PDFs from Large Momentum expansion of current-current correlations

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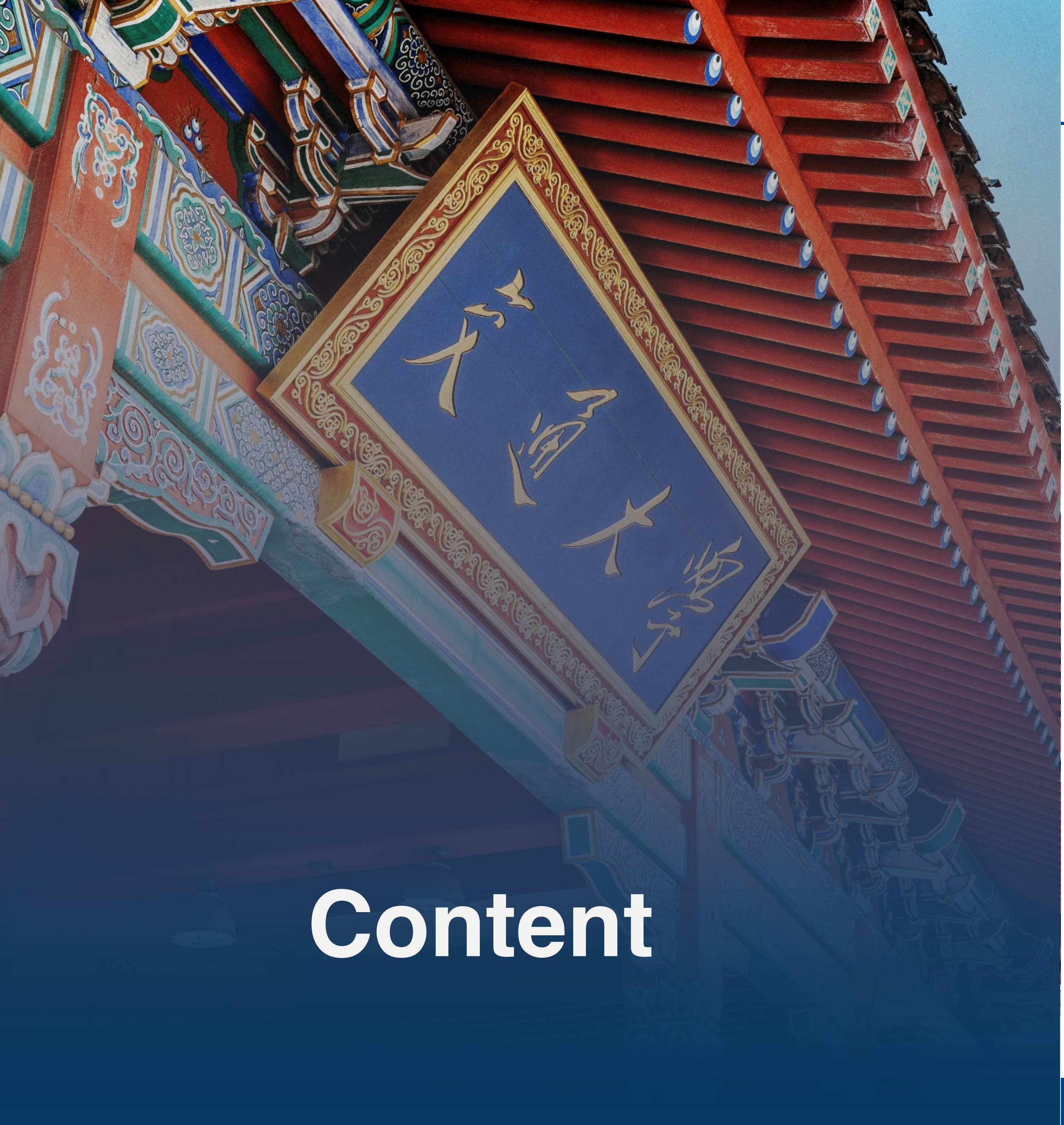

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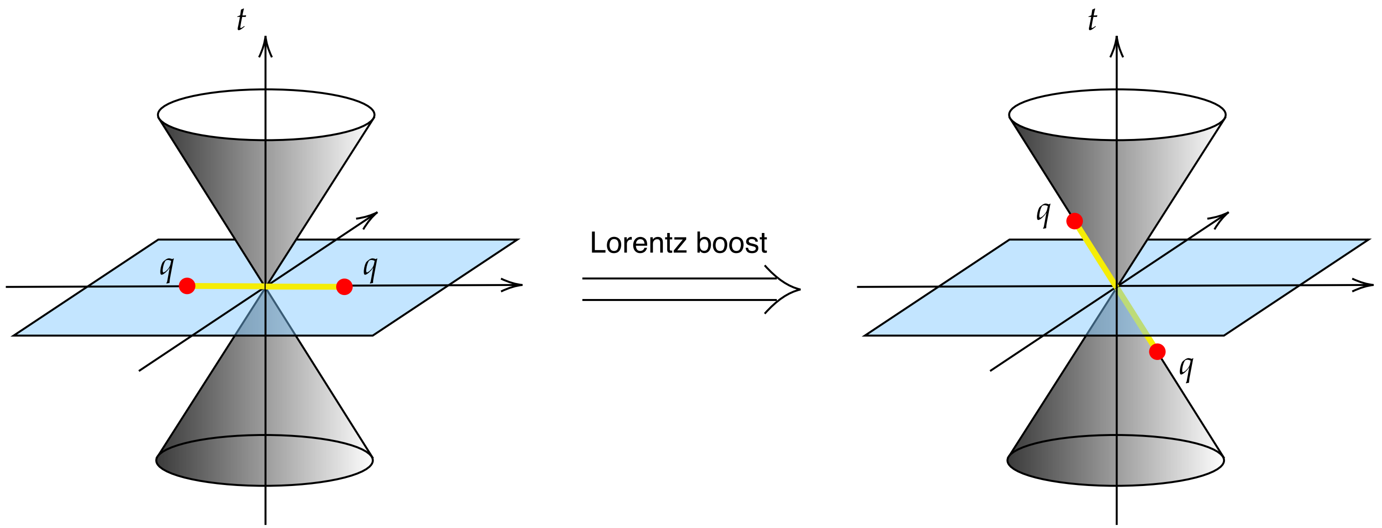
LaMET: From Equal-Time Correlation to PDFs

Large Momentum Expansion (LaMET)



Purposed by Prof. Ji (2013)

Phys.Rev.Lett. 110, 262002 (2013)

Sci. China PMA 57, 1407 (2014)

The starting point and most studied

Quasi-PDF → PDF

$$q(x) = C(\xi, \mu/(yP_z)) \otimes \tilde{q}(y, P_z) + \mathcal{O}\left(\frac{\Lambda_{QCD}^n}{(xP_z)^n}, \frac{\Lambda_{QCD}^n}{((1-x)P_z)^n}, \frac{M^n}{P_z^n}\right)$$

LaMET: From Equal-Time Correlation to PDFs

$$\tilde{q}(y, P_z) = \mathscr{F}\left[\langle P\,|\,\bar{\psi}(z)W(z,0)\psi(0)\,|\,P\rangle\right]$$

To deal with Wilson line: self-energy & renormalon ambiguity

- **Self renormalization**

  [Y.-K. Huo *et al.* (LPC), *Nucl. Phys. B* 969, 115443 (2021)]

- **Hybrid renormalization**

  [X. Ji *et al.*, *Nucl. Phys. B* 964, 115311 (2021)]

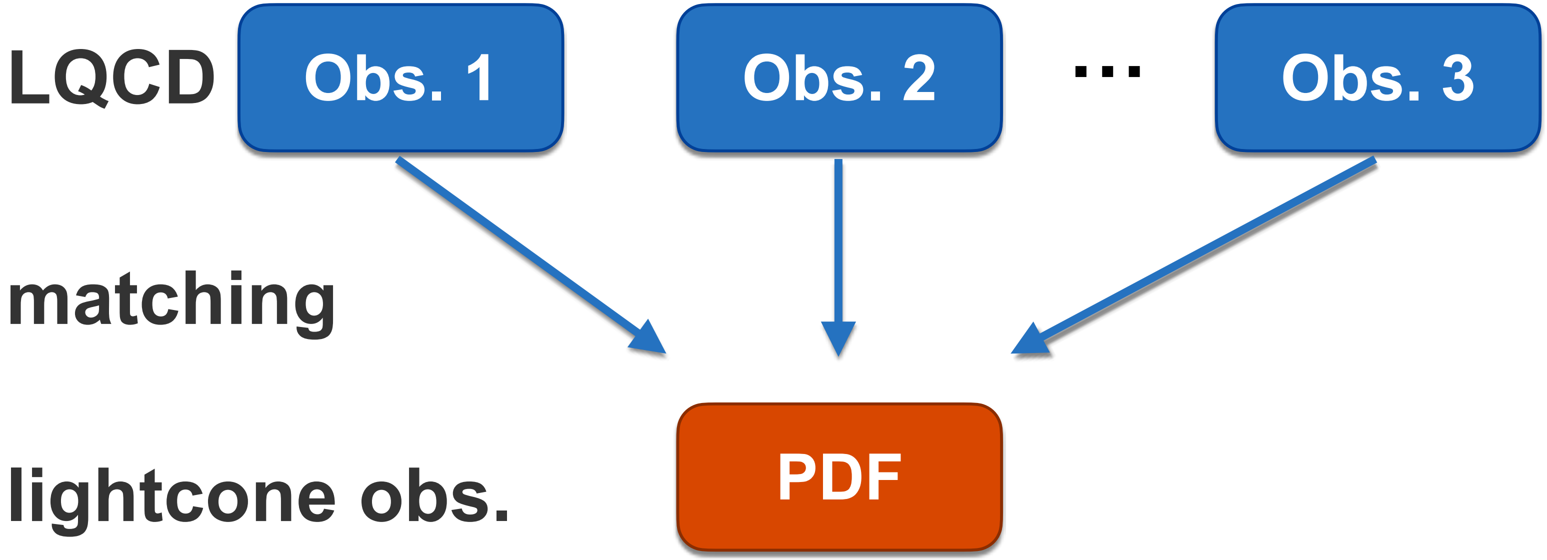
- **Leading renormalon resummation**

  [R. Zhang *et al.*, *Phys. Lett. B* 844, 138081 (2023)]

Can we get rid of the Wilson-line from the beginning?

LaMET: A Universality Class of Euclidean Observables

LaMET is not limited to Quasi-PDF



The lattice observables form a universal class which can be matched to one light-cone operator

[X. Ji *et al.*, Rev. Mod. Phys. 93, 035005 (2021)]

Criteria

- Have the same IR physics
- Differ only by UV physics
- Admit a factorization formula

Observables for Large Momentum Expansion

Quasi-PDF

$$\mathscr{F}\left[\langle P|\bar{\psi}(z)\gamma^{t}W(z,0)\psi(0)|P\rangle\right]$$

- Most studied
- Solid theoretical foundation
- All order factorization

- Complicated renormalization

Coulomb-gauge Quasi-PDF

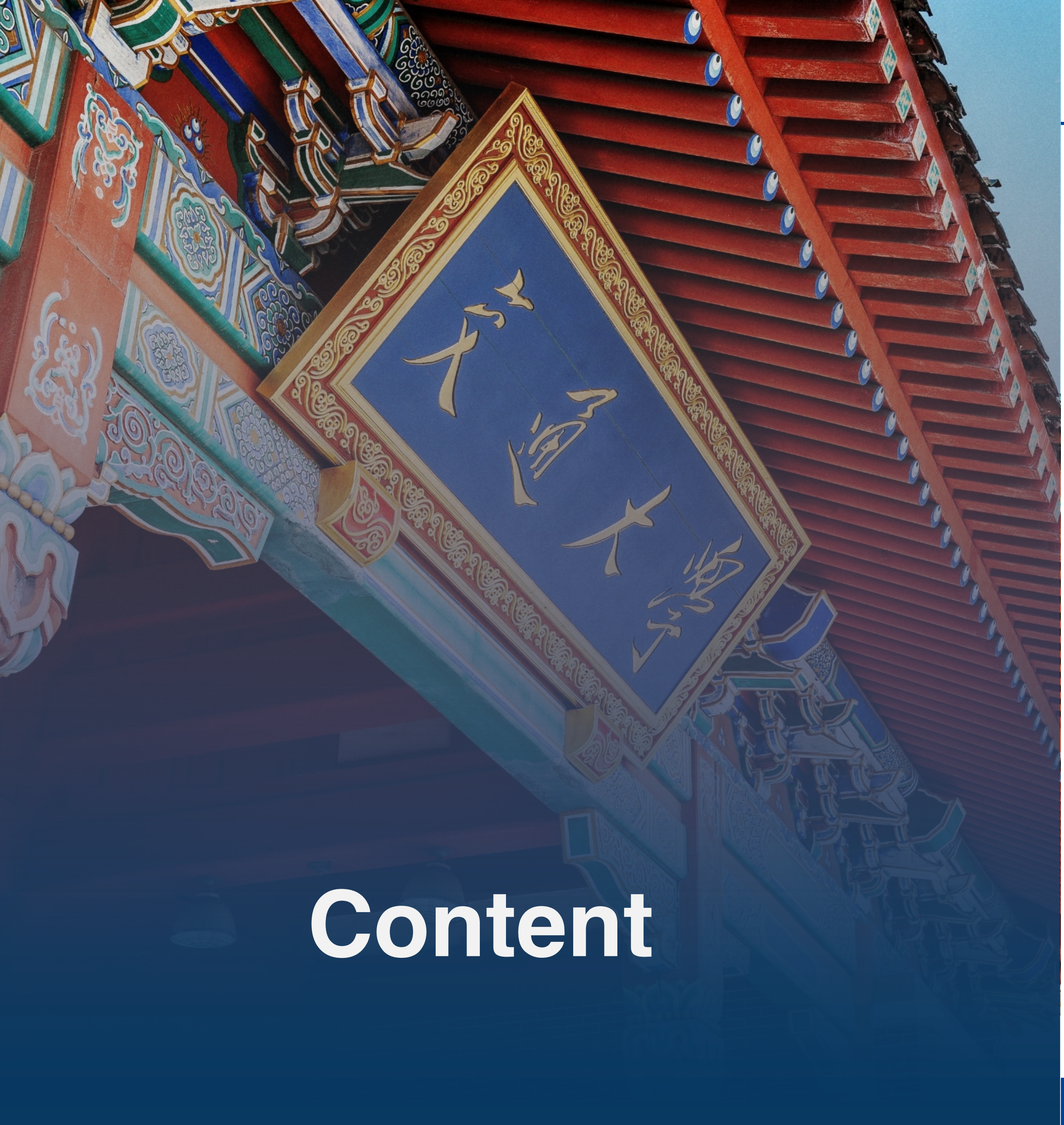
$$\mathscr{F}\left[\langle P|\bar{\psi}(z)\gamma^{t}\psi(0)\Big|_{\vec{\nabla}\cdot\vec{A}=0}|P\rangle\right]$$

- No wilson line, easier renormalization
- Better signal

- No all order factorization

Another operator sit in the same universal class

current-current correlation



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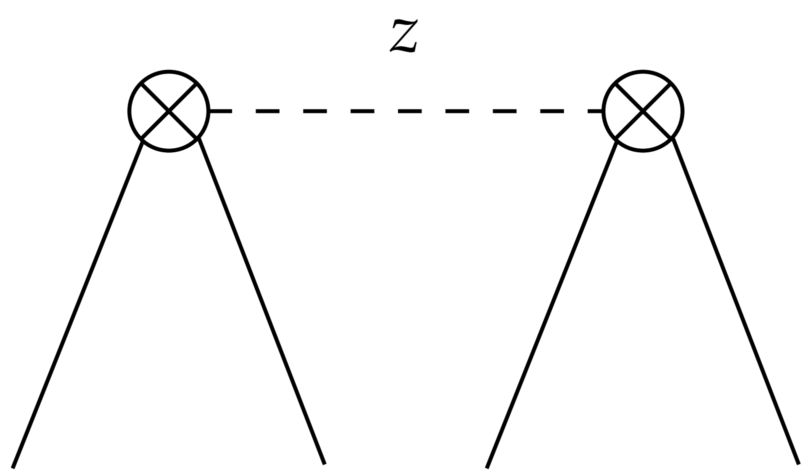
Current-current correlation

current-current correlation

$$\langle P \,|\, J^\dagger(z)\, J(0) \,|\, P \rangle$$

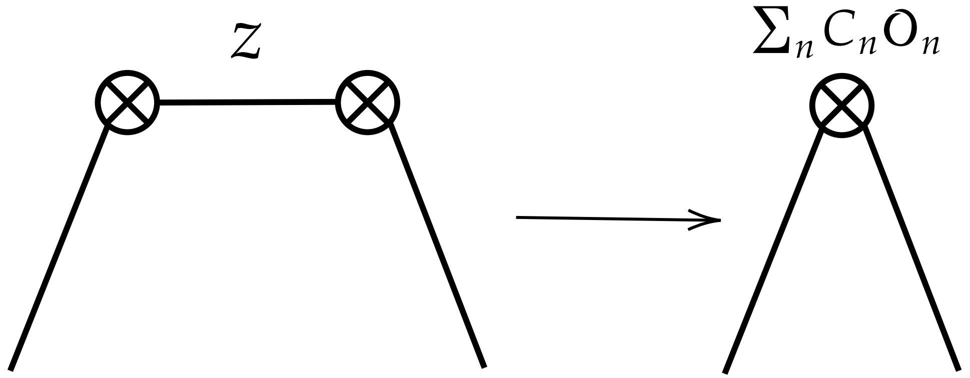


Choices of J:

- $J_S := \bar{\psi}_q \psi_q$
- $J_V^\mu := \bar{\psi}_q \gamma^\mu \psi_q$
- $J_A^\mu := \bar{\psi}_q \gamma^\mu \gamma_5 \psi_q$

…

The short-distance OPE of current-current correlations has been established



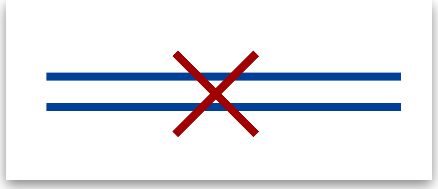
K-F Liu, *Phys.Rev.D* 62 074501 (2000)
Y.-Q. Ma and J.-W. Qiu, Phys. Rev. Lett. 120, 022003 (2018)
…

Why use current-current correlation

Advantages

No wilson line

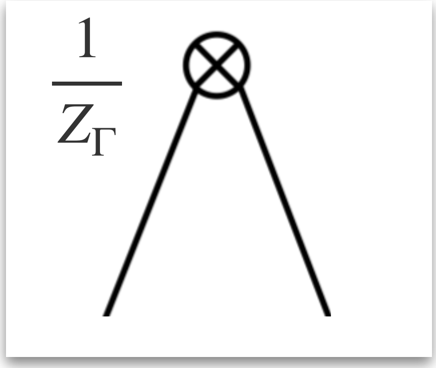
No Wilonson-line linear divergence and associated renormalon ambiguity



$$\mathscr{P}\left[exp\left(\int dtn \cdot A(nt)\right)\right]$$

Easier renormalization

Only need to do local composite operator renormalization

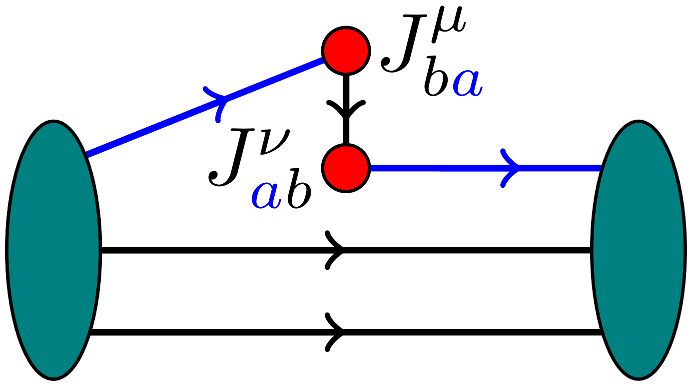


$$(\bar{\psi}\Gamma\psi)_R = \frac{1}{Z_\Gamma}(\bar{\psi}\Gamma\psi)_B$$

Drawback

It is a four point function

More expansive to get a competitive result



Current-current correlation

Current-current correlation used in this work

We consider the vector-vector (VV) and axial-axial (AA) correlations

$$M^{\mu\nu}_{XY,ab}(z,P) = \langle P \,|\, J^{\nu}_{X,ab}(z)\, J^{\mu}_{Y,ba}(0) \,|\, P\rangle, \qquad X, Y = V, A$$

$$J^{\mu}_{V,ab}(z) = \bar{q}_a(z)\gamma^{\mu} q_b(z), \qquad J^{\mu}_{A,ab}(z) = \bar{q}_a(z)\gamma^{\mu}\gamma_5 q_b(z)$$

- **a: Flavour of PDF**
- **b: Auxiliary field**

Leading contribution

For the unpolarized PDFs, the leading contribution is carried by transverse polarization

$$M^{\perp\perp}_{AA} \;\&\; M^{\perp\perp}_{VV}$$

Momentum-space distribution

LaMET is for momentum-space distributions

$$W^{\perp\perp}(z,P) := -\frac{i\pi^2 z^4}{\lambda} M^{\perp\perp}(z,P)$$

$$\tilde{h}(y,P_z) = \int_{-\infty}^{\infty} d\lambda\, M^{\perp\perp}(z,P)$$

Large momentum expansion of VV and AA currents

Large momentum expansion

$$f(x,\mu^2) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{|y|P_z}{\mu}\right) \tilde{h}(y,P_z) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^n}{(xP_z)^n}, \frac{\Lambda_{\rm QCD}^n}{[(1-x)P_z]^n}, \frac{M^n}{P_z^n}\right), \qquad n \geq 2$$

light-cone PDF

matching kernel

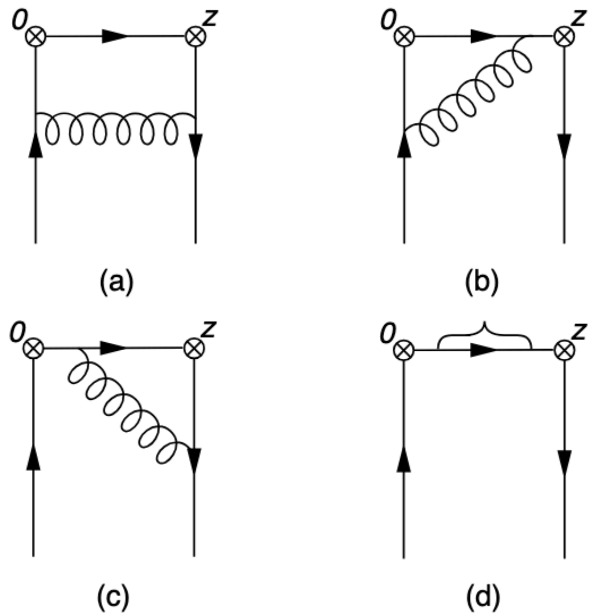
- **Iso-vector (u-d)**
  To avoid operator mixing
- **No Wilson-line**
  Power correction starts at n=2

Matching kernel at one-loop

At one-loop (identical for VV and AA)

$$C_{\rm ns}^{\overline{\rm MS}}\left(\xi, \frac{|y|P_z}{\mu}\right) = \delta(1-\xi) + \frac{\alpha_s C_F}{4\pi}\delta(1-\xi) + \frac{\alpha_s C_F}{2\pi} c^{(1)}\left(\xi, \frac{|y|P_z}{\mu}\right), \qquad \xi = \frac{x}{y}$$

$$c^{(1)} = \frac{\alpha_s C_F}{2\pi} \begin{cases} \left[\frac{(\xi^2+1)\log\left(\frac{\xi-1}{\xi}\right)+\xi+1/2}{1-\xi}\right]_{+(1)}^{[1,\infty)} & \text{if } \xi>1 \\ \left[\frac{2(\xi^2+3)\log(1-\xi)+2(\xi^2+1)\log(\xi)+2(\xi-4)\xi+7}{2(\xi-1)}\right]_{+(1)}^{[0,1]} + \left[\frac{1+\xi^2}{1-\xi}\right]_{+(1)}^{[0,1]} \log\left(\frac{\mu^2}{4y^2P_z^2}\right) & \text{if } 0<\xi<1 \\ \left[\frac{(\xi^2+1)\log\left(\frac{\xi-1}{\xi}\right)+\xi+1/2}{(\xi-1)}\right]_{+(1)}^{(-\infty,0]} & \text{if } \xi<0 \end{cases}$$



Large momentum expansion of VA and AV currents

Alternative choice

$$W_{\mathrm{VA}}^{\perp\perp} := \frac{\pi^2 z^3}{2E_P}\left[M_{\mathrm{VA}}^{xy}(z,P) + M_{\mathrm{AV}}^{xy}(z,P)\right]$$

OPE of VA correlation has been studied

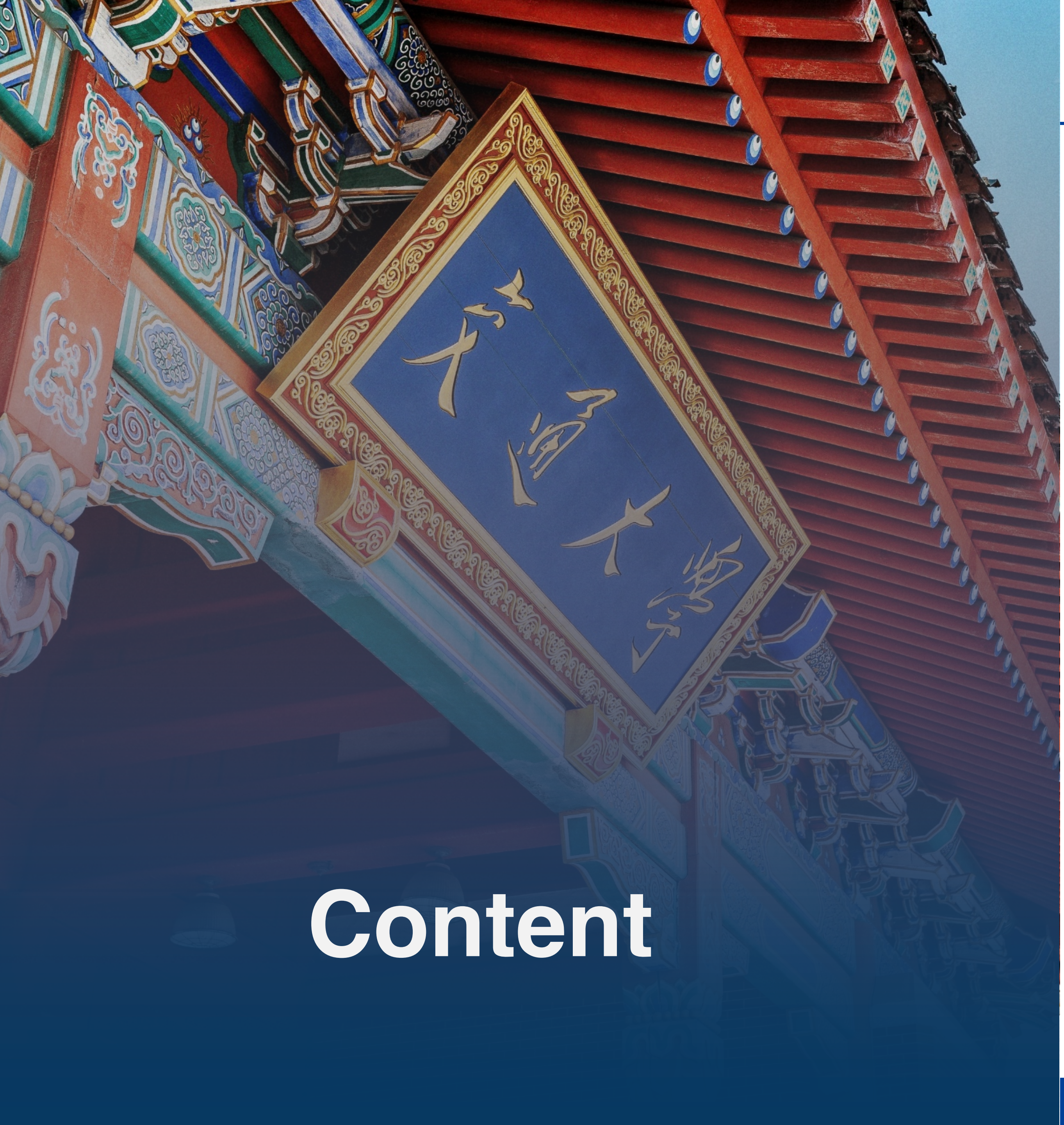
R. S. Sufian et al., Phys. Rev. D 99, 074507 (2019)

R. S. Sufian et al., Phys. Rev. D 102, 054508 (2020)

Matching kernel at one-loop

$$C_{\mathrm{VA,ns}}^{\overline{\mathrm{MS}}}(\xi) = C_{\mathrm{VV,ns}}^{\overline{\mathrm{MS}}}(\xi) + \frac{\alpha_s C_F}{\pi}[1-\xi]_{+(1)}^{[0,1]}\theta(\xi)\theta(1-\xi)\,.$$

Alternative for future study



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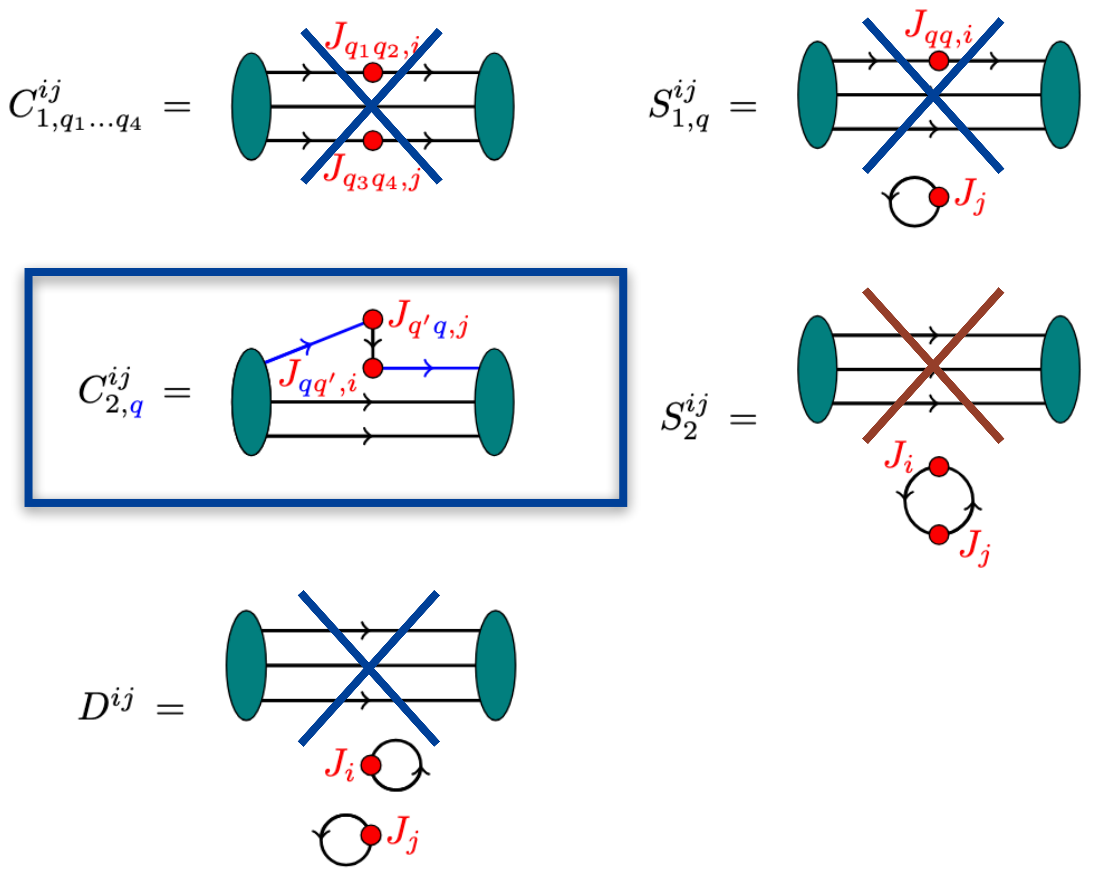
Lattice set-up

CLS H102 Ensemble

Nf=2+1 Wilson-clover

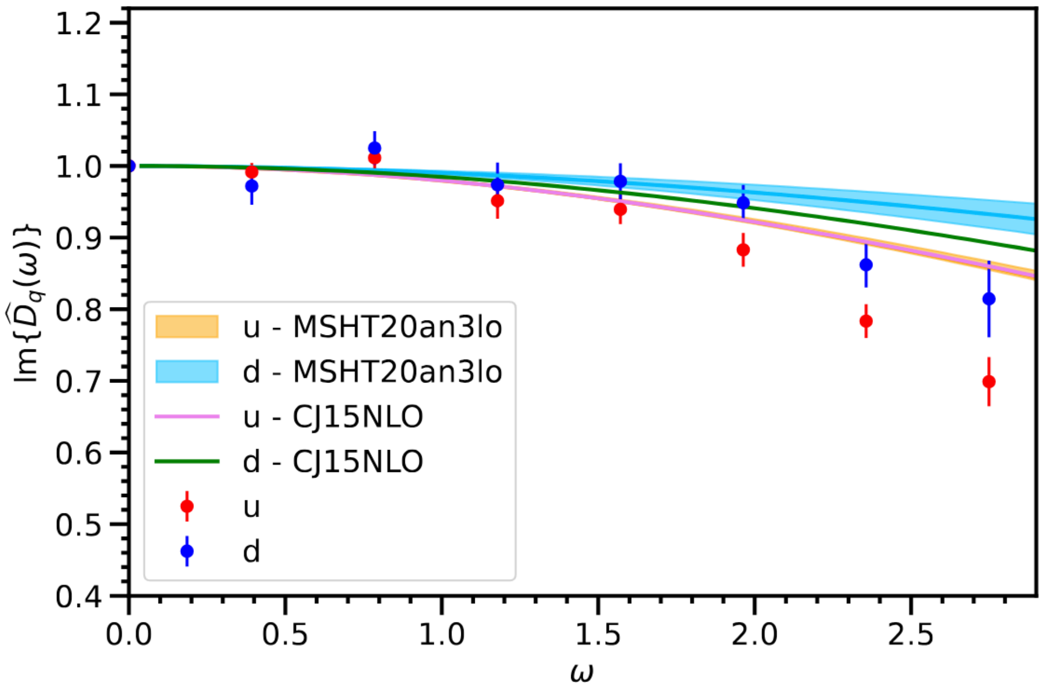
- **Lattice size:** $32^3 \times 96$
- **Lattice spacing** $a = 0.085\,\mathrm{fm}$
- **Pion mass** $m_\pi = 355\,\mathrm{MeV}$
- **Proton momentum** $\vec{P} = (\pm n, \pm n, \pm n)$
  $|\vec{P}|_{max} = 1.57\,\mathrm{GeV}$

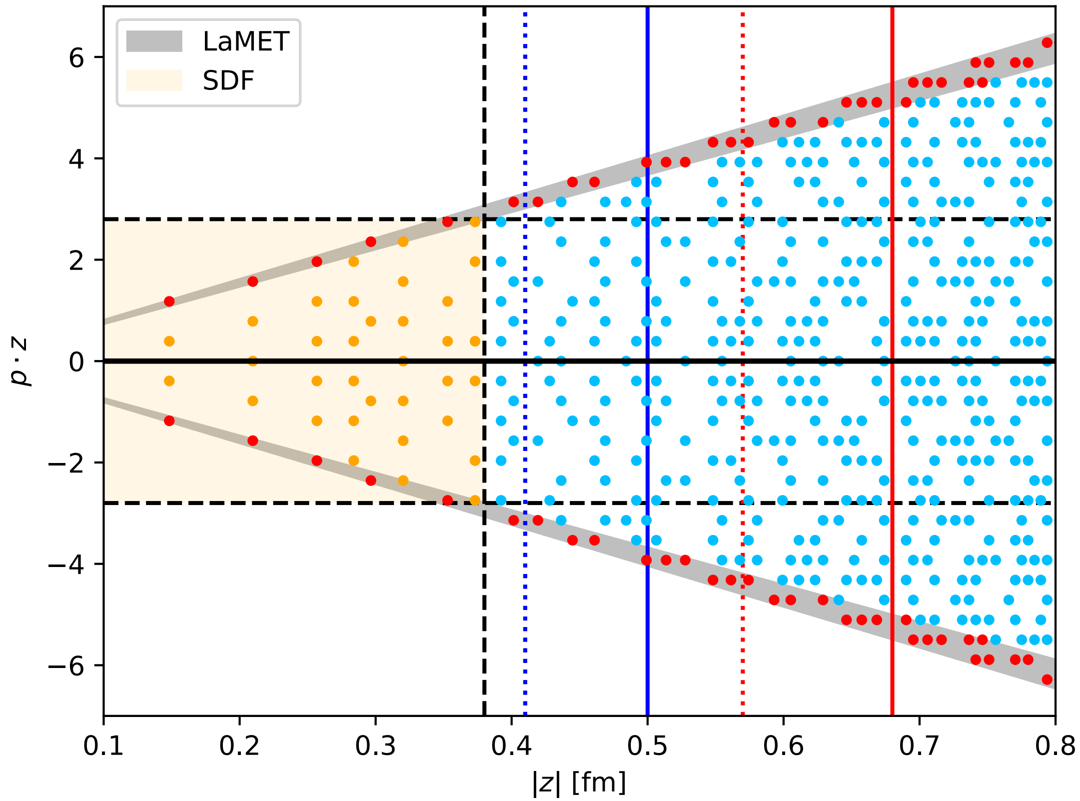
- **Flavour choice** $\bar{\psi}_a \gamma^\mu \psi_b \,, \bar{\psi}_b \gamma^\mu \psi_a$
- **Iso-vector** **(u-d)**



Different kinematic regime

[C. Zimmermann and A. Schäfer, Phys. Rev. D 110, 074503 (2024)]





- **LaMET uses a kinematic regime different from short-distance factorization $\vec{z} \parallel \vec{P}$**

- **To get more data points:** $|\vec{P}|(1-0.02) \leq \lambda/|\vec{z}| \leq |\vec{P}|(1+0.02)$

Lattice set-up

Quantity calculated

$$M^{\mu\nu}(z,P_z) = \frac{1}{2}\left(M^{\mu\nu}_{VV}(z,P_z) + M^{\mu\nu}_{AA}(z,P_z)\right)$$

The combination is chosen to suppress chiral-odd Wilson-fermion artifacts.

[G.S. Bali, et. Al., Phys. Rev. D 98, 094507 (2018)]

Wilson-fermion propagator

$$S_W(p) = \frac{-i\gamma_\mu \bar{p}_\mu + M_W(p)}{\bar{p}^2 + M_W^2(p)}$$

$$\bar{p}_\mu = \frac{1}{a}\sin(ap_\mu), \quad M_W(p) = m_0 + \frac{r}{a}\sum_\mu \left[1-\cos(ap_\mu)\right].$$

Isolating the transverse direction

Decomposition:

$$M^{\mu\nu}(z,P) = \frac{P^\mu P^\nu}{m^2}A(\lambda,z^2) - g^{\mu\nu}B(\lambda,z^2) + m^2 z^\mu z^\nu C(\lambda,z^2) + (P^\mu z^\nu + P^\nu z^\mu)D(\lambda,z^2),$$

$$M^{\perp\perp}(z,P_z) = B(\lambda^2,z) = P_{\mu\nu}M^{\mu\nu}(z,P)$$

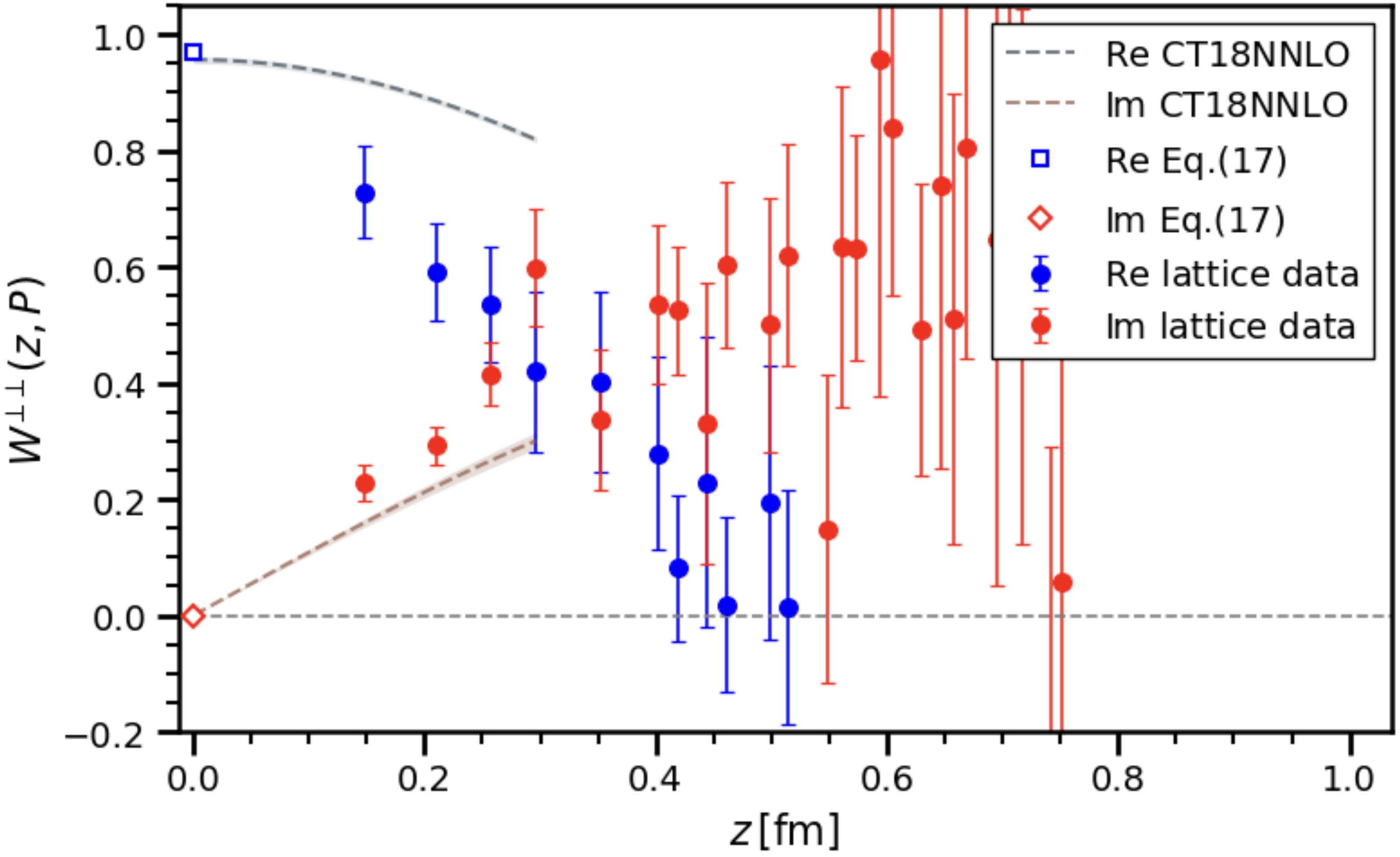
Projector:

$$P_{00} = 0, \quad P_{0j} = P_{j0} = 0$$

$$P_{jk} = \frac{1}{2m^2}\left(\frac{p^j p^k}{\vec{p}^{\,2}} - \delta_{jk}\right)$$

Numerical results

Gray bands are obtained from forward LaMET matching followed by a Fourier transform

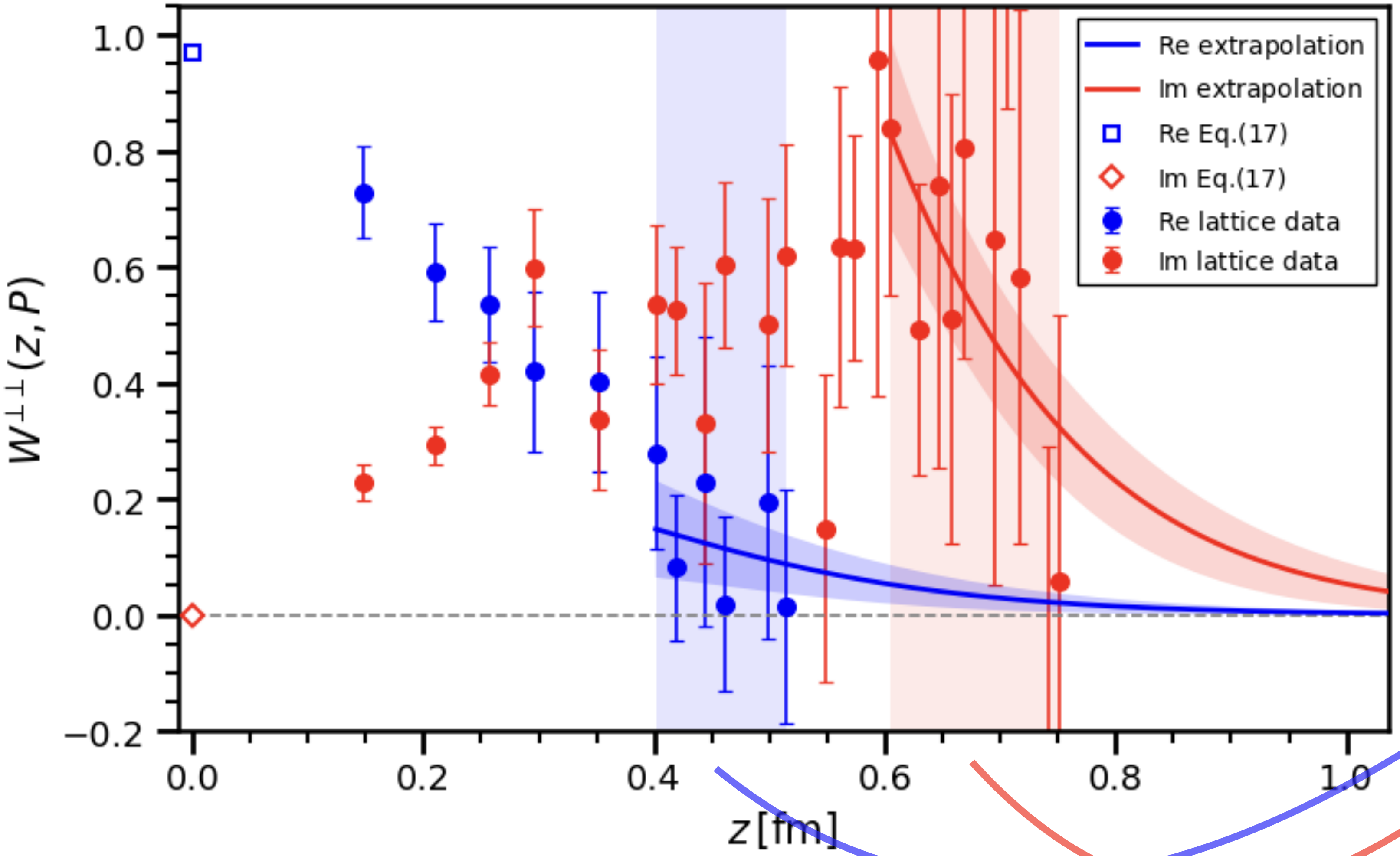


The moments are larger than CT18NNLO

Possible sources of the discrepancy

- **Pion mass**
- **Excited states**
- **Lattice spacing**

Numerical results



Large z Extrapolation

X. Ji, Y. Liu and Y. Su, arXiv:2601.12189 (2026).

$$\sum_X \langle P | J^\mu(z) | X \rangle \langle X | J^\nu(0) | P \rangle$$

$$W^{\perp\perp}(z,P) = A\,|z|^{5/2} e^{i\phi\,\mathrm{sign}(z)} e^{-m|z|}$$

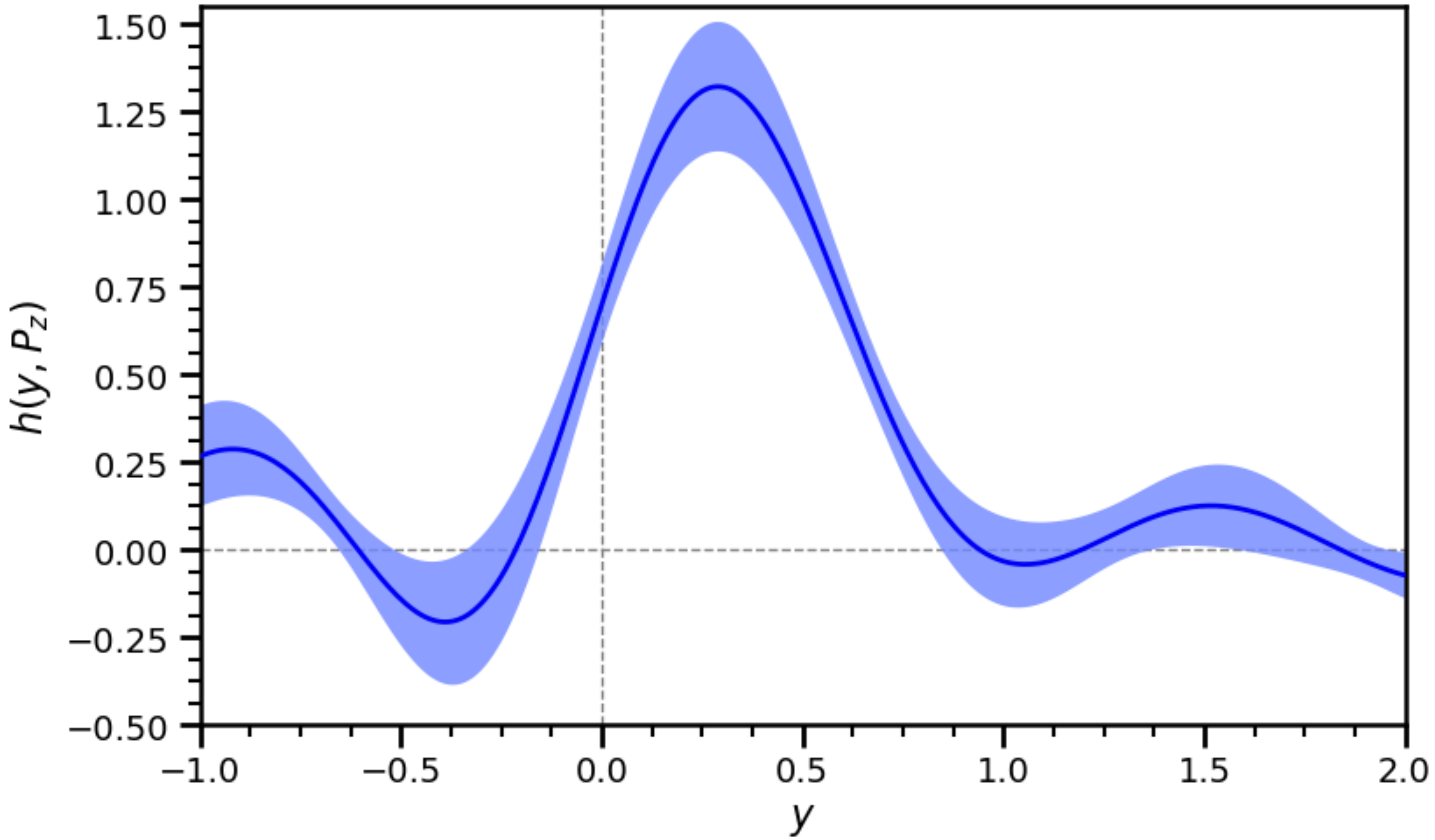
Fit range

Re: 0.4-0.51 fm

Im: 0.61-0.75 fm

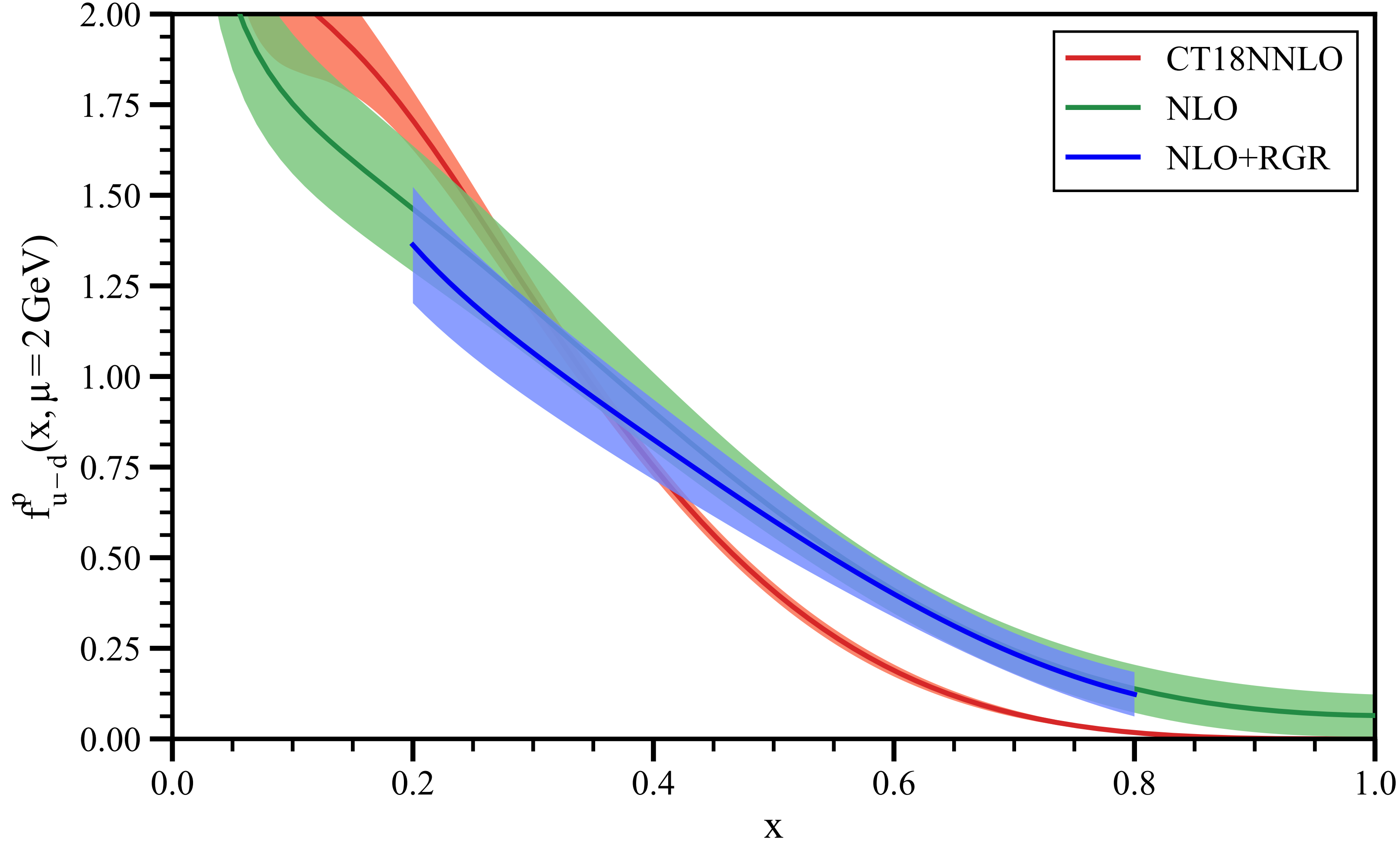
Numerical results

Momentum-space distribution $h(y, P_z)$



Numerical results

$$f(x,\mu^2)=\int_{-\infty}^{\infty}\frac{dy}{|y|}C\left(\frac{x}{y},\frac{|y|P_z}{\mu}\right)\widetilde{h}(y,P_z)+\mathcal{O}\left(\frac{\Lambda_{\rm QCD}^n}{(xP_z)^n},\frac{\Lambda_{\rm QCD}^n}{[(1-x)P_z]^n},\frac{M^n}{P_z^n}\right),\qquad n\geq 2$$

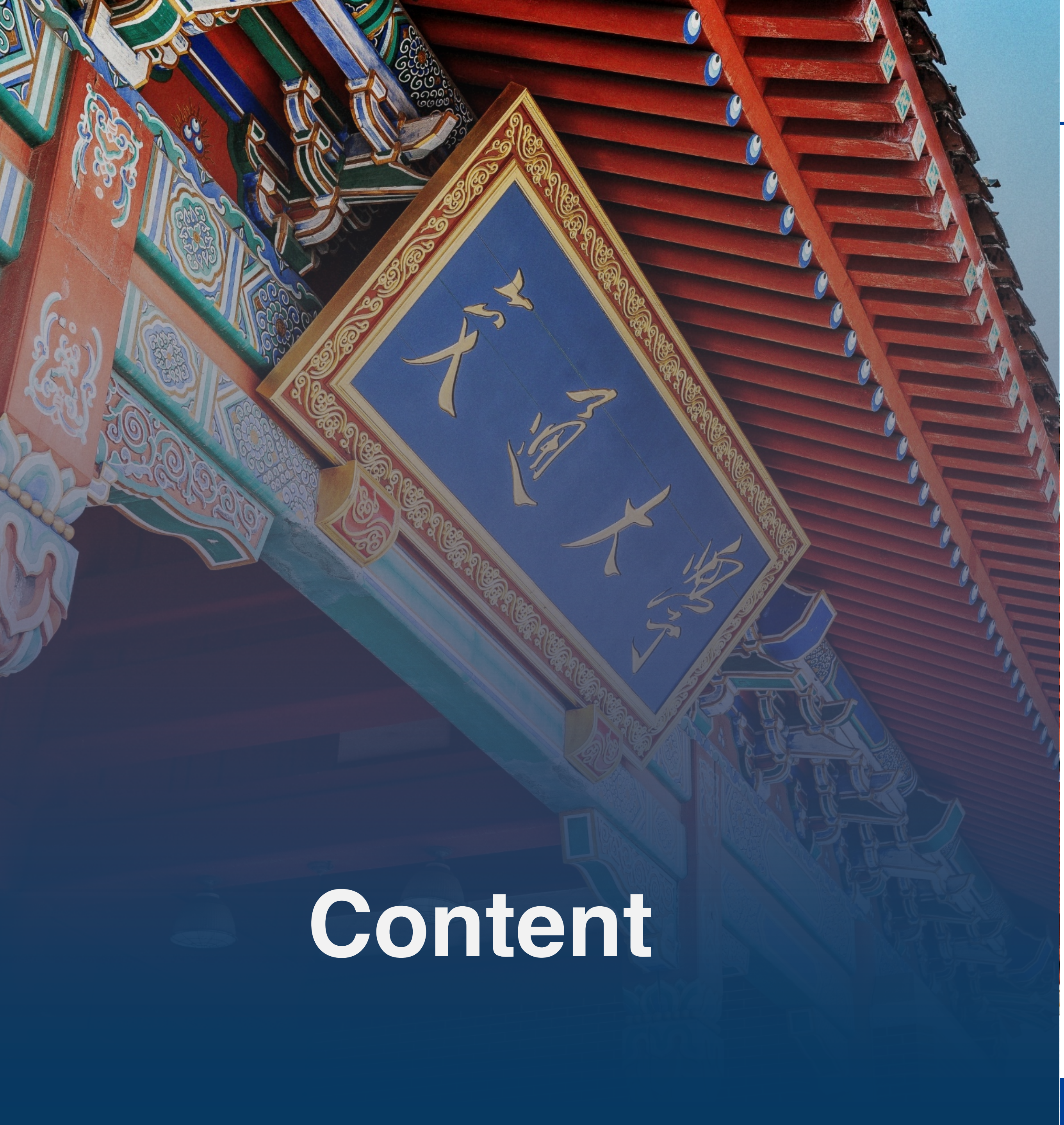


Red: CT18NNLO

Green: NLO

Blue: NLO+RGR

Y. Su et al., Nucl. Phys. B 991, 116201 (2023)



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Summary and Outlook

This work

- Established the factorization of VV (AA) correlation in LaMET framework
- Give the 1-loop kernel of Large momentum expansion for the first time
- Give the preliminary result for PDF from VV correlation

Outlook

- Improve precision: physical pion mass, large momentum, continuum limit…
- Extend to other current-current correlation: VA,….
- Flavour separated PDF from current-current correlations