Gluon Collins-Soper Kernel from lattice QCD

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July 6 - 9, 2026

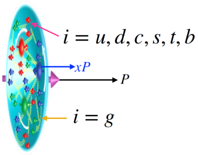
LaMET 2026, Jagiellonian University, Cracow, Poland

3D hadron structure: from PDF to TMD PDF

- Parton distribution function (PDF): $f_{i/h}(x)$

- probability of finding a parton i in hadron h

carrying momentum fraction $x \to$ longitudinal



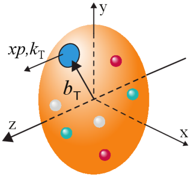
- Transverse-momentum-dependent PDF (TMD PDF):

$$f_{i/h}(x, \vec{k}_{\rm T}), \quad \text{or coordinate-space} \quad f_{i/h}(x, \vec{b}_{\rm T}) = \int {\rm d}^2\vec{k}_{\rm T}\, e^{i\vec{k}_{\rm T}\cdot\vec{b}_{\rm T}} f_i(x, \vec{k}_{\rm T})$$

- probability of finding parton i with fraction x and

transverse momentum $\vec{k}_{\rm T}$ $\to$ longitudinal and transverse

(or the Fourier conjugate $\vec{b}_{\rm T}$)

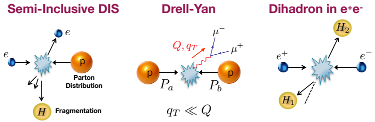


$\Rightarrow$ Rich hadron 3D internal structure in TMD PDFs!

Collins-Soper kernel

- TMD PDFs can be determined in various processes

need ability to relate different energy scales



- Evolution of TMD PDFs:

1. UV renormalization scale μ 2. rapidity scale ζ

The evolution kernels are universal (independent of external hadron h)

$$f_{i/h}(x, b_{\mathrm{T}}, \mu, \zeta) = f_{i/h}(x, b_{\mathrm{T}}, \mu_0, \zeta_0)$$

$$\times \exp\left[\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \underline{\gamma_\mu^i(\mu', \zeta_0)}\right] \exp\left[\frac{1}{2} \underline{\gamma_\zeta^i(\mu, b_{\mathrm{T}})} \ln \frac{\zeta}{\zeta_0}\right]$$

UV anomalous dimension rapidity anomalous dimension (Collins-Soper kernel)

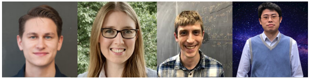
- UV anomalous dimension γ_μ^i is perturbative as long as scales are large

But CS kernel γ_ζ^i is always nonperturbative for $b_{\mathrm{T}} \gtrsim \Lambda_{\mathrm{QCD}}^{-1}$

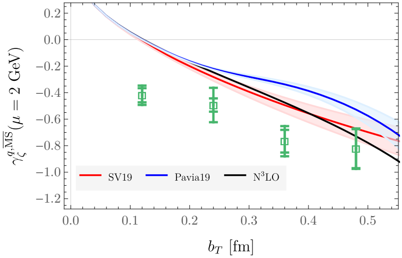
(even if the evolution variables μ, ζ are perturbative)

- Our group's LQCD results for quark CS kernel:

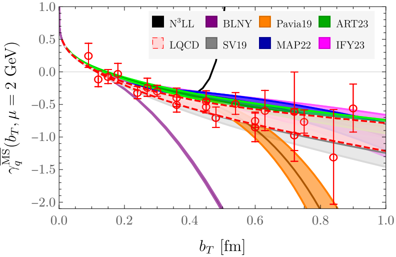
Artur Avkhadiev, Phiala Shanahan, Mike Wagman, Yong Zhao



Proof-of-concept: 2107.11930



Systematic control: 2307.12359, 2402.06725



- First such calculation with systematic control of quark mass, operator mixing, and discretization effects

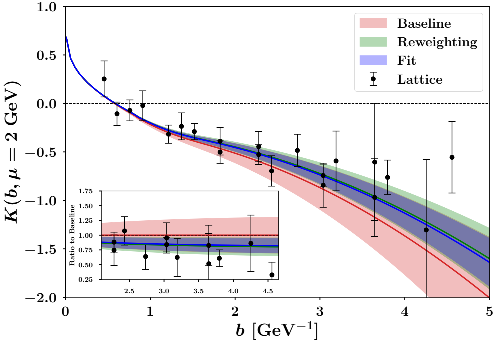
See also many CS/TMD talks today!

- Results are precise enough to discriminate between different models

Quark Collins-Soper kernel

- Joint analysis of experiment + lattice data for quark CS kernel

- Experimental data [482 pts] from full TMD fit by MAP, 2502.04166
- Lattice results [21 pts] from 3 ensembles, 2402.06725



Valerio Bertone | Chiara Bissolotti | Matteo Cerutti | Simone Rodini

in collaboration with MAP

Multi-dimensional Analyses of Partonic distributions

- Inclusion of lattice data can reduce the uncertainties by $\sim 40 - 50\%$

Gluon Collins-Soper kernel

What about gluon CS kernel?

- Experimentally:

lack of data for gluon TMDs. But can expect in the near future from EIC

- Theoretically:

- perturbative region: 4-loop calculation available and

$$\gamma_\zeta(\mu, b_T) = -\frac{\alpha_s}{\pi} \ln \frac{b_T^2 \mu^2}{4e^{-2\gamma_E}} \times \begin{cases} C_F, & \text{quark} \\ C_A, & \text{gluon} \end{cases} + \cdots$$

Casimir scaling holds through all known orders (C_A vs. C_F)

- non-perturbative region: nobody knows!

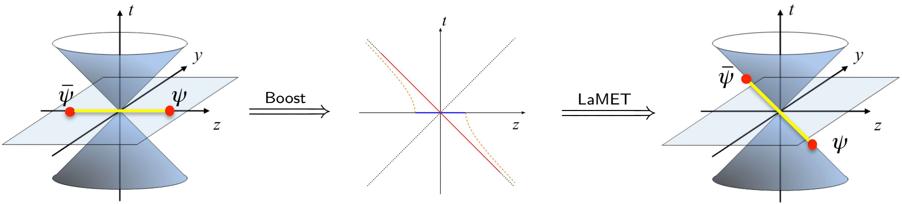
- This work: extend our calculation to the gluon CS kernel

Provides the first non-perturbative constraints for future experiments

- LQCD can not directly access parton physics defined on light-cone
- Large-Momentum Effective Theory (LaMET): Provides a framework to

link Euclidean equal-time correlation functions to light-cone one

X. Ji, PRL 110 (2013), SCPMA57 (2014)



Quasi: equal-time

Physical: light-cone

- Quasi distribution calculable on lattice, with same IR physics as light-cone

Differences in UV accounted for by perturbative matching

Quasi-TMDs

- Light-cone TMDs can be related to Quasi-TMDs via LaMET

Ebert, Schindler, Stewart & Zhao, JHEP 04, 178 (2022)

Quasi-TMDs | pert. matching | light-cone TMDs | CS kernel

$$\frac{\tilde{f}(x, b_{\mathrm{T}}, \mu, P^z)}{\sqrt{S_r(b_{\mathrm{T}}, \mu)}} = H(\mu, xP^z) f(x, b_{\mathrm{T}}, \mu, \zeta) \exp\left[\frac{1}{2}\gamma_\zeta(b_{\mathrm{T}}, \mu) \ln \frac{(2xP^z)^2}{\zeta}\right]$$

Soft factor

$$+ \mathcal{O}\left[\frac{1}{(xP^z b_T)^2}, \frac{M^2}{(xP^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(xP^z)^2} + (x \to 1-x)\right]$$

- CS kernel extracted from ratio with different momenta P_1 and P_2

$$\gamma_\zeta(b_T, \mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\tilde{f}(x, b_{\mathrm{T}}, \mu, P_1^z)}{\tilde{f}(x, b_{\mathrm{T}}, \mu, P_2^z)}\right] + \delta\gamma_\zeta(x, \mu, P_1^z, P_2^z) + \text{p.c.}$$

with $\delta\gamma_\zeta(x, \mu, P_1^z, P_2^z)$ computed from pert. matching kernel

- Power corrections need to be under control $\to$ x away from 0 and 1

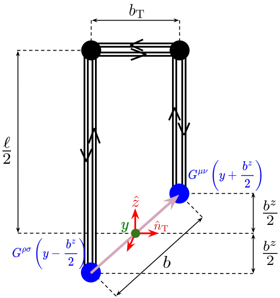
- Operators for the gluon quasi-TMDs

$$\mathcal{O}_g^{\mu\nu,\rho\sigma}(b) = G^{\mu\nu}\left(\frac{b}{2}\right) W_{\sqsupset}^{\text{adj}}(b,l) G^{\rho\sigma}\left(-\frac{b}{2}\right)$$

- For the unpolarized case, four operators are multiplicatively renormalizable

$$\text{Best StN} \rightarrow \; \mathcal{O}_g^{(1)} = \mathcal{O}_g^{0i,0i}, \quad \mathcal{O}_g^{(2)} = \mathcal{O}_g^{3i,3i}$$

$$\mathcal{O}_g^{(3)} = \frac{1}{2}(\mathcal{O}_g^{0i,3i} + \mathcal{O}_g^{3i,0i}), \quad \mathcal{O}_g^{(4)} = \mathcal{O}_g^{3\mu,3\mu}$$



Zhu et al, JHEP 02, 114 (2023)

$\Rightarrow$ renormalization cancels in the ratio

- Symmetry properties: by Hermiticity and translation invariance

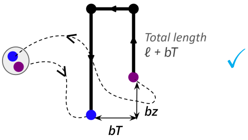
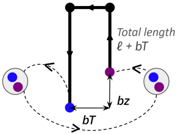
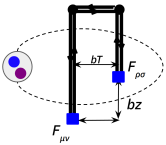
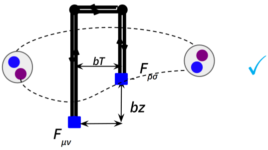
$$\mathcal{O}_g^{\mu\nu,\rho\nu}(b) = [\mathcal{O}_g^{\mu\nu,\rho\nu}(b)]^\dagger = \mathcal{O}_g^{\rho\nu,\mu\nu}(-b)$$

$\Rightarrow$ The matrix elements are real and symmetric under $b \rightarrow -b$

Quasi-TMDs

- Two observables can be used to compute the CS kernel on lattice

For quark and gluon we prefer to use different correlators

	quasi-TMD wavefunction	quasi-TMD beam function
quark	 less computational cost	 3pts need additional propagator
gluon	 quark disconnected diagram need self-to-self propagators	 correlating 2pts with gluon TMD almost the same cost as 2pts

Workflow

CS kernel from ratio:

$$\gamma_\zeta(b_T,\mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\int \frac{\mathrm{d}b^z}{2\pi}\, e^{ixP_1^z b^z} N(P_1^z) \lim_{\ell\to\infty} \tilde{B}(b^z, b_T, \ell, P_1^z)\tilde{\Delta}^S(b_T, l)}{\int \frac{\mathrm{d}b^z}{2\pi}\, e^{ixP_2^z b^z} N(P_2^z) \lim_{\ell\to\infty} \tilde{B}(b^z, b_T, \ell, P_2^z)\tilde{\Delta}^S(b_T, l)}\right] + \delta\gamma_\zeta(x,\mu,P_1^z,P_2^z) + \text{p.c.}$$

Quasi-soft factor $\tilde{\Delta}^S(b_T, l)$ is a Wilson loop to remove the linear divergence $\sim l + b_T$

1. Position-space MEs
   $\ell \to \infty$ extrapolation
2. x-space MEs
   integral range b^z_{cut}
3. Ratio of MEs
   with pert. matching

$\Longrightarrow$ CS kernel $\gamma_\zeta(b_T,\mu)$ ($x \sim 0.5$ with p.c. under control) $\Longrightarrow$ Repeat for each b_T

1-loop matching for gluon available in
Schindler et al., JHEP 08, 084 (2022)
Zhu et. al, JHEP 02, 114 (2023)

- Calculation carried out on a single MILC ensemble:

A. Bazavov et al. (MILC) PRD 87 (2013) 054505

$L^3 \times T = 32^3 \times 48$, $a = 0.15$ fm, $m_\pi = 170$ MeV

$N_{\text{meas}} \approx 6.5$ million

- CS kernel is universal — independent of hadronic state

Use pion state, suppressed power corrections $M^2/(xP^z)^2$

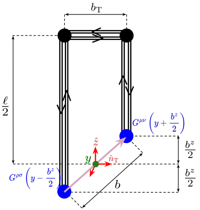
- Gluon TMD operator: $\mathcal{O}_g^{0i,0i}$ has the best signal

Staple length ℓ up to L/2 to control finite-ℓ effect

Boosted momentum $P^z = 1.03 - 2.05$ GeV

$(n^z = 4 - 8)$

Kinematically enhanced interpolating operators are used

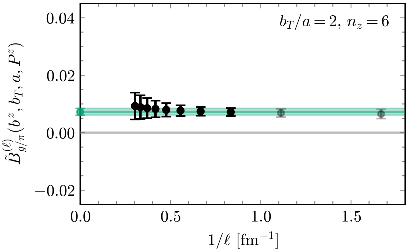


R. Zhang et al. PRD 112 (2025) L051502

- Position-space matrix elements:

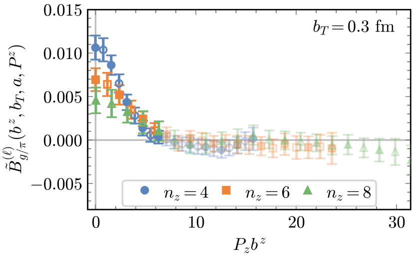
- ℓ-dependence

Mild with large enough P^z, constant fit is sufficient



- MEs as function of $P^z b^z$

Numerically real and symmetric after averaging over all orientations



- Most of the contribution is from the small $P^z b^z$ region

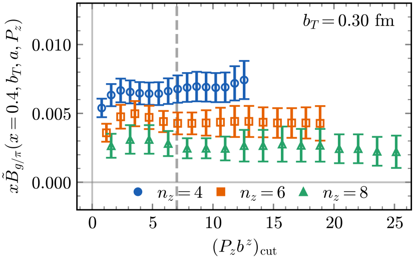
tails are not important within current precision (will show later)

- Fourier transform to x-space

$$x\tilde{B}(x, b_T, P^z) = \int \frac{\mathrm{d}b^z}{2\pi} e^{ixP^z b^z} N(P^z)\tilde{B}(b^z, b_T, P^z)$$

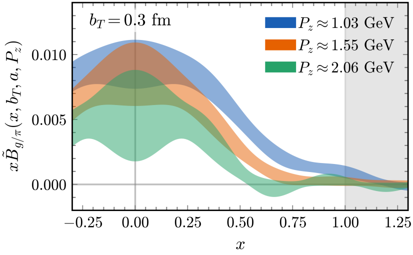
- Dependence on b^z_{cut}

Fourier transformation is saturated for $(P^z b^z)_{\text{cut}} \gtrsim 7$



- MEs as a function of x

tails outside physical range $x \in [-1, 1]$ are reduced as P^z increases

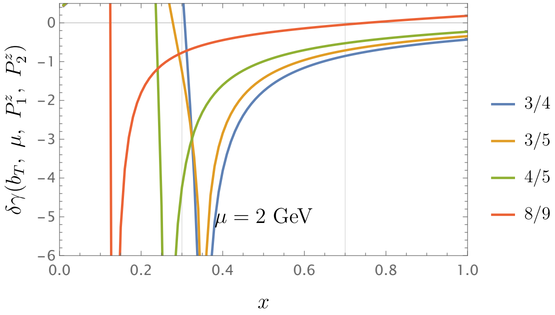


LaMET matching

- Matching for the gluon TMD was a problem at LaMET 2025

NLO $H_g = 1 + \frac{\alpha_s C_A}{2\pi} \times (\cdots)$ crossing zero near $x \sim 0.5$ with small P^z

Schindler et al., JHEP 08, 084 (2022)
Zhu et. al, JHEP 02, 114 (2023)



- Need larger momentum (noisy) and higher-order matching

Next-to-next-leading log matching

- Both the b_T-independent and "unexpanded" NNLL matching are derived

- Standard matching needs $P^z b_T \gg 1$, only the b_T-independent part kept

- But when $P^z b_T \sim 1$, matching is b_T-dependent and convolutional in x

$$\tilde{B}(x, b_T, P^z, \mu) = \int_0^1 \mathrm{d}y \; H_\phi(y, x, b_T, P^z, \mu) B(y, \mu)$$

at large P^z, can isolate the b_T-dependent multiplicative part

$$H_\phi(y, x, b_T, P^z, \mu) \xrightarrow{P^z b_T \gg 1} \delta(y-x) H_\phi^{\text{unexp}}(xP^z, b_T, \mu) + \cancel{\delta H_\phi(y, x, b_T, P^z, \mu)}$$

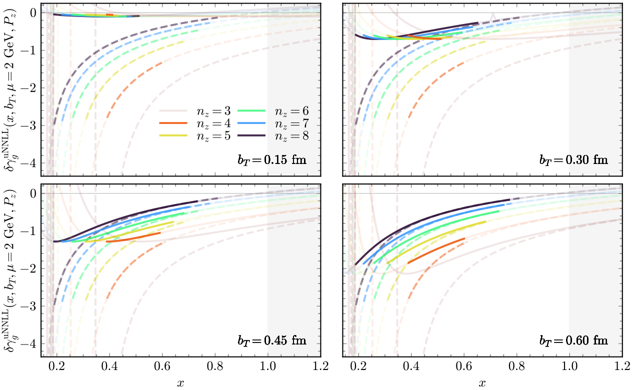
$\rightarrow$ convolutional piece dropped

then define the "unexpanded" matching

- This idea was used in our previous quark CS kernel calculation (2307.12359)

it reduced power correction in the small b_T region

Next-to-next-leading log matching



- Solid: uNNLL. Dashed: NNLL
- No zero-crossing singularity near $x \sim 0.5$
- Unexpanded matching partially mitigates b_T-dependent power corrections

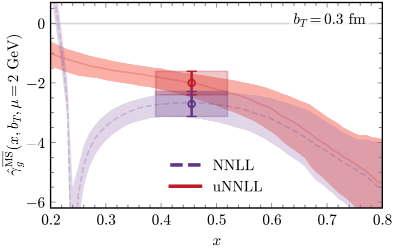
Full control will need convolutional matching

Gluon CS kernel

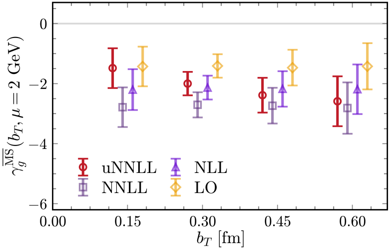
- CS kernel extracted by fitting

$$\tilde{B}_g\,(x, b_T, P_z)\,/H_g(x, b_T, \mu, P_z) = c(x, b_T)\exp\left[\gamma_g(x, b_T, \mu)\ln P_z\right]$$

- fitted γ_g as a function of x



- different PT matching



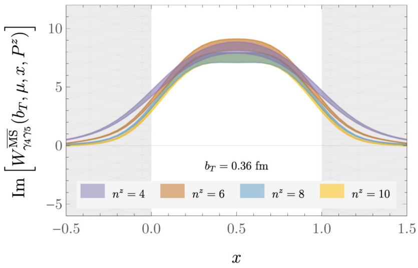
- Large b_T: consistent between different matchings
  Small b_T: power corrections → suppressed by unexp. matching

- Nonsymmetric x range supported by both the data and theory

(more on following slides)

- More on the x range: different symmetries for (quasi) TMD

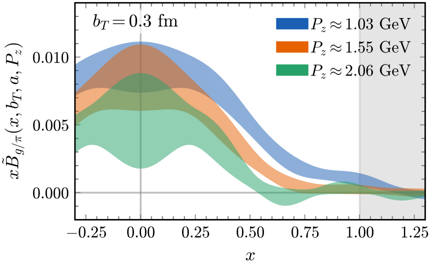
- Quark (with pion state)



Symmetric under $x \to 1 - x$

momentum fraction of two quarks

- Gluon



Symmetric under $x \to -x$

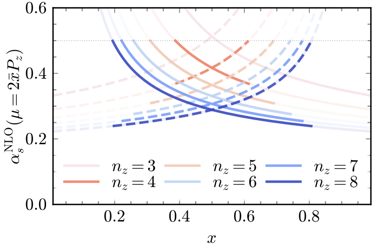
gluons are their own anti-particles

- Correspondingly, the $(1 - x)$ corrections for gluon TMD do not show up in the matching, but are expected nonperturbatively as spectator effects

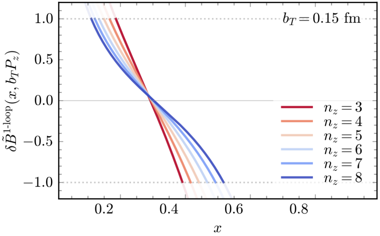
x range

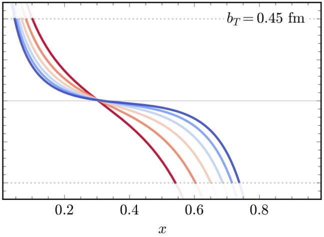
- x range is chosen to satisfy

- Strong coupling $\alpha_s(\mu) < 0.5$ for $\mu = 2xP^z$, $2(1-x)P^z$



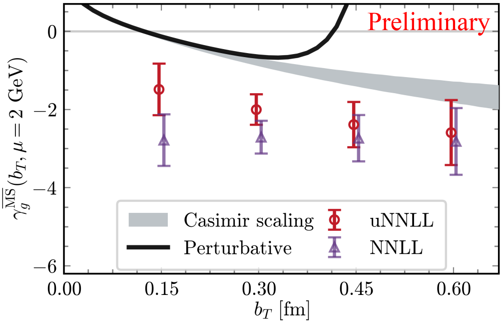
- Dominant b_T-dependent enhancement $|\delta\tilde{B}^{\text{1-loop}}(x, b_T P_z)| < 1$ (1901.03685)





- Our result of the gluon CS kernel

Compared with Casimir scaling of our quark kernel result $(C_A/C_F)\gamma_q(b_T,\mu)$



- First proof-of-principle calculation, systematics not fully quantified (!)

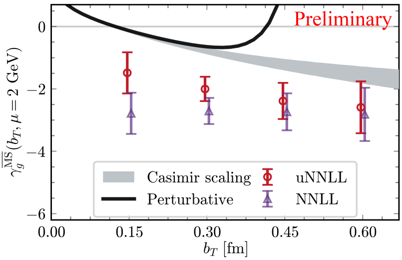
including lattice artifacts and residual power corrections (enhanced at small b_T)

- Is uNNLL good enough? Improving the matching would be important

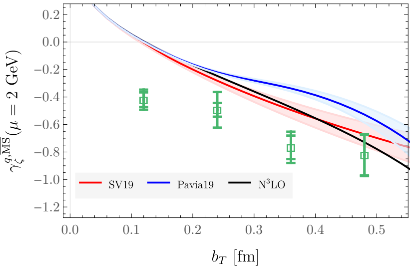
Gluon CS kernel

- Compare to quark CS kernel ∼ 5 years ago

- Gluon CS kernel



- Quark ∼ 5 years ago 2107.11930



- close-to-physical $m_\pi \approx 170$ MeV ✓
- NNLL matching
- ∼6M measurements !

- unphysical $m_\pi \approx 500$ MeV
- NLO matching
- ∼16K measurements

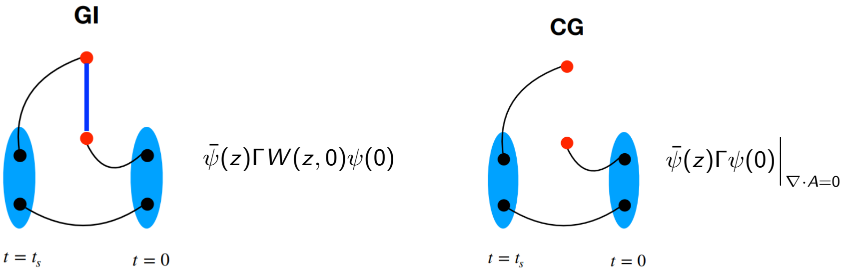
⇒ Similar stage, but gluon has significant statistics challenge

CG method

- Coulomb gauge method: eliminates the need for Wilson lines.

$\Rightarrow$ same universality class / IR physics in LaMET framework

Y. Zhao, 2311.01391



Many talks on CG method in this workshop!

Q. Shi, W. Good, R. Zhang, F. Yao, Y. Zhao, J. Lin, X. Gao, · · ·

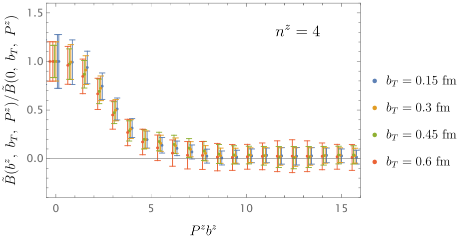
- Several advantages:

- Expect less gauge noise and less signal-to-noise degradation
- Use same quark propagators, almost free to calculate both
- TMD operator: staple-shaped Wilson lines with infinite extension

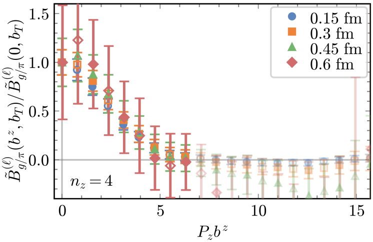
$\Rightarrow$ Absence of the Wilson line provides much convenience in computation

Comparison of the CG and GI methods:

- CG with $\sim$ 600K meas (prelim.)



- GI with $\sim$ 6M meas



- Much milder signal-to-noise degradation!
- For large b_T, $2 - 3\times$ smaller errors with $\sim 10\times$ less stats

$\Rightarrow$ Errors reduced by a factor of ~ 10, large b_T will be accessible

Gluon CS kernel:

- In contrast to quark TMDs, gluon TMDs are almost unknown
  (both experimentally and lattice QCD)

- We performed the first LQCD calculation of gluon CS kernel up to 0.6 fm
  systematics not fully controlled but result comparable with Casimir scaling

- Next steps: smaller lattice spacing to control discretization error,
  also Coulomb gauge method to reduce the statistical noise

Thank you!