Brookhaven National Laboratory

Skewness-dependent moments of the pion GPD from nonlocal quark-bilinear correlators

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Based on Phys.Rev.D 113 (2026) 1, 014505

Introduction

- Pion Generalized Parton Distributions (GPDs)

$$H^\pi(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \left\langle \pi(p_f) \middle| \bar\psi\left(-\frac{z^-}{2}\right)\gamma^+ \mathscr{L}\left(-\frac{z^-}{2},\frac{z^-}{2}\right)\psi\left(\frac{z^-}{2}\right) \middle| \pi(p_i)\right\rangle$$

x	$\Delta^\mu = (p_f - p_i)^\mu$	$-t = \Delta^2$	$\xi = \frac{(p_i^+ - p_f^+)}{(p_i^+ + p_f^+)}$	$P^\mu = \frac{(p_i^\mu + p_f^\mu)}{2}$
momentum fraction	momentum transfer	momentum transfer squared	skewness	average hadron momentum

$f_{\mathrm{GPD}}(x, \mathbf{b_T})$ → ($b_\perp \leftrightarrow \Delta_\perp$, FT) → $H(x,0,t)$ ← ($\xi = 0$) ← $H(x,\xi,t)$

Impact-Parameter Distributions

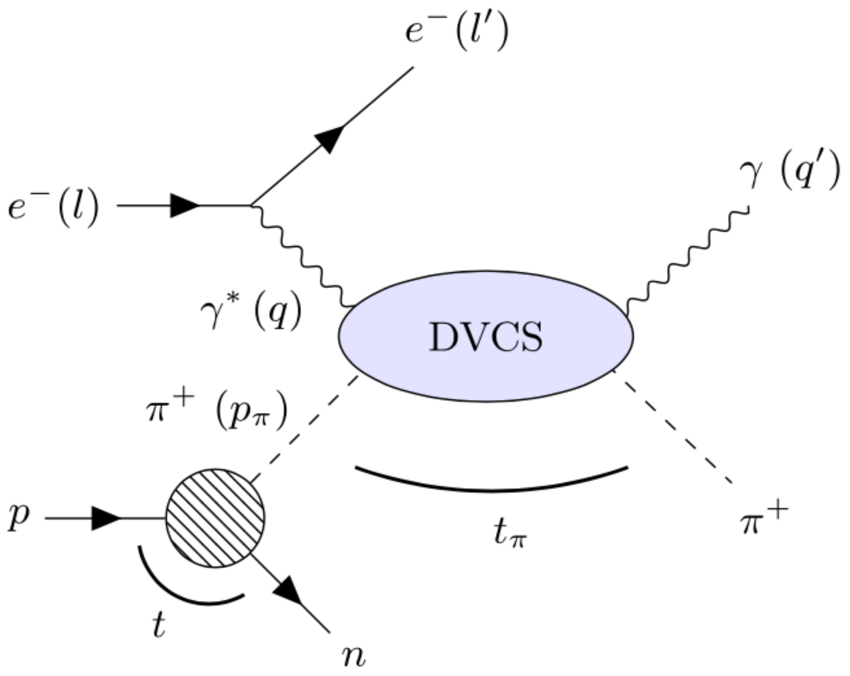
$H(x,0,t)$ → $\int dx$ → Form Factor $A_{1,0}$ (FF)

$H(x,\xi,t)$ → $\int_{-1}^{1} dx\, x^{n-1}$ → Generalized form factors $A_{n,k}$ (GFFs)

n-th Mellin moments

Experimental access to Pion GPDs

- No fixed pion target → no direct $e\pi$ scattering
- Pion GPDs are accessible only **indirectly** (via pion pole deep virtual Compton scattering)



Chávez et al., PRL 128 (2022) 202501

- No experimental data available yet.
- Future prospects: JLab 12 GeV, EIC, AMBER, EicC.

From first principles: lattice QCD

Pion GPDs from lattice QCD

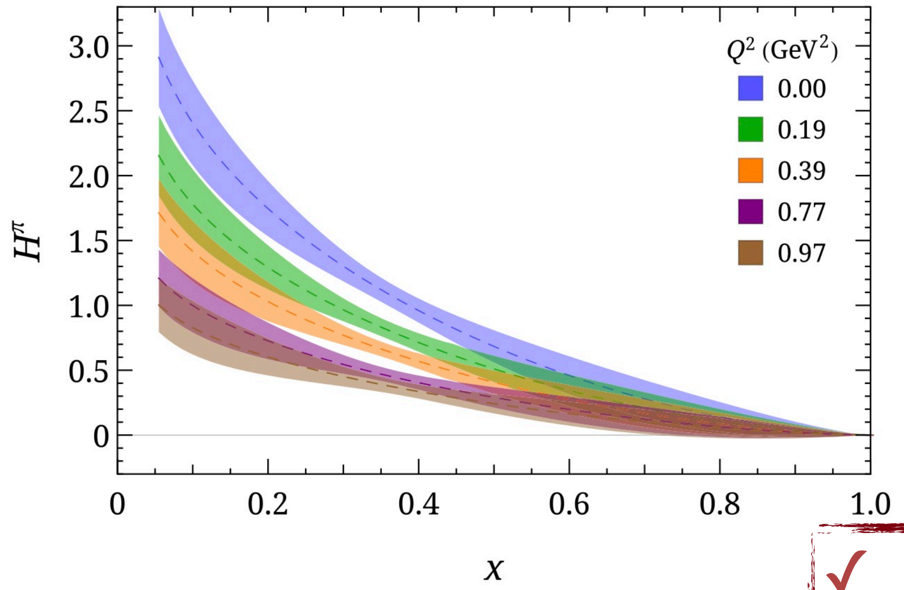
- Mellin moments of pion GPD from **local operators**

[1] QCDSF/UKQCD Collab., *PoS* LAT2005 (2006);
[2] QCDSF/UKQCD Collab., EPJC 51 (2007).
…

$$\langle \pi^+(p') | \mathcal{O}^{\mu\mu_1\mu_2\cdots\mu_n} | \pi^+(p)\rangle = \langle \pi^+(p') | \bar{u}(0)\gamma^{\{\mu} i\overleftrightarrow{D}^{\mu_1} i\overleftrightarrow{D}^{\mu_2}\cdots i\overleftrightarrow{D}^{\mu_n} u(0) | \pi^+(p)\rangle$$

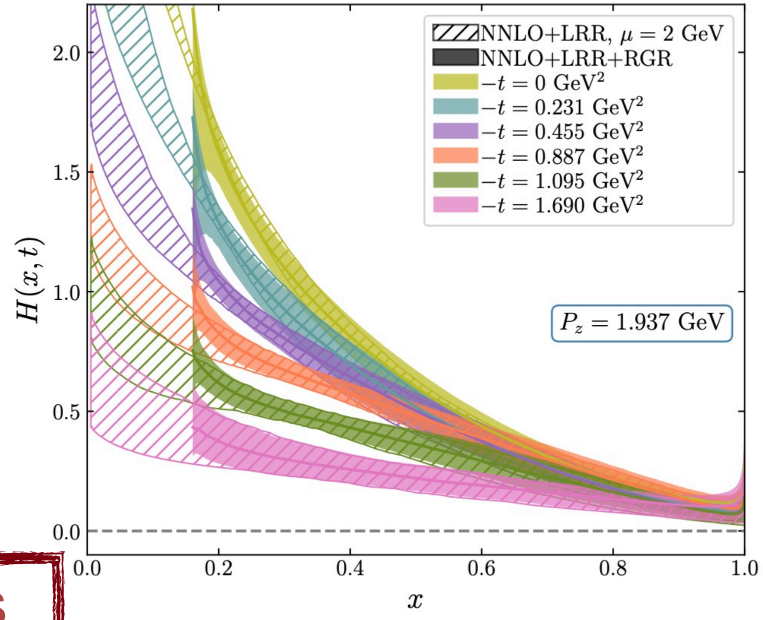
$$= 2P^{\{\mu}P^{\mu_1}\ldots P^{\mu_n\}}A_{n+1,0}(\Delta^2) + 2\sum_{i=1,odd}^{n} \Delta^{\{\mu}\Delta^{\mu_1}\ldots\Delta^{\mu_i}P^{\mu_{i+1}}\ldots P^{\mu_n\}}A_{n+1,i+1}(\Delta^2)$$

- LaMET approach to x-dependent GPDs



H.-W. Lin, Phys. Lett. B 846, 138181 (2023)

✓ Zero skewness



H.T. Ding et al., JHEP 02, 056 (2025)

Moments from nonlocal correlators

T. Izubuchi et al., PRD 98, 056004 (2018); Y.-S. Liu et al., PRD 100, 034006 (2019).

- Operator product expansion (OPE) of nonlocal operators:

☑ Pion and nucleon **PDFs**

X. Gao et al., *PRD* 109 (2024); X. Gao et al., *PRD* 107 (2023); X. Gao et al., *PRD* 104 (2021); J. Karpie et al., JHEP 11, 178 (2018); B. Joo et al., PRD 100, 114512 (2019) ; …

$$\tilde{Q}(\lambda, \mu^2 z^2) = \sum_n C_n(\mu^2 z^2) \frac{(-i\lambda)^n}{n!} a_{n+1}; \quad a_{n+1} = \int_{-1}^{1} dx\, x^n q(x, \mu)$$

Joshua's talk; Joseph's talk

☑ Pion and nucleon **GPDs** (zero skewness)

S. Bhattacharya et al., JHEP 01, 146 (2025);
X. Gao et al., PRD 109, 094506 (2024);
H.-T. Ding et al., PoS SPIN2023, 024 (2024);
X. Gao et al., PRD 104 (2021) ; …

$$\tilde{H}(z, P_z, t) = \sum_n C_n(\mu^2 z^2) \frac{(-i\lambda)^n}{n!} A_{n,0}(t); \quad A_{n,0}(t) = \int_{-1}^{1} dx\, x^{n-1} H(x, t, \mu)$$

☑ Only nucleon **GPDs** (**nonzero** skewness)

$$\widetilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^{k} H_n(\xi, t, \mu)\, \xi^{k-n} c_{k,k-n}(z^2\mu^2); \quad H_n(\xi, t, \mu) = \int_{-1}^{1} dx\, x^{n-1} H(x, \xi, t, \mu)$$

- $H_3 = A_{3,0} + \xi^2 A_{3,2}$ **becomes larger as** ξ **increases.**
- Only LL-resummed results were considered.

Opposite trend for pion case!!

Value at t = 0

GPD H^{u-d}		GPD E^{u-d}	
$A_{1,0}$: 0.974^{+12}_{-5}		$B_{1,0}$: 3.40^{+7}_{-1}	
$A_{2,0}$: 0.206^{+2}_{-6}		$B_{2,0}$: 0.370^{+9}_{-24}	
$A_{3,0}$: 0.064^{+2}_{-6}	$A_{3,2}$: 0.39^{+11}_{-3}	$B_{3,0}$: 0.063^{+24}_{-8}	$B_{3,2}$: 1.1^{+4}_{-8}
$A_{4,0}$: 0.065^{+5}_{-19}	$A_{4,2}$: 0.5^{+3}_{-3}	$B_{4,0}$: 0.06^{+16}_{-2}	$B_{4,2}$: > 1.1

HadStruc Collab., JHEP 02, 056 (2025)

Theoretical framework

- OPE of nonlocal operators:

$$\widetilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^{k} H_n(\xi, t, \mu)\, \xi^{k-n}\, c_{k,k-n}(z^2\mu^2) + \mathcal{O}\left(\Lambda_{\text{QCD}}^2 z^2\right)$$

✓ Renormalized matrix elements

✓ n-th moment of pion GPDs.

✓ Perturbative matching matrix (non-diagonal)

Valence-quark pion matrix element and its LI decomposition

$$\widetilde{H}^R(z,P_z,\xi,t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^{k} H_n(\xi,t,\mu)\,\xi^{k-n}\, c_{k,k-n}(z^2\mu^2) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2 z^2\right)$$

✓ Renormalized matrix elements

Ratio scheme:

$$\widetilde{H}^R(z,P_z,\xi,t) = \frac{\widetilde{H}(z,P_z,\xi,t)}{\widetilde{H}(z,0,0,0)}$$

A. Radyushkin, Phys. Rev. D 98, 014019 (2018)

- Valence-quark matrix element:

$$M^\mu(z,P,\Delta) = \left\langle \pi(p_f) \middle| \bar\psi(-z/2)\gamma^\mu L(-z/2,z/2)\psi(z/2) \middle| \pi(p_i)\right\rangle^{u-d}$$

- Lorentz-invariant (LI) amplitudes $A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$:

$$M^\mu(z,P,\Delta) = P^\mu A_1 + z^\mu m^2 A_2 + \Delta^\mu A_3$$

- Traditional quasi-GPD defined with γ^0 is **frame dependent**:

$$\widetilde{H}(z,P_z,\xi,t) = \frac{1}{P^0} M^0(z,P,\Delta) = A_1 + \frac{\Delta^0}{P^0} A_3$$

S. Bhattacharya et al., PRD 106 (2022)

- LI quasi-GPD: combining $M^0\,(\gamma^0)$ and $M^x\,(\gamma^x)$

$$\widetilde{H}_{\mathrm{LI}}(z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_1 + \frac{z\cdot\Delta}{z\cdot P} A_3$$

Consistency with the γ^0 definition will be shown later!

Polynomiality of GPD moments

$$\widetilde{H}^R(z,P_z,\xi,t) = \sum_{k=1}^{\infty}\frac{(-izP_z)^{k-1}}{(k-1)!}\sum_{n=1}^{k} H_n(\xi,t,\mu)\,\xi^{k-n}\,c_{k,k-n}(z^2\mu^2)+\mathcal{O}\left(\Lambda_{\text{QCD}}^2 z^2\right)$$

✓ n-th moment of pion GPDs.

Polynomiality of GPD moments (ξ-dependent):

$$H_n(\xi,t) = \sum_{\substack{k=0\\ \text{even}}}^{n-1} A_{n,k}(t)\,(2\xi)^k \pm \text{mod}(n+1,2)\,(2\xi)^n\,C_n(t)$$

GFFs

D-term contribution (for even n)

GFFs

H. B. O'Connell et al., PLB 354 (1995)

1. monopole-like parameterization

$$A_{n,k}(t) = \frac{A_{n,k}(0)}{1+t/M_{n,k}^2}$$

G. Lee et al., PRD 92 (2015)

2. z-expansion (model-independent)

$$A_{n,k}(t) = \sum_{l=0}^{2} a_{n,k,l}\,\mathbf{z}^l, \quad \mathbf{z}(t,t_{\text{cut}},t_0) = \frac{\sqrt{t_{\text{cut}}-t}-\sqrt{t_{\text{cut}}-t_0}}{\sqrt{t_{\text{cut}}-t}+\sqrt{t_{\text{cut}}-t_0}}$$

$\mathbf{z}$(t) is introduced to improve the convergence of the series expansion.

Matching and renormalization-group (RG) resummation

$$\widetilde{H}^R(z,P_z,\xi,t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^{k} H_n(\xi,t,\mu)\,\xi^{k-n}\, c_{k,k-n}(z^2\mu^2) + \mathcal{O}\left(\Lambda_{\rm QCD}^2 z^2\right)$$

✓ Perturbative matching matrix (non-diagonal)

RGE: $\mu^2 \frac{d}{d\mu^2} C(z^2\mu^2) = \gamma(\alpha_s(\mu))\, C(z^2\mu^2),$

with $\gamma(\alpha_s) = \gamma_0\,\alpha_s + \gamma_1\,\alpha_s^2,\ \ \beta(\alpha_s) \equiv \mu\frac{d\alpha_s}{d\mu} = -\frac{\beta_0}{2\pi}\alpha_s^2 - \frac{\beta_1}{2(2\pi)^2}\alpha_s^3.$ NLL-RGR

Resummation order	$\alpha^n L^k$ log resummed	Anomalous dimension	β-function	$c_{n,n-k}$
LL	$n=k$	1-loop ✓	1-loop ✓	tree-level ✓
NLL	$n-1 \le k \le n$	2-loop ✓	2-loop ✓	1-loop ✓
NNLL	$n-2 \le k \le n$	3-loop ❌	3-loop ✓	2-loop ❌

❌ No solution for $\xi \neq 0$ but $\xi = 0$ exists.

Matching and renormalization-group (RG) resummation

$$\widetilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^{k} H_n(\xi, t, \mu)\, \xi^{k-n}\, c_{k,k-n}(z^2\mu^2) + \mathcal{O}\left(\Lambda_{\rm QCD}^2 z^2\right)$$

✓ Perturbative matching matrix (non-diagonal)

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with $\gamma(\alpha_s) = \gamma_0\, \alpha_s + \gamma_1\, \alpha_s^2,\ \ \beta(\alpha_s) \equiv \mu \dfrac{d\alpha_s}{d\mu} = -\dfrac{\beta_0}{2\pi}\alpha_s^2 - \dfrac{\beta_1}{2(2\pi)^2}\alpha_s^3.$ NLL-RGR

$$c_{k,k-n} = \begin{pmatrix} c_{1,0} & 0 & 0 & 0 & 0 \\ 0 & c_{2,0} & 0 & 0 & 0 \\ c_{3,2} & 0 & c_{3,0} & 0 & 0 \\ 0 & c_{4,2} & 0 & c_{4,0} & 0 \\ c_{5,4} & 0 & c_{5,2} & 0 & c_{5,0} \end{pmatrix} \qquad \gamma_1^{ij} = \begin{pmatrix} 0.266 & 0 & 0 & 0 & 0 \\ 0 & 0.512 & 0 & 0 & 0 \\ -0.064 & 0 & 0.618 & 0 & 0 \\ 0 & -0.061 & 0 & 0.688 & 0 \\ -0.023 & 0 & -0.066 & 0 & 0.742 \end{pmatrix}$$

✓ Derived from Ref. [Y.Ji, **F.Yao** and J.H. Zhang, arXiv:2504.09367]

Lattice calculation

- **Asymmetric (non-Breit) approach**: computationally economical

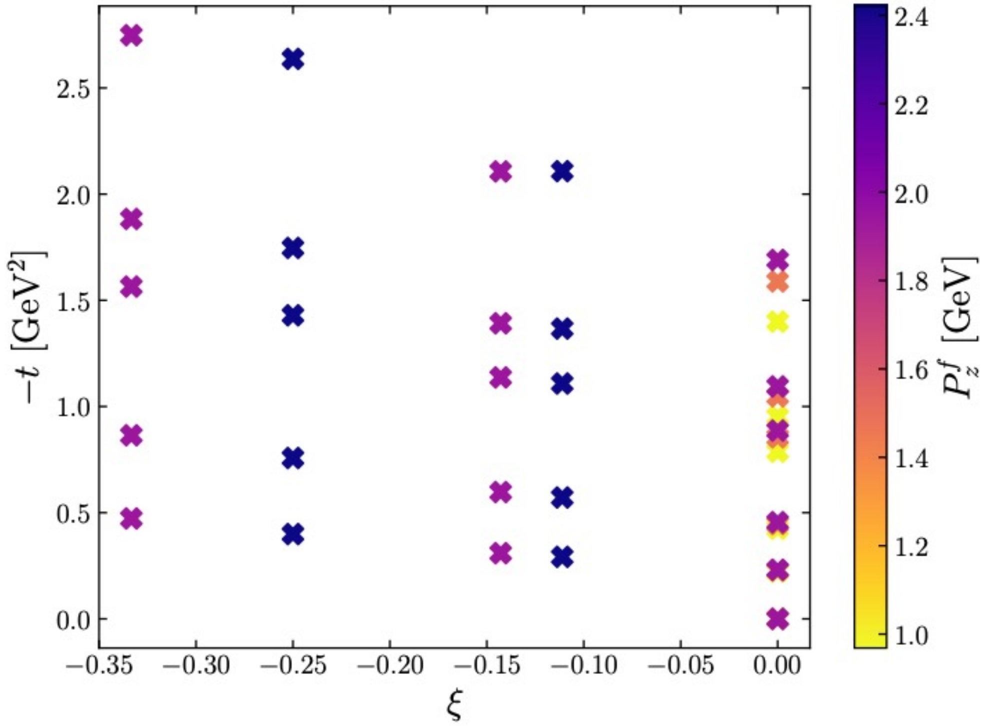
S. Bhattacharya et al., PRD 106 (2022)

$$\vec{p}_f = (0, 0, P_z), \;\; \vec{p}_i = \vec{p}_f - \vec{\Delta} = \left(-\Delta_x, -\Delta_y, P_z - \Delta_z\right)$$

- **Lattice setup:** HISQ gauge ensembles

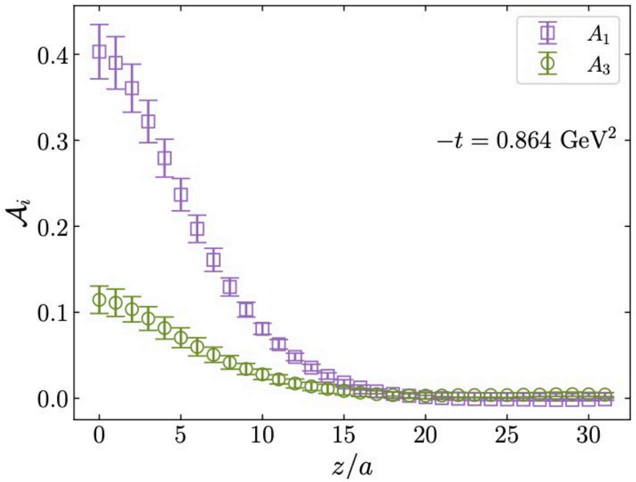
HotQCD collab., PRD 75 (2007)

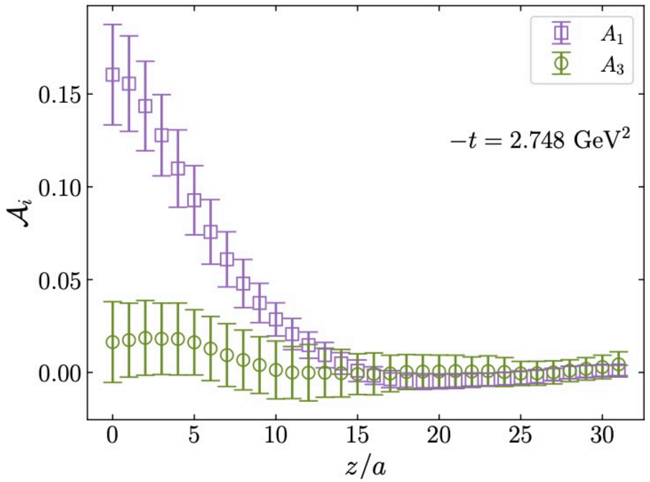
- $a = 0.04$ fm
- Pion mass: 300 MeV
- $P_z = \{0.968, 1.453, 1.937, 2.428\}$ GeV
- $\xi = \{-0.33, -0.25, -0.14, -0.11, 0\}$
- $-t$ up to 2.748 GeV^2



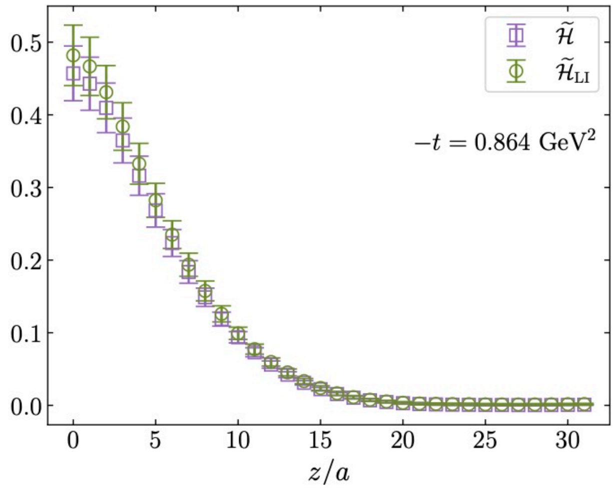
Comparison between $\widetilde{H}_{LI}$ and $\widetilde{H}$

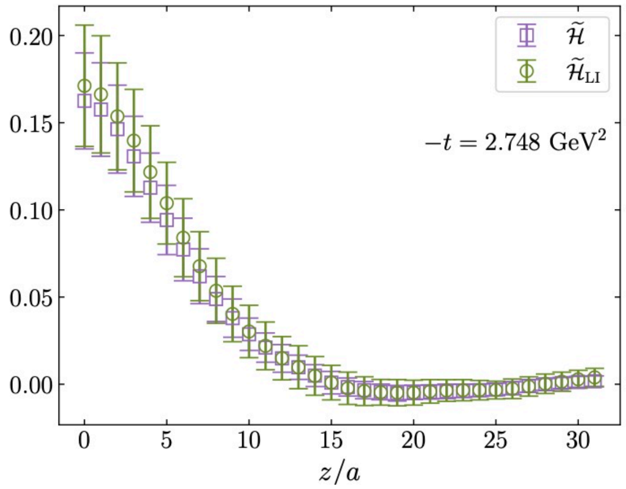
$P_z = 1.937$ GeV,
$\xi = -0.33$,
$-t = \{0.864, 2.748\}$ GeV^2





$$\widetilde{H}_{LI}(z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_1 + \frac{z\cdot\Delta}{z\cdot P}A_3$$



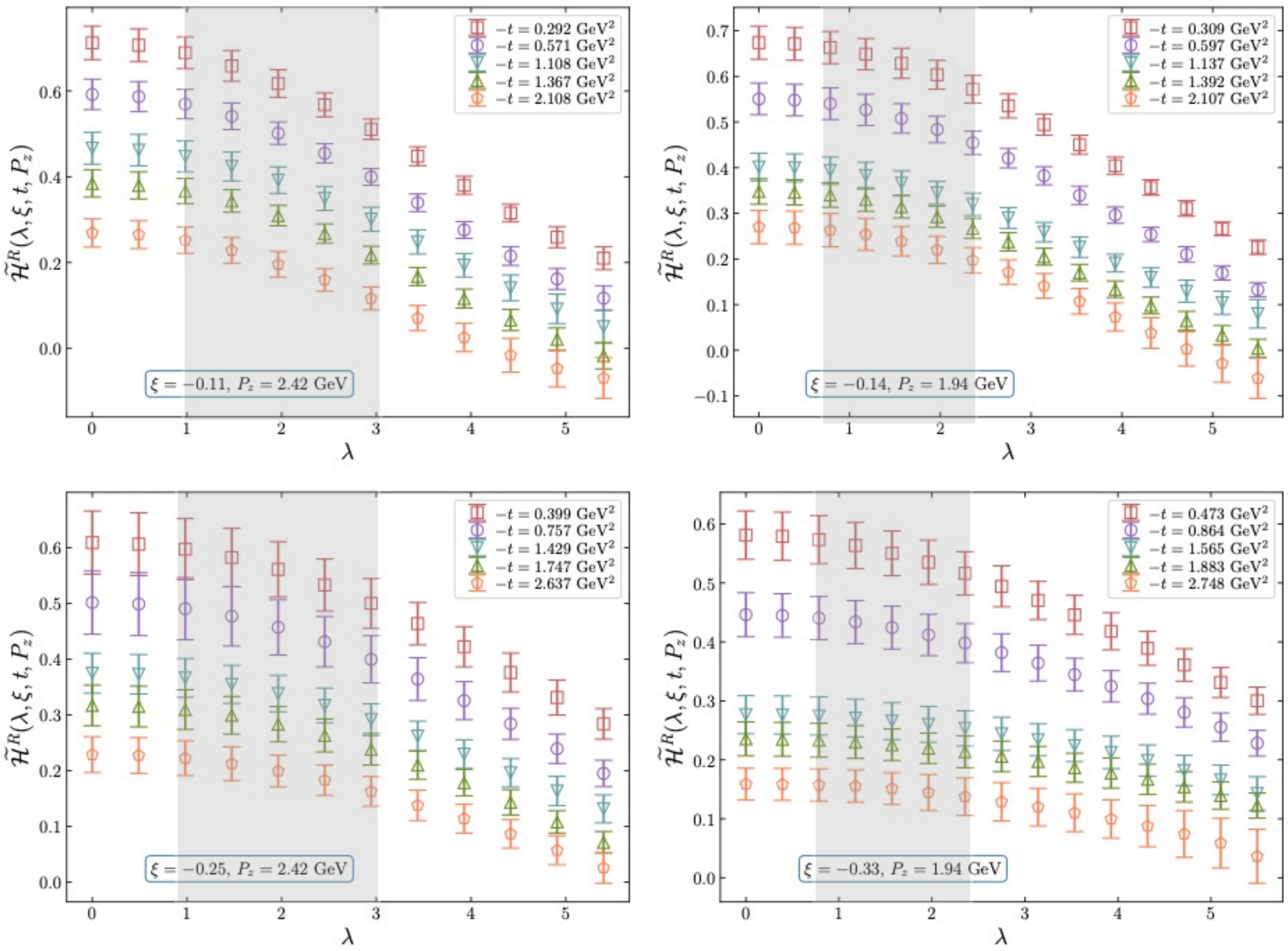


- Original definition by γ^0
  $\widetilde{H}(z, P_z, \xi, t) = \frac{1}{P^0}M^0(z, P, \Delta)$
- **Adopted in this work.**

Renormalized matrix elements

Fitting region used in this work:

$z = \{2a, 6a\} = \{0.08, 0.24\}$ fm



Choice of fitting region

Resummation order	$\alpha^n L^k$ log resummed	Anomalous dimension	β-function	$c_{n,n-k}$
LL	$n = k$	1-loop ✓	1-loop ✓	tree-level ✓
NLL	$n-1 \leq k \leq n$	2-loop ✓	2-loop ✓	1-loop ✓
NNLL	$n-2 \leq k \leq n$	3-loop ✗	3-loop ✓	2-loop ✗

✗ No solution for $\xi \neq 0$ but $\xi = 0$ exists.

$$\widetilde{H}^R(z, P_z, \xi = 0, t) = A_{1,0}(t)\, c_{1,0} + \frac{(-izP_z)^2}{2!} A_{3,0}(t)\, c_{3,0} + \frac{(-izP_z)^4}{4!} A_{5,0}(t)\, c_{5,0}$$

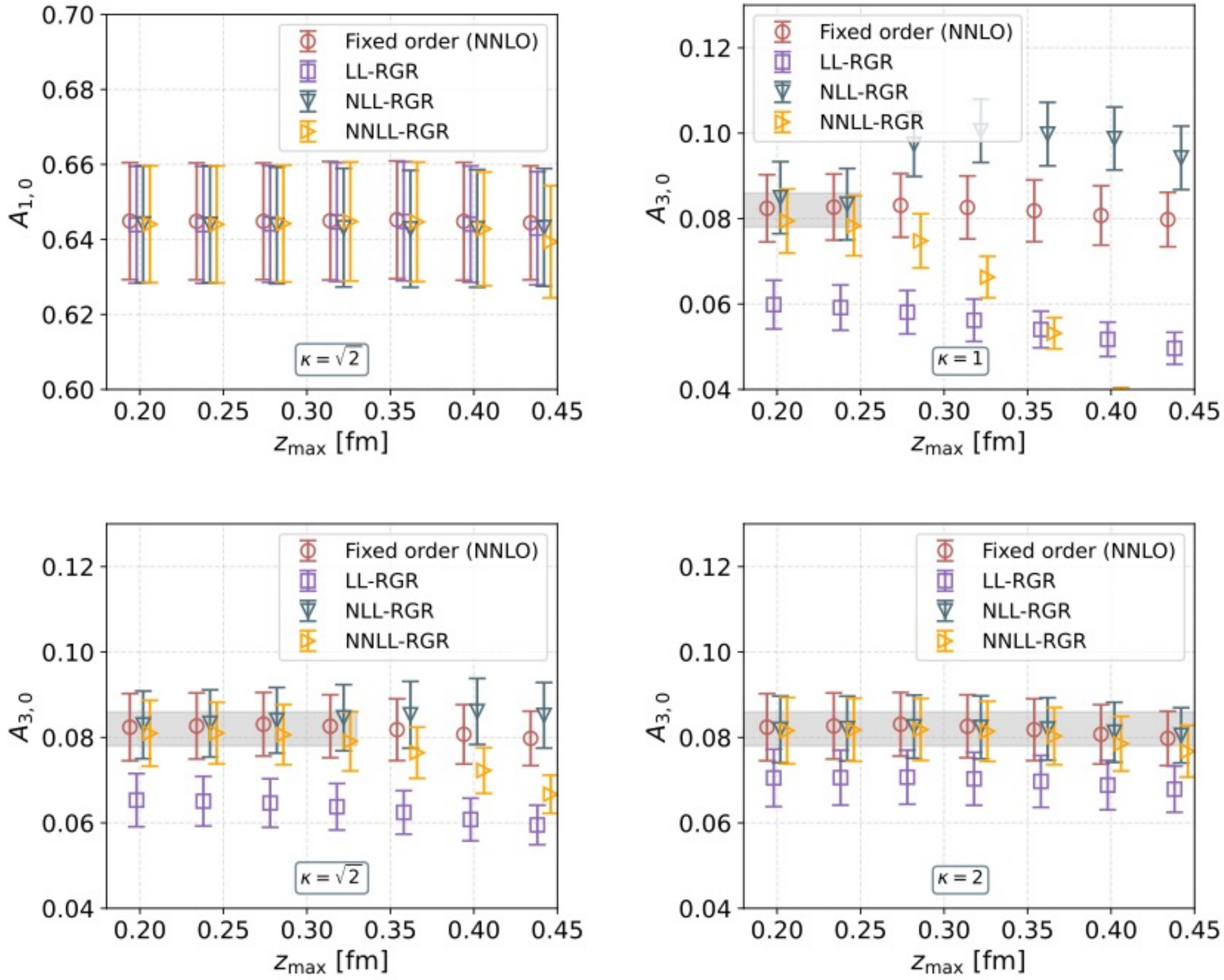
- Fixed $z_{\rm min} = 0.08$ fm, then varied $z_{\rm max}$.
- Considering **LL, NLL and NNLL resummation** effects.
- Evolve from $\mu_0 = (2\kappa e^{-\gamma_E})/z$ to $\mu = 2$ GeV, with $\kappa = \{1, \sqrt{2}, 2\}$.

Choice of fitting region

$P_z = 1.937$ GeV,

$\xi = 0$,

$-t = 0.446$ GeV^2



Numerical results for GPD moments

- **Extracting the moments through a joint fit of matrix elements over multiple ξ and $-t$,**

$$\widetilde{H}^R(z, P_z, \xi, t) = \sum_{k=1}^{\infty} \frac{(-izP_z)^{k-1}}{(k-1)!} \sum_{n=1}^{k} H_n(\xi, t, \mu^2)\, \xi^{k-n}\, c_{k,k-n}(z^2\mu^2)$$

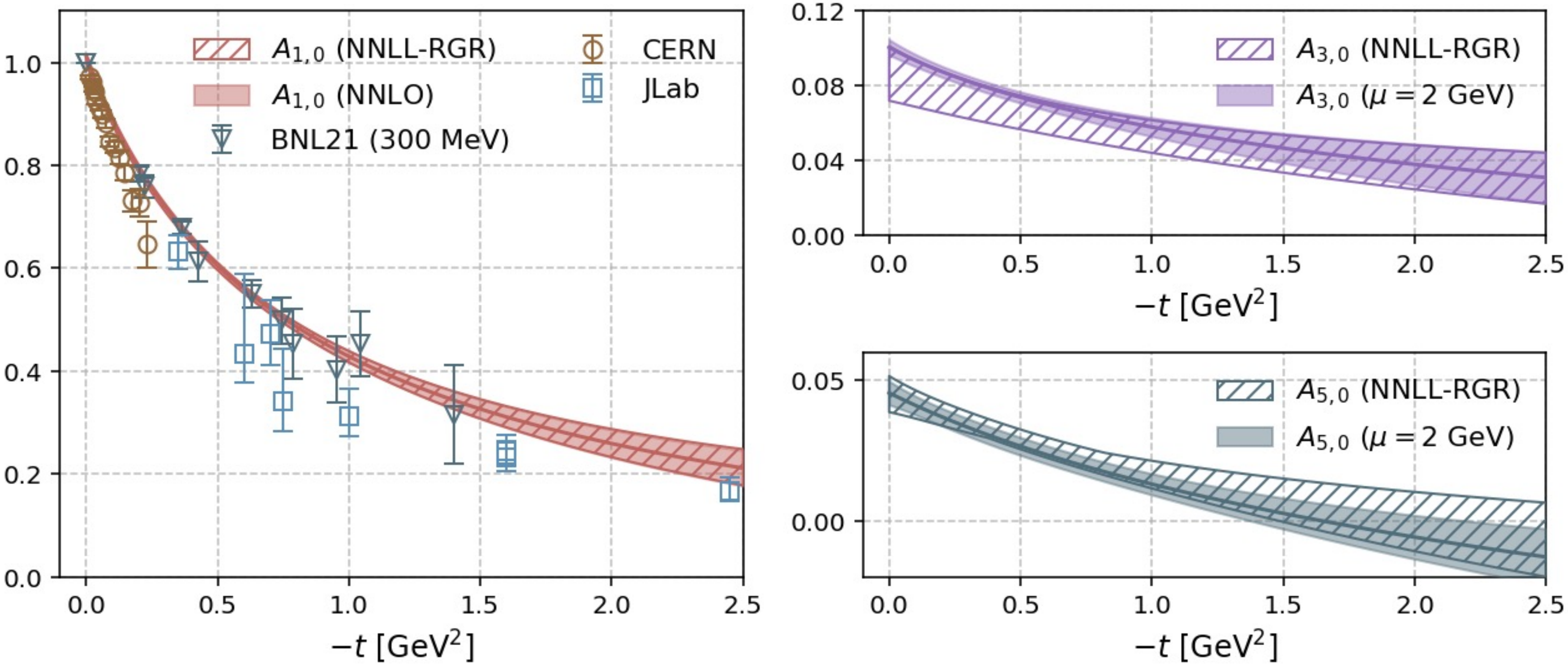
$$= H_1(\xi,t)\, c_{1,0} + \frac{(-izP_z)^2}{2!}\left(H_1(\xi,t)\,\xi^2\, c_{3,2} + H_3(\xi,t)\, c_{3,0}\right) + \frac{(-izP_z)^4}{4!}\left(H_1(\xi,t)\,\xi^4\, c_{5,4} + H_3(\xi,t)\,\xi^2\, c_{5,2} + H_5(\xi,t)\, c_{5,0}\right)$$

$$= A_{1,0}(t)\, c_{1,0} + \frac{(-izP_z)^2}{2!}\left[A_{1,0}(t)\,\xi^2\, c_{3,2} + \underbrace{\left(A_{3,0}(t) + A_{3,2}(t)(2\xi)^2\right)}_{H_3}\, c_{3,0}\right] + \frac{(-izP_z)^4}{4!}\Big[A_{1,0}(t)\,\xi^4\, c_{5,4} +$$

$$\left(A_{3,0}(t) + A_{3,2}(t)(2\xi)^2\right)\xi^2\, c_{5,2} + \underbrace{\left(A_{5,0}(t) + A_{5,2}(t)(2\xi)^2 + A_{5,4}(t)(2\xi)^4\right)}_{H_5}\, c_{5,0}\Big]$$

$$A_{n,k}(t) = \sum_{l=0}^{2} a_{n,k,l}\,\mathbf{z}^l, \quad \mathbf{z}(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut}-t} - \sqrt{t_{\rm cut}-t_0}}{\sqrt{t_{\rm cut}-t} + \sqrt{t_{\rm cut}-t_0}}$$

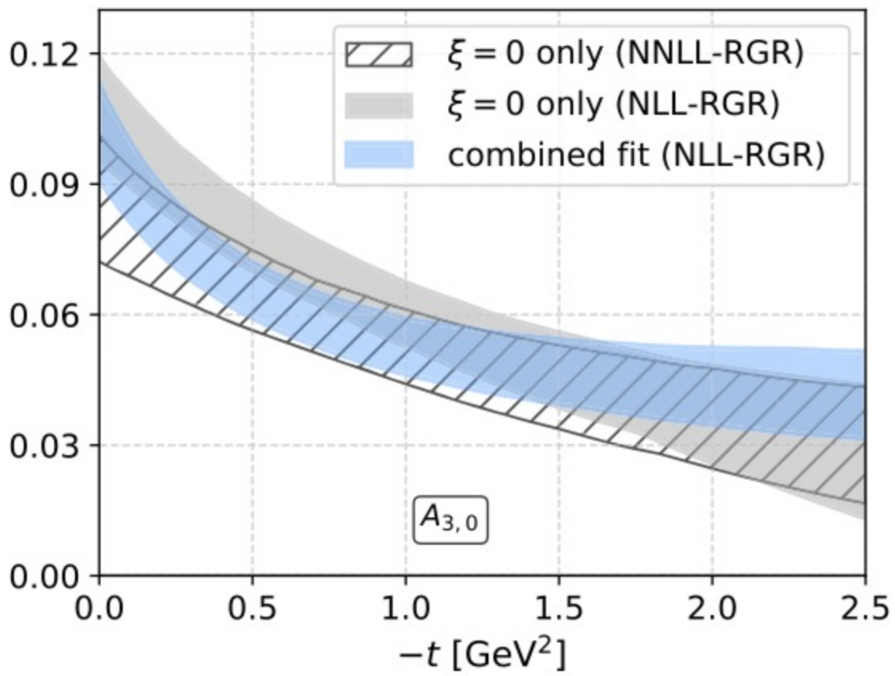
GFFs $A_{1,0}$, $A_{3,0}$ and $A_{5,0}$ ($\xi = 0$)

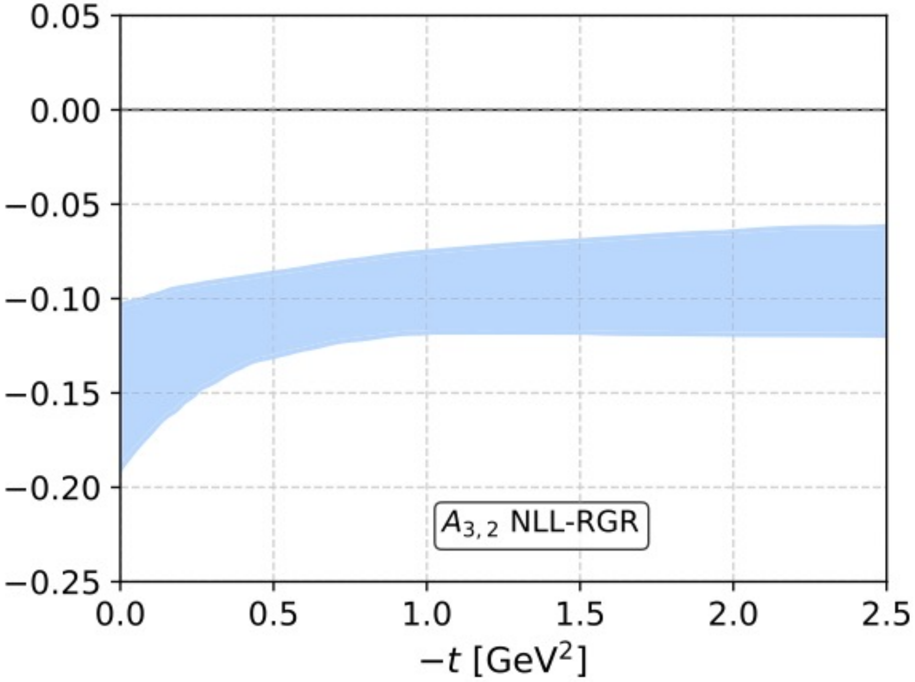


- The GFFs are extracted from **zero-skewness data**.
- Agree with experimental and BNL21 results. X. Gao et al., PRD 104 (2021)
- NNLL-RGR uncertainty: varying κ between 1 and 2.

GFFs $A_{3,i}$ and third Mellin moment $H_3(\xi, t)$

$$H_3(\xi, t) = A_{3,0} + (2\xi)^2 A_{3,2}$$

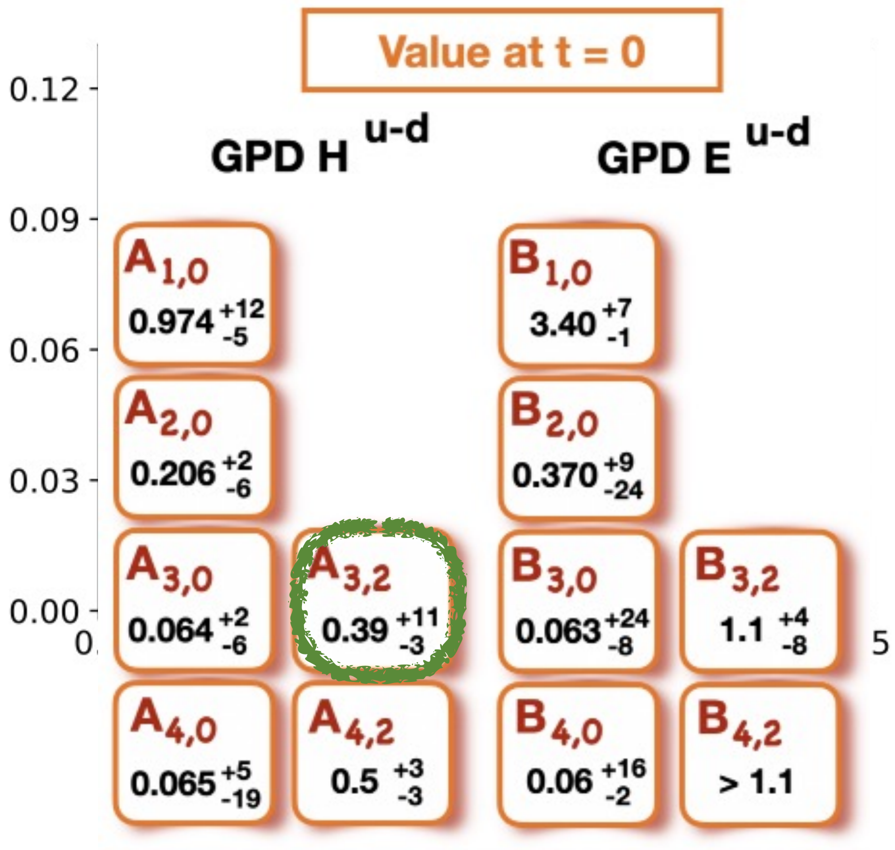




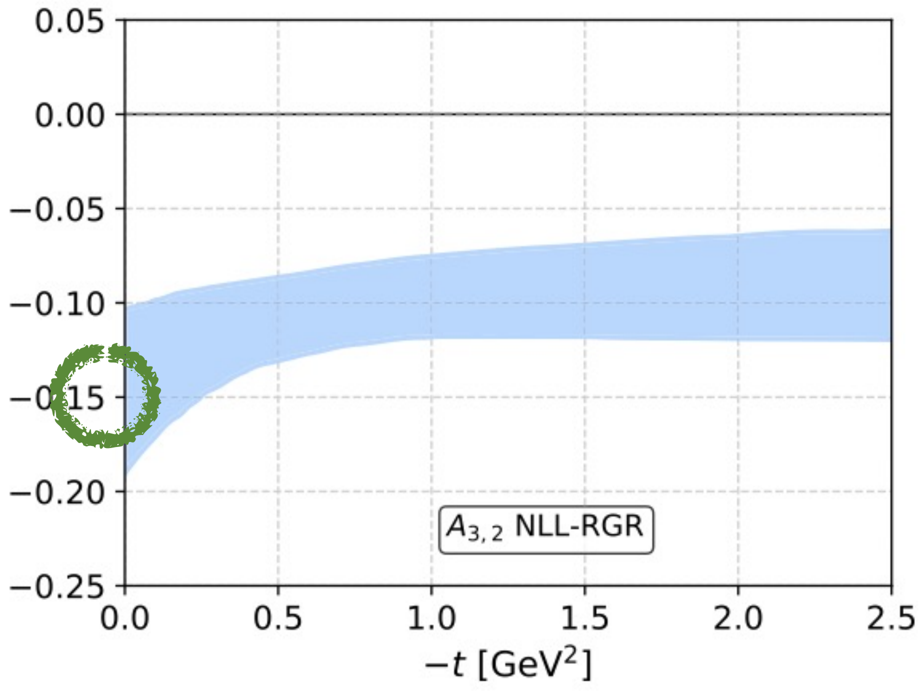
- Combined fit using both $\xi = 0$ and $\xi \neq 0$ data.
- $A_{3,2}$ is negative, giving visible skewness dependence.

GFFs $A_{3,i}$ and third Mellin moment $H_3(\xi, t)$

$$H_3(\xi, t) = A_{3,0} + (2\xi)^2 A_{3,2}$$



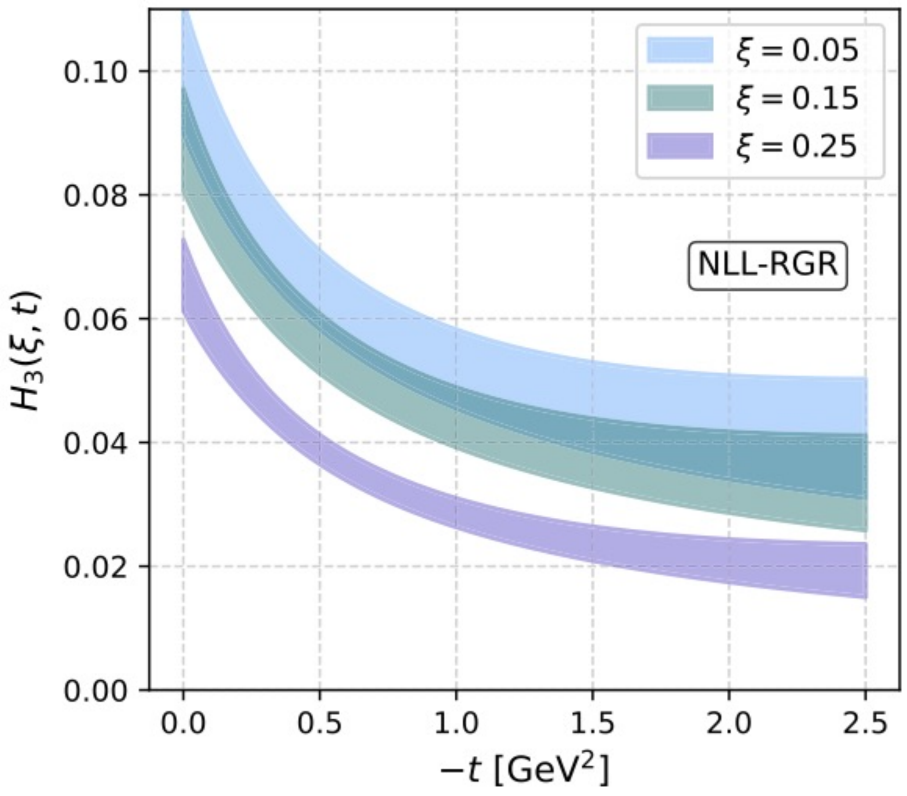
HadStruc Collab., JHEP 02, 056 (2025)

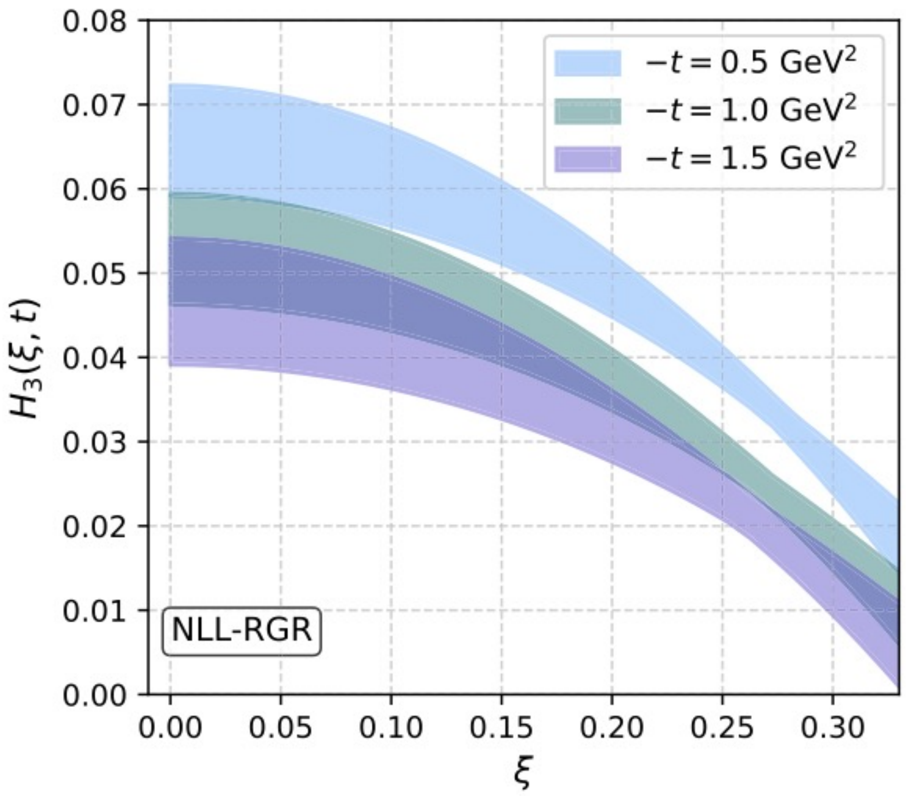


Opposite trend compared with the nucleon case.

GFFs $A_{3,i}$ and third Mellin moment $H_3(\xi, t)$

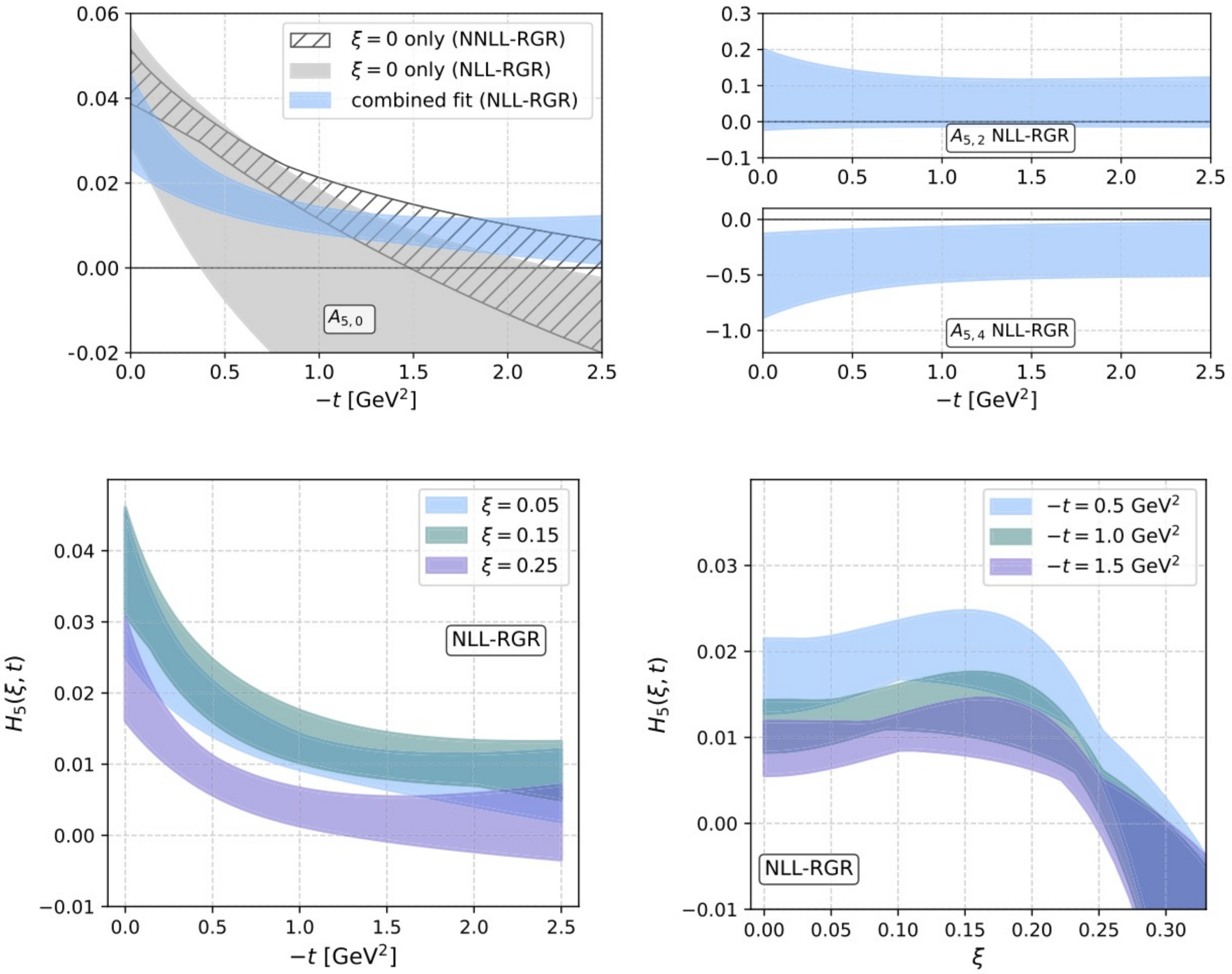
$$H_3(\xi, t) \;=\; A_{3,0} + (2\xi)^2\, A_{3,2}$$





- $H_3(\xi, t)$ decreases as $|\xi|$ increases.
- It also decreases with increasing $-t$.

GFFs $A_{5,i}$ and Fifth Mellin moment $H_5(\xi, t)$



$$H_5(\xi, t) = A_{5,0} + (2\xi)^2 A_{5,2} + (2\xi)^4 A_{5,4}$$

- $\xi = 0$ and $\xi \neq 0$ data combined fit.
- The fifth-moment GFFs $A_{5,i}$ are strongly suppressed.
- $H_5(\xi, t)$ decreases with increasing $|\xi|$ and $-t$.

Summary and outlook

Summary

- ☑ lattice QCD determination of pion GPD Mellin moments at **nonzero skewness.**
- ☑ Combined fits to $\xi = 0$ and $\xi \neq 0$ data $\rightarrow$ access to (ξ, t) dependence.
- ☑ Implemented **NLL resummation** to reduce perturbative matching uncertainties.

Outlook

- Extraction of x-dependent pion GPDs at nonzero skewness.
- D-term extraction from the imaginary part of matrix elements.
- Simulations at physical pion mass with higher statistics.

Thank you!

$$\int_{-1}^{1} dx\, x^{n-1} \begin{pmatrix} H^{u-d} \\ E^{u-d} \end{pmatrix}(x,\xi,t) = \sum_{k\,=\,0,\,even}^{n-1} \begin{pmatrix} A_{n,k}(t) \\ B_{n,k}(t) \end{pmatrix} \xi^k \pm \mathrm{mod}(n+1,2)\, \xi^n \left(C_n(t) \right).$$

- Solution (γ_0, γ_i do not commute)

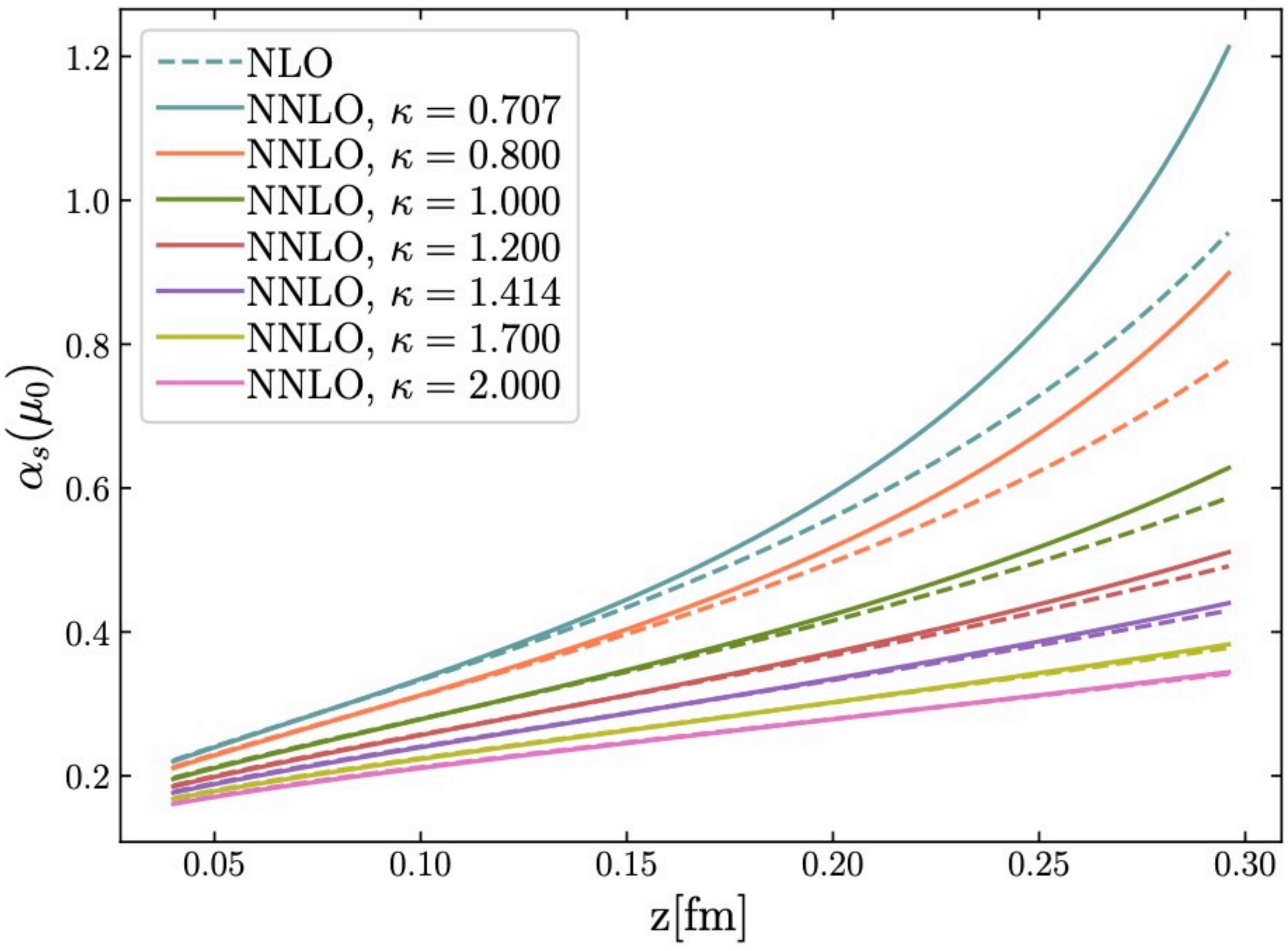
$$C(z^2\mu^2) = \mathscr{P} \exp\left[2 \int_{\alpha_s(\mu_0)}^{\alpha_s(\mu)} \frac{d\alpha}{\beta(\alpha)}\, \gamma(\alpha) \right] C(z^2\mu_0^2),$$

- LL-resummation

$$c_{i,j}(\alpha_s(\mu), z^2\mu^2) = c_{i,j}(\alpha_s(\mu_0), z^2\mu_0^2) \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{\frac{\gamma_0}{\beta_0}},$$

- NLL-resummation

$$c_{i,j}(\alpha_s(\mu), z^2\mu^2) = c_{i,j}(\alpha_s(\mu_0), z^2\mu_0^2) \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{\frac{\gamma_0}{\beta_0}} \exp\left\{ \frac{(\beta_0\gamma_1 - \beta_1\gamma_0)}{\beta_0\beta_1} \ln\left(\frac{\alpha_s(\mu_0)\beta_1 + (4\pi)\beta_0}{\alpha_s(\mu)\beta_1 + (4\pi)\beta_0} \right) \right\}.$$



NLO
NNLO, $\kappa = 0.707$
NNLO, $\kappa = 0.800$
NNLO, $\kappa = 1.000$
NNLO, $\kappa = 1.200$
NNLO, $\kappa = 1.414$
NNLO, $\kappa = 1.700$
NNLO, $\kappa = 2.000$
$\alpha_s(\mu_0)$
1.2
1.0
0.8
0.6
0.4
0.2
0.05
0.10
0.15
0.20
0.25
0.30
z[fm]