Proton isovector helicity PDF at NNLO and the twist-3 moment $\tilde{d}_2$ from lattice QCD at physical quark masses

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LaMET 2026 Workshop

Outline

Why the quark helicity PDF $g_1(x)$?

- Leading-twist quark PDFs of the nucleon: *unpolarized*, *transversity*, *helicity*.
- $g_1(x)$: difference of quark densities with spin aligned vs. anti-aligned with a longitudinally polarized proton.
- First moment $\Rightarrow$ quark-spin contribution to proton spin (in both Ji and Jaffe–Manohar decompositions).
- EMC $\rightarrow$ "proton spin puzzle".
- modern global fits: quark spin $\sim 30\%$ of proton spin. Confirmed by lattice QCD (χQCD and ETMC).

Ji, PRD (1997)
Jaffe & Manohar, NPB (2020)
JAM, PRD (2022)
Borsa, et.al, PRL (2024)
ETMC, PRD (2020)
chiQCD, PRD (2022)

Experimental probes

polarized DIS & SIDIS, RHIC *pp*, COMPASS, HERMES;
future: JLab-12 GeV, EIC, EicC.

This work

First-principles lattice QCD determination of the **isovector** $g_1^{u-d}(x)$, its **moments**, and the twist-3 moment $\tilde{d}_2$.

Beyond leading twist: the moment $\tilde{d}_2$

- $g_2(x)$ probes quark–gluon correlations.

Braun, Ji & Vladimirov, JHEP (2021)

- Related to transverse spin structure function $g_T(x) = g_1(x) + g_2(x)$
- Reduced twist-3 moment

$$\tilde{d}_2 \equiv \int_{-1}^{1} dx\, x^2 \left[3g_T(x) - g_1(x) \right] = \int [dx]\, S_-(x_1, x_2, x_3).$$

- Semiclassical interpretation: average transverse **color Lorentz force** on the struck quark.
- Experimentally demanding (g_2, $\tilde{d}_2$ less precise than leading twist).

Strategy of this work

Extract $\tilde{d}_2$ in the $\overline{\text{MS}}$ scheme *for the first time* via a short-distance factorization (SDF) of nonlocal correlators (Braun–Ji–Vladimirov framework), complementing local twist-3 determinations that suffer from power-divergent mixing.

The LaMET framework

Ji, PRL (2013)
Ji, SCPMA (2014)
Ji, et. al, RMP (2021)

- Light-cone PDFs $\Rightarrow$ not directly computable in Euclidean lattice QCD.
- **LaMET (Ji):** Universality class of Euclidean lattice correlators (quasi-distributions) sharing the same IR as light-cone PDFs.
- Matched perturbatively in Bjorken-x space; power corrections $\sim \Lambda_{\rm QCD}^2/(2xP_z)^2$.

$$g_1(x,\mu) = \int C(x,y,\mu,P_z)\,\tilde{g}_1(y,P_z,\mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(2xP_z)^2}\right)$$

Recent theory developments included

- Hybrid renormalization + leading renormalon resummation (LRR)
- NNLO matching kernel
- RG resummation (RGR) + threshold resummation (TR)
- SDF for z-space matrix elements

Lattice setup

- $n_f = 2+1$ HISQ sea (HotQCD), **physical** quark masses.
- Clover-Wilson valence quarks on HYP-smeared gauge fields.
- $m_\pi \approx 140$ MeV.
- Boosts $P_z = \frac{2\pi}{aN_\sigma}\{0,1,4,6\} = \{0, 0.25, 1.02, 1.53\}$ GeV.
- $t_{\rm sep}/a = 6, 8, 10, 12$ (12 only for g_A).
- AMA (exact + sloppy), multigrid QUDA, Coulomb-gauge momentum smearing.

β	a[fm]	m_π[GeV]	N_σ	N_τ	#conf
7.13	0.076	0.14	64	64	350

Table 1: Ensemble parameters.

Highlight

Computed at the **physical point** on a **fine** lattice ($a = 0.076$ fm) — achieving both simultaneously remains challenging.

Nonlocal matrix elements

Spatially separated quark bilinear with a straight Wilson line:

$$O_\Gamma(z) \equiv \bar{\psi}(z\hat{z})\,\Gamma\, W(z\hat{z},0)\,\psi(0).$$

Twist-2 helicity ($\Gamma = i\gamma_5\gamma_z$)

$$\tilde{h}_1^B(z, P_z, a) = \frac{1}{2E}\langle P, S|O_\Gamma|P, S\rangle$$

Twist-3 transverse ($\Gamma = i\gamma_5\gamma_x$)

$$\tilde{h}_T^B(z, P_z, a) = \frac{1}{2m_N}\langle P, S|O_\Gamma|P, S\rangle$$

- Isovector combination $u - d \Rightarrow$ no disconnected diagrams.
- Normalizations remove trivial boost factors from external spinors.
- Extracted from 3pt/2pt ratios; symmetry in $z \to -z$ (Re even, Im odd) used to average $\pm z$.

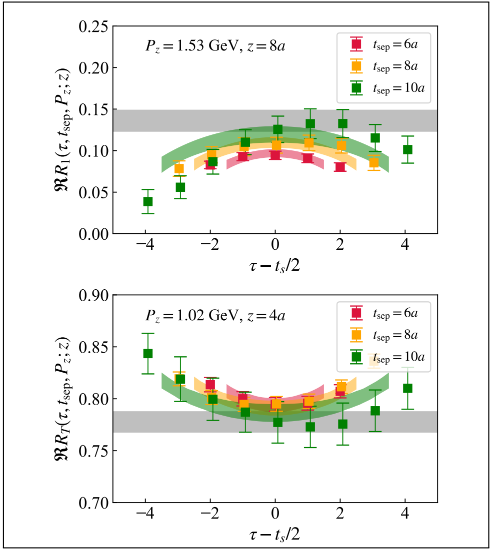
Extracting bare matrix elements: two-state fits

$$R_{1/T}(\tau, t_{\rm sep}, P_z; z) \equiv \frac{C^{\Gamma}_{\rm 3pt}}{C_{\rm 2pt}} \xrightarrow[t_{\rm sep}\gg\tau\gg 0]{} \tilde{h}^{B}_{1/T}(z, P_z)$$

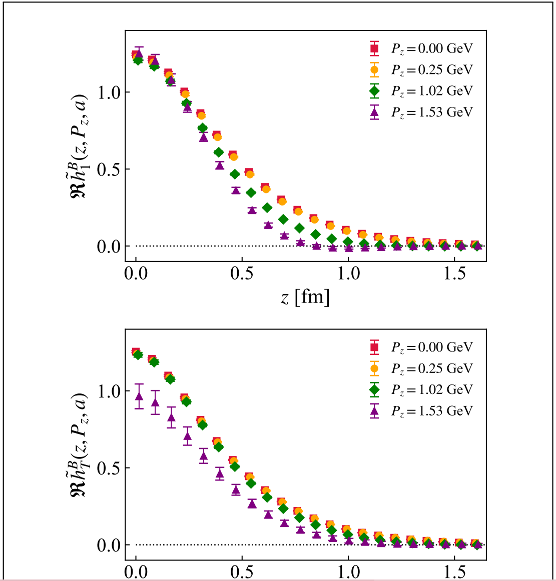
- E_n, amplitudes from 2-state 2pt fits, fed into 3pt analysis.
- Omit 2 time slices near source/sink to suppress excited-state contamination.
- Uncorrelated fits (large condition numbers), bootstrap errors.
- $-32a \leq z \leq 32a$.

Figure

R_1 at $P_z = 1.53$ GeV and R_T at $P_z = 1.02$ GeV — data well described by 2-state ansatz.



Bare matrix elements $\tilde{h}^B_{1,T}(z, P_z, a)$



- At $z = 0$: all boosts & channels agree $\Rightarrow$ local axial current, $g_A = Z_A \tilde{h}^B(0)$.
- Twist-3 signal degrades at large P_z: extra m_N/E kinematic suppression.
- Clear momentum dependence: $|\tilde{h}^B_{1,T}|$ falls faster in z as P_z increases.
- Axial charge: $g_A^{u-d} = 1.211(12)$.

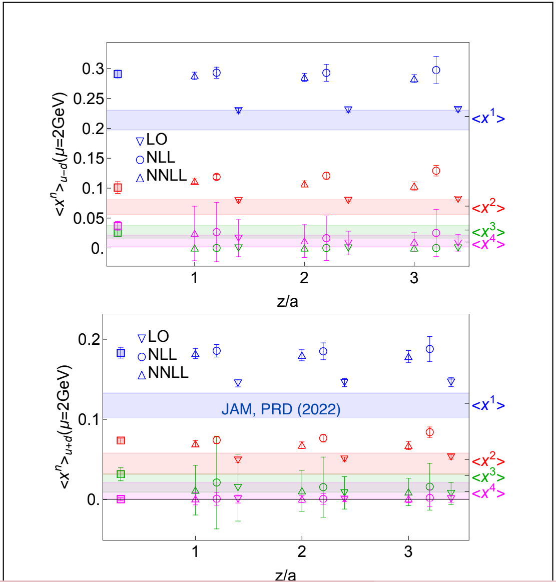
RG-invariant ratio & OPE

Multiplicative renormalizability $\Rightarrow$ RG-invariant ratio cancels UV:

$$\mathcal{M}_1(\lambda, z^2) \equiv \frac{\tilde{h}_1^B(z, P_z)}{\tilde{h}_1^B(z, P_z^0)} \frac{\tilde{h}_1^B(0, P_z^0)}{\tilde{h}_1^B(0, P_z)} = \frac{\tilde{h}_1^{\overline{\text{MS}}}(\lambda, z^2\mu^2)}{\tilde{h}_1^{\overline{\text{MS}}}(\lambda_0, z^2\mu^2)}, \quad P_z^0 = 0.$$

- OPE: $\mathcal{M}_1 = \dfrac{\sum_n C_n(\mu^2 z^2) \frac{(-izP_z)^n}{n!} \langle x^n \rangle}{\sum_n C_n(\mu^2 z^2) \frac{(-izP_z^0)^n}{n!} \langle x^n \rangle} + \mathcal{O}(\Lambda_{\rm QCD}^2 z^2)$.
- $\langle x^0 \rangle = g_A$; Re part $\rightarrow$ even moments, Im part $\rightarrow$ odd moments.
- Wilson coefficients: NLO known; NNLO from unpolarized γ_z.
- Fit moments near physical scale $\mu_0 \sim 2\kappa e^{-\gamma_E}/z$, then RG-evolve to $\mu = 2$ GeV (anomalous dims to 3-loop; β to 5-loop).

Helicity moments at $\mu = 2$ GeV



Results (fit region $z \in [1, 3]a$)

$$\langle x \rangle^{u-d} = 0.291(7), \quad \langle x^2 \rangle^{u-d} = 0.101(10)$$

$$\langle x \rangle^{u+d} = 0.183(6), \quad \langle x^2 \rangle^{u+d} = 0.073(3)$$

- NLL $\approx$ NNLL $\Rightarrow$ good perturbative convergence.
- Higher than moments from JAM22 global fits.
- Errors: statistics + scale variation $\kappa \in [1/\sqrt{2}, \sqrt{2}]$.

Short-distance factorization for $\tilde{d}_2$

Transverse RG-invariant ratio:

$$\mathcal{M}_T(\lambda, z^2) \equiv \frac{\tilde{h}_T^B(z, P_z)}{\tilde{h}_T^B(z, 0)} \frac{\tilde{h}_T^B(0, 0)}{\tilde{h}_T^B(0, P_z)}.$$

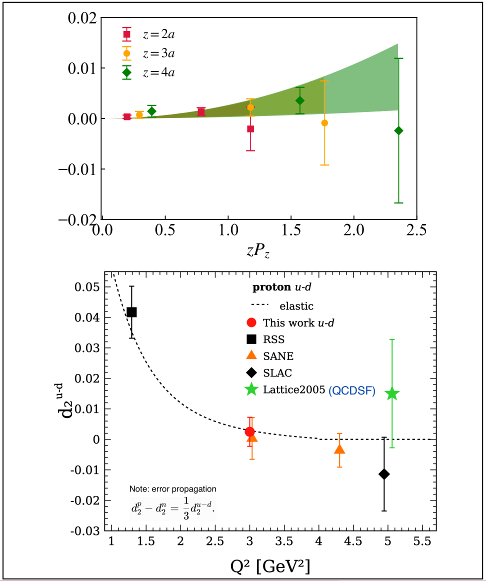
NLO matching (Braun–Ji–Vladimirov) — $\tilde{d}_2$ enters at $\mathcal{O}(\lambda^2)$:

$$\tilde{h}_T^{\overline{\text{MS}}}(\lambda, z^2\mu^2) = \int_0^1 d\alpha\, \tilde{h}_1^{\overline{\text{MS}}}(\alpha\lambda, z^2\mu^2) - \frac{\alpha_s C_F}{\pi} a_0 + i\lambda \frac{\alpha_s C_F}{3\pi} a_1$$
$$+ \frac{\lambda^2}{3}\left\{\tilde{d}_2 + \tfrac{7\alpha_s C_F}{24\pi} a_2 + \dots\right\} + \mathcal{O}(\lambda^3).$$

Braun, Ji & Vladimirov, JHEP (2021)

- Naive WW subtraction does *not* remove twist-2 at NLO ($C_T^{(1)} \neq C_1^{(1)}$) $\Rightarrow$ calculable a_n remainder.
- Genuine twist-3 $\tilde{d}_2$ appears with a perturbative coefficient $\Rightarrow$ clean lever arm.

Extracting $\tilde{d}_2$



Build the subtracted, twist-2–improved combination $\mathcal{M}_T^{\mathrm{tw3}}(\lambda, z^2\mu^2) \Rightarrow$ Re part $\propto \lambda^2$ at small λ, coefficient controlled by $\tilde{d}_2$.

First $\overline{\mathrm{MS}}$ result

$$\tilde{d}_2^{u-d}(\mu = 2\ \mathrm{GeV}) = 0.0024(46)$$

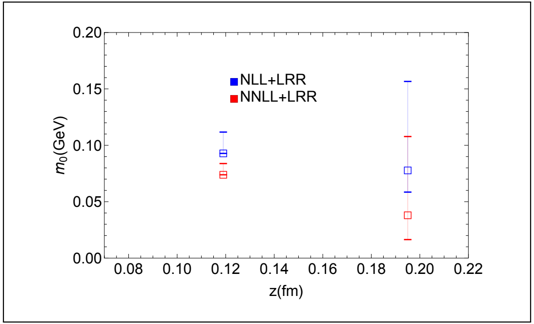
(statistical error; systematics to be studied)

- Combined fit over $z \in [2a, 4a]$, full NLO kernel.
- Consistent with zero $\Rightarrow$ strong suppression of the lowest chiral-even twist-3 correlation.
- Compatible with polarized-DIS extractions & model studies.

Hybrid renormalization + renormalon resummation

$$\tilde{h}_1^R(z, P_z) = \begin{cases} \dfrac{\tilde{h}_1^B(z, P_z, a)}{\tilde{h}_1^B(z, 0, a)}, & |z| \leq z_s \\ \dfrac{\tilde{h}_1^B(z, P_z, a)}{\tilde{h}_1^B(z_s, 0, a)} e^{(\delta m + m_0)(z - z_s)}, & |z| \geq z_s \end{cases}$$

- Ratio scheme at small z; linear + log UV divergence subtracted at large z.
- $a\,\delta m = 0.1597(16)$ from static potential.
- Renormalon ambiguity in δm removed via LRR (principal-value) $\Rightarrow m_0(\tau)$ in same scheme as matching.



Wilson coefficients $C_0(z^2, \mu^2)$ improved with LRR and RGR .

Huo, etl.al, NPB (2021)
Zhang, et.al, PLB (2023)
Su, et.al, NPB (2022)

Asymptotic extrapolation & Fourier transform

Form 1 (Regge / dispersive)

$$\tilde{h}_1^{\text{asy}}(\lambda, P_z) = \frac{A\, e^{-m_{\text{eff}}\lambda/P_z}}{|\lambda|^d}$$

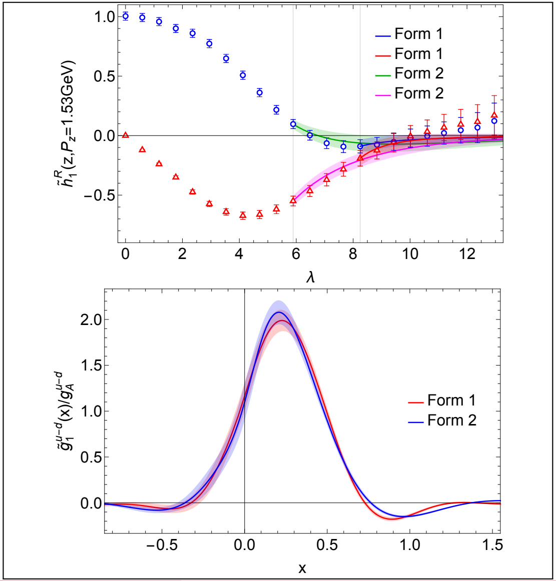
$m_{\text{eff}} \approx 0.77(60)$ GeV, $\chi^2/\text{dof} = 1.47$. Ji, et.al, NPB (2021)

Form 2 (rigorous large-z)

$$\tilde{h}_1^{\text{asy}} = \left(Ae^{i\phi\,\text{sgnz}} + \frac{A' e^{i\phi'\text{sgnz}}}{|z|}\right) e^{-|z|\Lambda}$$

$\Lambda \approx 0.47(11)$ GeV, $\chi^2/\text{dof} = 0.78$. Ji, et.al, (2026)

- Extrapolation uncertainty bounded, $\propto 1/x$.
- Discrepancy between Form 1 & 2 kept as systematic.



Perturbative matching with resummations

- NNLO matching kernel in $\overline{\mathrm{MS}}$; hybrid-scheme NNLO correction following the unpolarized case.
- Large logs require resummation:
  - **RGR** for $x \to 0$: $\mu_h = 2xP_z$ soft.
  - **Threshold (TR)** for $x \to 1$: $\mu_i = 2(1-x)P_z$ soft.
- LRR implicitly applied to cancel the linear power correction.

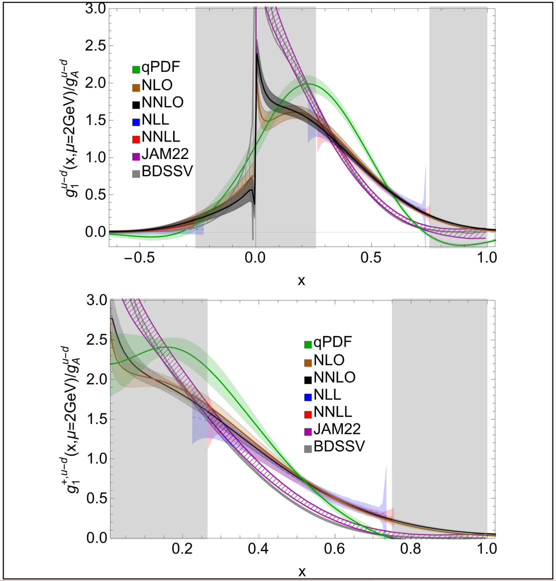
$$C(x, y, \mu, P_z) = \mathcal{P}_C(\mu, \mu_h) \otimes C(\mu_h, P_z), \quad C \xrightarrow{x \to y} S(\mu_h) \otimes H(\mu_h).$$

Reliable window

Both $2xP_z$ and $2(1-x)P_z$ perturbative $\Rightarrow$ trustworthy region $x \in [0.25, 0.75]$ ($x_0 \propto \Lambda_{\mathrm{QCD}}/2P_z$), improvable at higher P_z.

Zhang, et.al, PLB (2023)
Su, et.al, NPB (2022)
Ji, etl.al, JHEP (2023)
Ji, et.al, JHEP (2025)

x-dependent isovector helicity PDF



- NLL & NNLL consistent; NNLO reduces scale variation in perturbative region.
- More elevated distribution in mid-x ($x \in [0.4, 0.7]$) vs. global fits.
- Good agreement with Coulomb-gauge (no Wilson line) result for $x < 0.6$; discrepancy for $x > 0.6$ likely power corrections at $P_z \approx 1.5$ GeV.

Higher P_z (e.g. kinematically enhanced interpolators) needed to extend the reliable range & reduce power corrections.

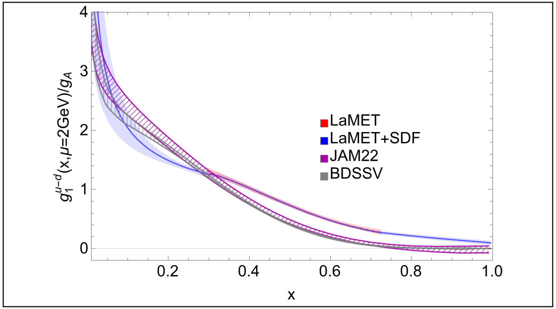
Constraining endpoints with SDF

Model endpoints, keep LaMET prediction in mid-x:

Ji, Research (2025)
Holligan, et.al, NPB (2023)

$$g_1(x) = \begin{cases} f(x_0)\, x^a / x_0^a, & x < x_0 \\ f(x), & x_0 < x < 1 - x_0 \\ f(1-x_0)(1-x)^b/(1-x_0)^b, & x > 1 - x_0 \end{cases}$$

- Fit RG-invariant ratio at $z = \{0.076, 0.152, 0.228\}$ fm, $x_0 = 0.3$, NNLO kernel.
- g_1^+: $a = -0.38(9)$, $b = 1.68(87)$, $\chi^2/\text{dof} = 0.16$.
- g_1^-: $a = -0.10(83)$, $b = 0.15(22)$, $\chi^2/\text{dof} = 1.21$.



Endpoint regions still subject to model / cutoff-x_0 uncertainties (not yet quantified).

Summary

- Physical-point, fine-lattice ($a = 0.076$ fm) study of the proton spin-dependent quark structure.
- **Helicity moments** via OPE of RG-invariant ratios (NNLO):

$$\langle x \rangle^{u-d} = 0.291(7), \quad \langle x^2 \rangle^{u-d} = 0.101(10).$$

- **First SDF extraction** of the twist-3 moment:

$$\tilde{d}_2^{u-d}(2\text{ GeV}) = 0.0024(46) \ \ \text{(consistent with 0)}.$$

  SDF isolates a genuine twist-3 moment from nonlocal correlators, avoiding power-divergent mixing of local operators.
- x-**dependent** $g_1^{u-d}(x)$ in LaMET with hybrid+LRR renormalization, NNLO matching, RGR+TR $\Rightarrow$ reliable $x \in [0.25, 0.75]$; endpoints modeled with SDF.

Outlook

- Tighter control of lattice systematics:
  - Multiple lattice spacings (continuum limit).
  - Increased statistics & improved excited-state control (See Yushan's talk).
- Higher P_z (kinematically enhanced interpolators, see Daniel's talk) to expand the LaMET window in x and reduce power corrections.
- Cross-check systematics in the LaMET calculation with other operators such as the CG method and the current-current operator (See Jialu's talk), and with momentum space method (See Rui's talk)
- Higher precision and full quantification of systematics for $\tilde{d}_2$.
- More flexible endpoint parametrizations & finer lattices to reduce model dependence.

Thank you!