From Benchmark of Local & Nonlocal Operators to Precise Light Meson LCDAs

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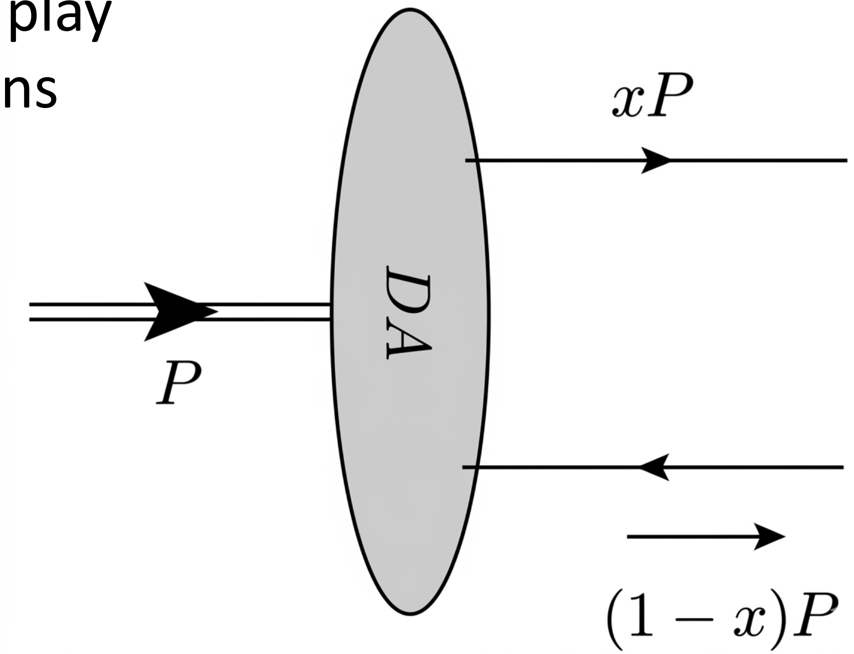
Date: July 7th @ LaMET 2026

Meson light-cone distribution amplitudes

Meson light-cone distribution amplitudes (LCDAs) play an important role in hadron hard exclusive reactions

- $\pi \to \gamma\gamma^*$ Transition Factor
- Deeply Virtual Meson Production
- Heavy Meson Decay
- Exclusive Photoproduction
- ……



$$i f_\pi \phi_\pi(x,\mu) = \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle 0|\bar{\psi}_d(\xi^-)\,\gamma^+\gamma_5\,\mathcal{W}(\xi^-,0)\,\psi_u(0)|\pi(P)\rangle$$

Light meson LCDAs are also the simplest quantities among parton distributions.

Therefore, LCDAs provide a good laboratory to test the capability of lattice QCD in studying parton physics.

Extract LCDAs on lattice

LaMET(non-local operator):

Calculate the quasi-DA on the lattice and match it to the LCDA through LaMET factorization.

$$\langle\Omega|\bar{\psi}_i(z)\,\gamma^t\gamma_5\,\mathcal{W}(z,0)\,\psi_i(0)|M(\vec{P})\rangle \xrightarrow{P_z\to\infty} \langle\Omega|\bar{\psi}_i(\xi^-)\,\gamma^+\gamma_5\,\mathcal{W}(\xi^-,0)\,\psi_i(0)|M(\vec{P})\rangle$$

OPE(local operator):

Calculate moments on lattice and reconstruct the LCDA.

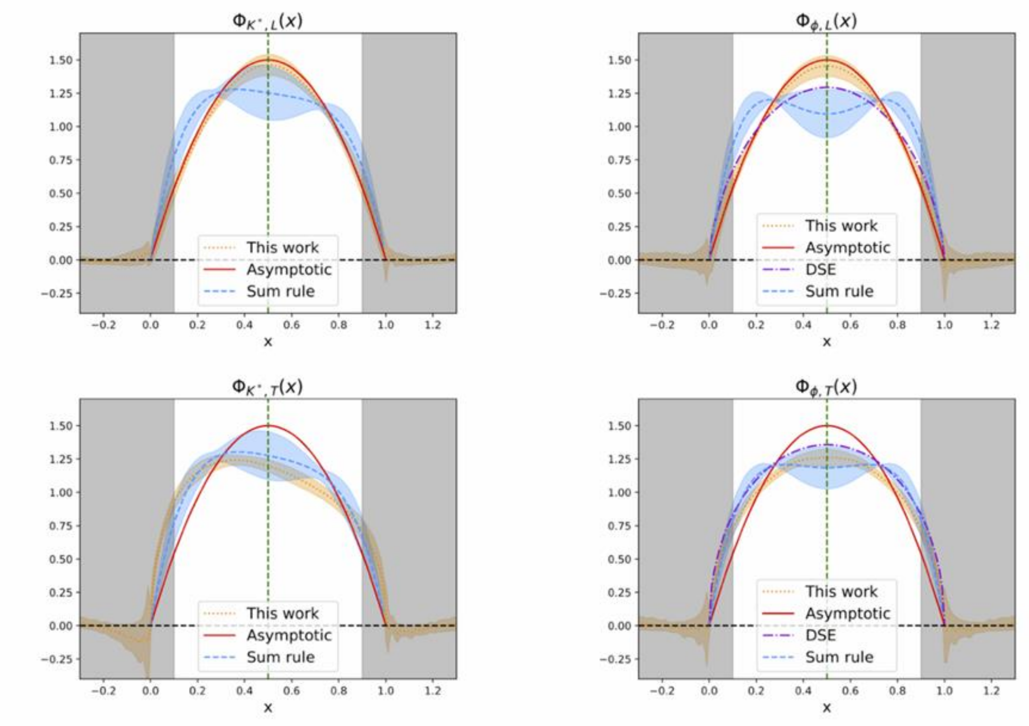
$$\bar{\psi}_i(\xi^-)\,\gamma^+\gamma_5\,\mathcal{W}(\xi^-,0)\,\psi_i(0) \to \frac{1}{n!}\sum_n^{\infty}\bar{\psi}_i(0)\gamma_{(\mu_1}\gamma_5\overleftrightarrow{D}_{\mu_2}...\overleftrightarrow{D}_{\mu_N)}\psi_i(0)n^{\mu_1}n^{\mu_2}...n^{\mu_N}(\xi^-)^n$$

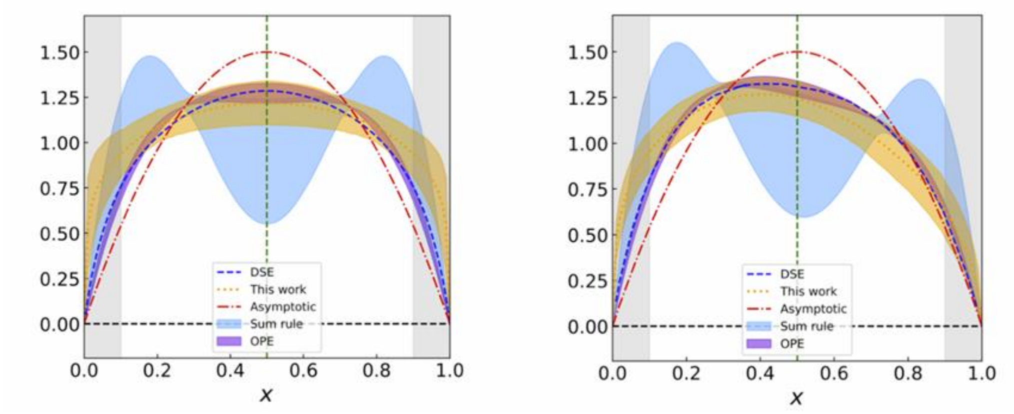
Moments:

$$\langle 0|\bar{\psi}_i\gamma_{(\mu_1}\gamma_5\overleftrightarrow{D}_{\mu_2}...\overleftrightarrow{D}_{\mu_N)}\psi_i|M(\vec{P})\rangle = (-i)^n f_\pi\langle\xi^n\rangle p_{(\mu_1}p_{(\mu_2}...p_{\mu_N)}$$

$$\langle\xi^n\rangle = \int_0^1 (2x-1)^n\phi(x)dx$$

Previous results of LCDA





LaMET:

Phys.Rev.Lett. 129 (2022) 13, 132001, Jun Hua et al.
Phys.Rev.Lett. 127 (2021) 6, 062002, Jun Hua et al.

$$a_\pi^{(2)} = 0.258(70)(52)$$
$$a_K^{(1)} = 0.108(14)(15)$$
$$a_K^{(2)} = 0.170(14)(44)$$

OPE(form RQCD):

J. High Energ. Phys. 2019, 65 (2019) Bali, G.S. et al.

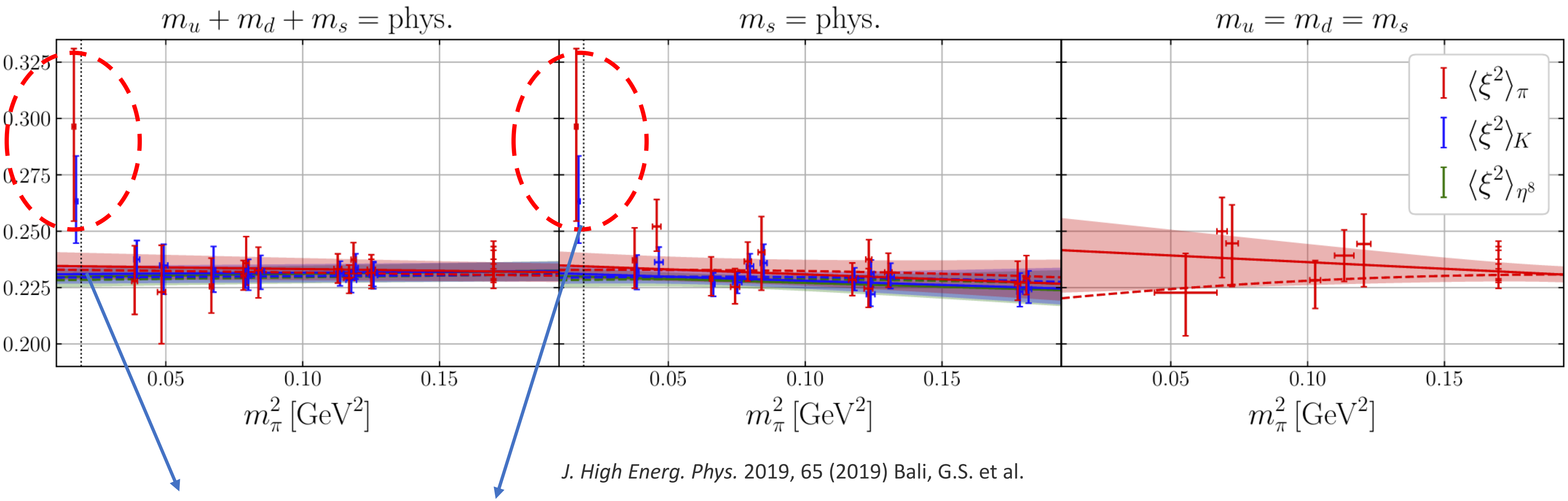
$$a_\pi^{(2)} = 0.116^{+19}_{-20}$$
$$a_K^{(1)} = 0.0525^{+31}_{-33}$$
$$a_K^{(2)} = 0.106^{+15}_{-16}$$

Tension of the DA moments in the literatures.

Previous results of LCDA

In order to make a better comparison, we calculate on MILC ensembles using OPE method.:

OPE results form RQCD (on CLS ensembles)



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ξ^2 on physical point much larger than others

Moments form local operator

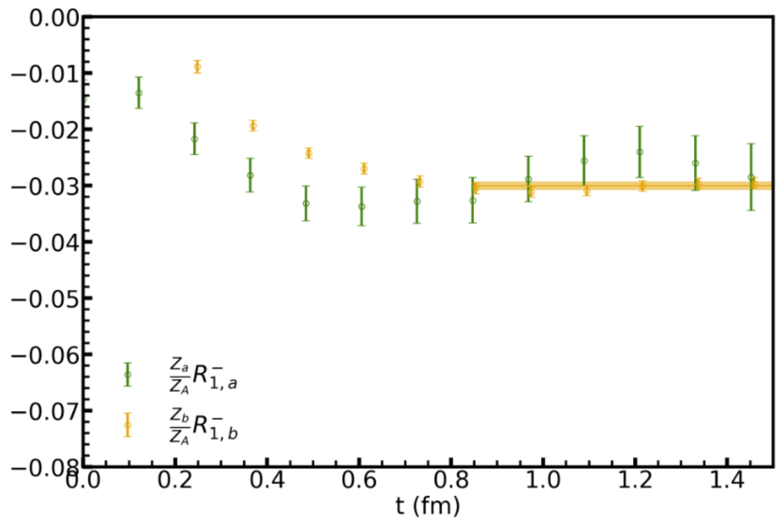
Lattice Formulation:

$$\langle 0|\bar{\psi}_i \gamma_{(\mu_1}\gamma_5 \overleftrightarrow{D}_{\mu_2}\ldots\overleftrightarrow{D}_{\mu_N)}\psi_i|M(\vec{P})\rangle = (-i)^n f_\pi \langle \xi^n\rangle p_{(\mu_1} p_{(\mu_2}\ldots p_{\mu_N)}$$

$$\langle \xi^n\rangle = \int_0^1 (2x-1)^n \phi(x)dx$$

$$P = \bar{\psi}_1\gamma_5\psi_2,$$
$$A_\mu = \bar{\psi}_1\gamma_\mu\gamma_5\psi_2,$$
$$A^-_{\mu\nu} = \bar{\psi}_1\gamma_{(\mu}\gamma_5[\overrightarrow{D}_\nu - \overleftarrow{D}_{\nu)}]\psi_2,$$
$$A^\pm_{\mu\nu\rho} = \bar{\psi}_1\gamma_{(\mu}\gamma_5[\overrightarrow{D}_\nu\overrightarrow{D}_\rho \pm 2\overrightarrow{D}_\nu\overleftarrow{D}_{\rho)} + \overleftarrow{D}_\nu\overleftarrow{D}_{\rho)}]\psi_2.$$

First moments of kaon on a12m130



- **First moments:**

$$R^-_{1,a} = \frac{i}{3}\sum_{i=1}^{3}\frac{1}{p_i}\frac{C_{A^-_{4i}P}(\mathbf{p},t)}{C_{A_4P}(\mathbf{p},t)}, \qquad R^-_{1,b} = \frac{4E}{3E^2+p^2}\frac{C_{A^-_{44}P}(\mathbf{p},t)}{C_{A_4P}(\mathbf{p},t)}.$$

- **Second moments:**

$$R^\pm_{2,a_1} = -\frac{1}{3}\sum_{i\neq j}^{3}\frac{1}{p_ip_j}\frac{C_{A^\pm_{4ij}P}(\mathbf{p},t)}{C_{A_4P}(\mathbf{p},t)}, \qquad R^\pm_{2,a_2} = \frac{1}{3}\sum_{i=1}^{3}\frac{p_i}{p_1p_2p_3}\frac{C_{A^\pm_{123}P}(\mathbf{p},t)}{C_{A_iP}(\mathbf{p},t)}.$$

$$C_{\mathcal{O}_1\mathcal{O}_2}(\mathbf{p},t) = \sum_x e^{i\mathbf{p}\cdot\mathbf{x}}\langle \mathcal{O}_1(\mathbf{x},t)\mathcal{O}_2(\mathbf{0},0)\rangle$$

Moments form local operator

Lattice Set up:

MILC 2+1+1 (HISQ)

Ensembles	a (fm)	$L^3 \times T$	m_π^{v}	$m_{\eta_s}^{\mathrm{v}}$	m_π^{sea}	$m_{\eta_s}^{\mathrm{sea}}$	$N_{\mathrm{conf}} \times N_{\mathrm{tsrc}} \times N_{\mathrm{multi\text{-}src}}$
a04m310	0.0425(5)	$64^3 \times 192$	300	636	315	693	$187 \times 4 \times 2$
a06m310	0.0574(5)	$48^3 \times 144$	322	656	329	725	$83 \times 4 \times 4$
			162	718			$176 \times 4 \times 2$
a09m310	0.0882(7)	$32^3 \times 96$	313	647	316	697	$129 \times 6 \times 2$
			152	690			$129 \times 12 \times 2$
a12m310	0.1213(9)	$24^3 \times 64$	310	612	305	676	$200 \times 2 \times 2$
			190	659			$200 \times 8 \times 2$
a12m220S	0.1213(9)	$24^3 \times 64$	222		220	675	$86 \times 8 \times 2$
a12m220	0.1213(9)	$32^3 \times 64$	225		220	675	$85 \times 8 \times 2$
a12m130	0.1213(9)	$48^3 \times 64$	131	679	132	675	$165 \times 32 \times 2$

Four lattice spacing (a04m310, a06m310, a09m310, a12m310)

Finite volume (a12m220S, a12m220)

Physical pion mass (a12m130)

1-step HYP-smeared clover fermion for the valence quark

Moments form local operator

Extrapolation form for moments:

- **First moments:**

$$\langle\xi\rangle_M(m_{\pi,n_s}^{\nu,\text{sea}},a,)=\langle\xi\rangle_M^{\text{phys}}[1+\delta(c_{1/2,M}^{(1)},a)][1-c_{3,M}^{(1)})\delta m_q^{\nu}+c_{3,M}^{(1)}\delta m_q^{\text{sea}}]/\delta m_q^{\text{phys}}.$$

- **Second moments:**

$$\langle\xi^2\rangle_M(m_{\pi,\eta_s}^{\nu,\text{sea}},a)=\langle\xi^2\rangle_0[1+\delta(c_{1/2,M}^{(2)},a)]+\sum_{X=\nu,\text{sea}}[c_3^{(2),X}\bar{m}_q^X+n_M c_4^{(2),X}\delta m_q^X],$$

Continuous extrapolation **Partially quench**

$$\delta(c_{1/2/3},a,1/L)=c_1\bar{\alpha}_s\Lambda_\chi a+c_2\Lambda_\chi^2a^2$$

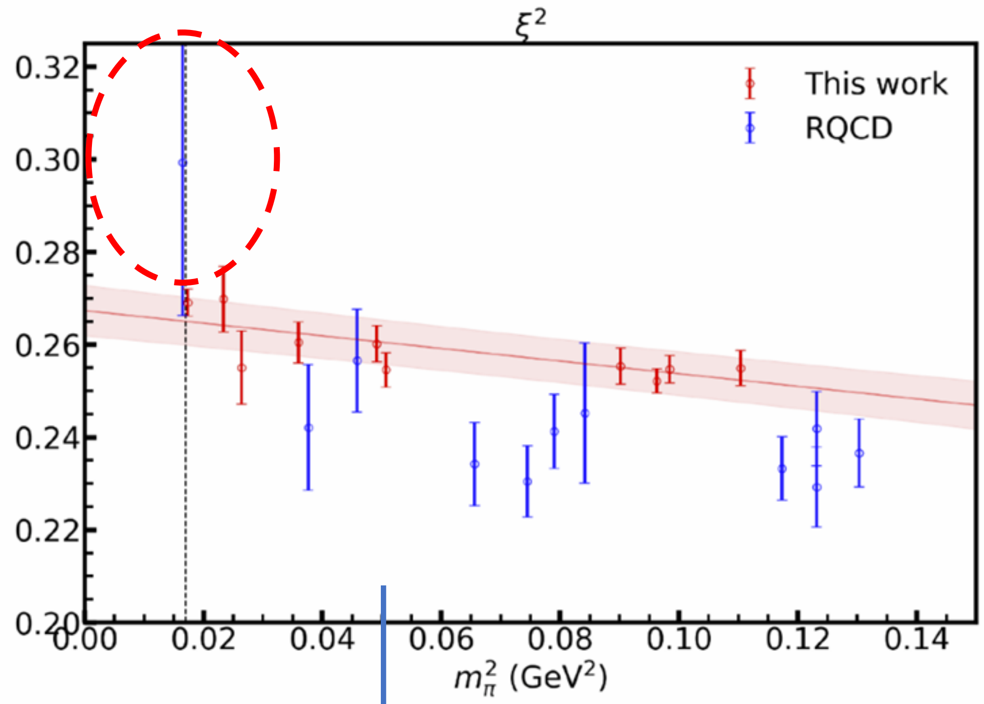
$$\bar{\alpha}_s\equiv-\tfrac{4}{3}\ln u_0\qquad\Lambda_\chi=1\ \text{GeV}$$

Moments form local operator

Moments from MILC ensembles:

$$\langle \xi^2 \rangle_M (m_{\pi,\eta_s}^{\nu,\text{sea}}, a) = \langle \xi^2 \rangle_0 [1 + \delta(c_{1/2,M}^{(2)}, a)] + \sum_{X=\nu,\text{sea}} [c_3^{(2),X} \bar{m}_q^X + n_M c_4^{(2),X} \delta m_q^X],$$

	$a_\pi^{(2)}$	$a_K^{(1)}$	$a_K^{(2)}$	
RBC/UKQCD [22]	0.23(3)(6)	0.036(1)(2)	0.18(3)(6)	
RQCD [24]	0.116^{+19}_{-20}	0.0525^{+31}_{-33}	0.106^{+15}_{-16}	
This work	0.187(15)	0.0481(23)	0.158(14)	
	$a_\rho^{(2)}$	$a_{K^*}^{(1)}$	$a_{K^*}^{(2)}$	$a_\phi^{(2)}$
RBC/UKQCD [22]	0.20(3)(6)	0.037(1)(2)	0.15(6)(6)	0.15(6)(3)
RQCD [25]	0.132(27)			
This work	0.102(18)	0.0609(15)	0.085(17)	0.069(17)
	$a_\rho^{(2)T}$	$a_{K^*}^{(1)T}$	$a_{K^*}^{(2)T}$	$a_\phi^{(2)T}$
RQCD [25]	0.101(22)			
This work	0.133(20)	0.0301(16)	0.087(19)	0.041(19)



- **Higher precision compared with RQCD.**
- **Consistent with RQCD within 2σ (except for $a^2_{\pi/K}$)**

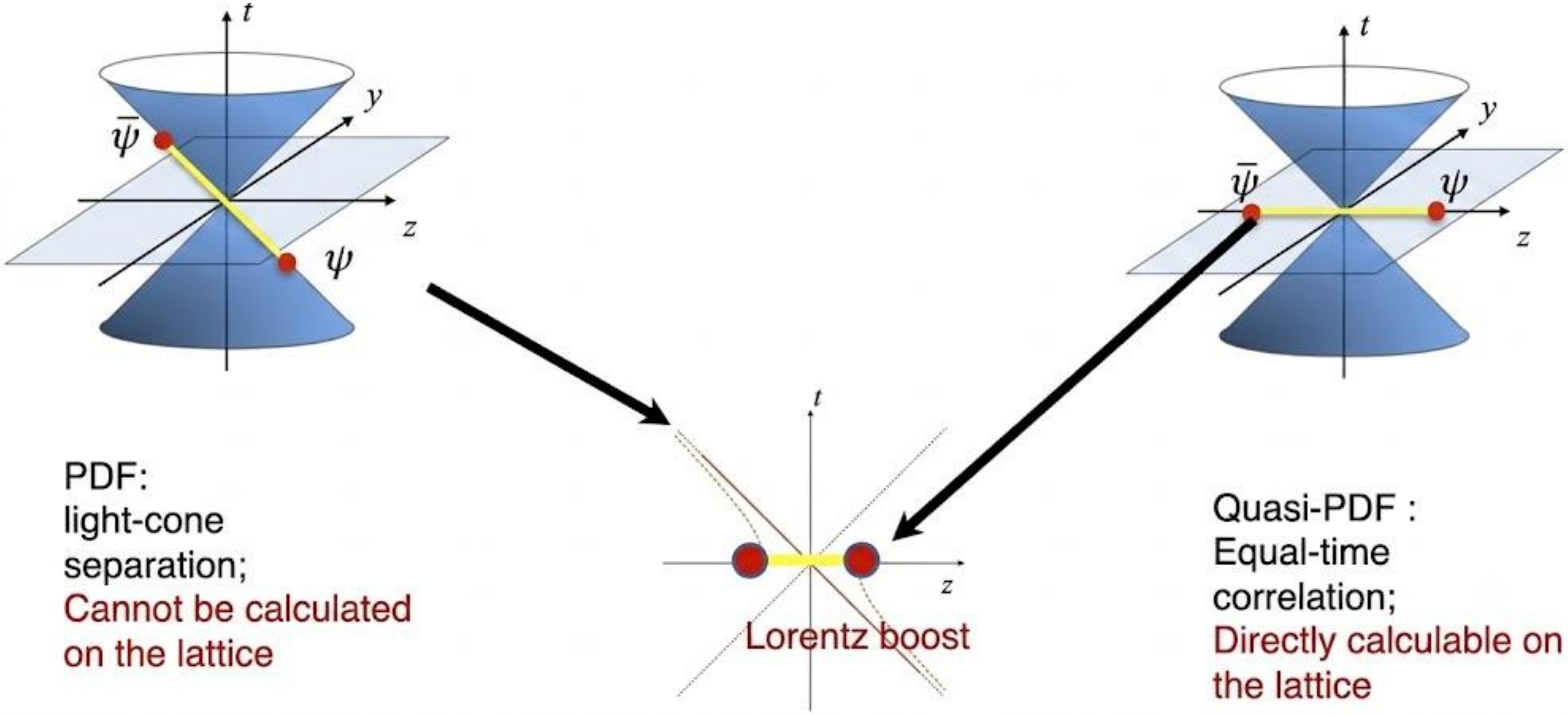
More accurate pion mass dependence

Moments benchmark for Quasi-DA

Quasi-DA: $$i f_M \phi_M^{\text{Quasi}}(x,\mu) = \int \frac{dz}{2\pi}\, e^{-ixP_z z}\, \langle 0|\bar{\psi}_i(z)\,\gamma^+\gamma_5\,\mathcal{W}(z,0)\,\psi_j(0)|M(P)\rangle$$

$P_z \to \infty$ **and perturbative matching**

LCDA: $$i f_M \phi_M(x,\mu) = \int \frac{d\xi^-}{2\pi}\, e^{-ixP^+\xi^-}\, \langle 0|\bar{\psi}_i(\xi^-)\,\gamma^+\gamma_5\,\mathcal{W}(\xi^-,0)\,\psi_j(0)|M(P)\rangle$$



PDF:
light-cone
separation;
Cannot be calculated
on the lattice

Quasi-PDF :
Equal-time
correlation;
Directly calculable on
the lattice

Moments benchmark for Quasi-DA

$$\begin{aligned}\langle\xi^n\rangle_M &= \int_0^1 dx(2x-1)^n\phi_M(x,\mu), \quad \xi = 2x-1 \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dx d\lambda (2x-1)^n e^{ix\lambda}\mathcal{I}(\lambda,\mu) \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dx d\lambda (-2i)^n \frac{d^n}{d\lambda^n}(e^{\frac{i}{2}(2x-1)\lambda})e^{i\frac{\lambda}{2}}\mathcal{I}(\lambda,\mu) \\ &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dx d\lambda (2i)^n e^{\frac{i}{2}(2x-1)\lambda}\frac{d^n}{d\lambda^n}(e^{i\frac{\lambda}{2}}\mathcal{I}(\lambda,\mu)) \\ &= (2i)^n\int_{-\infty}^{+\infty} d\lambda\delta(\lambda)e^{-i\frac{\lambda}{2}}\frac{d^n}{d\lambda^n}(e^{i\frac{\lambda}{2}}\mathcal{I}(\lambda,\mu)) \\ &= (2i)^n\frac{d^n}{d\lambda^n}(e^{i\frac{\lambda}{2}}\mathcal{I}(\lambda,\mu))|_{\lambda=0}\end{aligned}$$

Light-cone correlation (LCDA in coordinate space):

$$if_M P_+\mathcal{I}(\lambda,\mu) = \langle\Omega|\bar{\psi}_1(0)\gamma_+\gamma_5 U(0,\xi^-)\psi_2(\xi^-)|M(\vec{P})\rangle, \quad \lambda = \xi^- P_+$$

$$\langle\xi^n\rangle = \int_0^1 (2x-1)^n\phi(x)dx \qquad\qquad \langle\xi^n\rangle_M = (2i)^n\frac{d^n}{d\lambda^n}(e^{i\frac{\lambda}{2}}\mathcal{I}(\lambda,\mu))|_{\lambda=0}$$

Integral in momentum space ⟺ Derivative in coordinate space

Integral in momentum space

- **Renormalization:**

  Different renormalization schemes can lead to different long-distance behavior.

- **Matching :**

  Large power corrections near x=0 and x=1

- **λ Extrapolation and Fourier transformation :**

  The shape cthe model used in λ Extrapolation an depend on

Derivative in coordinate space

- **Renormalization:**

  Moments only depend on the short-distance behavior.

- **Matching :**

  Power corrections is small in short-distance

- **λ Extrapolation and FT:**

  Without λ Extrapolation and FT

We can check the moments on coordinate space!

Moments benchmark with non-local operator

Extract Light-Cone correlation on lattice:

Quasi correlation from 2pt

$$R_{\text{quasi}}(z, P_z, t) = \frac{C_2(t, z, P_z)}{C_2(t, 0, P_z)} \qquad R_{\text{quasi}}(z, P_z, t) \xrightarrow{t\to\infty} H(z, P_z)$$

⇩

Self renormalization

$$R^R_{\text{quasi}}(z, P_z, t) = \frac{R_{\text{quasi}}(z, P_z, t)}{Z_{\text{self}}(z) H^{\overline{\text{MS}}}(z)}$$

⇩

Matching in coordinate space

$$R^R_{\text{quasi}}(\lambda, P_z, t) = \int_0^\lambda d\lambda' \; C(\lambda, \lambda', P_z, \mu) R_{\text{light-cone}}(\lambda', P_z, t) \quad \text{where } \lambda = zP_z$$

$$\Rightarrow R_{\text{light-cone}}(\lambda, P_z, t) = \int_0^\lambda d\lambda' \; C^{-1}(\lambda, \lambda', P_z, \mu) R^R_{\text{quasi}}(\lambda', P_z, t)$$

Moments benchmark with non-local operator

Gamma structure: **(dynamical enhance considered)**

$$\sum_x e^{iP\cdot x}\langle\bar{\psi}_i(x+z,t)\,\Gamma_{\rm snk}\,\mathcal{W}(x+z,x)\,\psi_j(x,t)\bar{\psi}(0,0)\Gamma_{\rm src}\psi(0,0)\rangle$$

☐ **: previous choices**

	$\Gamma_{\rm Snk}$	$\Gamma_{\rm Src}$
Pseudo Saclar	$\gamma_t\gamma_5$, [$\gamma_z\gamma_5$]	$\gamma_t\gamma_5$, $\gamma_z\gamma_5$, [γ_5]
Vector L	[γ_t], γ_z	γ_t, [γ_z], $\gamma_z\gamma_t$
Vector T	[σ_{yz}], $\gamma_y\gamma_t$	σ_{yz}, $\gamma_y\gamma_t$, [γ_y]

Better Choices →

$\Gamma_{\rm Snk}$	$\Gamma_{\rm Src}$
$\gamma_t\gamma_5$	$\gamma_t\gamma_5$, $\gamma_z\gamma_5$,
γ_z	γ_t, γ_z
σ_{yz}, $\gamma_y\gamma_t$	σ_{yz}, $\gamma_y\gamma_t$

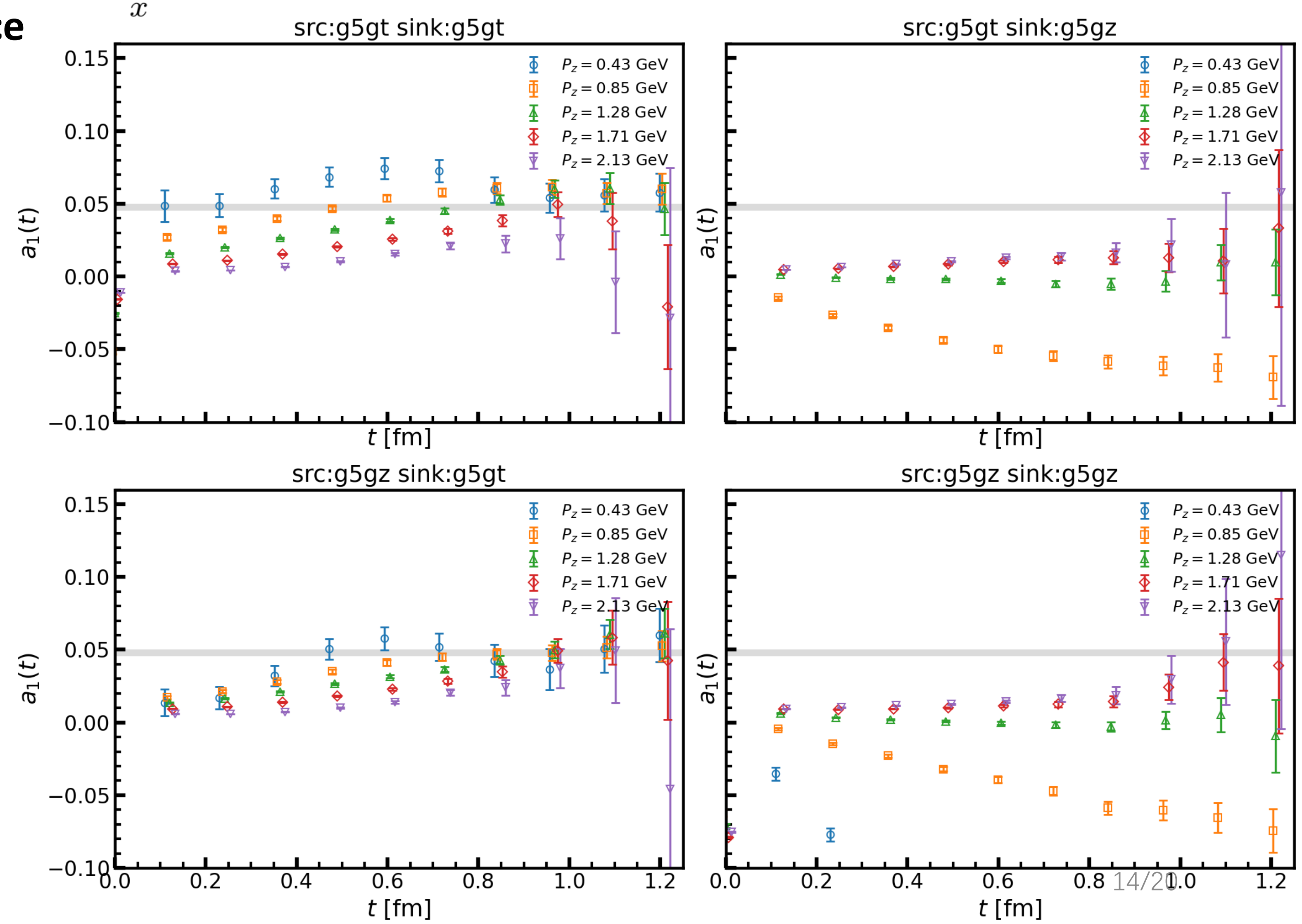
- 6 choices for each case **to find the better choices**
- **Take the self renormalization from Wilson lines (to avoid zero-momentum matrix elements in some cases)**

Moments benchmark with non-local operator

$$\sum_x e^{iP\cdot x} \langle \bar{\psi}_i(x+z,t)\,\Gamma_{\text{snk}}\,\mathcal{W}(x+z,x)\,\psi_j(x,t)\bar{\psi}(0,0)\Gamma_{\text{src}}\psi(0,0)\rangle$$

Excited contamination and Pz dependence of DA on coordinate space:

- **The imaginary part show a excited state contamination**
- **Different gamma matrix can have very different Pz dependence**



Moments benchmark with non-local operator

$a \to 0, P_z \to \infty$ limit (kaon)

Second Moments:

Source: Γ_1

Sink: Γ_2

benchmark with a_2 by the curvature of real part of non-local quasi-DA in coordiante space

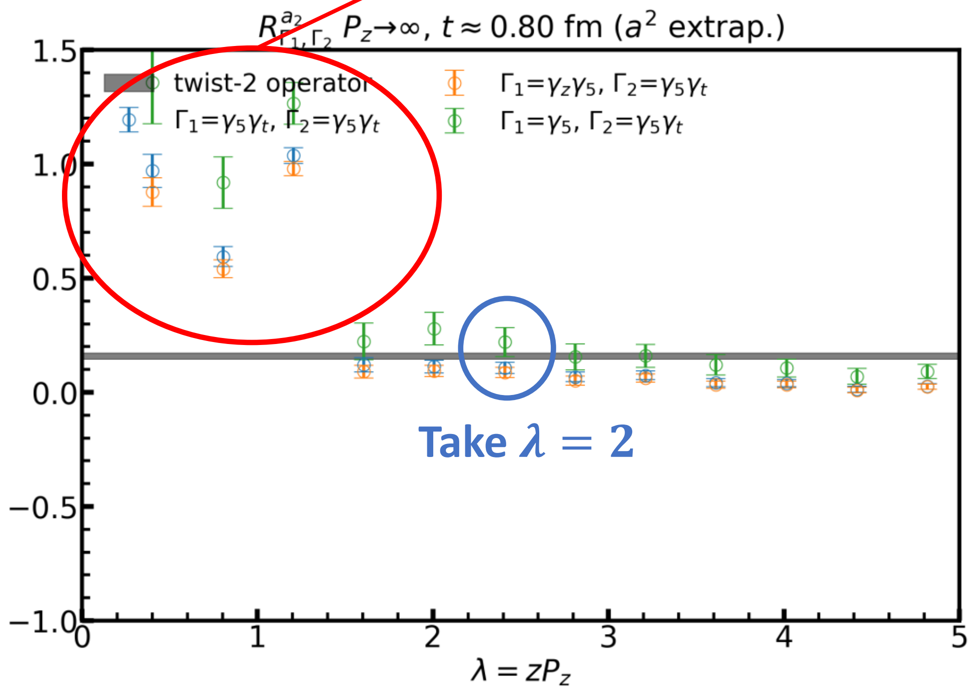
$$R_{\langle \xi^2 \rangle_M}(t, P_z) = -8\,\mathrm{Re}\left(e^{\frac{\lambda}{2}} R_{\text{light-cone}}(\lambda, P_z, t) - R_{\text{light-cone}}(0, P_z, t)\right)/\lambda^2$$

$$R_{a_2}(t, P_z) = \frac{7}{12}\left(5R_{\langle \xi^2 \rangle_M}(t, P_z) - 1\right)$$

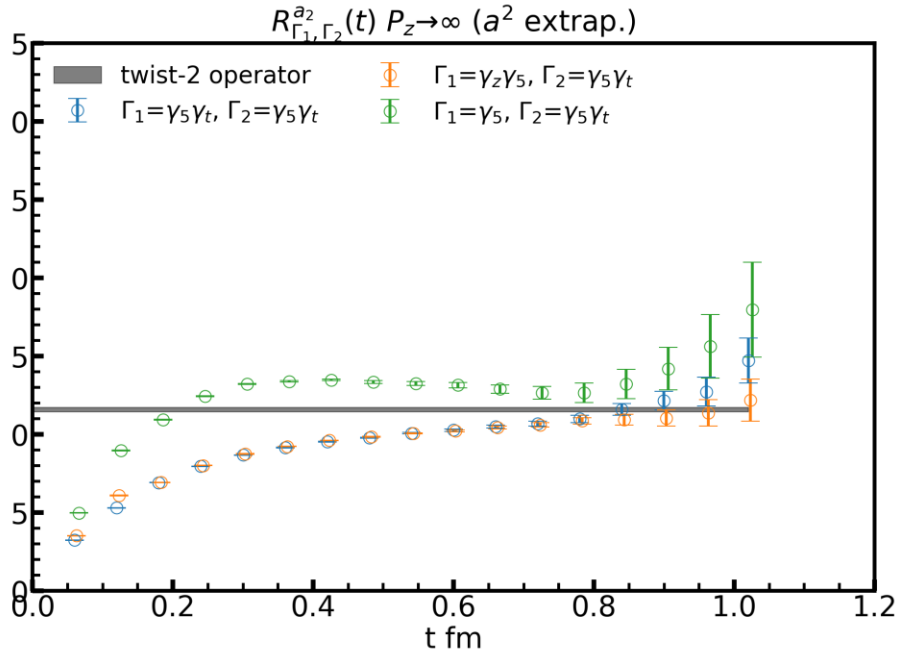
$\lambda \to 0$

However discreate effects ↑ at small z(λ)

Extracted a_2 at fixed $t \approx 0.8\mathrm{fm}$, $P_z \to \infty$, with different z(λ)



t dependence of a_2 using $\lambda = 2$



Moments benchmark with non-local operator

$a \to 0, P_z \to \infty$ **limit (kaon)**

benchmark with a_1 by the slope of imaginary part of non-local quasi-DA in coordiante space

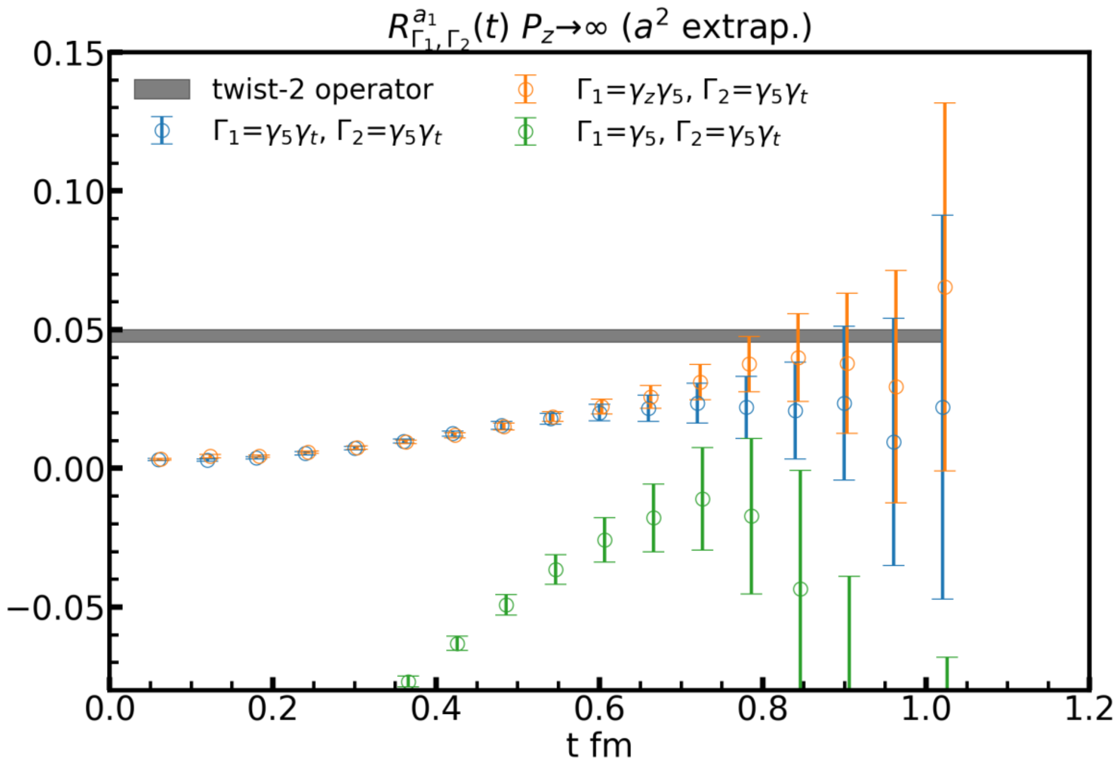
First Moments:

$$R_{\langle\xi\rangle_M}(t,P_z) = 2\,\mathrm{Im}\left(e^{\frac{\lambda}{2}} R_{\text{light-cone}}(\lambda,P_z,t) - R_{\text{light-cone}}(0,P_z,t)\right)/\lambda$$

$$R_{a_1}(t,P_z) = \frac{5}{3} R_{\langle\xi\rangle_M}(t,P_z)$$

Source: Γ_1
Sink: Γ_2

t dependence of a_1 using $\lambda = 2$



$\Gamma_1 = \gamma_5$ suffer a large excited state contamination.

$\Gamma_1 = \gamma_t\gamma_5$ or $\gamma_5\gamma_z$ are better choices.

Moments benchmark with non-local operator

Extract LCDA on lattice:

bare correlation $R_{\text{quasi}}(z, P_z, t) = \frac{C_2(t, z, P_z)}{C_2(t, 0, P_z)}$ $\quad R_{\text{quasi}}(z, P_z, t) \xrightarrow{t\to\infty} H(z, P_z)$

⇩

self renormalization with Wilson line $H^R(z, P_z) = \frac{H(z, P_z)}{Z_{\text{self}}(z) H^{\overline{\text{MS}}}(z)}$

⇩

coordinate space matching $H^R(\lambda, P_z) = \int_0^\lambda d\lambda' \; C(\lambda, \lambda', P_z, \mu) h(\lambda', P_z)$

⇩

extrapolation (a, P_z) in coordinate space ⇨ **benchmark with OPE moments**

⇩

large lambda extrapolation $h(\lambda) \sim \frac{c_1}{(i\lambda)^a} + e^{-i\lambda} \frac{c_2}{(-i\lambda)^b} \quad \lambda \to \infty.$

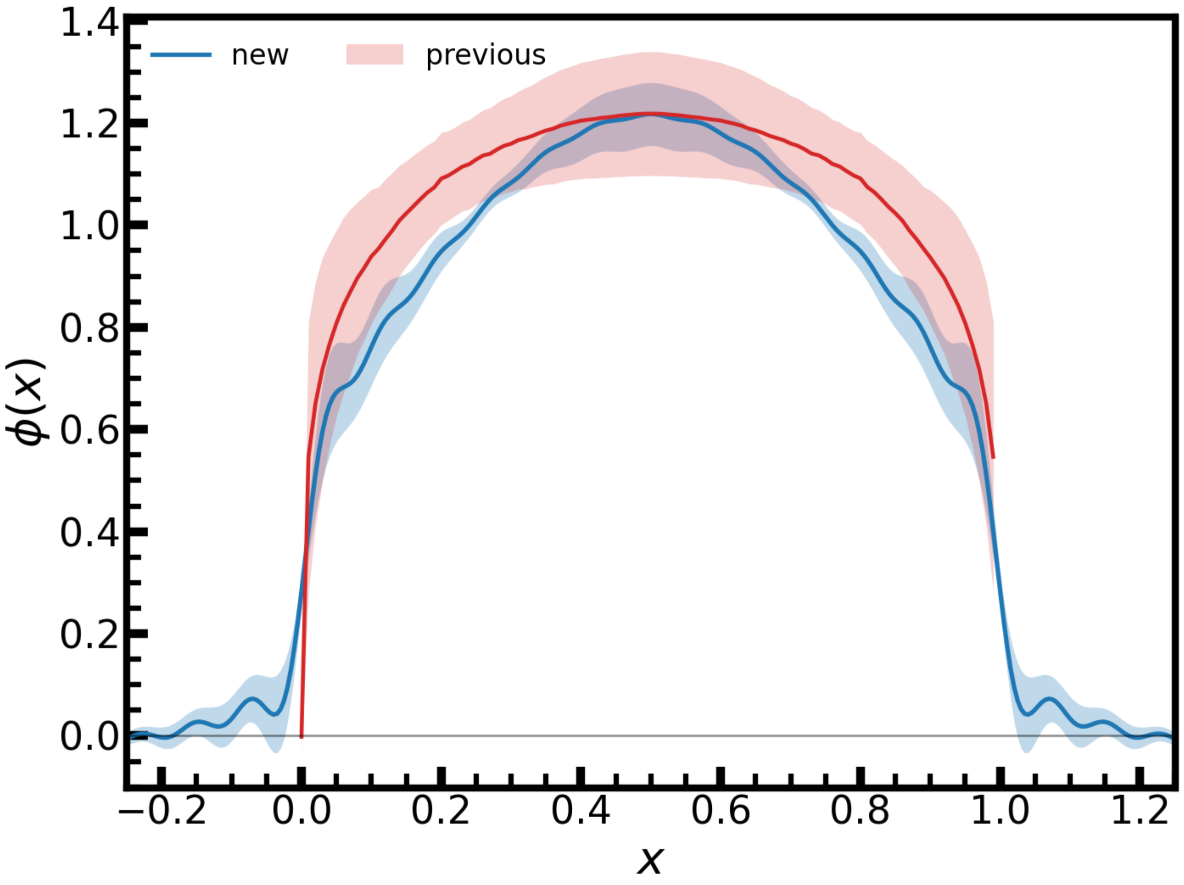
⇩

FT to final results

Moments benchmark with non-local operator

Final numerical results of pion DA in momentum space

Pion



Previous:

$$a_{2,\pi}^{\text{LaMET,intgral}} = 0.258(70)(52)$$

$$a_{2,\pi}^{\text{OPE(RQCD)}} = 0.116^{+19}_{-20}$$

New:

$$a_{2,\pi}^{\text{LaMET,derviative}} = 0.174(62)$$

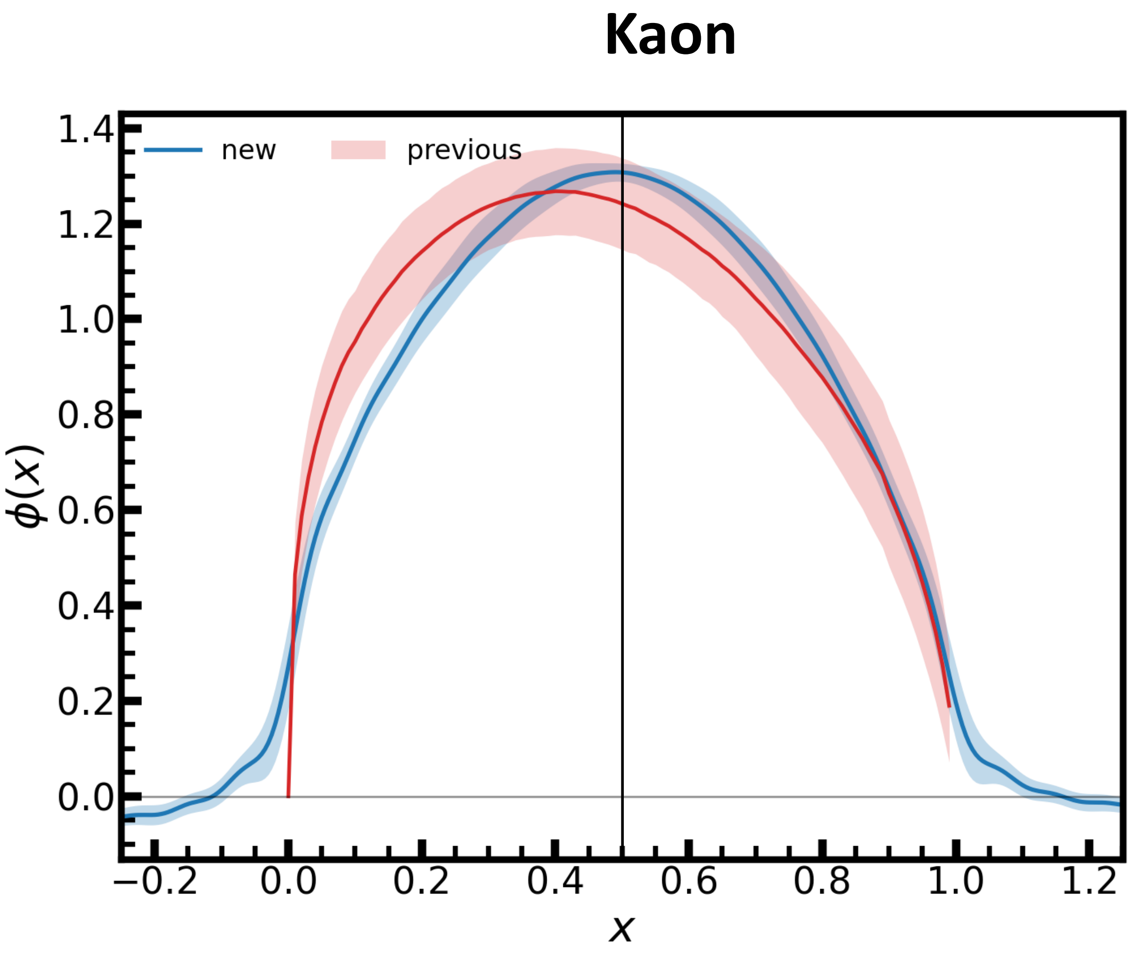
$$a_{2,\pi}^{\text{LaMET,intgral}} = 0.186(46)$$

$$a_{2,\pi}^{\text{OPE}} = 0.187(15)$$

Consistent

Moments form non-local operator

Final numerical results of kaon DA in momentum space



Previous:

$a_{2,K}^{\text{LaMET,intgral}} = 0.170(14)(44)$ $\quad a_{1,K}^{\text{LaMET,intgral}} = 0.108(14)(15)$

$a_{2,K}^{\text{OPE(RQCD)}} = 0.106^{+15}_{-16}$ $\quad a_{1,K}^{\text{OPE(RQCD)}} = 0.0525^{+31}_{-32}$

New:

$a_{2,\pi}^{\text{LaMET,derviative}} = 0.158(38)$ $\quad a_{1,K}^{\text{LaMET,,integral}} = 0.037(10)$

$a_{2,K}^{\text{LaMET,integral}} = 0.126(11)$ $\quad a_{1,K}^{\text{LaMET,,integral}} = 0.034(26)$

$a_{2,K}^{\text{OPE}} = 0.158(14)$ $\quad a_{1,K}^{\text{OPE}} = 0.0477(23)$

Consistent

Summary

✓: The results of OPE and LaMET are consisent(within 2σ)
✗: The results of OPE and LaMET are not consisent

- We calculate the **Pseudo scalar and Vector meson 1st and 2nd moments** with MILC ensembles
- With the benchmark of lower moments, we systemically control the **excited contamination and Pz dependence** of Quasi-DA in coordinate space
- With **dynamic enhanced operator, NNLO matching kernel**, we update **most precise light meson LCDAs**
- Some tensions still exist in vector mesons due to possible heavier masses of K^*, ϕ

☐ LRR & RGR may be tested, new extrapolation will be adopted in next step

$a_\pi^{(2)}$	$a_K^{(1)}$	$a_K^{(2)}$	
✓	✓	✓	
$a_\rho^{(2)}$	$a_{K^*}^{(1)}$	$a_{K^*}^{(2)}$	$a_\phi^{(2)}$
✓	✗	✓	✓
$a_\rho^{(2)T}$	$a_{K^*}^{(1)T}$	$a_{K^*}^{(2)T}$	$a_\phi^{(2)T}$
✗	✓	✗	✗