Brookhaven National Laboratory

One-loop matching for Coulomb-gauge quasi-GPDs

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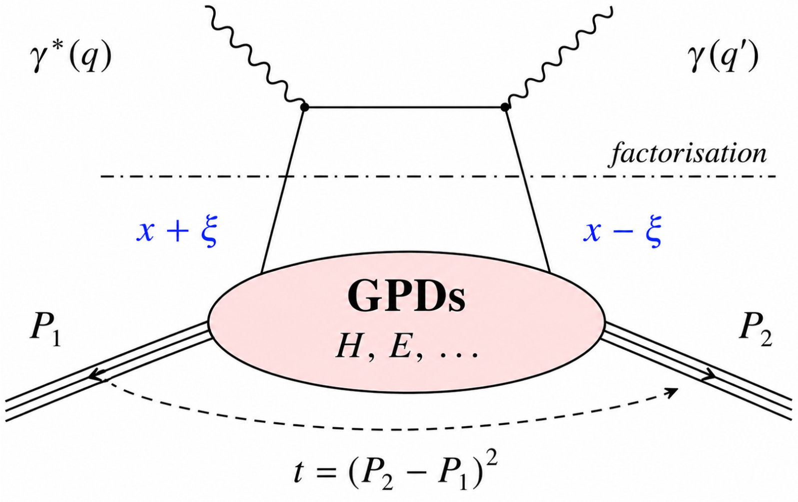
Introduction

GPDs describe the quark and gluon structure of hadrons
in terms of three kinematic variables



x : longitudinal quark momentum fraction

ξ : skewness (longitudinal momentum transfer)

t : momentum transfer to the hadron

GPDs depend on three variables:
$H(x,\xi,t),\ E(x,\xi,t),\ \ldots$

In appropriate limits, GPDs reduce to familiar quantities:

- $\xi=0,\ t=0$ ➔ PDFs: $H(x,0,0) = q(x)$
- $\int dx\, H(x,\xi,t)$ ➔ **Form factors $F(t)$**

➔ **A first-principles lattice determination** requires **quasi-GPDs** and **perturbative matching**.

Why consider Coulomb-gauge quasi operators?

A complementary realization of LaMET without an explicit Wilson line

Gauge-invariant quasi

$$\tilde{O}^{\mathrm{GI}}_{\Gamma}(z) = \bar{\psi}\left(-\frac{z}{2}\right)\Gamma L\left(-\frac{z}{2},\frac{z}{2}\right)\psi\left(\frac{z}{2}\right)$$

- Explicit Wilson line along the spatial direction
- Wilson-line contributions in perturbation theory
- Wilson-line renormalization

Coulomb-gauge quasi

$$\tilde{O}^{\mathrm{CG}}_{\Gamma}(z) = \bar{\psi}\left(-\frac{z}{2}\right)\Gamma\psi\left(\frac{z}{2}\right)\Big|_{\nabla\cdot\mathbf{A}=0}$$

- Avoids Wilson-line power **divergence / renormalon** issue
- No explicit Wilson-line diagrams
- Simpler multiplicative renormalization

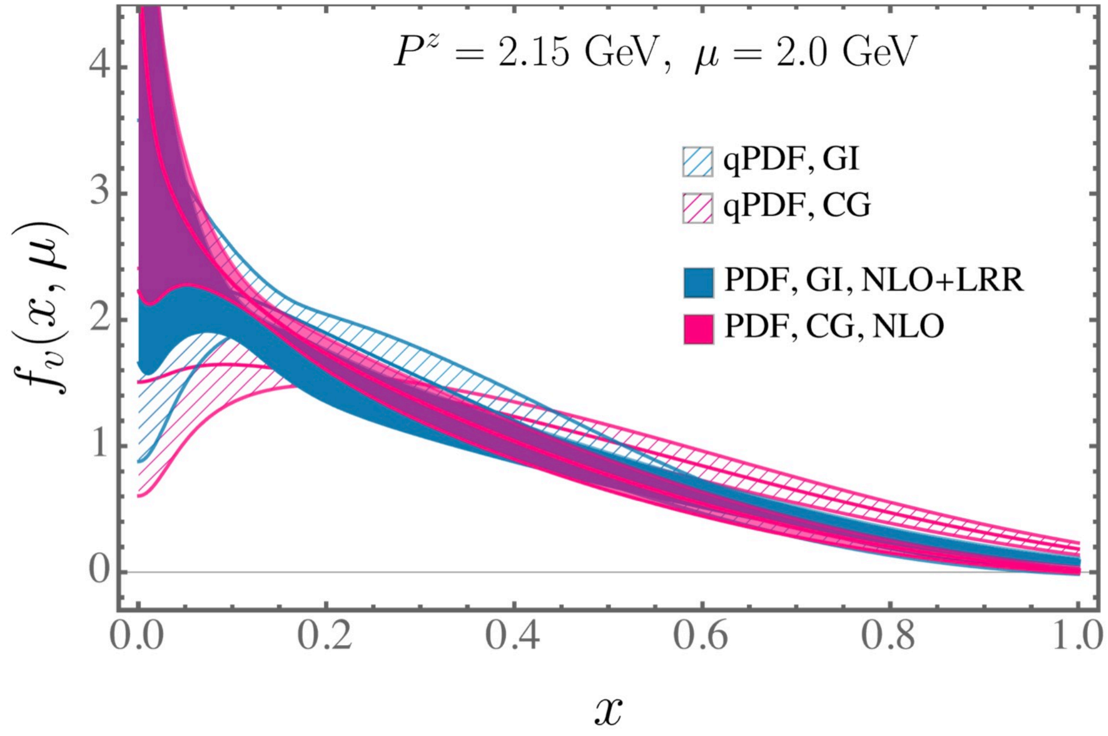
$$\bar{\psi}_0(z)\Gamma\psi_0(0) = Z^R_{\psi}(a)\left[\bar{\psi}_R(z)\Gamma\psi_R(0)\right].$$

Different quasi operators → Same infrared behavior → One-loop matching → The same light-cone GPD

Current status of the Coulomb-gauge program

Pion quark-PDFs: lattice results available

- ✓ One-loop matching established for quark PDFs.
- ✓ Lattice extractions have been carried out.
- ✓ CG and GI extractions give consistent pion valence PDFs.

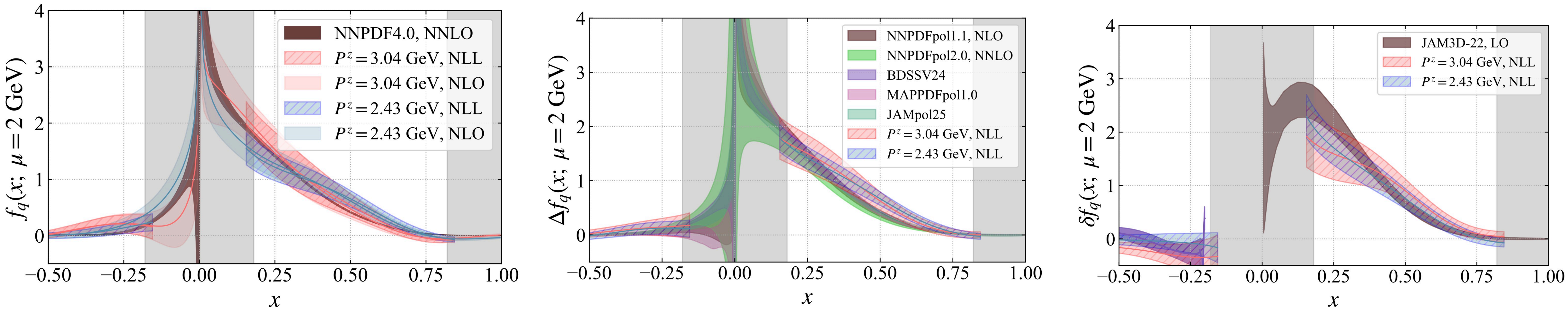


X. Gao, W. Y. Liu and Y. Zhao, PRD 109 (2024)

CG quasi-PDF ➡ validated for quark PDFs

Toward a broader Coulomb-gauge collinear program

- Recent progress includes nucleon unpolarized, helicity, and transversity PDFs.



X. Gao et al., ArXiv: 2602.11283

- The nucleon unpolarized PDF at the physical point. ***See Qi's talk***
- Gluon PDF in Coulomb-gauge. ***See W. Good's talk***

GPD/DA matching in this work

Theoretical framework

* Taking the unpolarized quark-GPDs as an example, it is defined as

$$F(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-}\langle P_2, S_2 | \bar{\psi}\Big(-\frac{z^-}{2}\Big)\gamma^+ L\Big(-\frac{z^-}{2},\frac{z^-}{2}\Big)\psi\Big(\frac{z^-}{2}\Big) | P_1, S_1\rangle$$

$$= \frac{1}{2P^+}\bar{u}(P_2,S_2)\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E(x,\xi,t)\right]u(P_1,S_1)$$

$P = \frac{P_1+P_2}{2}$: average hadron momentum

$\Delta = P_2 - P_1$: momentum transfer

$t = \Delta^2$: momentum transfer squared

$\xi = -\frac{\Delta^+}{2P^+}$: skewness

* In Coulomb-gauge, we study the equal time correlators

$$\tilde{F}_\Gamma(x,\xi,t) = \frac{1}{2}\int \frac{dz}{2\pi} e^{ixP^z z}\langle P_2, S_2 | \bar{\psi}(-z/2)\,\Gamma\,\psi(z/2) | P_1, S_1\rangle, \quad \text{with} \quad \Gamma = \gamma^z, \gamma^t, \gamma^z\gamma_5, \gamma^t\gamma_5, i\sigma^{z\perp}$$

Goal: relate the Coulomb-gauge quasi-GPD to the universal light-cone GPD

Theory strategy: coordinate space and momentum space

- We compute the Coulomb-gauge quasi-GPD matching in two complementary languages

Short-distance factorization (SDF)

Quasi-LF correlation

$$\tilde{h}^R(z,\lambda,\xi,t) = \int_0^1 d\alpha\, d\beta\, C(\alpha,\beta,\mu_z^2 z^2)\, h(\alpha,\beta,\lambda,\xi,t,\mu) + h.t., \qquad \lambda = zP_z\,.$$

Factorization in LaMET

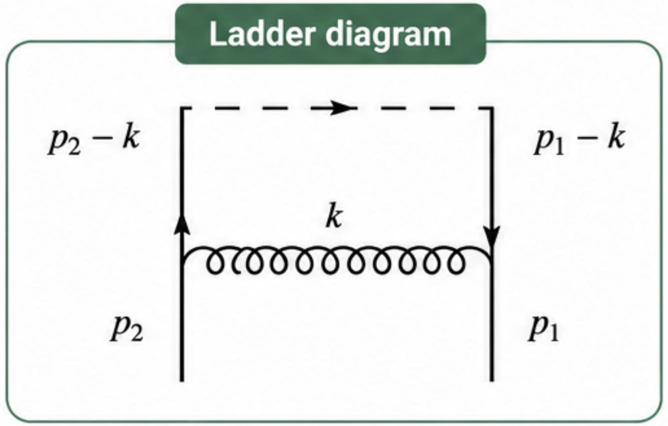
Quasi-GPD

$$\tilde{F}(x,\xi,t,P_z) = \int dy\, C(x,y,\xi,t,\mu/P_z)\, F(y,\xi,t,\mu) + h.t.$$

Consistency checks: PDF limit ($\xi \to 0$) $\longleftrightarrow$ DA limit from GPD kinematics

	Momentum-space matching	Coordinate-space matching
PDFs	✓ Available for one-loop unpolarized / helicity / transversity *X. Gao et al., ArXiv: 2602.11283*	■ Requires clarification Establish in $[-\infty,\infty]$ region *X. Gao, W. Y. Liu and Y. Zhao, PRD 109 (2024)*
GPDs	★ **This work** unpolarized / helicity / transversity	★ **This work** Matching kernels for SDF
DAs	★ From GPDs	★ Same with GPDs

Perturbative calculation in momentum space



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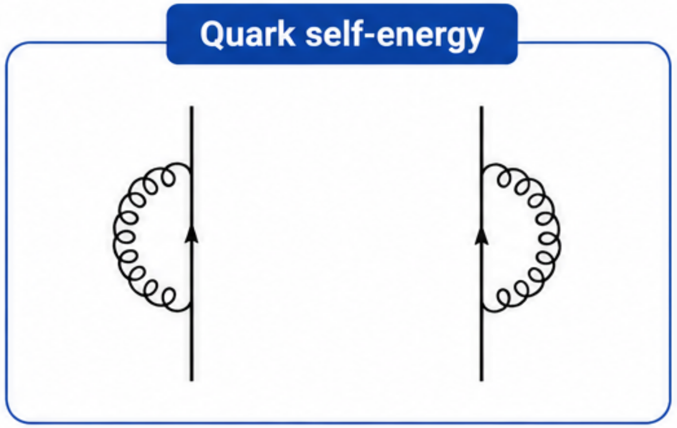


$p_{1z} = x_1 P_z = (x + \xi) P_z$

$p_{2z} = x_2 P_z = (x - \xi) P_z$

One-loop ladder contribution

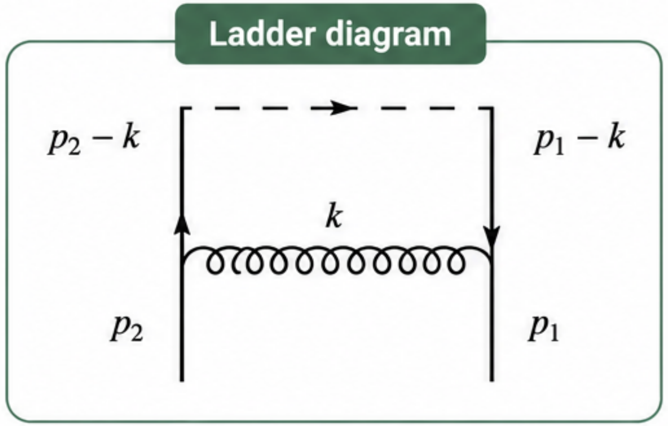
$$\tilde{f}_\Gamma^{(1)}\Big|_{\rm ladder} = ig^2 C_F \int \frac{d^d k}{(2\pi)^d} \frac{\bar{u}(p_1)\gamma^\mu (\not{p}_1 - \not{k})\,\Gamma\,(\not{p}_2 - \not{k})\gamma^\nu u(p_2)}{(p_1 - k)^2\, k^2\, (p_2 - k)^2} \mathcal{G}_{\mu\nu}(k)\, \delta[(x_1 - y_1)P^z - k^z]\,,$$

$\Gamma = \gamma^z, \gamma^t, \gamma^z\gamma_5, \gamma^t\gamma_5, i\sigma^{z\perp}$

Coulomb-gauge gluon propagator (numerator)

$$\mathcal{G}_{\mu\nu}(k) = \underbrace{-g_{\mu\nu}}_{\text{Contribution in GI case}} + \underbrace{\frac{(n\cdot k)\,(k_\mu n_\nu + k_\nu n_\mu) - k_\mu k_\nu}{-k^2 + n^2\,(n\cdot k)^2}}_{\text{Complicated part in this calculation}}\,.$$

Perturbative calculation in momentum space



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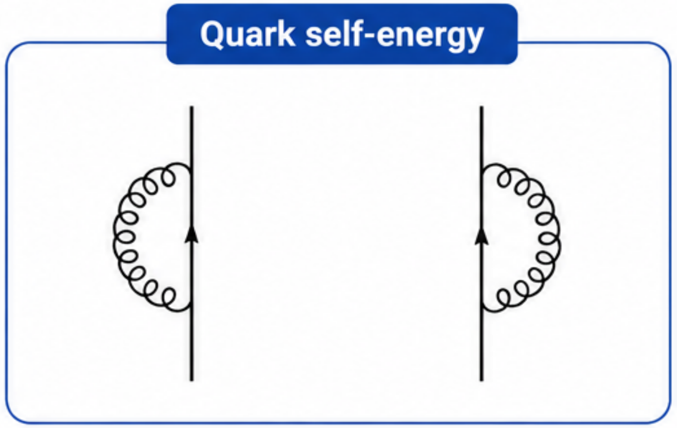


$p_{1z} = x_1 P_z = (x + \xi) P_z$

$p_{2z} = x_2 P_z = (x - \xi) P_z$

$$\delta Z_\psi^{(1)}$$

$$\tilde{f}_\Gamma^{(1)}\Big|_{\text{self-energy}} = -\frac{\alpha_s C_F}{4\pi}\left[\left(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}\right) - 2\left(\frac{1}{\epsilon_{IR}^2} + \frac{3}{\epsilon_{IR}} + \frac{1}{\epsilon_{IR}}\ln\frac{\mu^2}{4p_z^2} + 12 - \frac{7\pi^2}{12} + 3\ln\frac{\mu^2}{4p_z^2} + \frac{1}{2}\ln^2\frac{\mu^2}{4p_z^2}\right)\right]\delta(x-y)$$

$$\tilde{f}_\Gamma^{\overline{\text{MS}}} = \delta(x-y) + \tilde{f}_\Gamma^{(1)}\Big|_{\text{ladder}} + \tilde{f}_\Gamma^{(1)}\Big|_{\text{self-energy}}$$

Should yield a plus prescription

Plus prescription and IR cancellation

- We reconstruct the plus prescription by using the coordinate-space limit.

1. Kernel

$\tilde{f}^{(1)}(x) = \tilde{f}^{(1)}_{\text{ladd.}}(x) + \tilde{f}^{(1)}_{\text{self}}(x)$

$\tilde{h}^{(1)}(z, p_z) = \int dx\, e^{i x z p_z} \tilde{f}^{(1)}(x)$

$z \to 0 \;\Rightarrow\; \tilde{h}^{(1)}(0, P_z) = \int dx\, \tilde{f}^{(1)}(x)$

$z \to 0$ ⟶

2. Coordinate-space check

$\tilde{h}(z=0, P_z) = \tilde{h}(z, P_z = 0)$

$\int dx \left[\tilde{f}^{(1)}_{\text{ladd.}}(x) + \tilde{f}^{(1)}_{\text{self}}(x)\right]$

= zero-mementum matrix

Subtract ⟶

3. Plus-prescription

FT of $\tilde{h}(z, P_z = 0)$ is well-known.

$\tilde{f}^{(1)}_{\text{ratio}}(x) = \tilde{f}^{(1)}(x) - \tilde{f}^{(1)}(x, p_z = 0)$

is a complete plus.

Therefore, $\tilde{f}^{(1)}_{\text{ladd.}}(x) + \tilde{f}^{(1)}_{\text{self}}(x)$, after subtracting the zero-mementum matrix elements, form a complete plus!

✓ The IR/evolution kernel of the quasi-GPD matches the light-cone one.

Matching kernel in momentum space

* Ratio-scheme results:

$$\mathbb{C}^{(1)}_{\text{ratio},\Gamma}\left(x_1,x_2,y_1,y_2;\frac{\mu}{P^z}\right)=\frac{\alpha_s C_F}{2\pi}\left\{\left(A_{1,\Gamma}\frac{|x_1-y_1|}{4x_1x_2}+\frac{1}{2|x_1-y_1|}\frac{y_1}{x_1}\right)L_{x_1y_1}+\left(A_{1,\Gamma}\frac{|y_1|}{2(x_1-x_2)x_1}+\frac{|y_1|}{2(x_1-y_1)x_1}\right)L_{y_1}-\frac{3}{4|x_1-y_1|}\right.$$

$$\left.-A_{2,\Gamma}\left(\frac{|x_1-y_1|}{2x_1x_2}+\frac{|y_1|}{2(x_1-x_2)x_1}\right)+\frac{x_1-3y_1}{2(x_1-y_1)}\frac{\tan^{-1}\left(\sqrt{x_1(x_1-2y_1)/y_1^2}\right)}{\sqrt{x_1(x_1-2y_1)}}+(x_1\leftrightarrow x_2,y_1\leftrightarrow y_2)\right\}_+$$

where

$$A_{1,\gamma^0/\gamma^0\gamma_5}=1,\qquad A_{1,\gamma^z/\gamma^z\gamma_5}=1,\qquad A_{1,\sigma^{z\perp}}=0$$

$$A_{2,\gamma^0/\gamma^0\gamma_5}=1,\qquad A_{2,\gamma^0/\gamma^z\gamma_5}=-1,\qquad A_{2,\sigma^{z\perp}}=0$$

* Hybrid-scheme results:

$$\mathbb{C}^{(1)}_{hyp.,\Gamma}\left(x_1,x_2,y_1,y_2;z_s,\frac{\mu}{P^z}\right)=\mathbb{C}^{(1)}_{\text{ratio},\Gamma}\left(x_1,x_2,y_1,y_2;\frac{P^z}{\mu}\right)-A_{1,\Gamma}\left[\frac{\text{Si}\left[(x_1-y_1)z_sP^z\right]}{\pi(x_1-y_1)}-\frac{1}{2|x_1-y_1|}\right]_+^{(-\infty,\infty)}$$

Known limits of matching coefficients

Our general result: Coulomb-gauge quasi GPD matching kernel

$$\mathbb{C}\left(x_1, x_2, y_1, y_2;\, \mu/P_z\right)$$

Forward limit (PDF)

$\xi \to 0$

$$\mathbb{K}\left(x, y;\, \frac{\mu}{P_z}\right) = \mathbb{C}\left(x, x, y, y;\, \frac{\mu}{P_z}\right)$$

The kernel reduces to the Coulomb-gauge quasi-PDF matching kernel and **agrees** with previous work.

[X. Gao et al., ArXiv: 2602.11283]

Distribution-amplitude (DA) limit

Crossing limit ($\xi \to 1$)

$$\mathbb{V}\left(x, y;\, \frac{\mu}{P_z}\right) = \mathbb{C}\left(x, -\bar{x}, y, -\bar{y};\, \frac{\mu}{P_z}\right)$$

We obtain the DA matching kernel in Coulomb gauge.

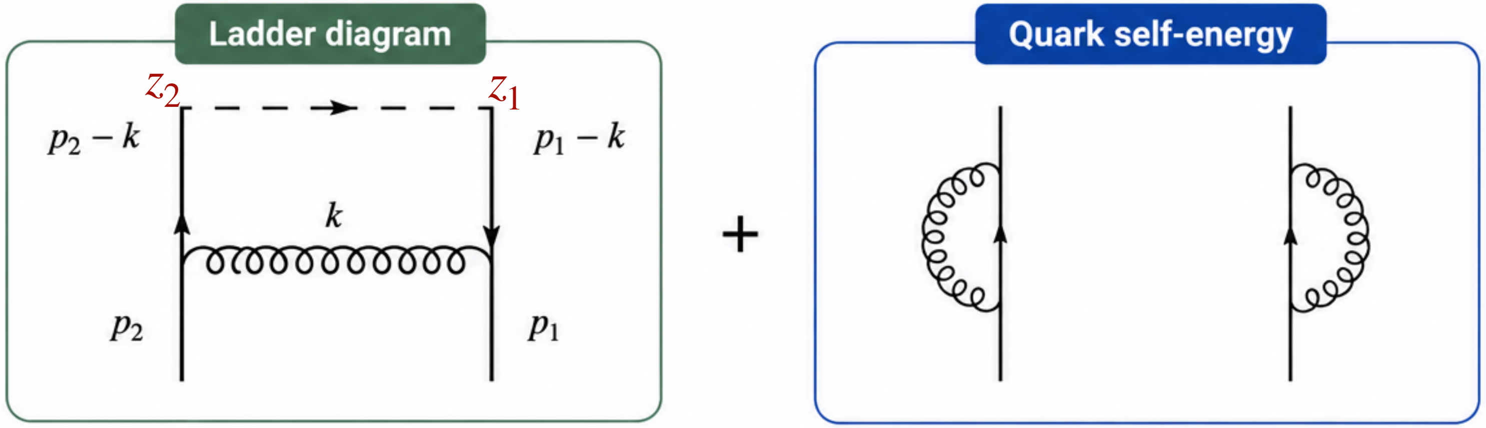
This is a new result.

New result

Our general kernel **reproduces** the known PDF limit and **obtains** the DA limit in Coulomb gauge.

Perturbative calculation in coordinate space

- Same diagrams as in momentum space, but keep the coordinate-space phase factor explicitly.



$$\tilde{h}_\Gamma^{(1)}\Big|_{\text{ladder}} = ig^2 C_F \int \frac{d^d k}{(2\pi)^d} \frac{\bar{u}(p_1)\gamma^\mu(\not{p}_1 - \not{k})\,\Gamma\,(\not{p}_2 - \not{k})\gamma^\nu u(p_2)}{(p_1-k)^2\, k^2\, (p_2-k)^2}\, \mathcal{G}_{\mu\nu}(k)\; e^{-i(p_1\cdot z_1 - p_2\cdot z_2 - z_{12}\cdot k)}$$

$$= \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta\, C_{\text{ladd.}}(\alpha,\beta,\mu^2 z^2, z p_{1z}, z p_{2z})\, e^{-i(p_1\cdot z_{12}^\alpha - p_2\cdot z_{21}^\beta)}, \qquad z_{12}^\alpha = \bar{\alpha} z_1 + \alpha z_2$$

Coordinate-space calculation

- The noncovariant Coulomb-gauge propagator turns standard coordinate-space loop integrals into **nontrivial Bessel-K kernels**.

standard covariant case: $$\int \frac{d^d\ell}{(2\pi)^d}\frac{e^{i\ell\cdot z}}{[\ell^2]^n} = \frac{(-1)^{-n} i}{4^{n-d/2}(4\pi)^{d/2}}\frac{\Gamma(d/2-n)}{\Gamma(n)}(-z^2)^{n-d/2}.$$

CG-induced integral: $$\int \frac{d^d\ell}{(2\pi)^d}\frac{e^{i\ell\cdot z}}{[\ell^2+\Omega^2]^n} = \frac{(-1)^{-n}2^{1+d/2-n} i(-\Omega^2)^{d/4-n/2}(-z^2)^{n/2-d/4}}{(4\pi)^{d/2}\Gamma(n)}K_{n-d/2}\left(\sqrt{-\Omega^2}\sqrt{-z^2}\right)$$

zp_{1z}, zp_{2z} **dependence**

$\vec{k}^2$ in the CG propagator $\Rightarrow$ nonstandard Fourier integrals $\Rightarrow$ zp_z dependence.

Explicit zP_z dependence in the coordinate-space result

- Ratio-scheme results:

$$C^{(1)}_{\text{ratio},\Gamma}(\alpha,\beta,z^2\mu^2) = \frac{\alpha_s C_F}{2\pi}\Bigg[-\left(B_{1,\Gamma}+\frac{\alpha}{\bar\alpha}\delta(\bar\beta)\right)L_z - \frac{2\ln(\bar\alpha)}{\bar\alpha}\delta(\bar\beta) + B_{2,\Gamma} + \left(\frac{3}{2\sqrt{\bar\alpha}}-\frac{2}{\bar\alpha}\right)K_0\left(zp_{1z}\sqrt{\alpha\bar\alpha}\right)$$

$$-\frac{2}{\bar\alpha}\mathcal{K}_{\text{rem}}(\bar\alpha, zp_{1z}) + (\alpha\leftrightarrow\beta, p_{1z}\leftrightarrow p_{2z})\Bigg]_+$$

$zp_{1z} = x_1\, zP_z$

where

$$B_{1,\gamma^0/\gamma^0\gamma_5} = 1/2,\qquad B_{1,\gamma^z/\gamma^z\gamma_5} = 1/2,\qquad B_{1,\sigma^{z\perp}} = 0$$

$$B_{2,\gamma^0/\gamma^0\gamma_5} = 1/2,\qquad B_{2,\gamma^z/\gamma^z\gamma_5} = 3/2,\qquad B_{2,\sigma^{z\perp}} = 0$$

And

$$\mathcal{K}_{\text{rem}}(\alpha, zp_z) = \int_0^\infty dx\ \cos(zp_z\, x)\left[\frac{1-\alpha}{x+\sqrt{\alpha x^2+\alpha^2\bar\alpha}} - \frac{1-\sqrt{\alpha}}{\sqrt{x^2+\alpha\bar\alpha}}\right].$$

match with momentum-space result after FT.

Moment expansion in Coulomb-gauge

- Standard local OPE would suggest:

$$\mathcal{M}(z,p_z) = \sum_{n=0}^{\infty} \frac{(izp_z)^n}{n!} C_n(\mu^2 z^2) \langle x^n \rangle + \ldots$$

with C_n independent of the external momentum.

- In Coulomb-gauge, the coordinate space coefficient contains $K_0\left(z\, p_z \sqrt{\alpha\, \bar{\alpha}}\right)$, which generates non-analytic terms as $(z\, p_z)^n \ln(\mathrm{z\, p_z})$.

The moment expansion is not a standard local-OPE expansion.

$$C_n^{\text{moment}} = \left[\langle x^n \rangle_{\text{quasi}} - \langle x^n \rangle_{\text{LC}} \right]_{\text{IR cancel}}$$

$\Rightarrow$ $C_n^{\text{moment}} = C_n(\mu^2 z^2, \mu^2/p_z^2)$, a finite-$p_z$ LaMET-type matching coefficient.

Summary & Outlook

Summary

- ☑ **One-loop matching** for Coulomb-gauge quasi-GPDs is established for unpolarized, helicity and transversity cases.

- ☑ The **general kernels** reproduce the known **quasi-PDF limit** and give the Coulomb-gauge **quasi-DA** matching kernel.

Outlook

- Apply the matching kernel to lattice Coulomb-gauge quasi-GPD calculations.
- Clarify the coordinate-space SDF and finite-zP_z moment interpretation.

Thank you!