The Collins-Soper kernel from a vacuum soft function

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with

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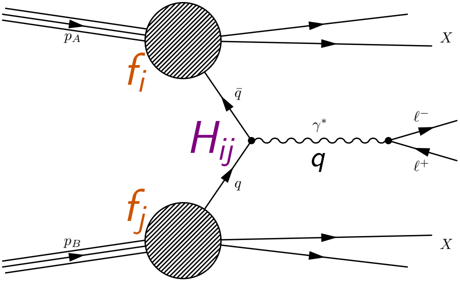
TMD factorization in Drell-Yan scattering



- $\Lambda_{\mathrm{QCD}} \lesssim |\vec{q}_\perp| \ll Q = \sqrt{q^2}$
- Rapidity divergences in TMDPDFs need to be regulated
- Collins-Soper scale, $\zeta_{a,b}$, dependence

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y\,\mathrm{d}^2\vec{q}_\perp} = \sum_{i,j} H_{ij}(Q,\mu) \int \mathrm{d}^2\vec{b}_\perp e^{i\vec{b}_\perp\cdot\vec{q}_\perp} f_i\left(x_a, \vec{b}_\perp, \mu, \zeta_a\right) f_j\left(x_b, \vec{b}_\perp, \mu, \zeta_b\right)$$

$$\times \left[1 + \mathcal{O}\left(\frac{q_\perp^2}{Q^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)\right] \qquad \zeta_{a,b} = 2(x_{a,b}P^+e^{-y_n})^2$$

TMDPDF, soft factor, and CS kernel

$$f_i\left(x_a, \vec{b}_\perp, \mu, \zeta_a\right) = \lim_{\substack{\epsilon\to 0 \\ y_B\to-\infty}} Z_{\mathrm{UV}}\left(\mu, \zeta_a, \epsilon\right) \frac{f_i^{\mathrm{bare}}\left(x_a, \vec{b}_\perp, \epsilon, y_B, x_a P^+\right)}{\sqrt{S_i\left(b_\perp, \epsilon, 2y_n - 2y_B\right)}}$$

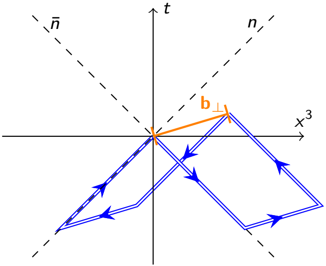
Evolution kernel:

$$\gamma_\mu^q(\mu, \zeta) = \frac{\mathrm{d}\log f_q\left(x, \vec{b}_\perp, \mu, \zeta\right)}{\mathrm{d}\log\mu}$$

Collins-Soper (CS) kernel:

$$\gamma_q(b_\perp, \mu) = \frac{\mathrm{d}\log f_q\left(x, \vec{b}_\perp, \mu, \zeta\right)}{\mathrm{d}\log\sqrt{\zeta}}$$

$$S_q\left(b_\perp, y_A, y_B, \mu\right) = S_I\left(b_\perp, \mu\right) e^{\gamma_q(b_\perp, \mu)(y_A - y_B)}\left(1 + \mathcal{O}\left(e^{-2(y_A - y_B)}\right)\right)$$



Regulate rapidity divergence with spacelike Wilson lines:

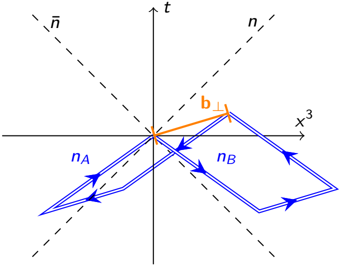
$$y_A, -y_B \to \infty,$$

$$n_A \equiv n - e^{-y_A}\bar{n},$$
$$n_B \equiv \bar{n} - e^{+y_B} n$$

Soft function defined on the lightcone:

$$n^2 = \bar{n}^2 = 0$$

$$n = (1, 0, 0, 1),$$
$$\bar{n} = (1, 0, 0, -1)$$



In the LaMET framework:

$$
\begin{aligned}
&\sqrt{S_I\left(b_\perp,\mu\right)}\,\tilde{f}_\Gamma\left(x,b_\perp,P^z,\mu\right) \\
&\quad = f\left(x,b_\perp,\mu,\zeta\right) H_f\left(x,P^z,\mu\right)\exp\left[\frac{1}{2}\log\frac{\left(2xP^z\right)^2}{\zeta}\gamma_q\left(b_\perp,\mu\right)\right] \\
&\qquad + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{xP^z},\frac{1}{b_\perp(xP^z)}\right)
\end{aligned}
$$

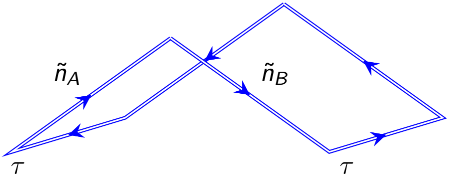
[Ebert, *et. al.*, 2019], [Ji, Liu, Liu, 2019], [Ebert, *et. al*, 2022]

- Intrinsic soft function: S_I [Ji, Liu, Liu, 2020]
- quasi-TMD beam function: $\tilde{f}_\Gamma$
- TMDPDF: f
- Perturbative hard kernel: H_f
- CS kernel: γ_q

Motivated by a proposal by Liu in [Ji, Liu, Liu, 2019]

Euclidean space directional vectors with purely imaginary time components

$$\tilde{n}_A = \left(in_A^0, 0, 0, n_A^3\right), \quad \tilde{n}_B = \left(in_B^0, 0, 0, n_B^3\right)$$



$$r_a = \frac{n_A^3}{n_A^0} = \frac{1+e^{-2y_A}}{1-e^{-2y_A}}, \quad r_b = \frac{n_B^3}{n_B^0} = \frac{1+e^{2y_B}}{1-e^{2y_B}}$$

Auxiliary field definition of the Wilson line

Write Wilson line in terms of one dimensional 'fermions' that live along the path:

$$P\exp\left\{-ig\int_{s_i}^{s_f} \mathrm{d}s n^\mu A_\mu(y(s))\right\}$$
$$= Z_\psi^{-1}\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\,\psi\bar{\psi}\exp\left\{ig\int_{s_i}^{s_f}\mathrm{d}s\bar{\psi}i\partial_s\psi - \bar{\psi}n\cdot A\psi\right\}$$

[Gervais, Nevau 1980], [Aref'eva 1980]

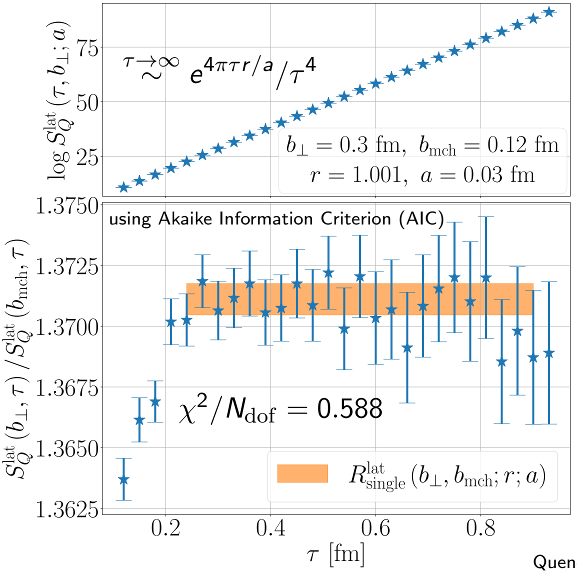
Auxiliary field propagator:

$$in\cdot DH_n(y) = \delta(y) \xrightarrow{\text{Euclidean space}} -i\tilde{n}\cdot D_E H_{\tilde{n}}(y) = \delta(y), \qquad \tilde{n} = (in_0, \vec{n})$$

Meaningful solution only obtained with a UV cutoff [Aglietti, *et. al.* 1992], [Aglietti, 1994]

Perturbative computation of soft factor with auxiliary field propagators requires further investigation [Liu, 2022]

Single ratio, time dependence



$$R^{\rm lat}_{\rm single}(b_\perp, b_{\rm mch}, r, a) = \lim_{\tau\to\infty} \frac{S_Q^{\rm lat}(b_\perp, \tau; r, r; a)}{S_Q^{\rm lat}(b_{\rm mch}, \tau; r, r; a)}$$

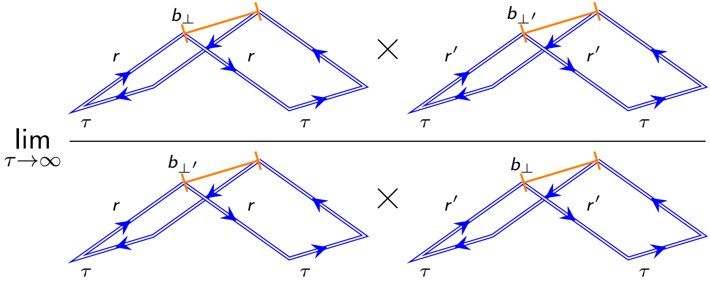
$$b_\perp = 0.3 \text{ fm}$$
$$b_{\rm match} = 0.12 \text{ fm}$$
$$r = 1.001$$

Quenched configurations from [Detmold, Endres, 2018]

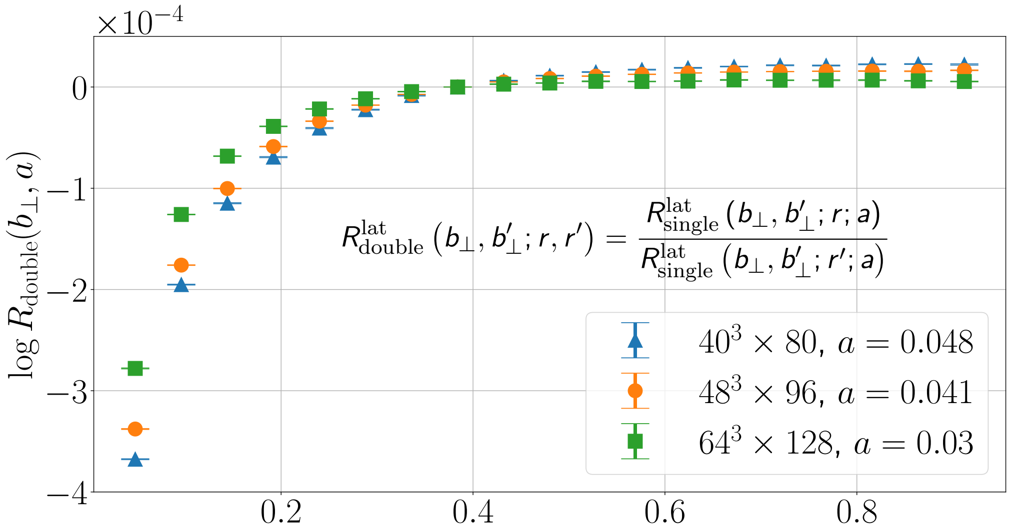
Treatment of divergences

$$R^{\text{lat}}_{\text{double}}\left(b_\perp, b'_\perp; r, r'\right) = \frac{R^{\text{lat}}_{\text{single}}\left(b_\perp, b'_\perp; r; a\right)}{R^{\text{lat}}_{\text{single}}\left(b_\perp, b'_\perp; r'; a\right)} = e^{\Delta\gamma_q(b_\perp, b'_\perp)\Delta Y(r,r')}$$

$$\Delta\gamma_q(b_\perp, b'_\perp) = \gamma_q\left(b_\perp, a\right) - \gamma_q\left(b'_\perp, a\right), \qquad \Delta Y(r, r') = 2\log\left(\frac{r-1}{r+1}\Big/\frac{r'-1}{r'+1}\right)$$



Interpolated double ratio, $b_\perp$ dependence



$b_{\mathrm{match}} = 0.384$ fm, $r_1 = 1.002$, $r_2 = 1.001$

Matching procedure for double ratio

$$\gamma_q\left(b_{\perp,1},\mu\right) = \gamma_q\left(b_{\perp,2},\mu\right) + \frac{\frac{1}{2}\log\left(R_{\text{double}}\left(b_{\perp,1},b_{\perp,2},\mu,r_1,r_2\right)\right)}{\log\left(\frac{r_1+1}{r_1-1}\Big/\frac{r_2+1}{r_2-1}\right)}$$

- Implies that we need at least one perturbative value of γ_q.
- Another perturbative value of γ_q is needed, because $r_{1,2}$ is renormalized due to $O(4)$ symmetry breaking. [Aglietti, *et. al.* 1992], [Aglietti, 1994]

Model for the double ratio

- Model the double ratio as a piecewise function:

$$\log R^{\text{lat}}_{\text{double}}\left(b_\perp, b_{\text{mch}}; r, r'\right) =$$
$$\Delta Y^{\text{ren}}_i \left(\begin{Bmatrix} \gamma_q^{\overline{\text{MS}}}\left(b_\perp, \mu\right) - \gamma_q^{\text{mch}} & , & b_{\text{cut}} \le b_\perp \le b_{\text{th}} \\ \{d_\ell\} & , & b_\ell = b_\perp > b_{\text{th}} \end{Bmatrix} + c_1 \left(\frac{a^2}{b_\perp^2} - \frac{a^2}{b_{\text{mch}}^2} \right) \right),$$

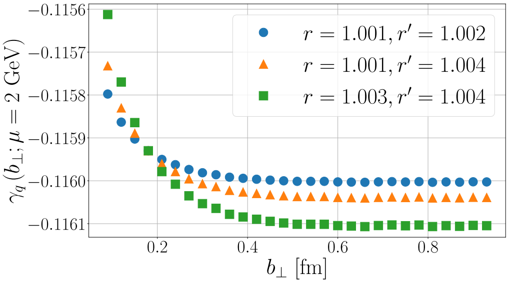
- Combined fit on three ensembles
- Model imposes a constraint on the large-$b_\perp$ region

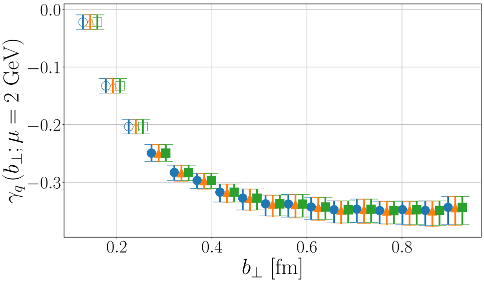
$$b_{\text{cut}} = 0.144 \text{ fm}, \quad b_{\text{th}} = 0.24 \text{ fm}, \quad b_{\text{mch}} = 0.384 \text{ fm} > b_{\text{th}}$$

$$d_\ell = \gamma_q\left(b_\ell, \mu\right) - \gamma_q\left(b_{\text{mch}}, \mu\right), \qquad \gamma_q^{\text{ext}}\left(b_\ell, \mu\right) = \gamma_q^{\text{mch}} + d_\ell$$

- Model does not adequately estimate the lattice spacing dependence

Bare vs renormalized rapidity factor





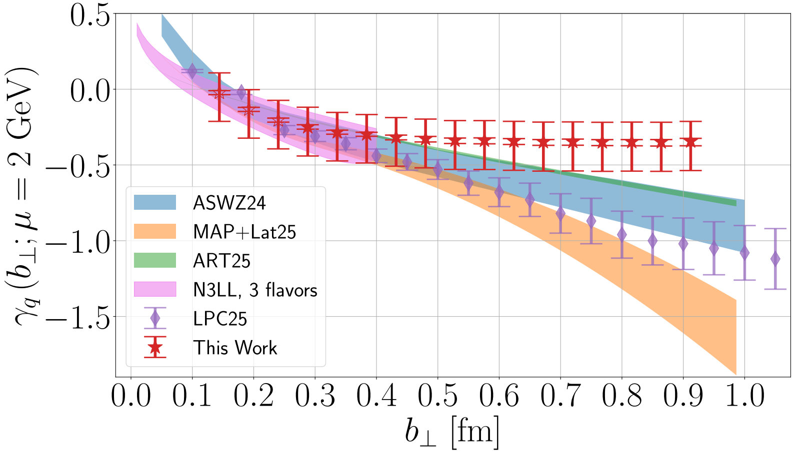
The CS kernel determined on the $a = 0.03$ fm ensemble with three sets of bare rapidities.

- Left plot uses bare rapidity:

$$2(y - y') = \log\left(\frac{r+1}{r-1}\Big/\frac{r'+1}{r'-1}\right)$$

- Right plot from model fit with renormalized rapidity
- Included statistical, truncation and discretization error.

Result



- Our result is pure gauge. Comparison plots have $n_f = 3, 4$.
- Inner error bars include statistical, truncation and discretization errors.
- Outer error bars include UV scale variation uncertainty, $\sqrt{2}\mu$.

- Euclidean space soft function with complex directional vectors maps directly to Minkowski space result
- Lattice data at high statistical precision, low computational cost
- Double ratio method gives: $\gamma_q\left(b_\perp, \mu\right) - \gamma_q\left(b'_\perp, \mu\right)$
- Error primarily from UV scale uncertainty
- Our results are competitive with existing results

Open questions and future direction:

- Ongoing exploration of perturbative results for auxiliary field computation
- Improve systematic uncertainties by simulating at finer lattices or with an alternative scheme for the rapidity renormalization

Thank you!