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The Chinese University of Hong Kong, Shenzhen

LaMET 2026



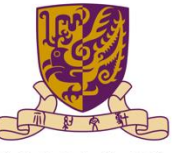
Gluon Helicity PDF in Continuum Limit from Lattice QCD

Speaker: Shi-Yi Zhong

2026 July 6th

In collaboration with Jian-Hui Zhang, Peng Sun,
Liu-Ming Liu, Yi-Bo Yang et al.

Background

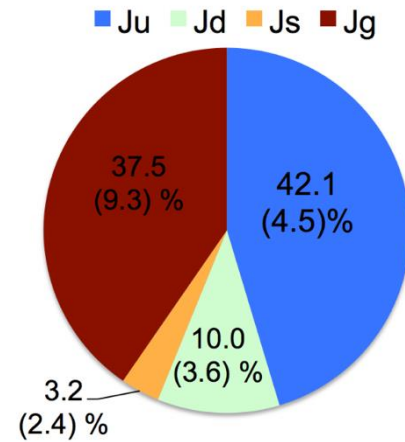


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Proton spin decomposition

Ji Decomposition

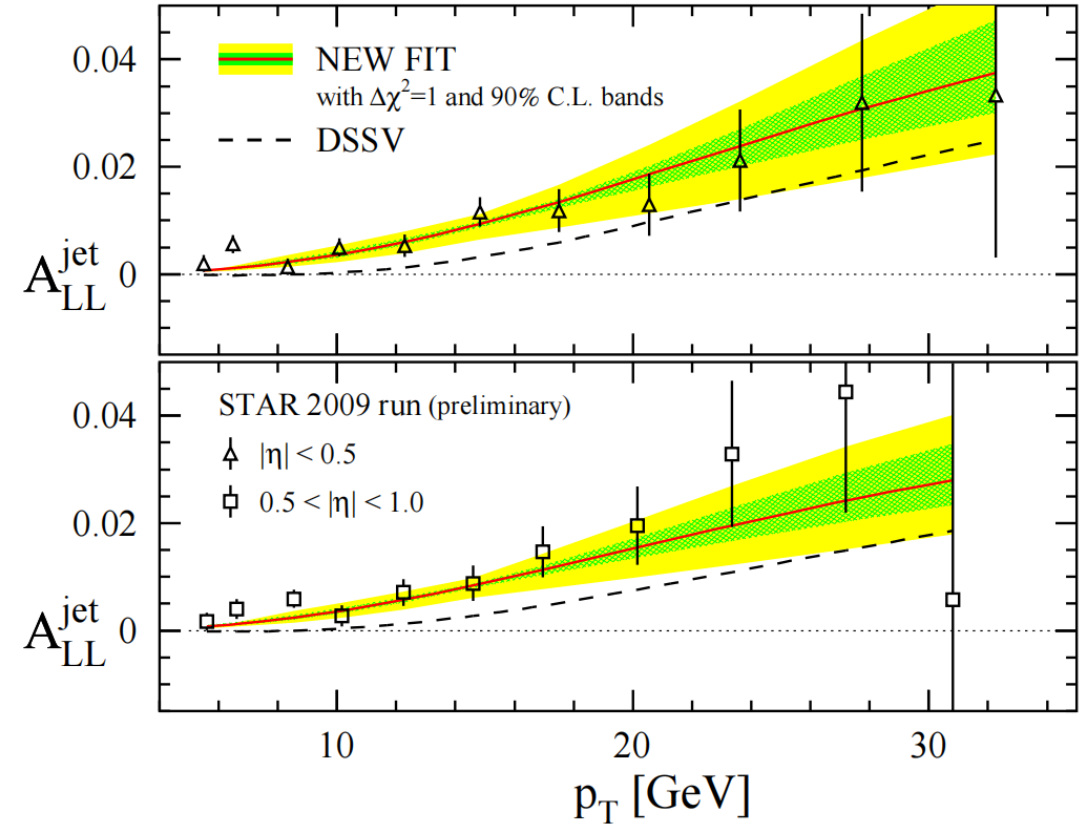
$$\frac{1}{2}\Delta\Sigma + L_q^z + J_g = \frac{\hbar}{2}$$



Jaffe-Manohar decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

Double-spin asymmetry

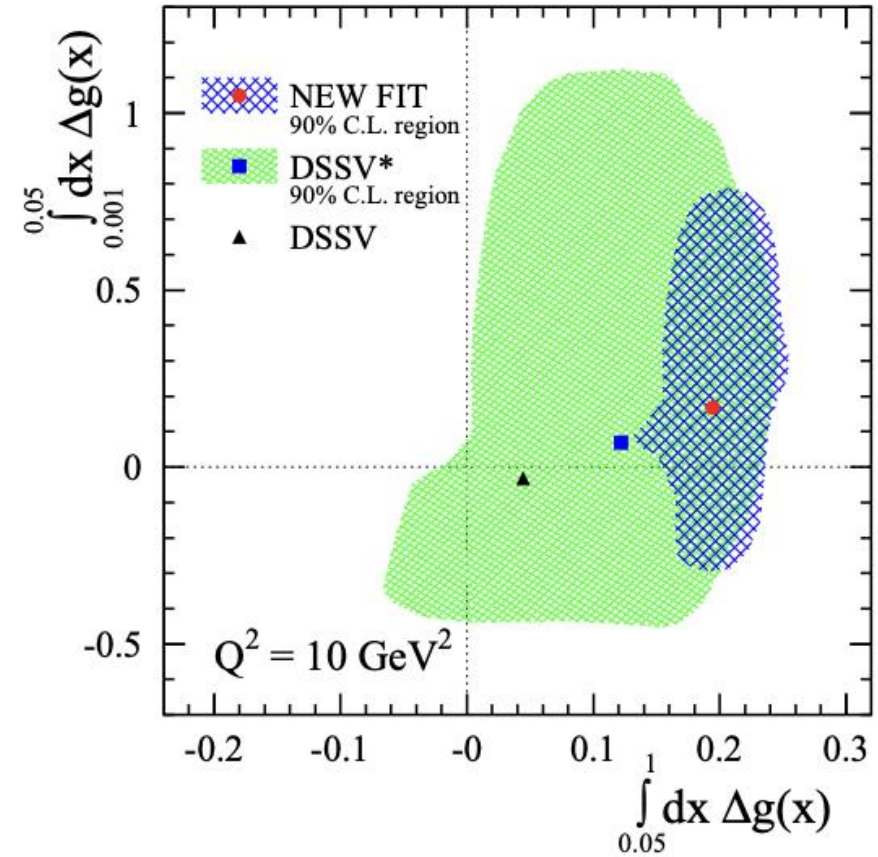
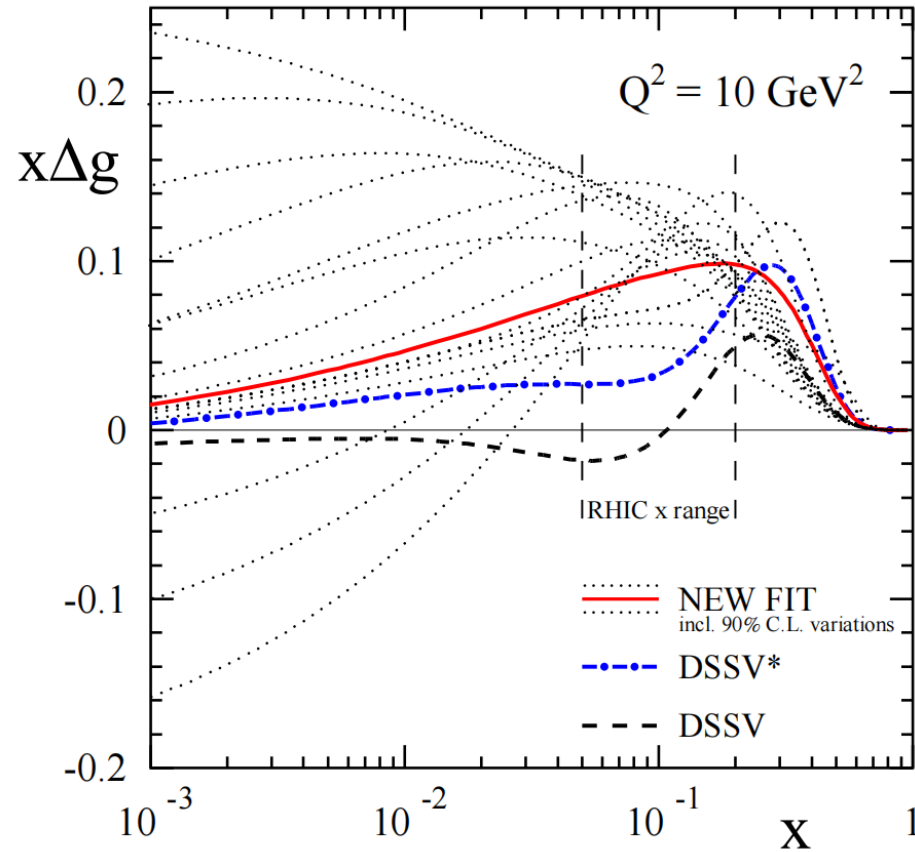


C. Alexandrou, et al. *Phys. Rev. D* 101, 094513, **2020**.
D. de Florian, et al., *Phys. Rev. Lett.* 113, 012001 (**2014**).

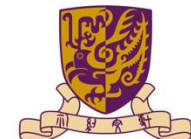


Background

Gluon helicity distribution from global fit

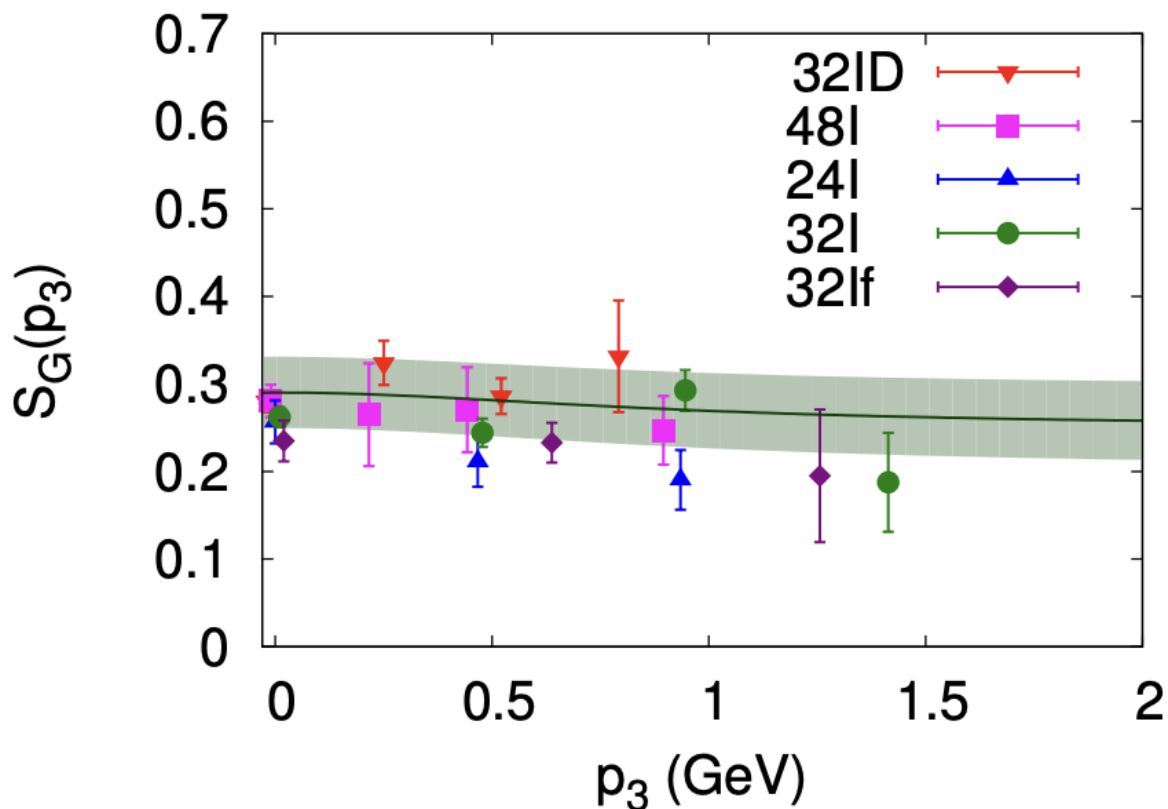


Previous Work



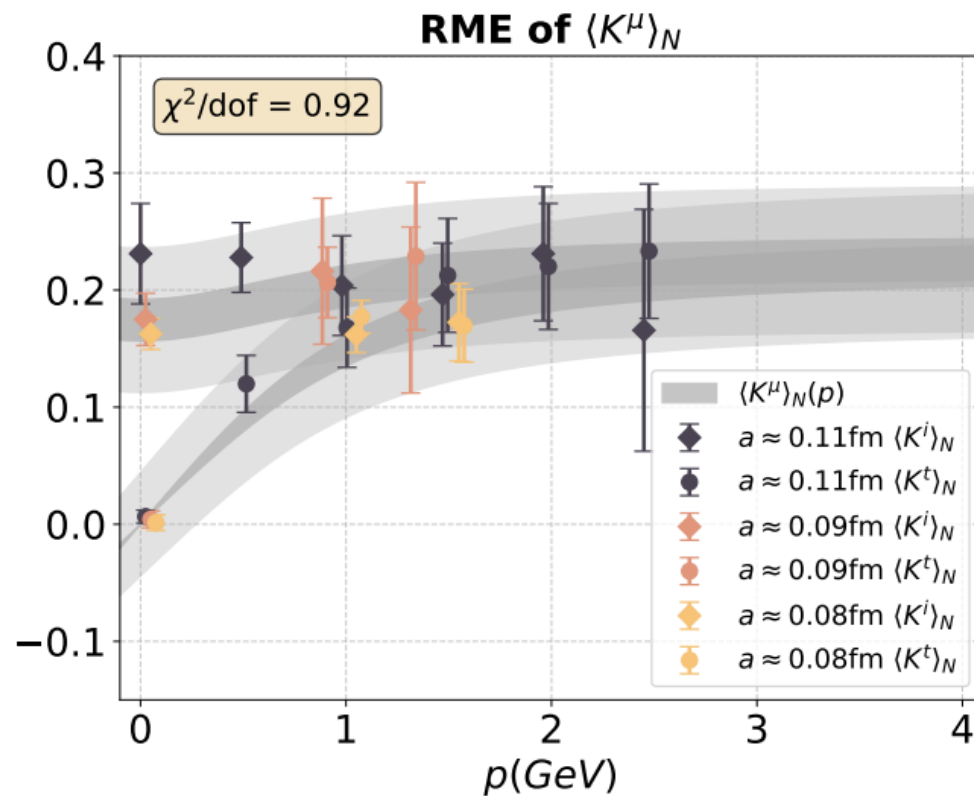
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Total gluon spin



$$\Delta G \propto \langle \vec{E} \times \vec{A} \rangle_{\text{C.G.}} = S_G|_{\text{IMF}} = 0.251(47)^{\text{sta.}} (16)^{\text{sym.}}$$

Y.-B. Yang, et al., Phys. Rev. Lett. 118, 102001 (2017).



$$\Delta G^{\overline{\text{MS}}}, 10^{-3} \text{ GeV}^2 = 0.231(17)^{\text{sta.}} (44)^{\text{sym.}}$$

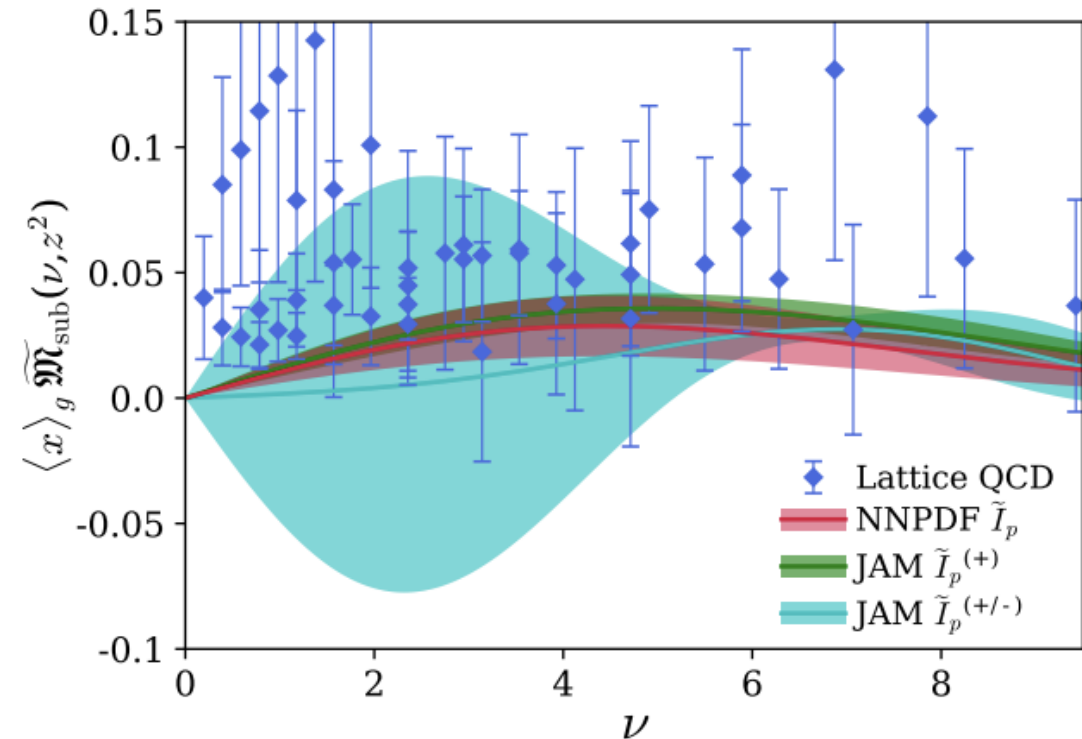
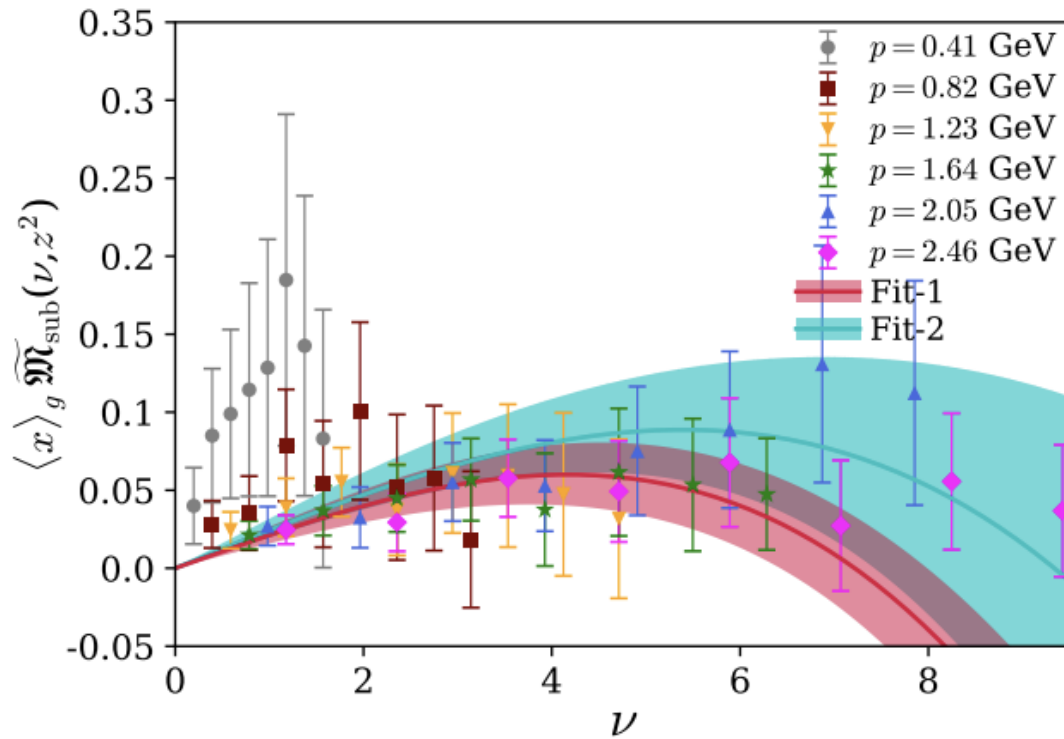
D.-J. Zhao, et al. arXiv preprint arXiv:2512.24315 (2025).

Previous Work



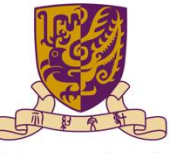
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loffe-time pseudo-distribution



ID	a (fm)	m_π (MeV)	$L^3 \times N_t$	N_{cfg}	N_{srcs}
<i>a094m358</i>	0.094(1)	358(3)	$32^3 \times 64$	1901	64

Gluon Helicity PDF



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Quasi PDF

$$\tilde{h}_B(z, P_z, 1/a) = \langle PS | F^{0i} W(z, 0) \tilde{F}_{0i} + F^{ij} W(z, 0) \tilde{F}_{ij} - \{(z \leftrightarrow -z)\} | PS \rangle$$

Gluon helicity PDF

$$\Delta\tilde{g}(x, P_z, 1/a) = \frac{i}{2xP_z} \int \frac{dz}{2\pi} e^{ixzP_z} \tilde{h}(z, P_z, 1/a)$$

Total gluon helicity


$$\Delta\tilde{G}(P_z, 1/a) \equiv \int dx \Delta\tilde{g}(x, P_z, 1/a) = \frac{1}{2P_z} \int_0^\infty dz \tilde{h}(z, P_z, 1/a).$$

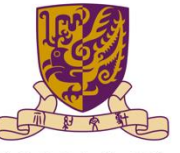
Balitsky, et al., *J. High Energ. Phys.* 193 (2022).

Pang, Z., Yao, F. & Zhang, JH. *J. High Energ. Phys.* 222 (2024).

Lattice Setup



 CLQCD	Sea Type	$N_L^3 \times N_T$	Lattice Spacing (fm)	$m_\pi^{(s)}$ (MeV)	Valence Type	Ncfg	Nsource
C32P29	Clover (2+1) + TITLS	$32^3 \times 64$	0.1052	292.4(1.1)	Clover	980	64
E32P29		$32^3 \times 64$	0.0897	287.3(2.5)		1700	64
F32P30		$32^3 \times 96$	0.0775	300.4(1.2)		800	96



Bare Matrix

Feynman-Hellmann method

2pt
$$C_2(t_f; \mathcal{H}) = \left\langle \sum_{\vec{y}} \mathcal{H}(t_f, \vec{y}) \sum_{\vec{x}} \mathcal{H}^\dagger(0, \vec{x}) \right\rangle$$

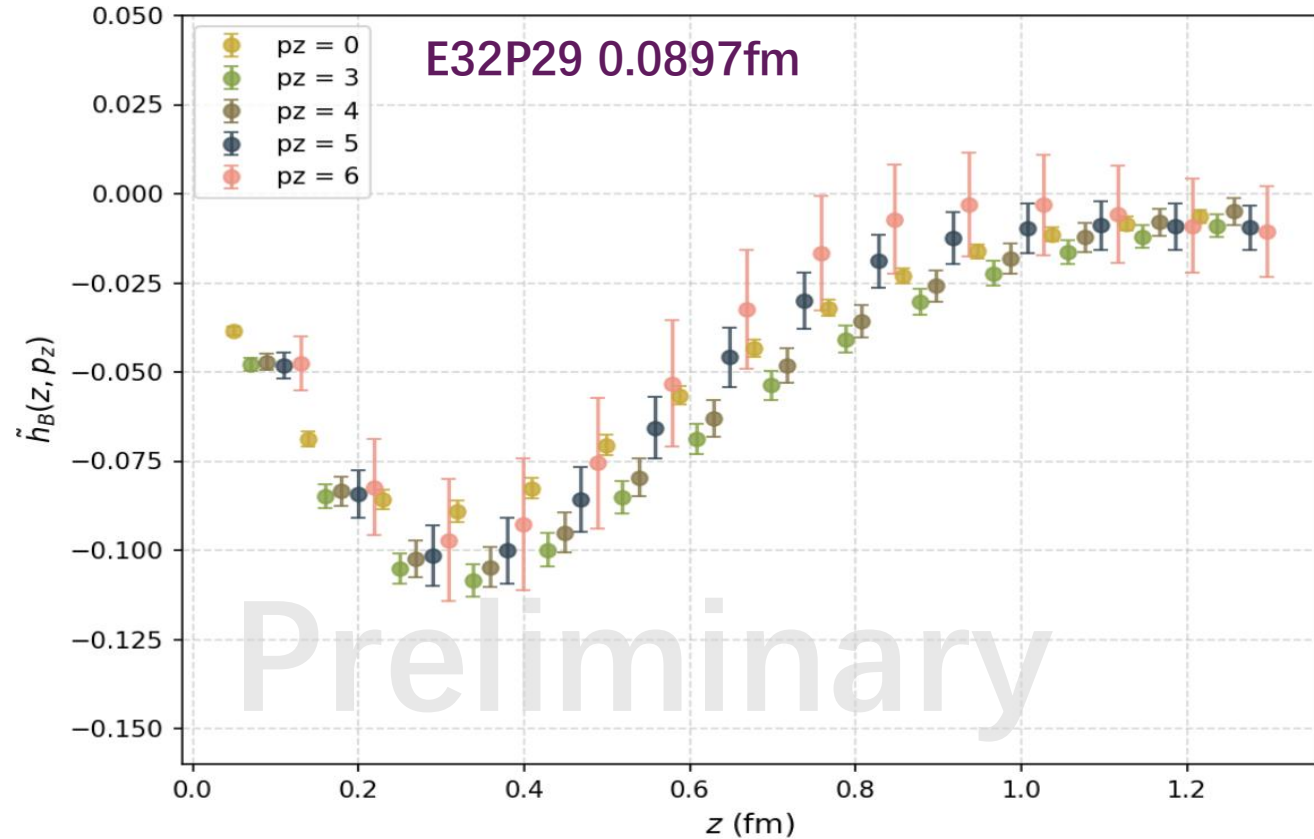
3pt
$$SC_3(t_f; \mathcal{H}) = \left\langle \sum_{\vec{y}} \mathcal{H}(t_f, \vec{y}) \sum_{t \in (t_f, 0), \vec{z}} O(t, \vec{z}) \sum_{\vec{x}} \mathcal{H}^\dagger(0, \vec{x}) \right\rangle$$

Bare matrix

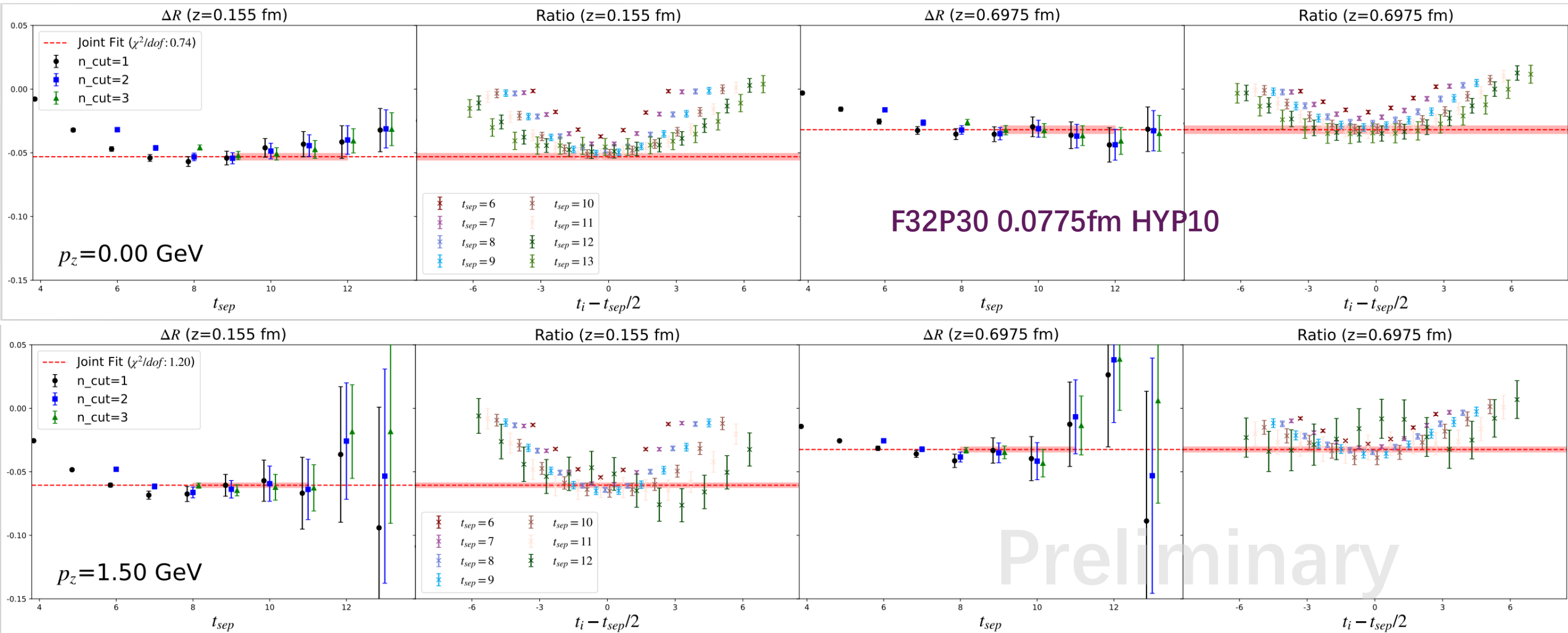
$$\tilde{h}_B(z, p_z) = \langle O \rangle_H = \Delta R(t_f) + \mathcal{O}(e^{-\delta m t_f})$$

$$\Delta R(t_f) \equiv \frac{SC_3(t_f; H)}{C_2(t_f; H)} - \frac{SC_3(t_f - 1; H)}{C_2(t_f - 1; H)}$$

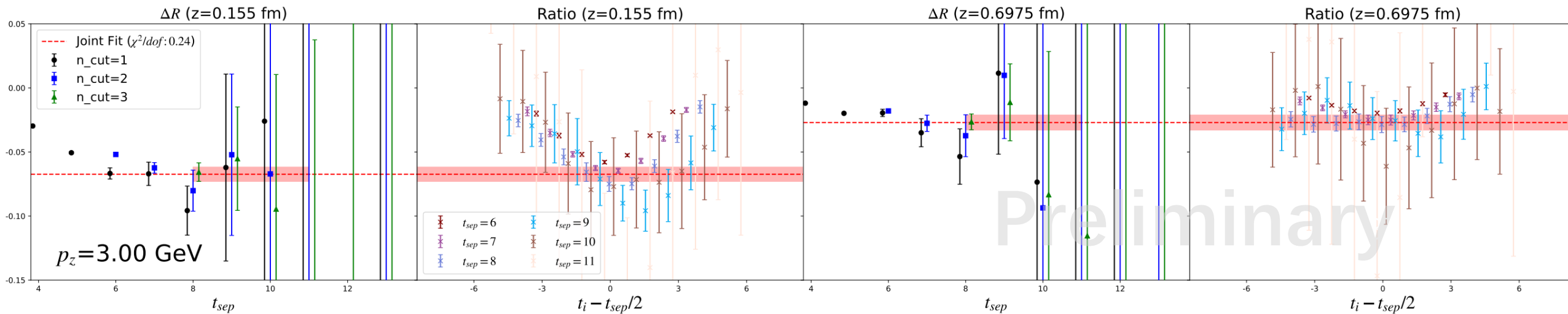
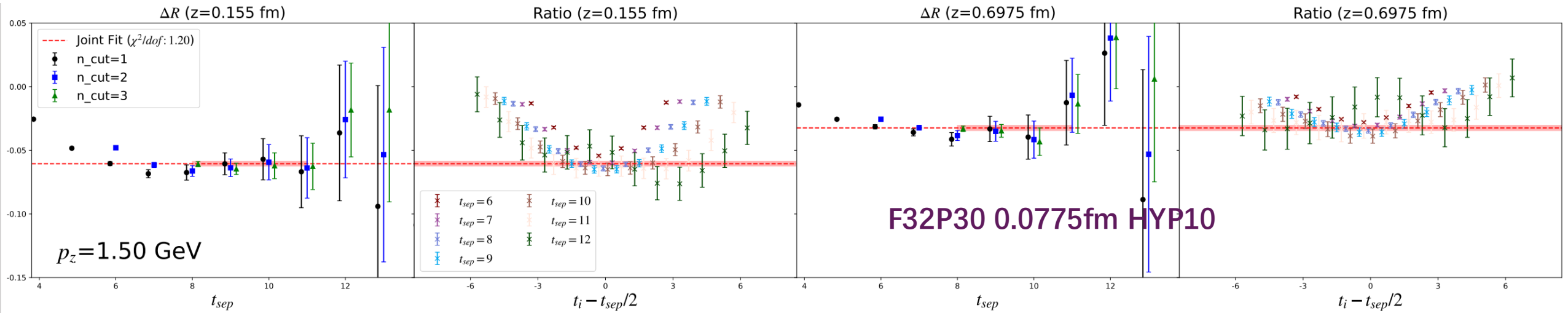
Bare matrix



Bare Matrix



Bare Matrix



Self Renormalization



Bare matrix

Symmetrization
$$\tilde{h}_{\text{sym}}(z, P_z, 1/a) = \frac{\tilde{h}_B(z, P_z, 1/a)/z}{[\tilde{h}_B(z, P_z, 1/a)/z]_{z \rightarrow 0}}$$

Renormalization

$$\ln \tilde{h}_{\text{sym}}(z, P_z = 0, 1/a) = \ln [Z_R(z, 1/a, \mu; k, \Lambda_{\text{QCD}}, m_0, d)]$$

$$+ \begin{cases} \ln [\tilde{h}_{\text{sym}, \overline{\text{MS}}}^{\text{pert}}(z, \mu)] & \text{if } z_0 \leq z \leq z_1 \\ g(z) - m_0 z & \text{if } z_1 < z \end{cases}$$

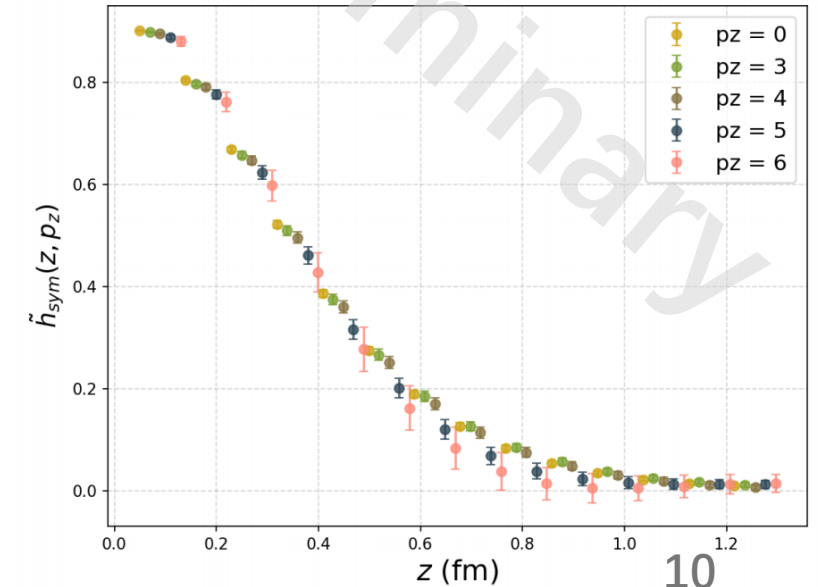
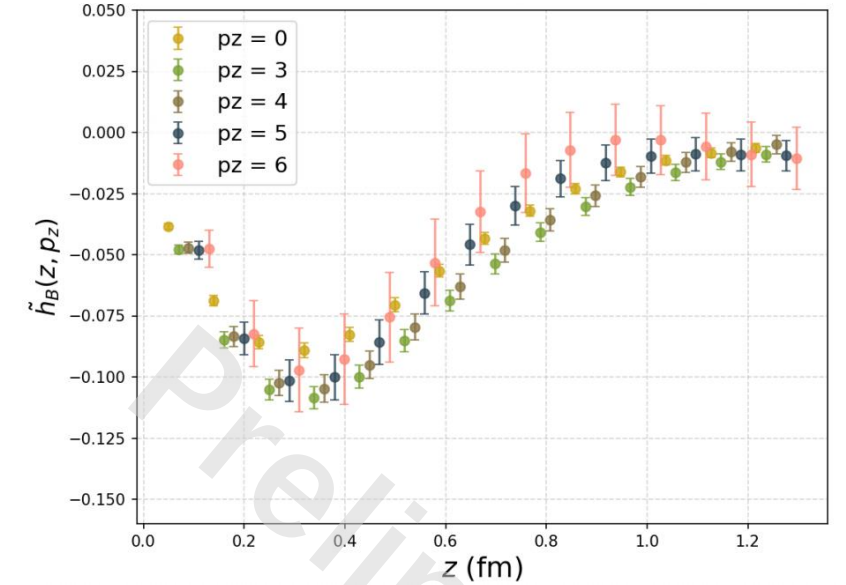
$$\tilde{h}_{\text{sym}, \overline{\text{MS}}}^{\text{pert}}(z, \mu) = 1 + \frac{\alpha_s(\mu) C_A}{4\pi} \left(\frac{19}{6} \ln \frac{z^2 \mu^2}{4e^{-2\gamma_E}} - \frac{5}{6} \right)$$

$$Z_R(z, 1/a, \mu; k, \Lambda_{\text{QCD}}, m_0, d) = \exp \left\{ \frac{kz}{a \ln[a\Lambda_{\text{QCD}}]} + m_0 z + \frac{14C_A}{3b_0} \ln \left[\frac{\ln(1/a\Lambda_{\text{QCD}})}{\ln(\mu/\Lambda_{\text{QCD}})} \right] + \frac{1}{2} \ln \left[\left(1 + \frac{d}{\ln[a\Lambda_{\text{QCD}}]} \right)^2 \right] \right\}$$

Yao, F., Ji, Y. & Zhang, JH. *J. High Energ. Phys.* 21 (2023).

Ji et al. *Nucl. Phys. B*, 964, 115311 (2021),

Deng et al. In progress.

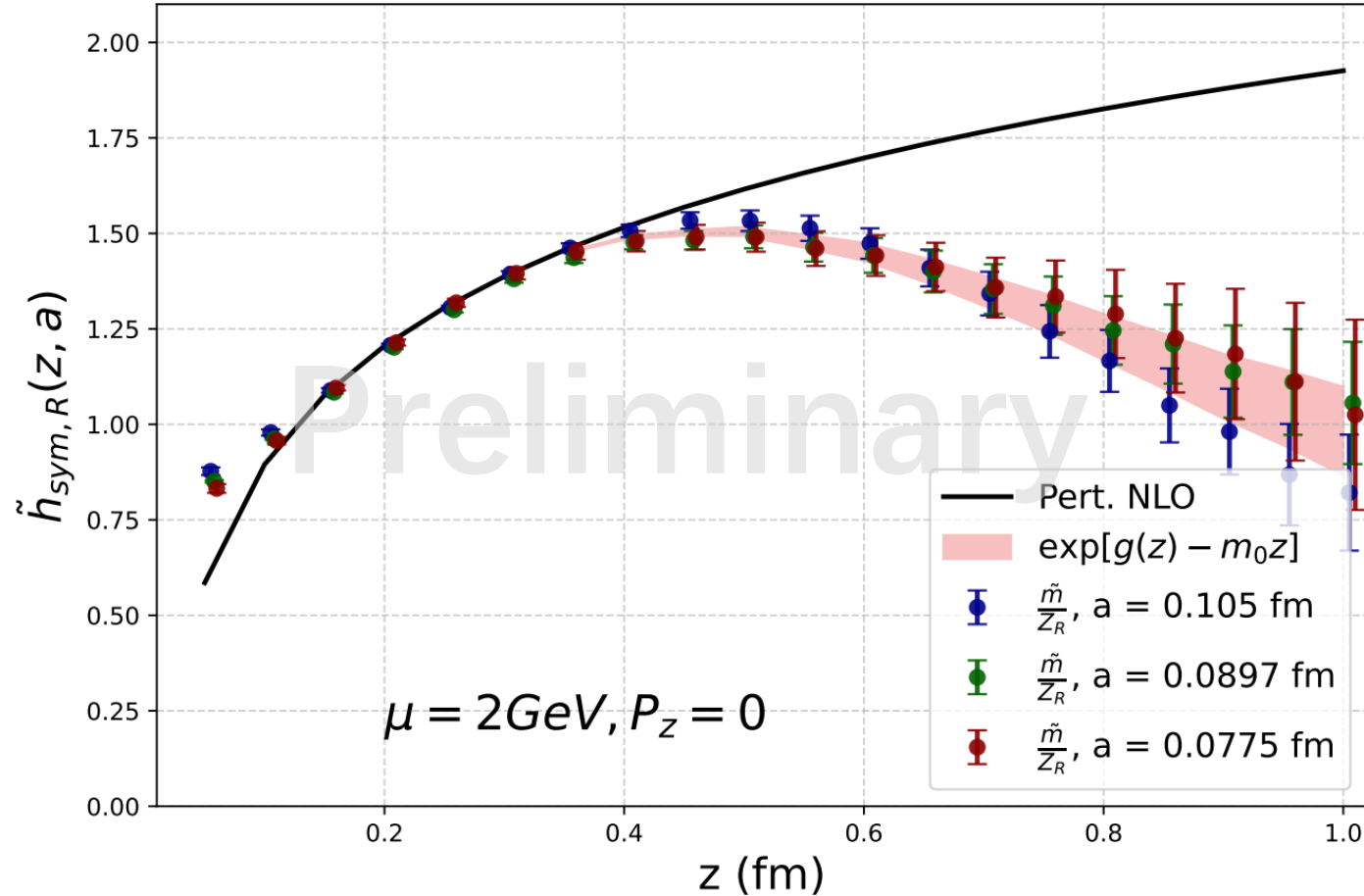


Renormalization



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Renormalized Matrix



$$\mu = 2\text{GeV}$$

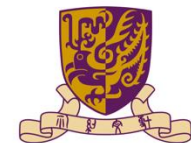
$$\alpha_s(\mu = 2\text{GeV}) = 0.296$$

k	1.61(0.24)
$\Lambda_{\text{QCD}}(\text{GeV})$	0.032(0.043)
$m_0(\text{GeV})$	-0.0086(0.23)
d	-1.66(0.79)
$\chi^2/\text{d.o.f.}$	0.57

$$\tilde{h}_{\text{sym}}(z, P_z, 1/a) = \frac{\tilde{h}_B(z, P_z, 1/a)/z}{[\tilde{h}_B(z, P_z, 1/a)/z]_{z \rightarrow 0}}$$

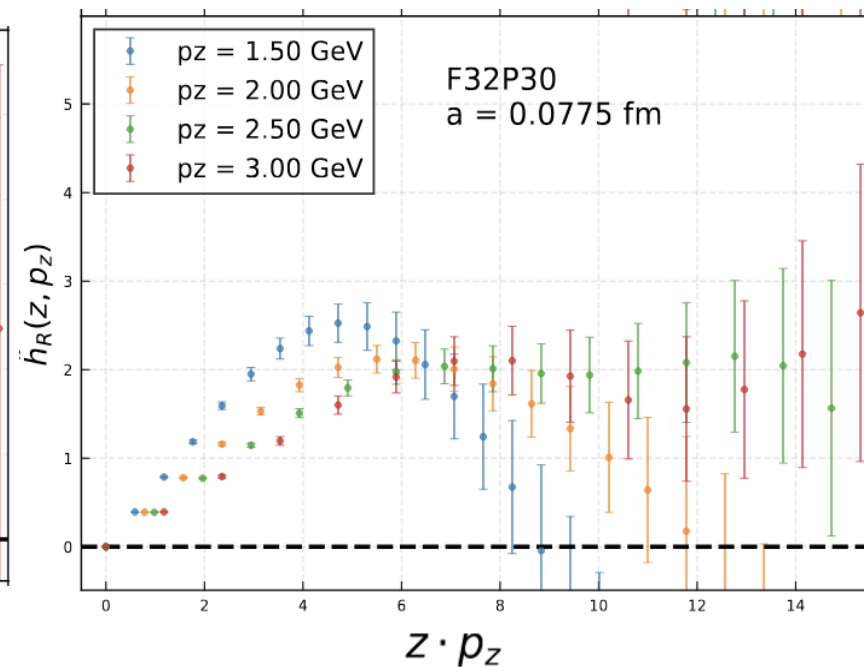
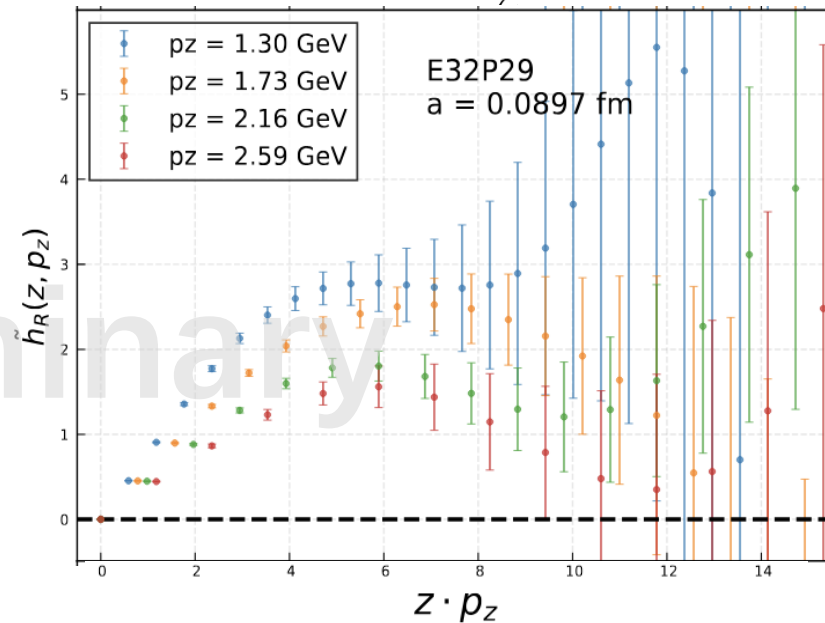
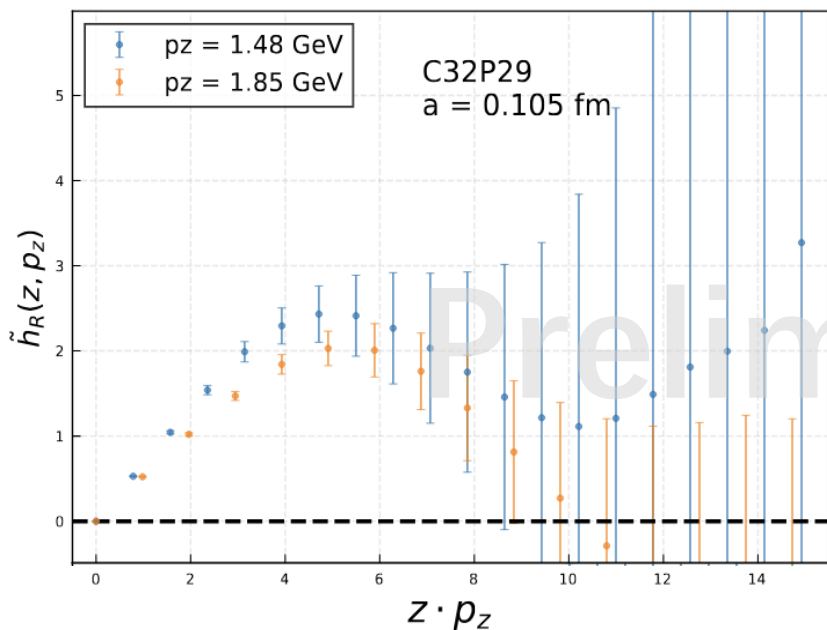
$$\tilde{h}_{\text{sym}}^R = \frac{\tilde{h}_B(z, p_z, 1/a)}{z_R}$$

Renormalized Matrix



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$$\tilde{h}_R^{\text{ratio}}(z, P_z) = z \left[\frac{\tilde{h}_{\text{sym}}(z, P_z, 1/a)}{\tilde{h}_{\text{sym}}(z, P_z = 0, 1/a)} \theta(z_1 - |z|) + \frac{\tilde{h}_{\text{sym}}(z, P_z, 1/a)}{Z_R(z, 1/a) \tilde{h}_{\text{sym}, \overline{\text{MS}}}^{\text{pert}}(z, \mu)} \theta(|z| - z_1) \right]$$

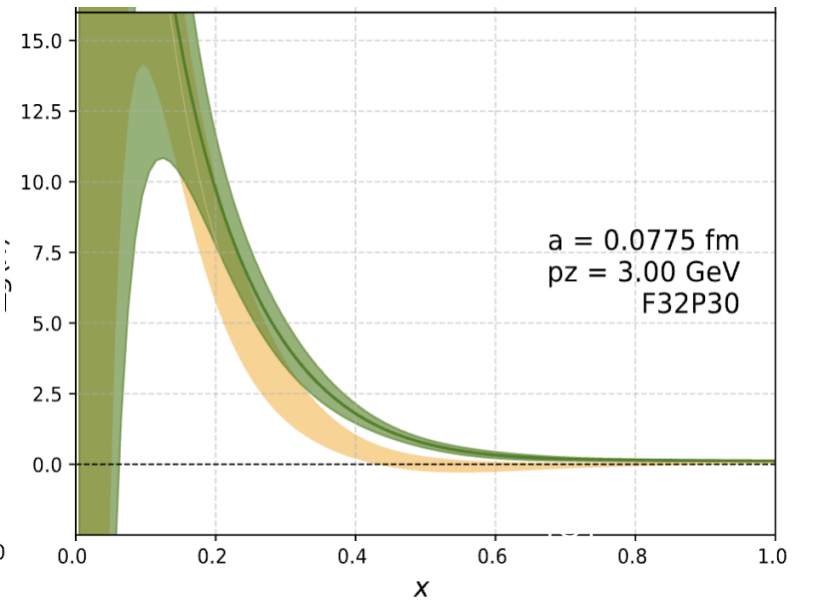
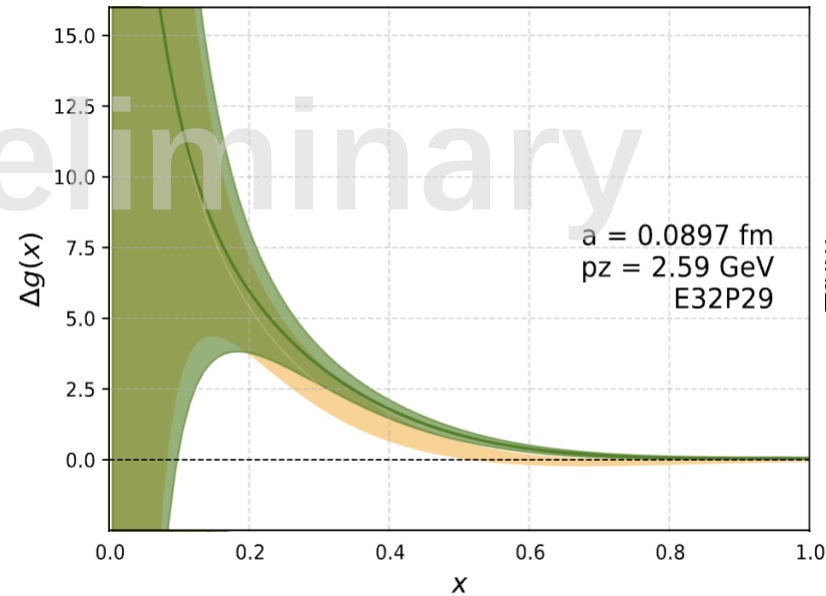
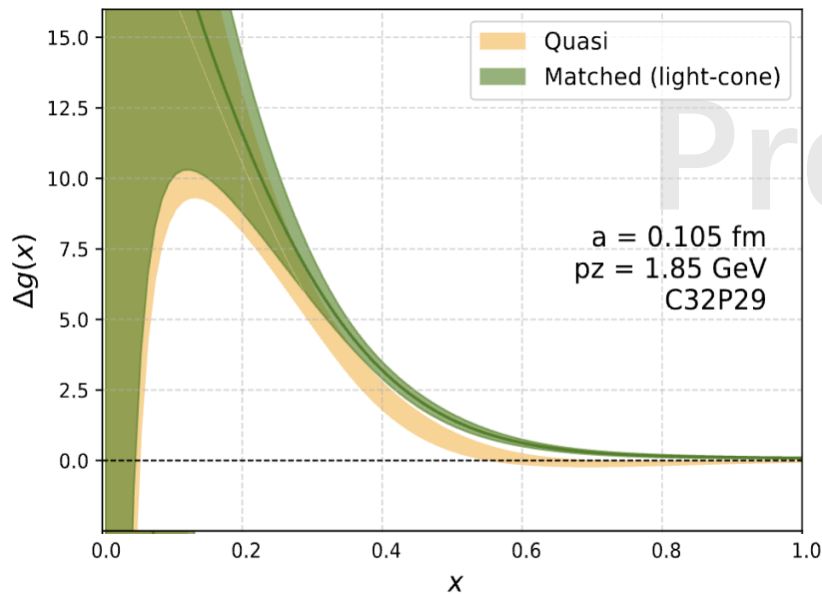


Matching



$$C_{\text{ratio}}\left(\xi, \frac{\mu}{yP_z}\right) = \delta(1-\xi) + \frac{\alpha_s(\mu)C_A}{2\pi} \begin{cases} \left[\left(\frac{2\xi^2}{1-\xi} + 4\xi(1-\xi) \right) \ln \frac{\xi}{\xi-1} - \frac{7-12\xi+12\xi^2}{3(1-\xi)} \right]_+ & \xi > 1 \\ \left[\left(\frac{2\xi^2}{1-\xi} + 4\xi(1-\xi) \right) \left(-\ln \frac{\mu^2}{4p_z^2} + \ln(\xi(1-\xi)) \right) + \frac{7-36\xi+69\xi^2-42\xi^3}{3(1-\xi)} \right]_+ & 0 < \xi < 1 \\ \left[- \left(\frac{2\xi^2}{1-\xi} + 4\xi(1-\xi) \right) \ln \left(\frac{-\xi}{1-\xi} \right) + \frac{7-12\xi+12\xi^2}{3(1-\xi)} \right]_+ & \xi < 0, \end{cases}$$

$$C_{\text{hybrid}}\left(\xi, \lambda_s, \frac{\mu}{yP_z}\right) = C_{\text{ratio}}\left(\xi, \frac{\mu}{yP_z}\right) + \frac{\alpha_s(\mu)C_A}{2\pi} \frac{13}{6} \left(-\frac{1}{|1-\xi|} + \frac{2\text{Si}((1-\xi)|y|\lambda_s)}{\pi(1-\xi)} \right)_+,$$



Infinite Momentum & Continuum Limit

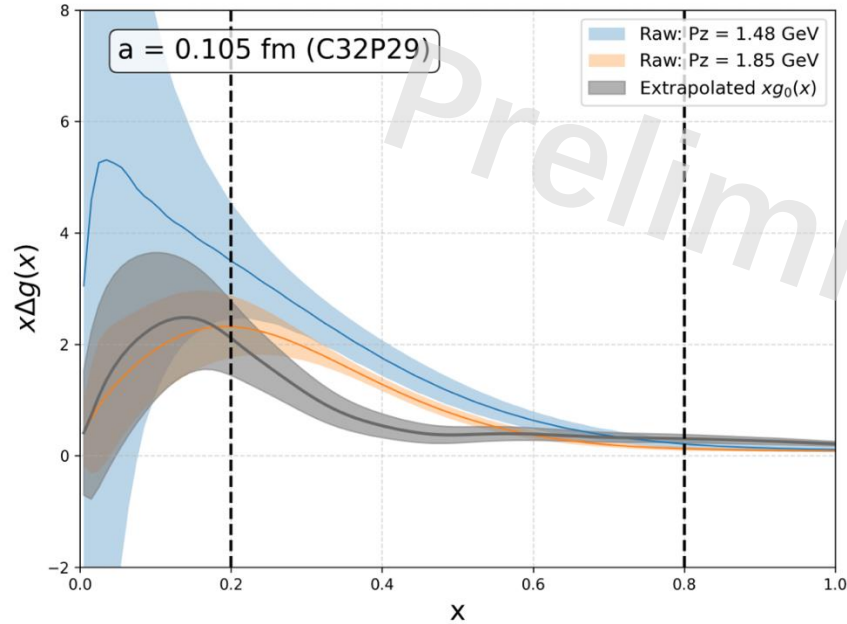


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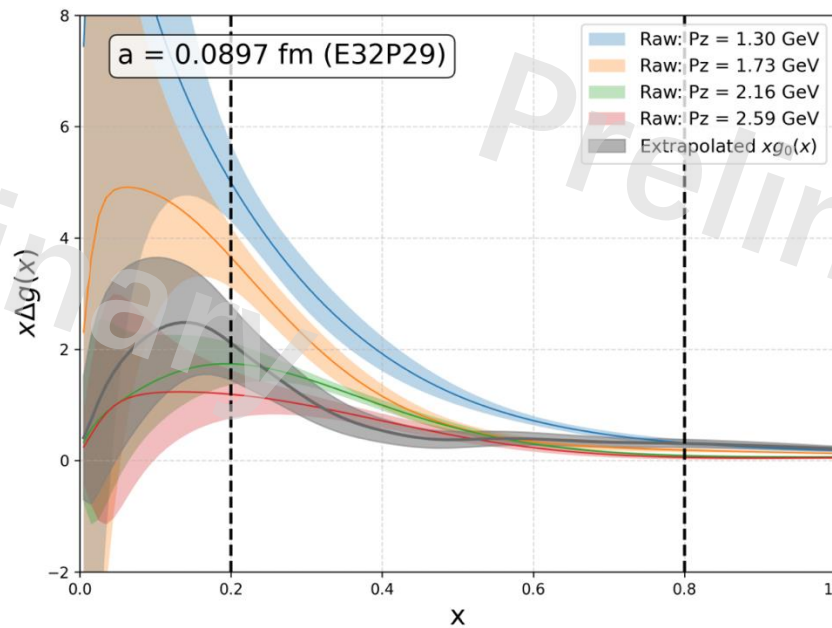
Extrapolation
$$xg(x, P_Z, a) = xg_0(x) + a^2 f(x) + a^2 P_z^2 h(x) + \frac{d(x)}{P_z^2}$$

Distribution after Fourier transformation and matching

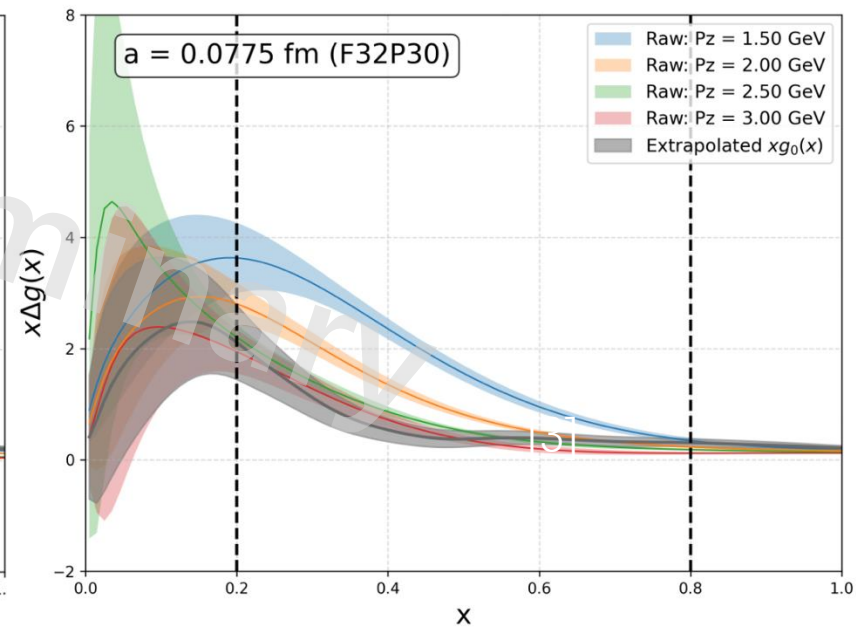
C32P29 0.105 fm



E32P29 0.0897 fm

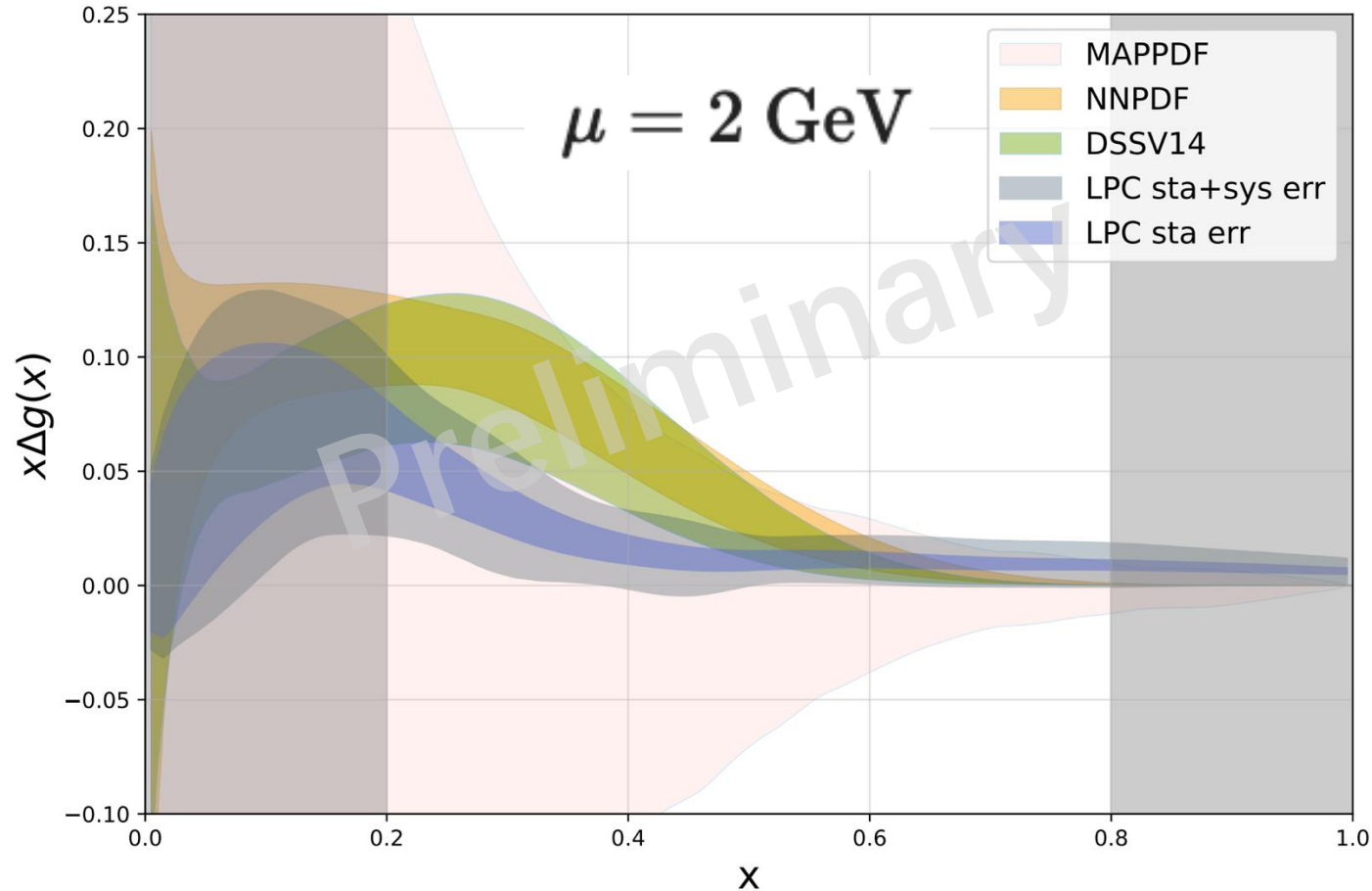


F32P30 0.0775 fm



Infinite Momentum & Continuum Limit

Final PDF



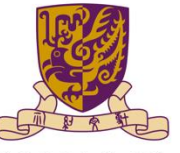
Systematic error

- Renormalization
- Lambda extrapolation
- Infinite momentum
- Delta G matched

Cruz-Martinez, J., et al. *J. High Energ. Phys.* **2025**, 168 (2025).
D. de Florian, et al., *Phys. Rev. D* 100, 114027 (2019).

MAPPDF: 2025
DSSV14: 2019
NNPDF: 2025

Summary and Outlook

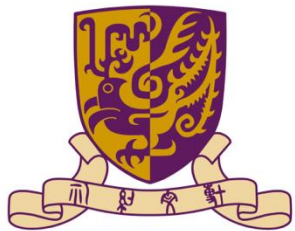


Summary

- Extracted the first *ab initio* gluon helicity PDF entirely from Lattice QCD and compared it with global fits.
- Achieved continuum and large-momentum limits using gauge configurations at three distinct lattice spacings.
- Implemented a self-renormalization scheme for purely imaginary, antisymmetric observables.

Outlook

- Investigate the pion mass dependence of the gluon helicity PDF.
- Further enhance the signal-to-noise ratio and systematically reduce uncertainties.



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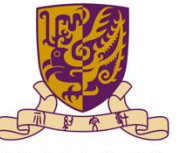


Thank you !

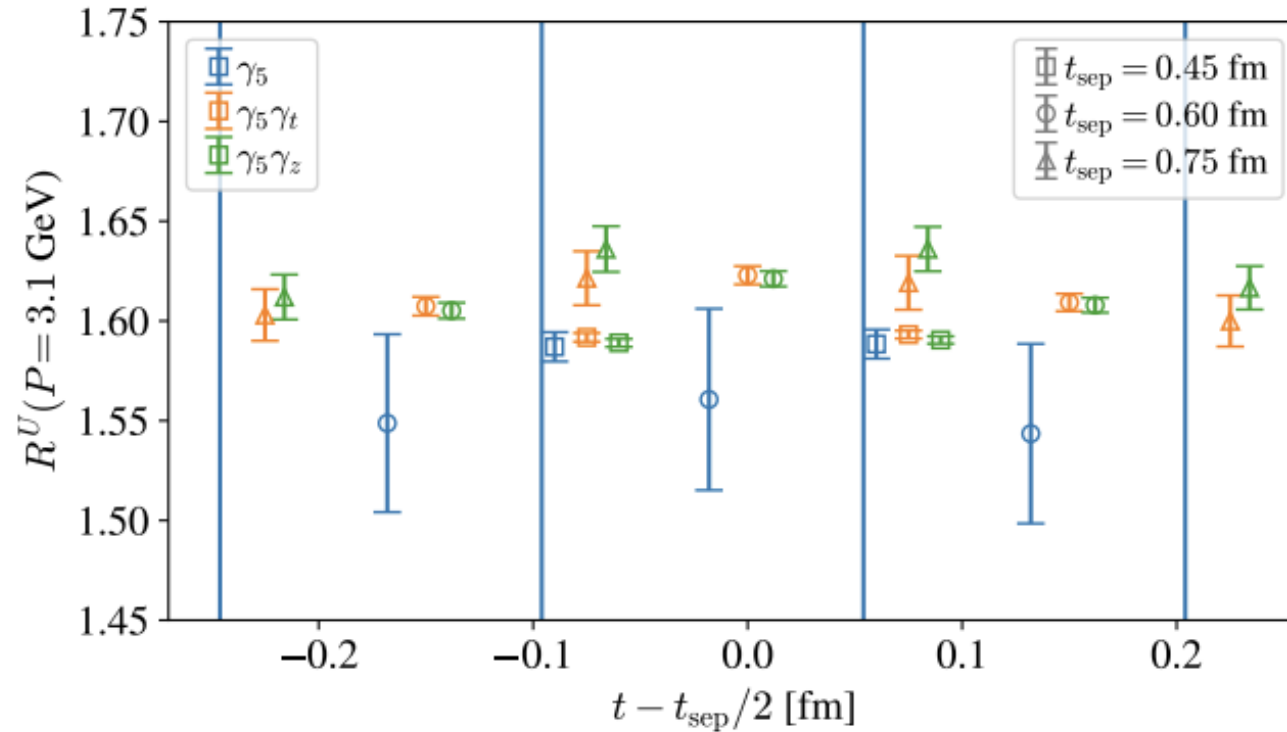
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Backup



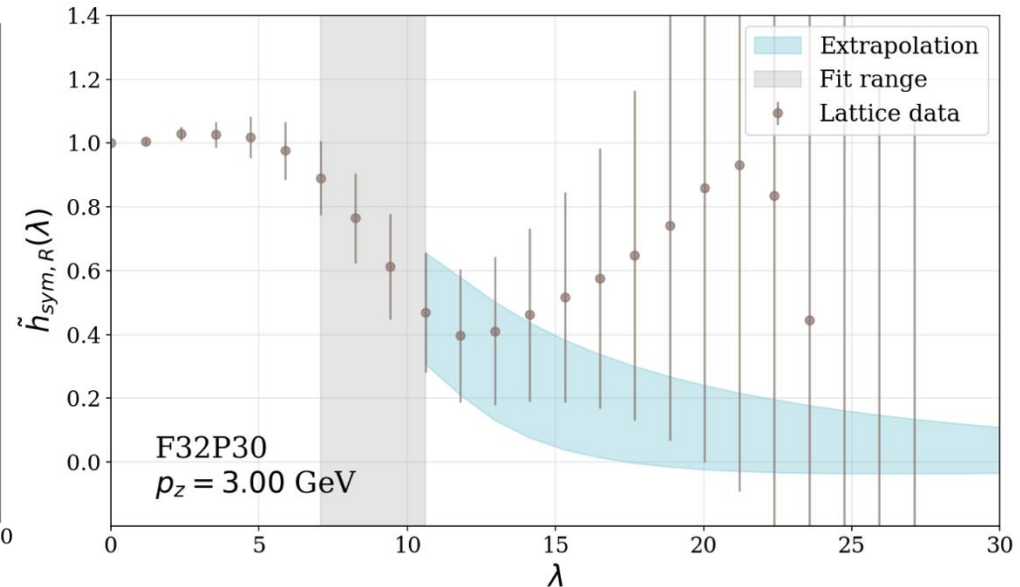
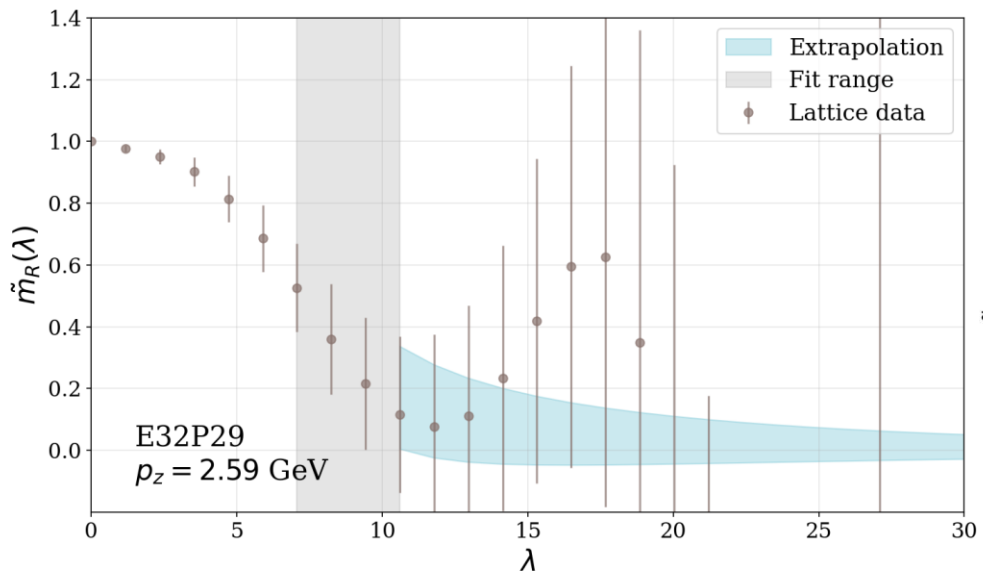
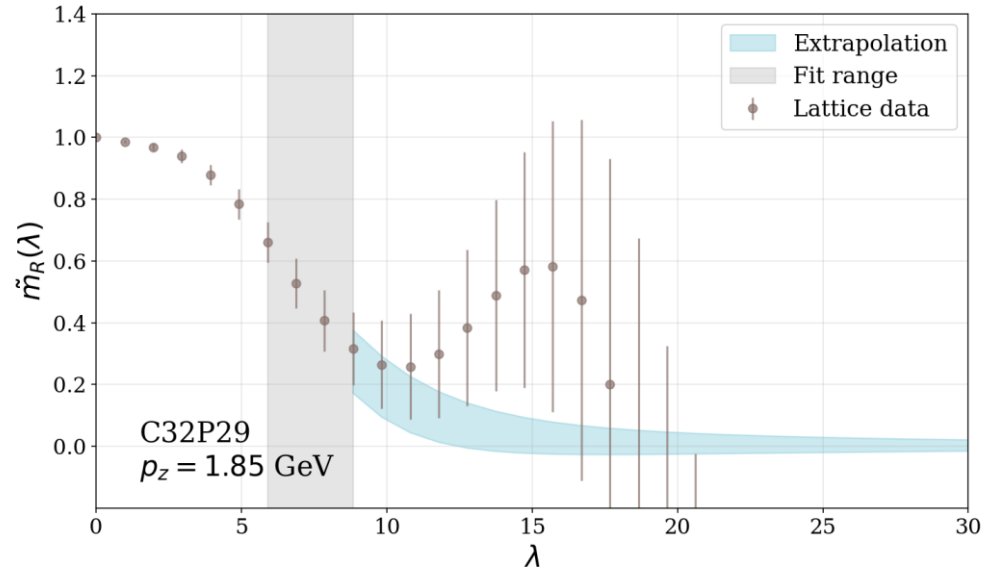
Vector current in the nucleon state



$$C_{3\text{pt}}(\vec{P}, t, t_{\text{sep}}) = \sum_{\vec{x}} e^{i\vec{x}\cdot\vec{P}} \text{Tr} \left[\mathcal{P}_+ \langle N_{\Gamma}(t_{\text{sep}}, \vec{x}) \right. \\ \left. \times \bar{u}(t) \gamma_t u(t) \bar{N}_{\Gamma}(0) \rangle \right],$$

Backup

$$\tilde{m}_R(\lambda) = l_1 \lambda^{-a_1} e^{-\lambda/\lambda_0},$$



Ensemble	P_z (GeV)	default range (aP_z)	new range (aP_z)
C32P29	1.48	[9, 13]	[7, 13]
	1.85	[8, 12]	[6, 12]
E32P29	1.3	[9, 13]	[7, 13]
	1.73	[9, 13]	[7, 13]
	2.16	[6, 10]	[4, 10]
	2.59	[6, 10]	[4, 10]
F32P30	1.50	[9, 13]	[7, 13]
	2.00	[9, 13]	[7, 13]
	2.50	[8, 12]	[6, 12]
	3.00	[7, 11]	[5, 11]

TABLE III. λ extrapolation with the baseline and an adjusted fitting window; the resulting systematic error is given as the difference.

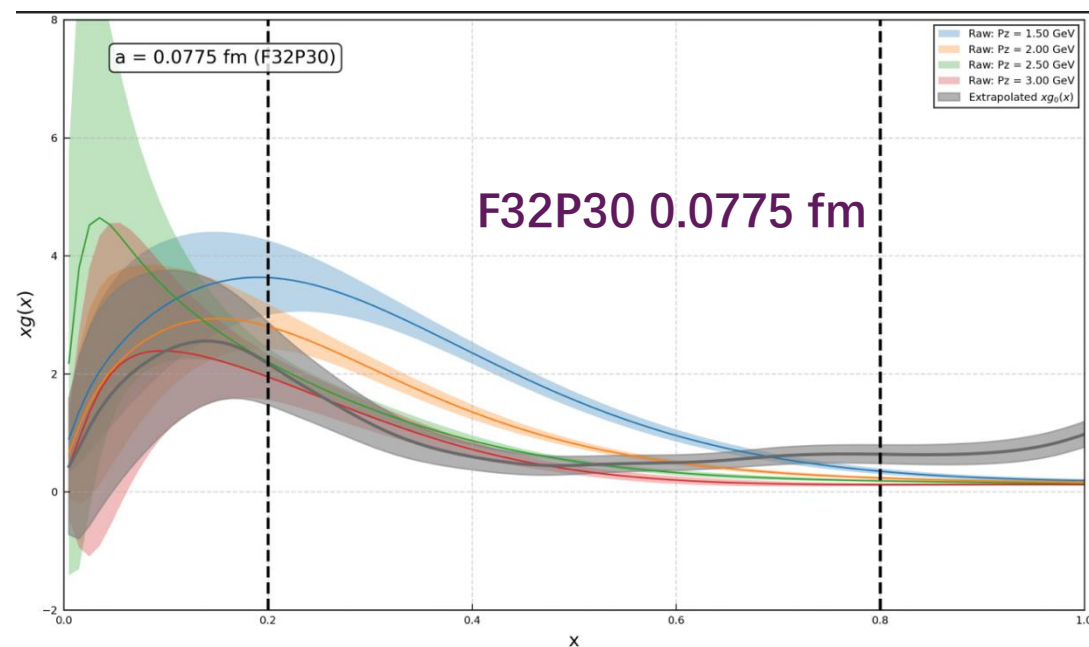
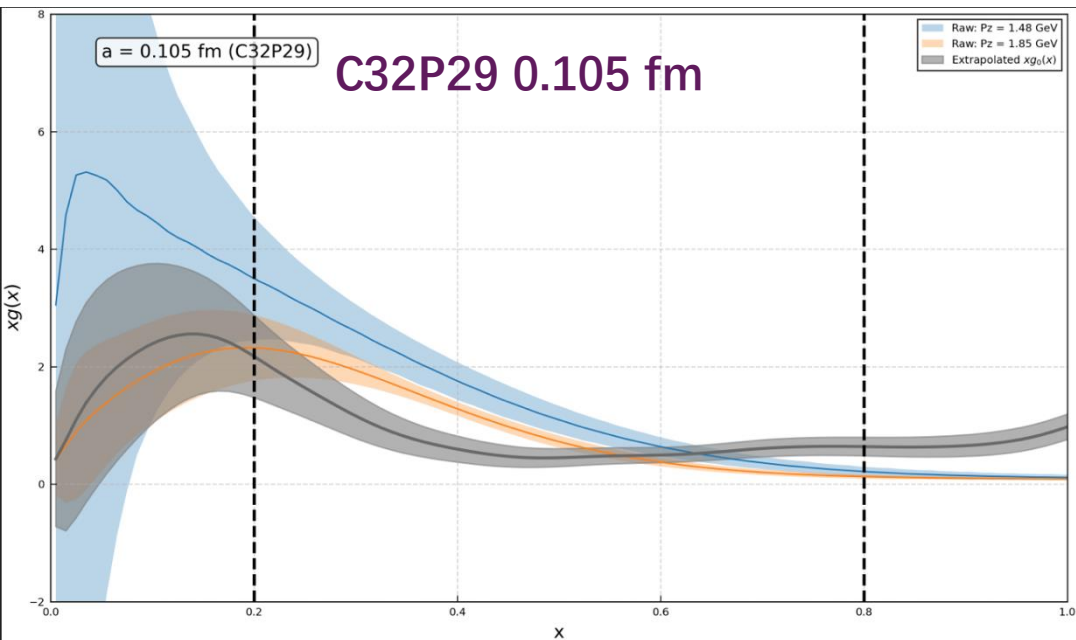
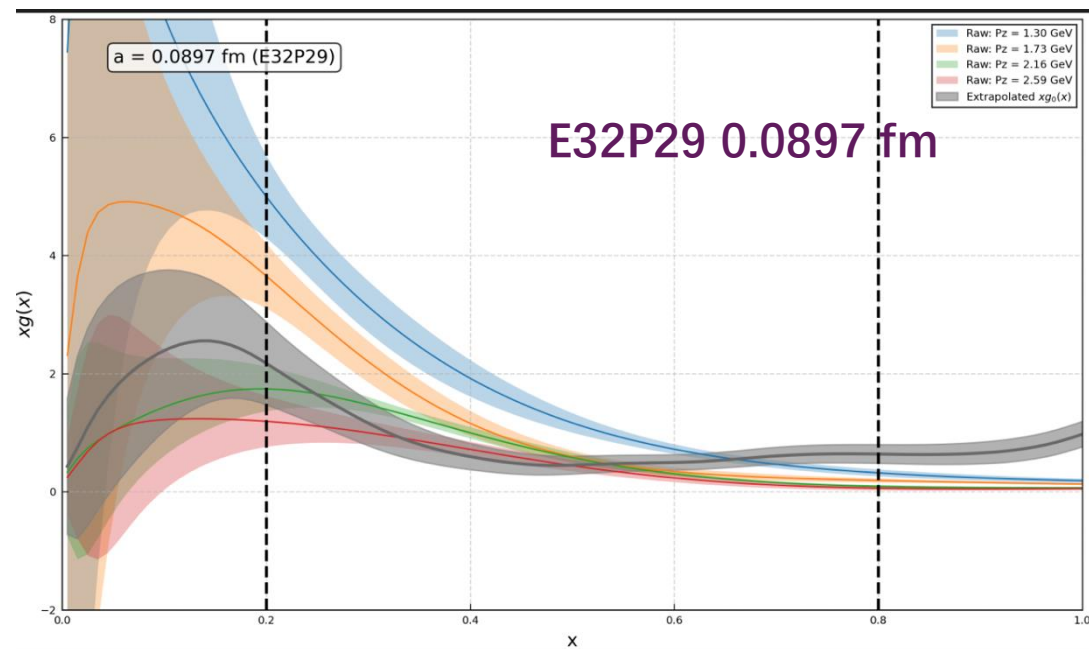
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Extrapolation

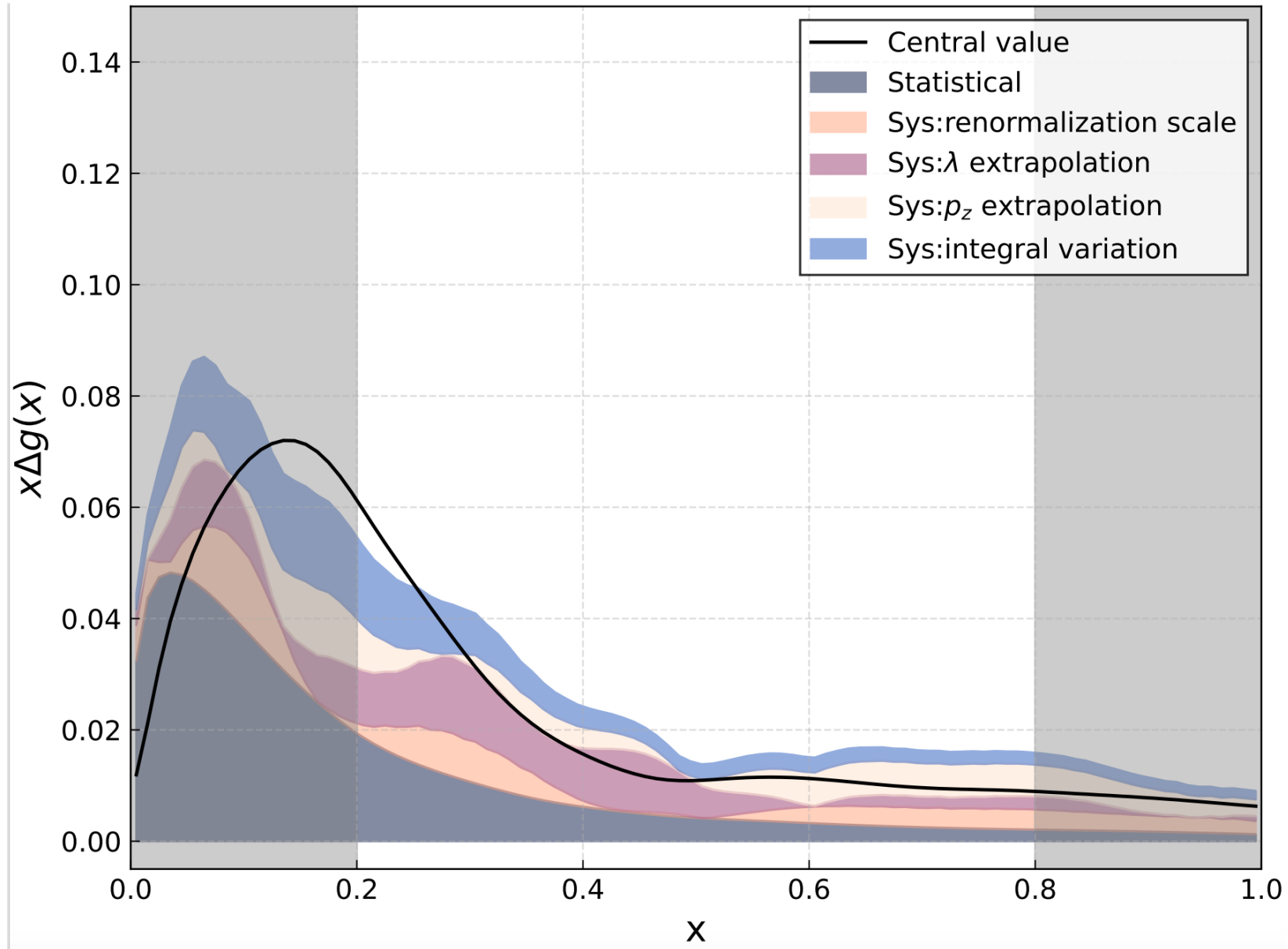
$$xg(x, P_Z, a) = xg_0(x) + a^2 f(x) + ah(x) + \frac{d(x)}{P_z^2}$$



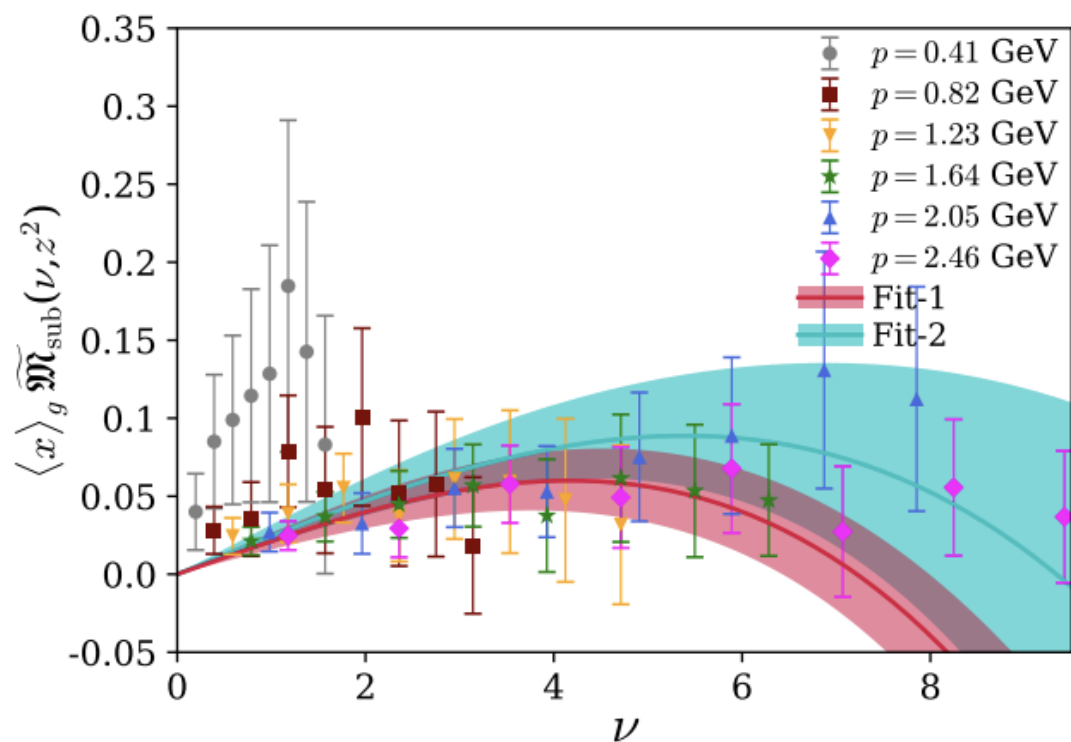
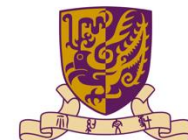
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ID	a (fm)	m_π (MeV)	$L^3 \times N_t$	N_{cfg}	N_{srCs}
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	1901	64

