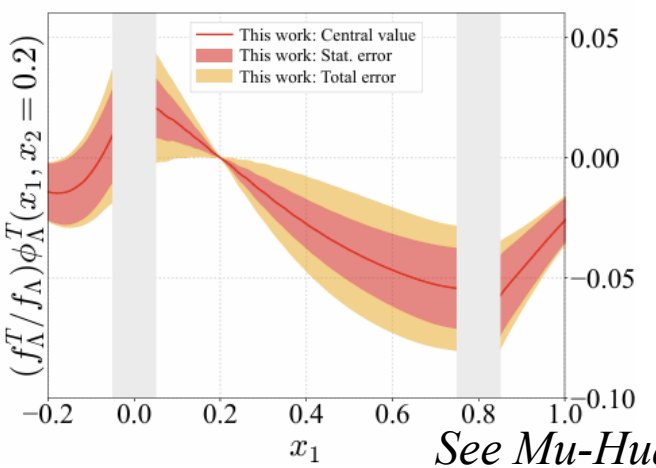






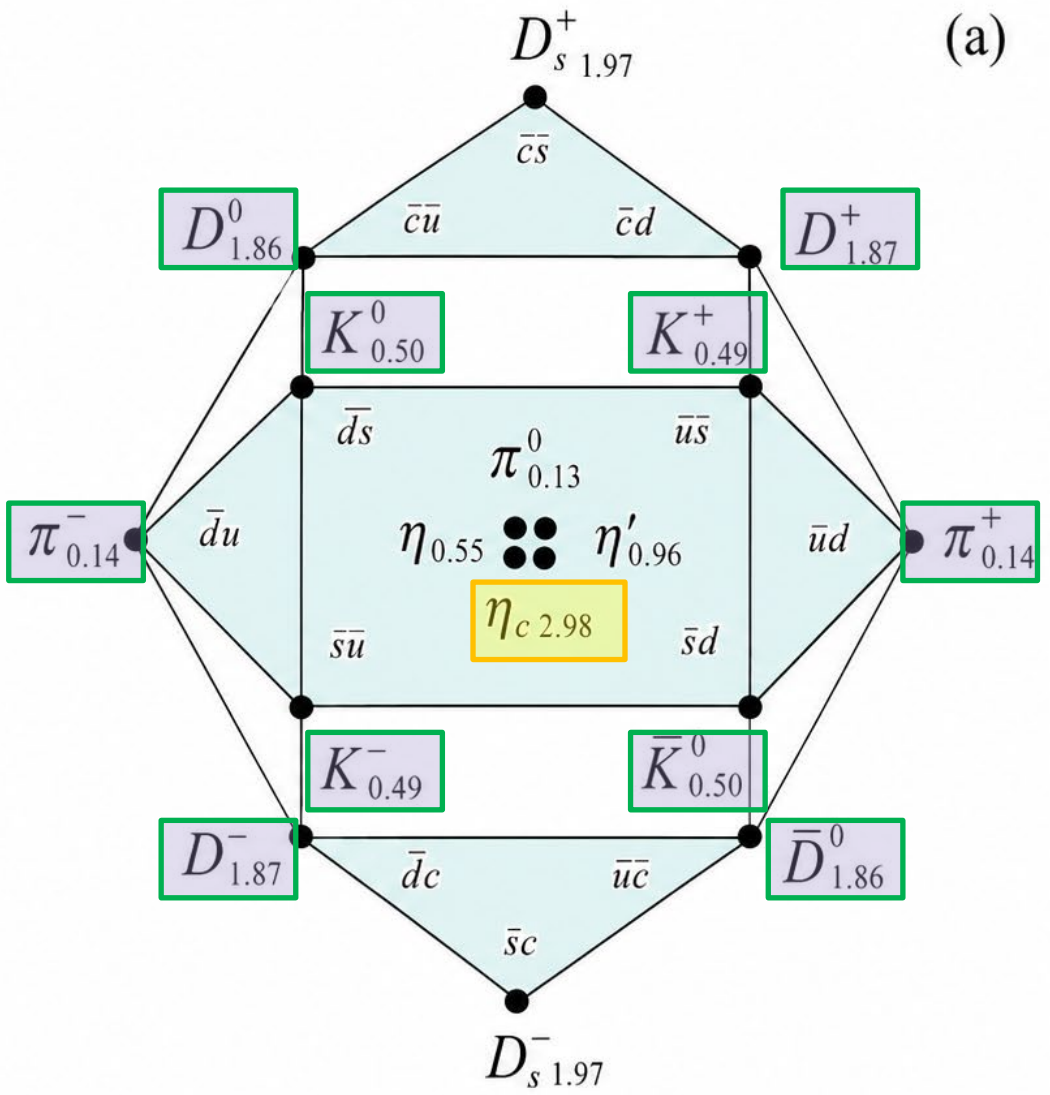


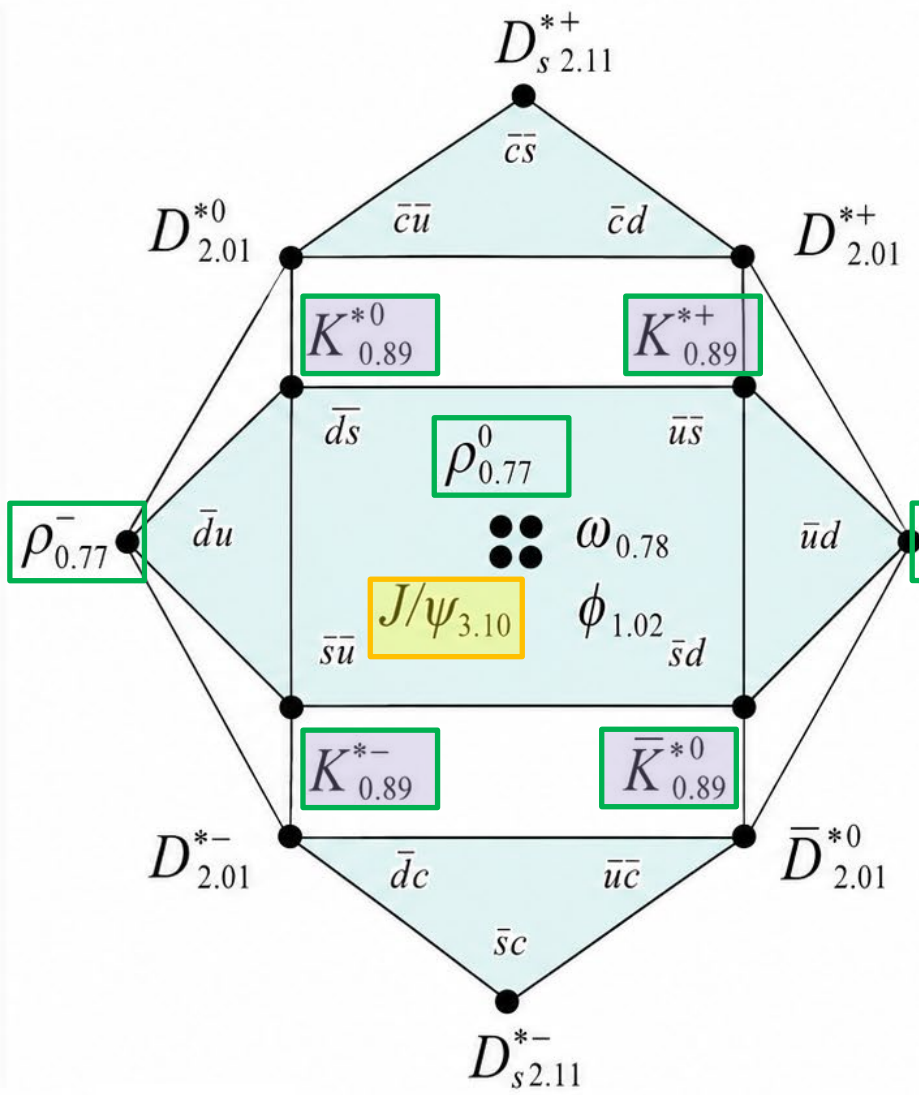




See Mu-Hua's Talk

Light Meson Map —— LaMET

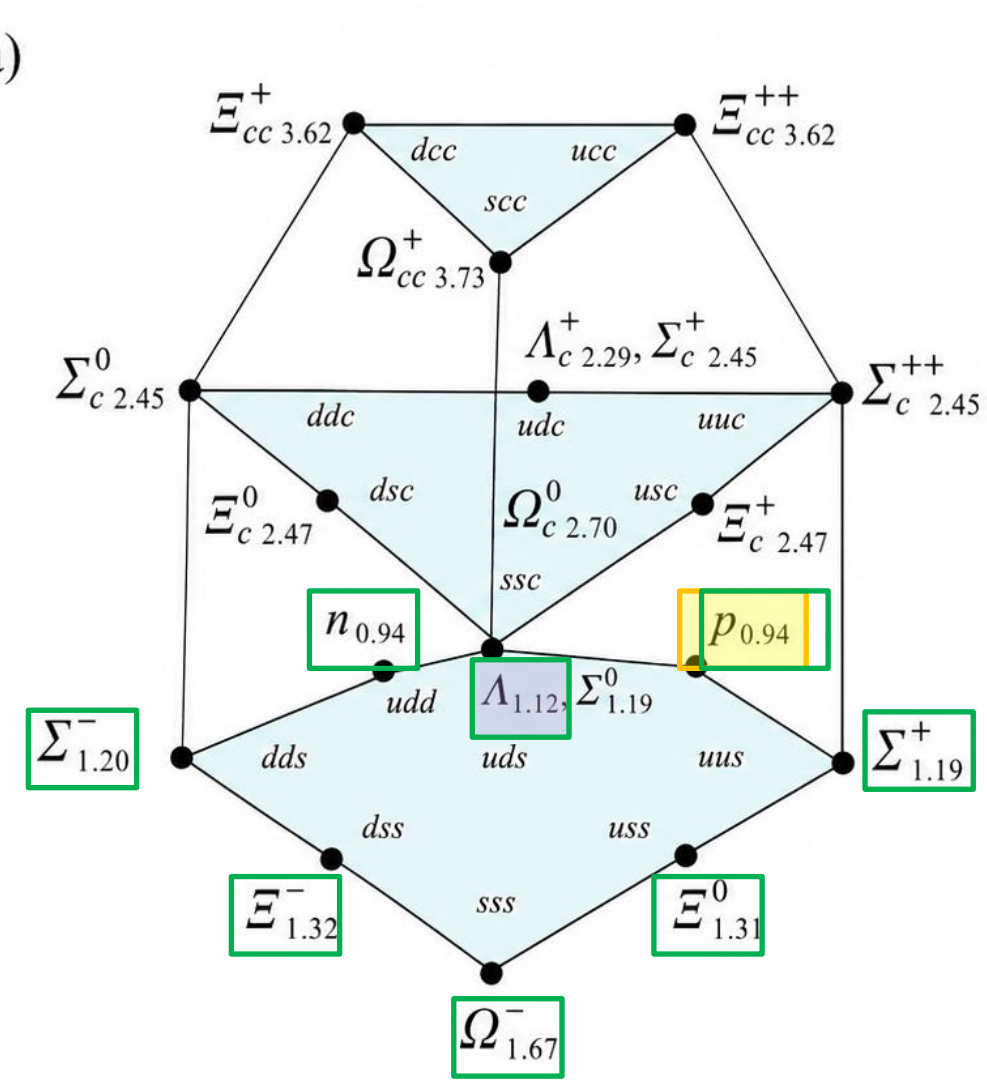


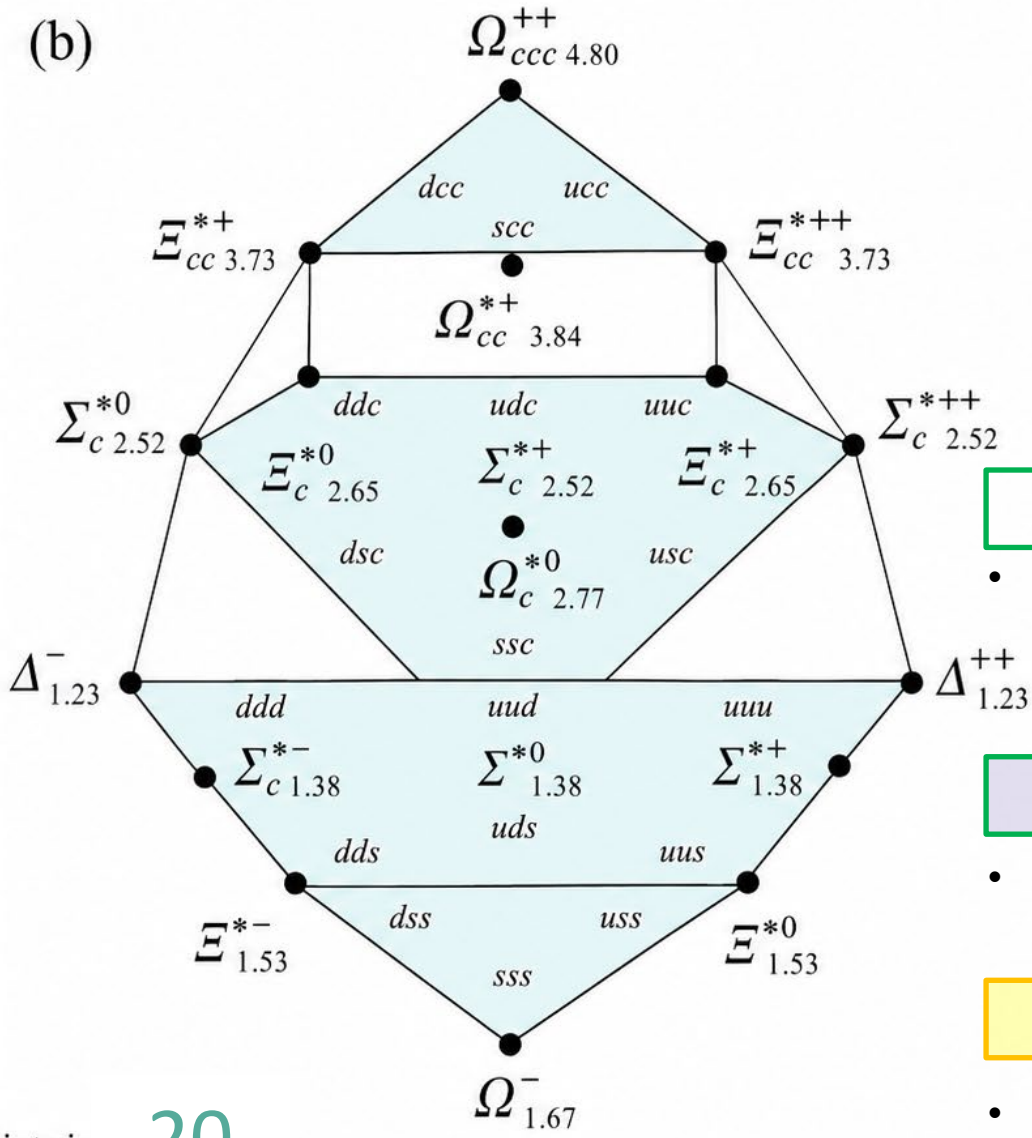


- Second moment done in OPE (LaMET under going)
- Leading twist almost done (OPE & LaMET)
- Leading twist moments in Plan/Process

mass subscripts in

Light Baryon Map —— LaMET





- First moment done in OPE
- Leading twist almost done (OPE & LaMET)
- Leading twist LaMET in Plan/Process

mass subscripts in

Summary & Outlook

1. **LQCD observables**
   equal-time quasi-DAs
   local moments for checks
2. **EFT bridge**
   LaMET / HQLaMET
   renorm + matching
3. **LCDA outputs**
   light meson 1D / baryon 2D
   heavy-meson HQET
4. **Physics use**
   exclusive processes
   flavor and CP observables

Light mesons

precision benchmark stage

- direct x-dependent pseudoscalar/vector LCDAs
- hybrid/self renorm + RGR/LRR/threshold resum.
- NNLO matching + OPE-LaMET moment benchmark

Light baryons

direct 2D LCDA stage

- beyond moments/models: x1-x2 VAT distributions
- SNR enhancement + PyQUDA + CLQCD ensembles
- hybrid renorm + large-λ extrapolation

Heavy mesons

HQLaMET / two-step matching

- QCD quasi-DA -> QCD LCDA -> HQET LCDA
- dynamic heavy quark; avoid direct static-HQET Euclidean operator
- continuum-limit B/HQET LCDA for heavy-flavor decays

Long-term goal: *From individual LCDAs to a systematic first-principles LCDA atlas*

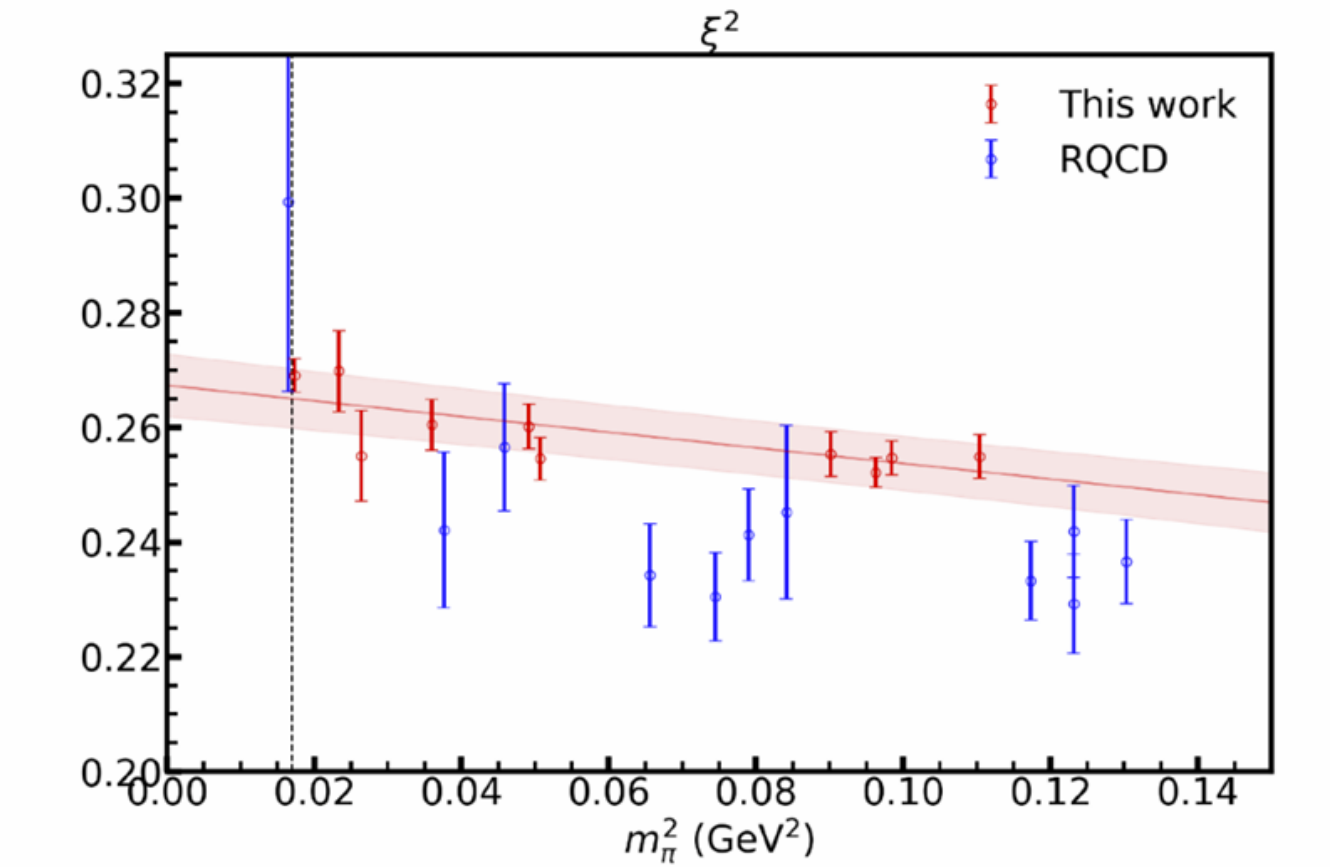
Outlook

- **Precision**
  multi-a/m_pi/Pz data; higher-order matching; stable tails/endpoints
- **Wider LCDA map**
  light mesons, baryon octet, heavy mesons and shape functions
- **Phenomenology**
  exclusive form factors; B-decay uncertainties; flavor/CP tests

Thanks for Your Attention !

Back Up

Pion mass extrapolation for $\pi(\xi_1)$:



Physical point problem within RQCD