Nucleon Unpolarized PDF at Physical Point with High Boost

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Why PDF matter & How they are obtained

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LHC:
Precision Standard Model tests

Drell-Yan & SIDIS:
Nucleon sea flavor decomposition

↑ **Why**

PDF: probability of finding a quark or gluon carrying x of the hadron

↓ **How**

Global QCD Analyses:
Fit DIS, Drell-Yan and Jet data

Lattice QCD: first-principles
Independent cross-check of global fits

LaMET: Two Requirements in Balance

PRL 110 (2013) 262002
SCPMA 57 (2014) 1407
RMP 93 (2021) 035005

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Boost the nucleon to a large finite momentum: Quasi-PDF ⇒ Light-cone PDF

Large Momentum P_z

- Power corrections scale as

$$\left(\frac{\Lambda_{\rm QCD}}{xP_z}\right)^2 \text{ and } \left(\frac{m}{xP_z}\right)^2$$

Fine Lattice Spacing a

- Discretization artifacts grow with aP_z and become uncontrolled as $aP_z \to 1$

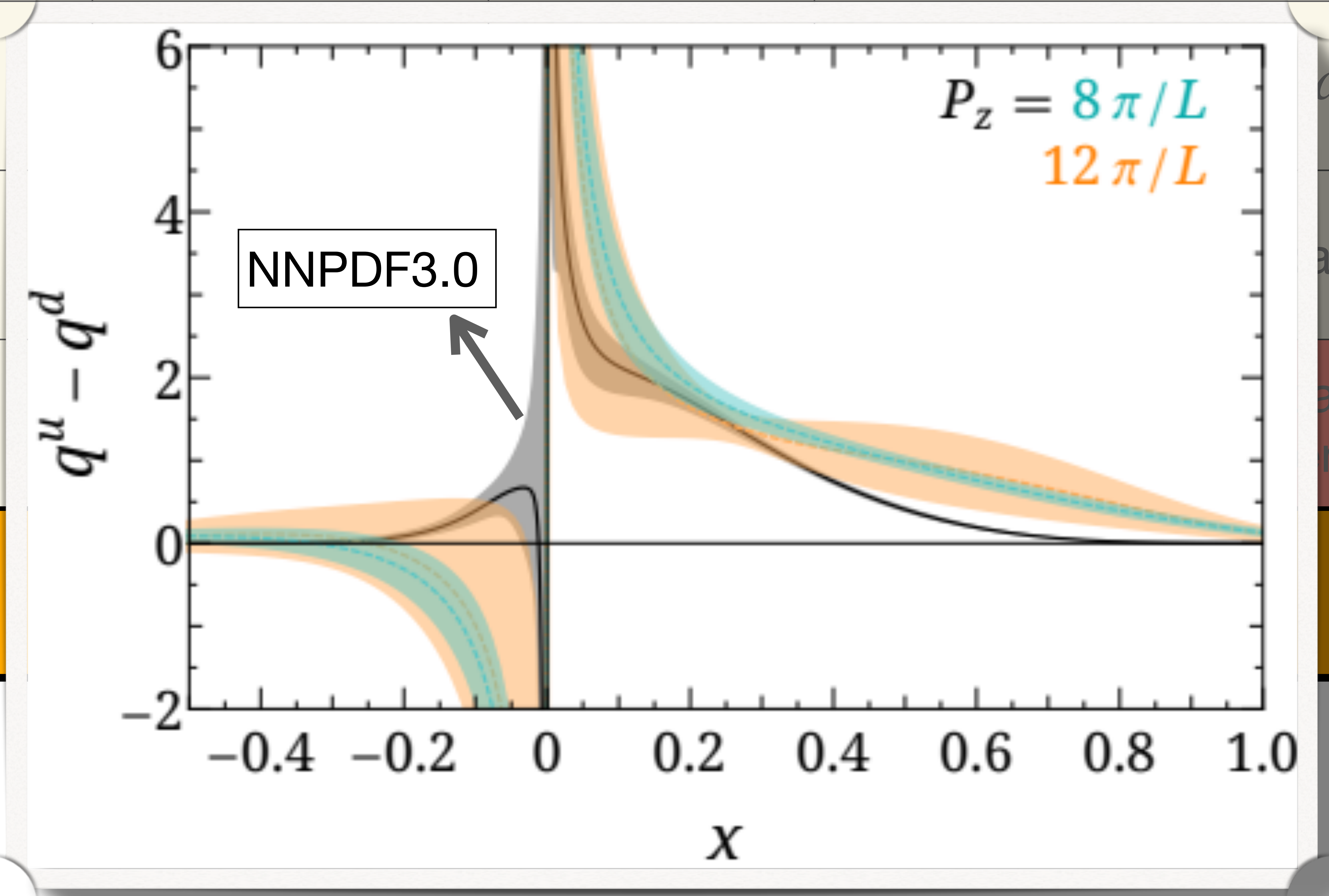
$$aP_z < 1$$

Complementary approaches: pseudo-PDF, lattice cross-section method, HOPE

arXiv:1708.05301 [hep-lat].
Phys. Rev. Lett. 121, 242003 (2018), arXiv:1807.07431 [hep-lat].
C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, A. S... F. Steffens, Phys. Rev. D 99, 114504 (2019), arXiv:1902.00587 [hep-lat].
Z. Fan, X. Gao, R. Li, H.-W. Lin, N. Karthik, S. Mukherjee, P. Petreczky, S. Syritsyn, Y.-B. Yang, and R. Zhang, Phys. Rev. D 102, 074504 (2020), arXiv:2005.12015 [hep-lat].
F. Manigrasso, A. Scapellato, and F. Steffens, Phys. Rev. D 103, 094512 (2021),
...nd H.-W. Lin, Phys. Lett. B 854, 138731 (2024), arXiv:2405.18238 [hep-lat].
...Hanlon, S. Mukherjee, P. Petreczky, H.-T. Shu, F. Yao, R. Zhang, and Y. Zhao, ...arXiv:2604.00143 [hep-lat].
X. Gao, J. He, J. Lin, S. Mukherjee, P. Petreczky, R. Zhang, and Y. Zhao, (2026), arXiv:2602.11283 [hep-lat].

Lattice nucleon unpolarized PDF: P_z & m_π

Study	a [fm]	m_π [MeV]	P_z [GeV]	aP_z	Outcome
PRD 98 (2018) 5, 054054	0.09	Physical	1.29	< 0.6	Higher P_z needed
arXiv: 1803.04393	0.09	Physical			aP_z
PRD 107 (2023) 7, 074509	0.076	Physical			al fits
arXiv: 2602.11283	0.06	300			al fits; nce
This work	0.076	Physical			



Lattice nucleon unpolarized PDF: P_z & m_π

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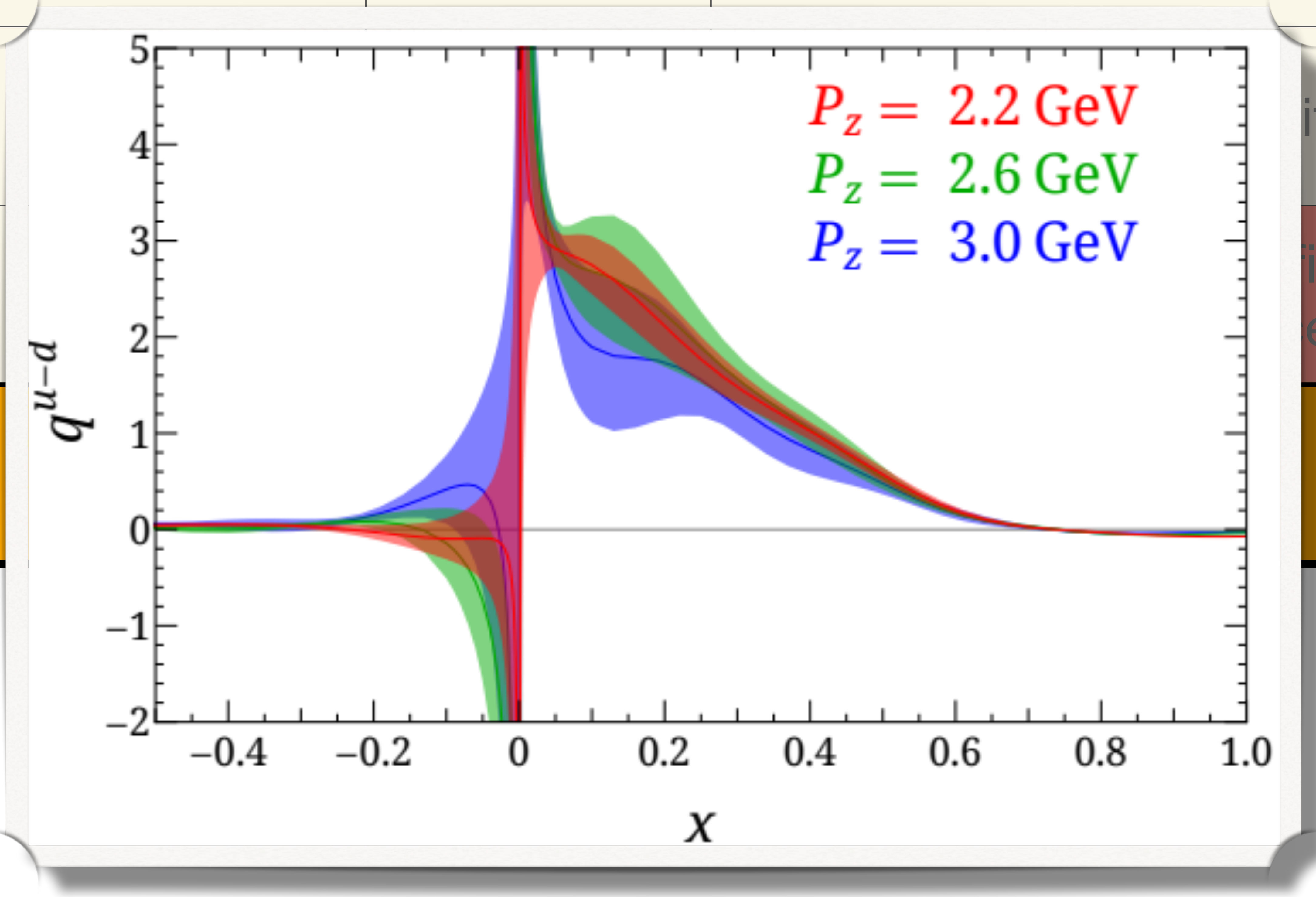
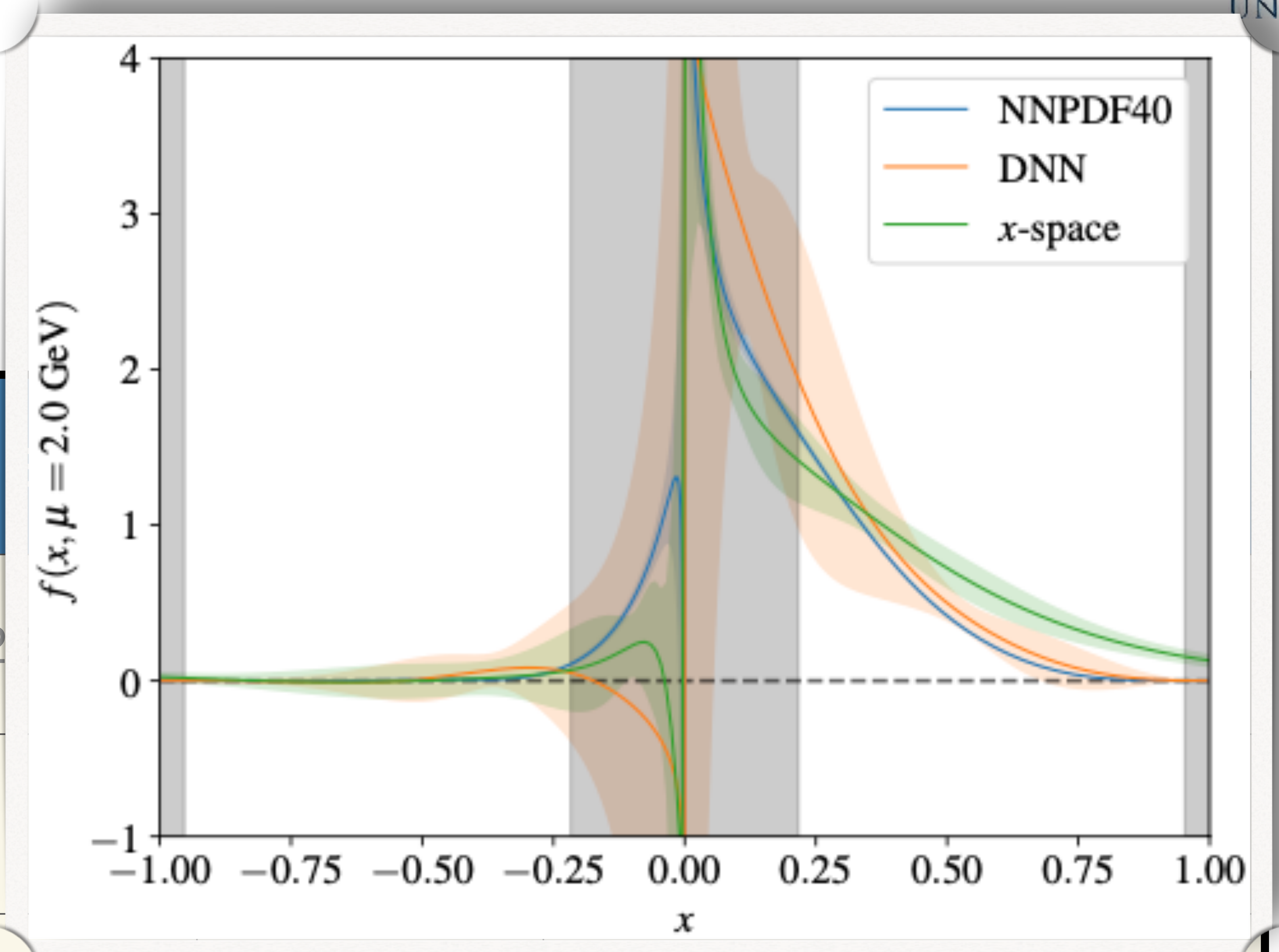
PRD 98 (2018) 5, 054054	0.09	Physical	1.29	< 0.6	Higher P_z needed
arXiv: 1803.04393	0.09	Physical	3	> 1.3	Uncontrolled aP_z
PRD 107 (2023) 7, 074509	0.076	Physical			its
arXiv: 2602.11283	0.06	300			its; e
This work	0.076	Physical			

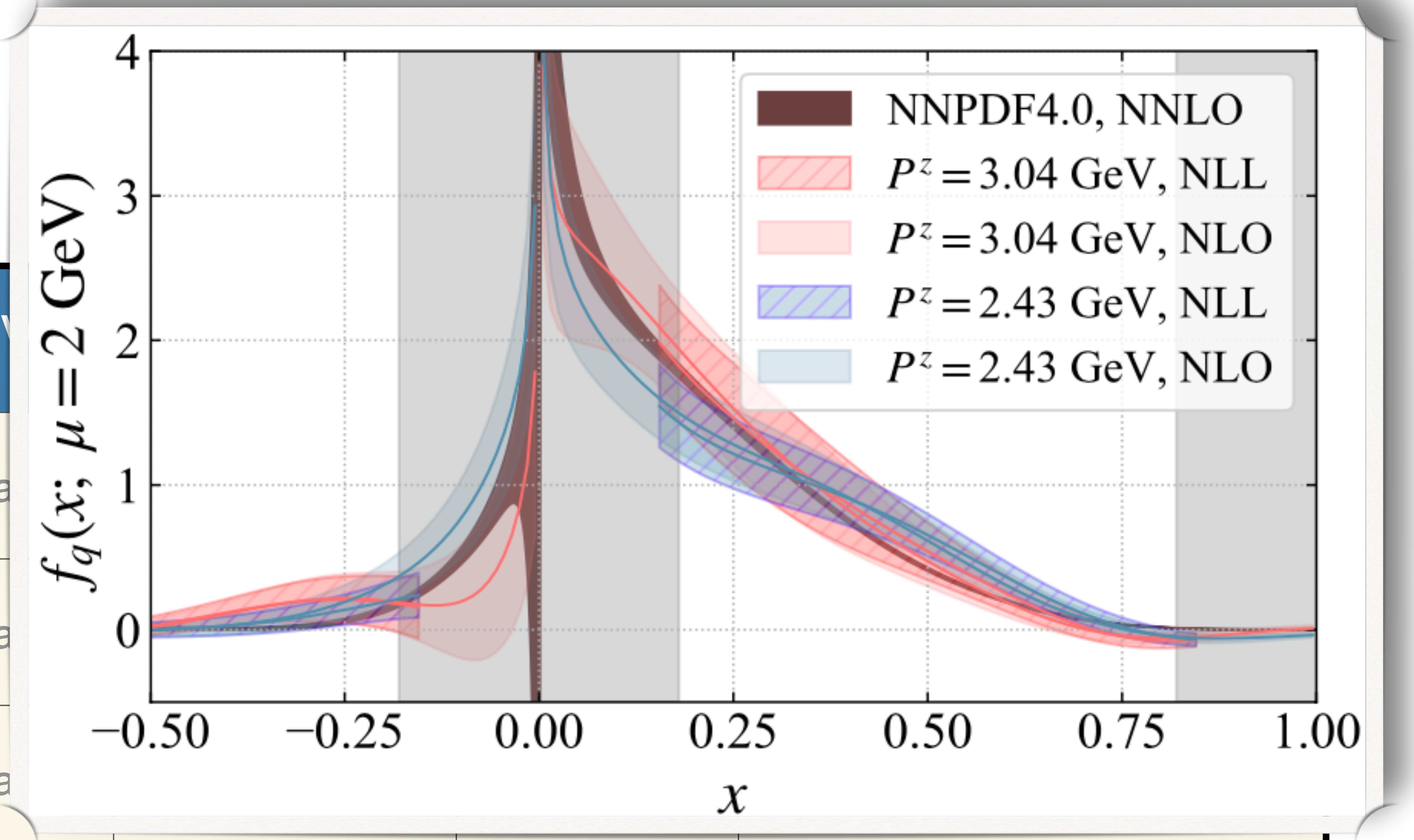


Lattice nucleon unpolarized PDF: P_z & m_π



Study	a [fm]	m_π [MeV]	P_z [G		
PRD 98 (2018) 5, 054054	0.09	Physical	1.2		
arXiv: 1803.04393	0.09	Physical	3		
PRD 107 (2023) 7, 074509	0.076	Physical	1.53	< 0.6	Deviates from global fits
arXiv: 2602.11283	0.06	300	3	< 1	Compatible w. Global fits; Good P_z convergence
This work	0.076	Physical	2.29	< 0.9	?

Lattice nucleon unpolarized PDF: P_z & m_π



Study	a [fm]	m_π [Me			
PRD 98 (2018) 5, 054054	0.09	Physica			
arXiv: 1803.04393	0.09	Physica			
PRD 107 (2023) 7, 074509	0.076	Physica			
arXiv: 2602.11283	0.06	300	3	< 1	Compatible w. Global fits; Good P_z convergence
This work	0.076	Physical	2.29	< 0.9	?

Motivation

Study	a [fm]	m_π [MeV]	P_z [GeV]	aP_z	Outcome
PRD 98 (2018) 5, 054054	0.09	Physical	1.29	< 0.6	Higher P_z needed
arXiv: 1803.04393	0.09	Physical	3	> 1.3	Uncontrolled aP_z
PRD 107 (2023) 7, 074509	0.076	Physical	1.53	< 0.6	Deviates from global fits
arXiv: 2602.11283	0.06	300	3	< 1	Compatible w. Global fits; Good P_z convergence
Dominance of P_z or Accidental coincidence ?					
This work	**0.076**	**Physical**	**2.29**	**< 0.9**	**?**

A Universality Test:

The lattice determination reflects the underlying physics rather than a particular choice of ensemble or extraction method

Large P_z with Controlled Systematics

- $a = 0.076$ fm, $L_s \times L_t = 64^3 \times 64$; Physical Point
- Isovector ($u - d$) & connected isoscalar ($u + d$)

$$\Rightarrow \; P_z = \{1.78, 2.29\} \text{ GeV}, \; aP_z < 0.9$$

Coulomb-Gauge quark bilinears

$$C^q_{\mathrm{3pt}}(\mathbf{P}, \tau, t_s; z) = \sum_{\mathbf{x},\mathbf{y}} e^{-i\mathbf{P}\cdot\mathbf{x}} \mathscr{P}^{\mathrm{unpol}}_{\alpha\beta} \langle N_\beta(\mathbf{x}, t_s) \bar{q}(z)\gamma^t q(0) \bar{N}_\alpha(\mathbf{0},0)\rangle$$

- Avoids the divergence in the Wilson line
- Simpler renormalization, better SNR at large z
- Belongs to universality class in LaMET

PRD 109 (2024) 094506

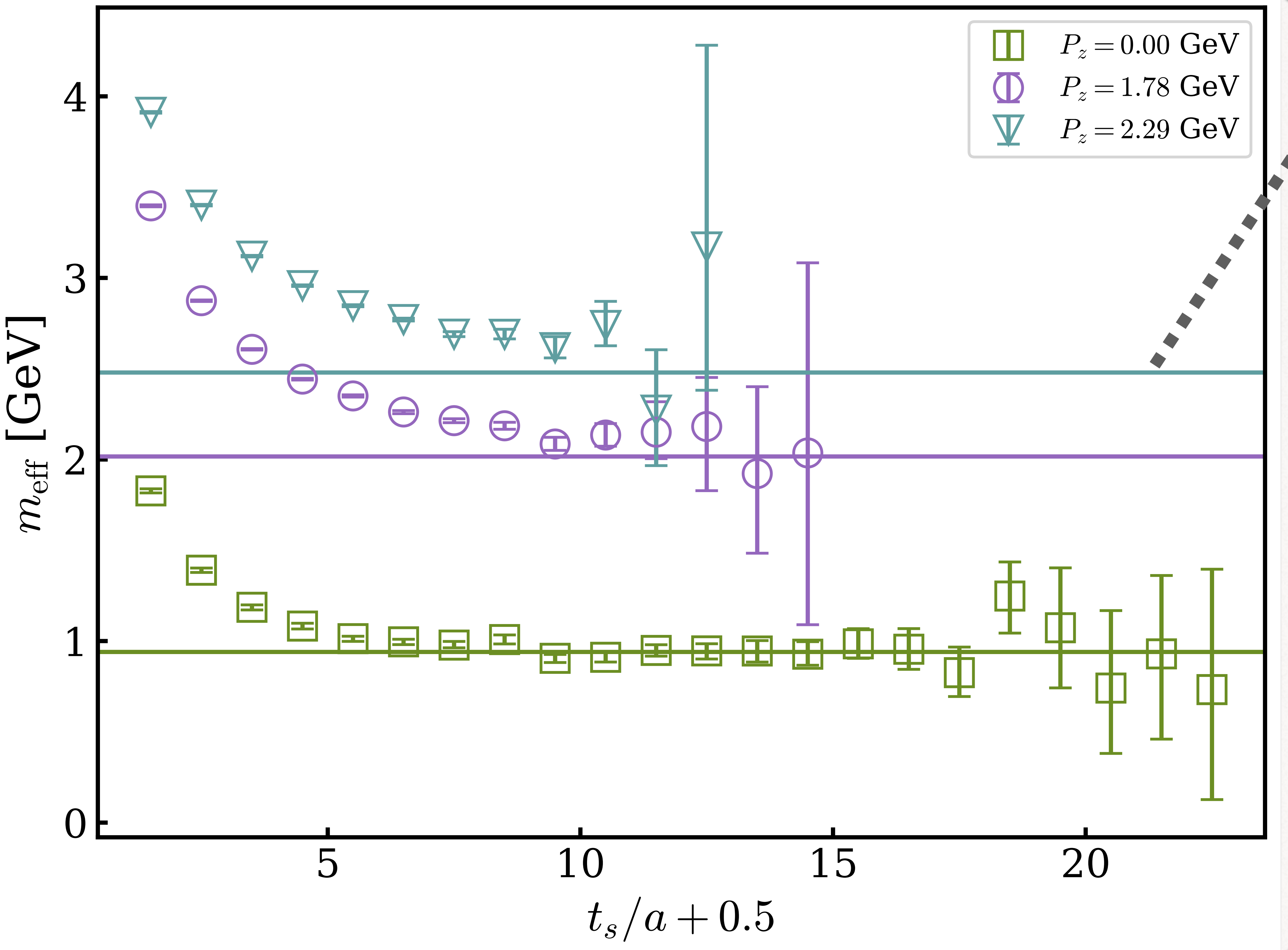
Enhanced interpolating operators

$$N_\alpha(\mathbf{x}, t) = \epsilon_{abc}\, u_{a,\alpha}(\mathbf{x}, t) \left[u_b^T(\mathbf{x}, t) C\gamma_t\gamma_5 d_c(\mathbf{x}, t) \right]$$

- Ground-state overlap grows with boost
- SNR improves at large P_z

PRD 112 (2025) L051502

Bare Matrix Element h^{B}



$$E(P_z) = \sqrt{m_{\mathrm{N}}^2 + P_z^2}\,,\ m_{\mathrm{N}} = 0.94\ \mathrm{GeV}$$

$$\begin{cases} C_{\mathrm{2pt}}(\mathbf{P}, t_{\mathrm{s}}) = \sum_{n=0}^{\infty} |A_n|^2 e^{-E_n t_{\mathrm{s}}}, \\ C_{\mathrm{3pt}}(\mathbf{P}, \tau, t_{\mathrm{s}}; z) = \sum_{m,n=0}^{\infty} A_m h_{mn} A_n^* e^{-E_m(t_{\mathrm{s}}-\tau)} e^{-E_n \tau} \end{cases}$$

$$\Rightarrow \quad R(\mathbf{P}; \tau, t_{\mathrm{s}}; z) \equiv \frac{C_{\mathrm{3pt}}(\mathbf{P}, \tau, t_{\mathrm{s}}; z)}{C_{\mathrm{2pt}}(\mathbf{P}, t_{\mathrm{s}})}$$

$$\downarrow\ t_{\mathrm{s}} \to \infty$$

$$h^{\mathrm{B}} = h_{00}$$

Two Fit Strategies

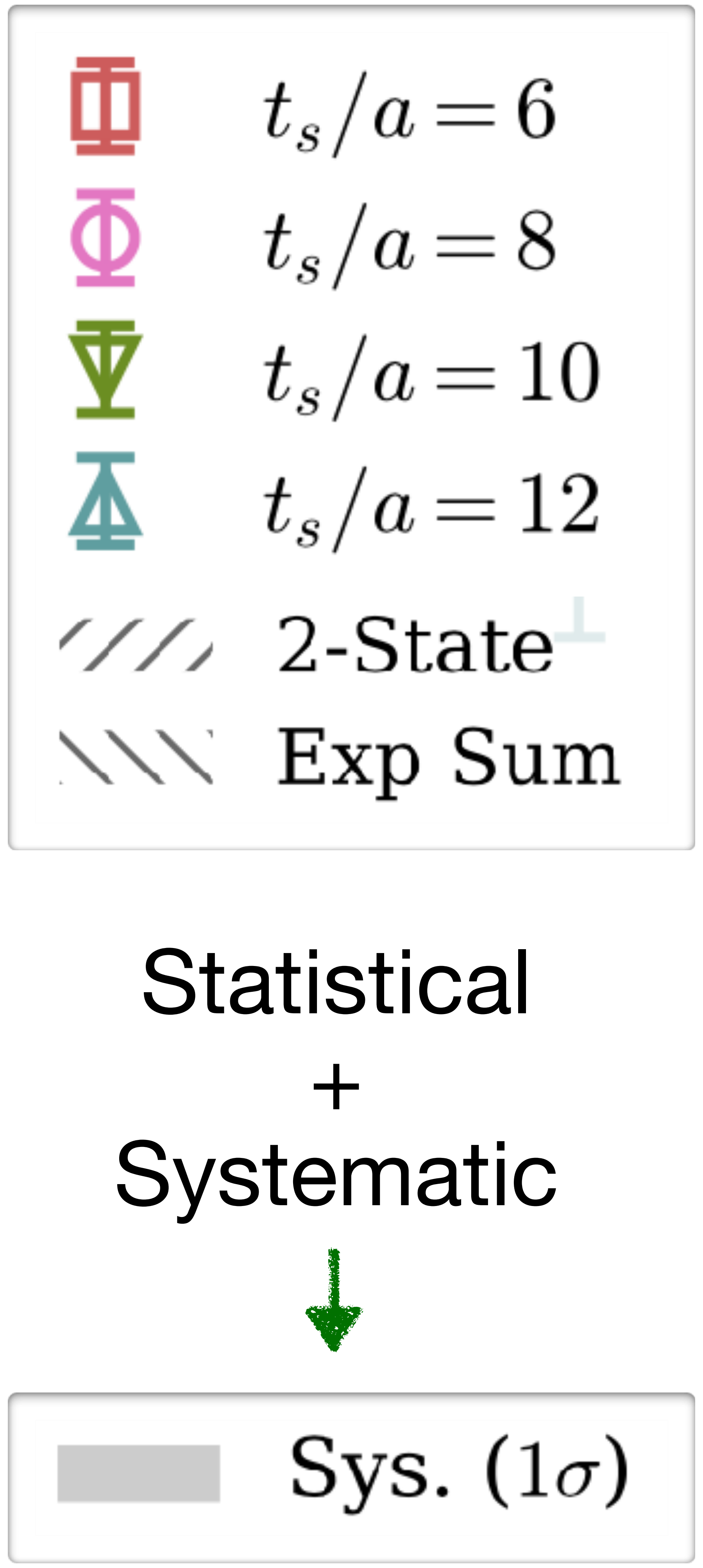
- Two-state Fit:

$$R(\tau, t_s) \approx \frac{h_{00} + h_{11}\frac{|A_1|^2}{|A_0|^2}e^{-\Delta E t_s} + h_{01}\frac{|A_1|}{|A_0|}e^{-\Delta E \tau} + h_{10}\frac{|A_1|}{|A_0|}e^{-\Delta E (t_s - \tau)}}{1 + \frac{|A_1|^2}{|A_0|^2}e^{-\Delta E t_s}}, \quad \Delta E = E_1 - E_0$$

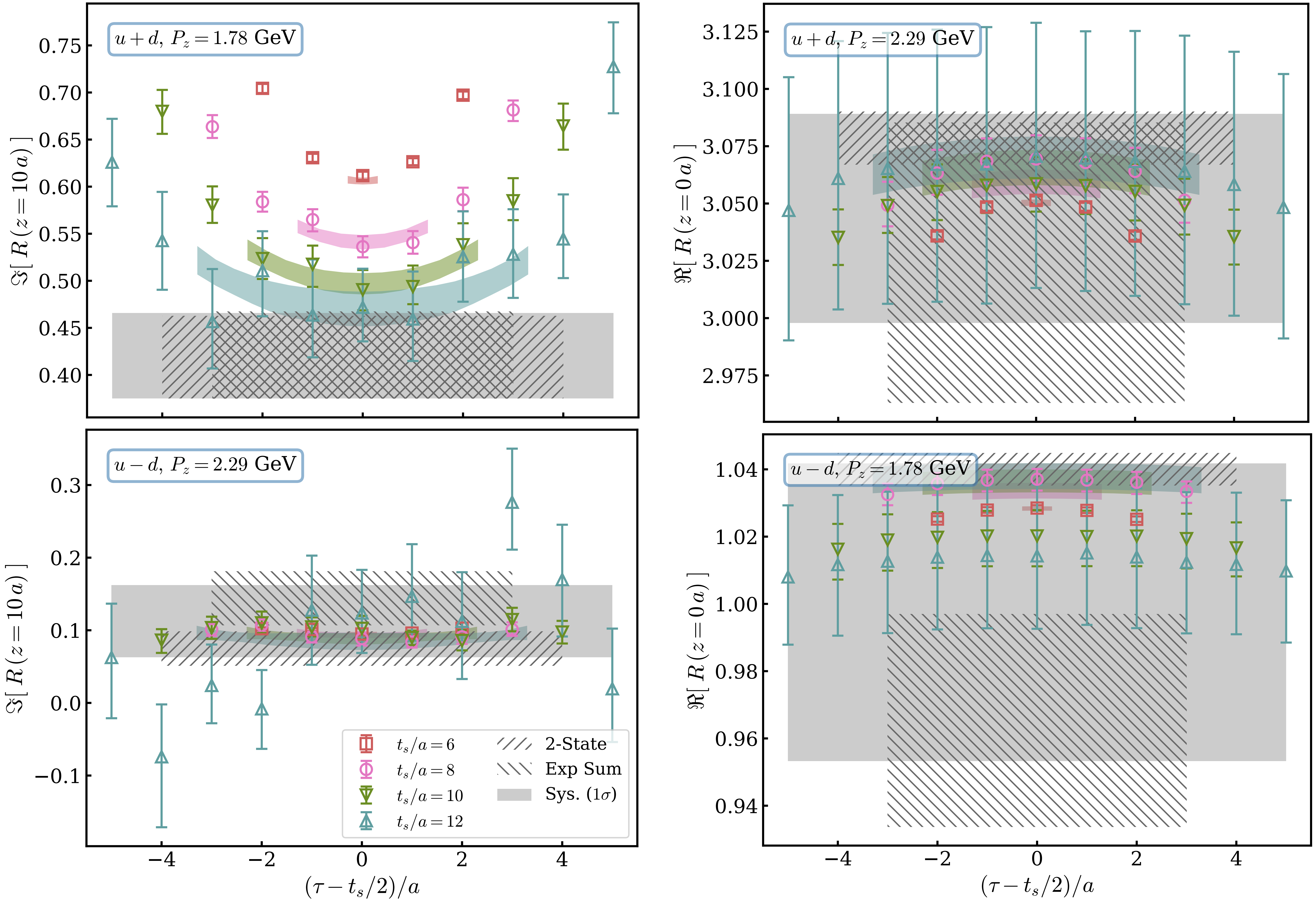
- Exponential Summation Fit:

$$R_{sum}(t_s) = \sum_{\tau=\tau_{skip}}^{t_s - \tau_{skip}} R(\tau, t_s) \approx n h_{00} + n B_1 e^{-\Delta E t_s} + B_0 , \quad n = (t_s - 2\tau_{skip})/a + 1$$

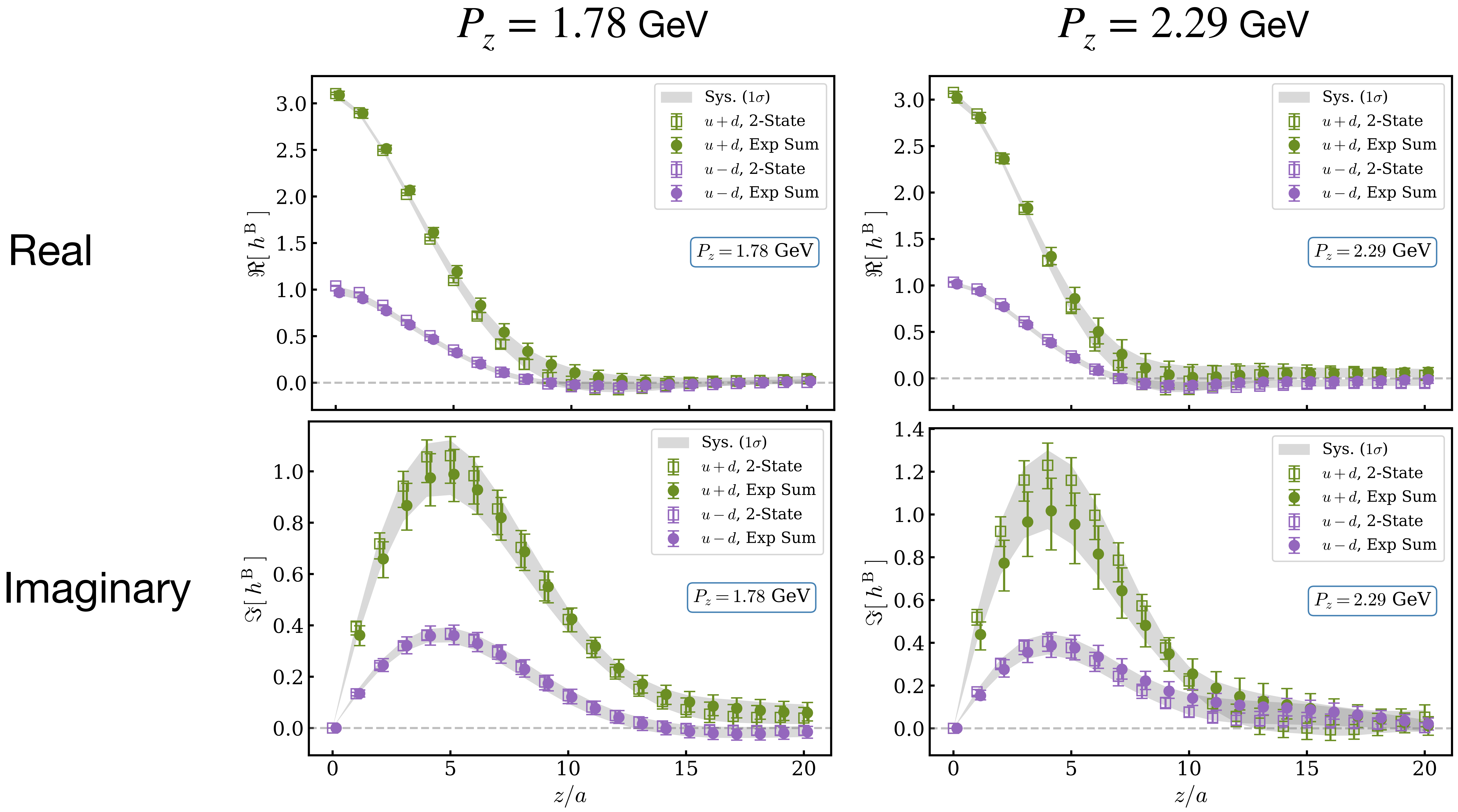
Representative Examples



Statistical
+
Systematic
↓



h^{B} v.s. z/a



$P_z = 1.78$ GeV
$P_z = 2.29$ GeV
Real
Imaginary
Sys. (1σ)
u + d, 2-State
u + d, Exp Sum
u − d, 2-State
u − d, Exp Sum
$P_z = 1.78$ GeV
$P_z = 2.29$ GeV
ℜ[h^B]
ℑ[h^B]
z/a

Renormalization & Extrapolation

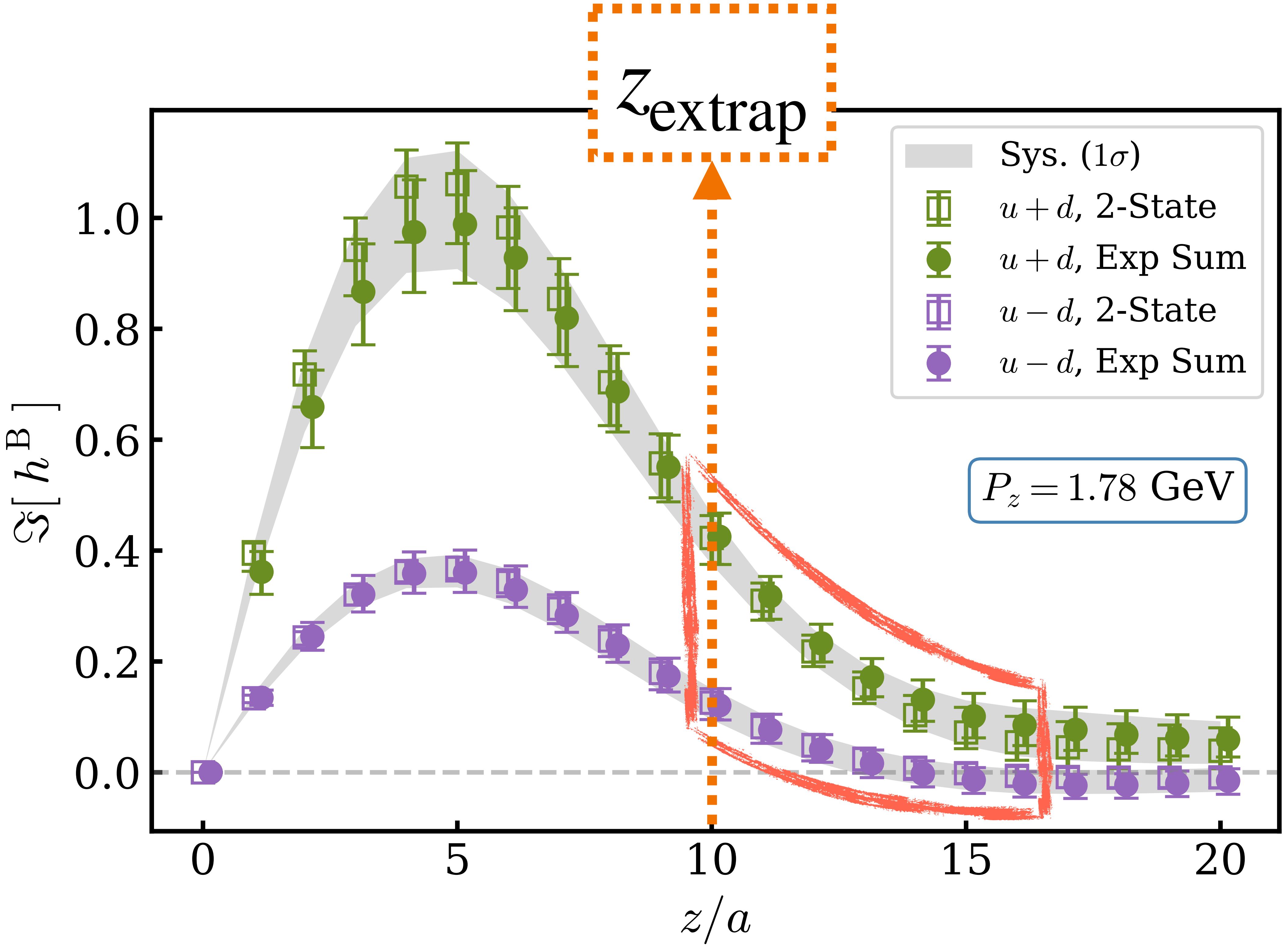
NPB 964 (2021) 115311

- Hybrid renormalization: $h^{\mathrm{R}}(z, z_{\mathrm{s}}, P_z) = N\dfrac{h^{\mathrm{B}}(z, P_z, a)}{h^{\mathrm{B}}(z,0,a)}\theta(z_{\mathrm{s}} - |z|) + N\dfrac{h^{\mathrm{B}}(z, P_z, a)}{h^{\mathrm{B}}(z_{\mathrm{s}},0,a)}\theta(|z| - z_{\mathrm{s}})$

arXiv: 2601.12189

- Next-to-leading-asymptotic (NLA) model: $h^{\mathrm{NLA}}(z, P_z) = \left[A_2 e^{i\,\mathrm{sgn}(\lambda)\phi_2} + \dfrac{A_2'}{|\lambda|}e^{i\,\mathrm{sgn}(\lambda)\phi_2'}\right]\dfrac{e^{-m|z|}}{|\lambda|^n}$

$\lambda = zP_z$, $\{A_2, A_2', \phi_2, \phi_2', m, n\}$ fit params

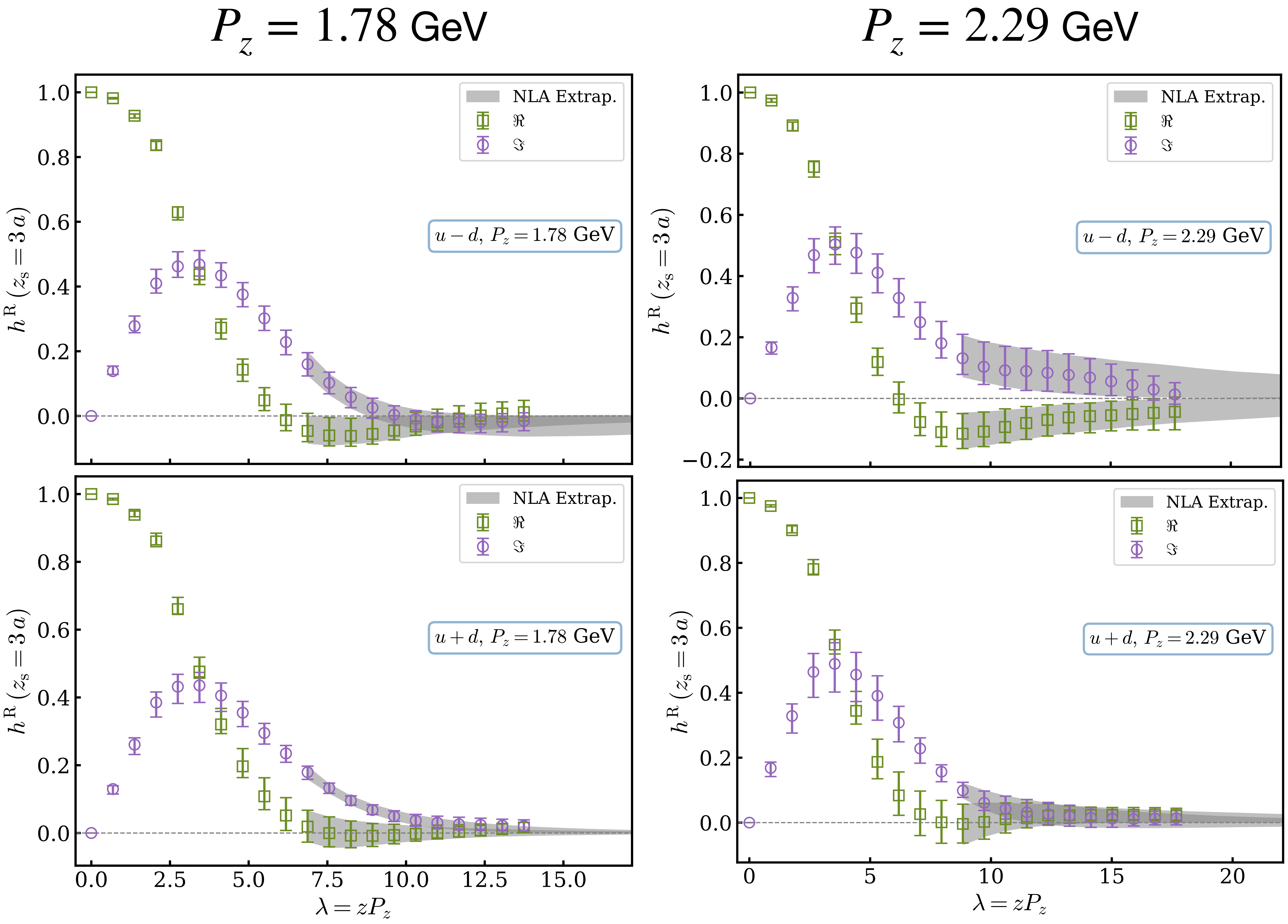


$$h(z, P_z) = \begin{cases} h^{\mathrm{R}}(z, P_z), & |z| \le z_{\mathrm{extrap}}, \\ h^{\mathrm{NLA}}(z, P_z), & |z| > z_{\mathrm{extrap}}. \end{cases}$$

h^{R}

Isovecter

Isoscalar



PDF

☆ Quasi-PDF: $\tilde{q}(x, P_z, z_s) = P_z \int \frac{dz}{2\pi} e^{-ixP_z z} h^R(z, P_z, z_s)$

☆ NLO: $\propto \ln\left(\mu^2 / (2yP_z)^2\right)$

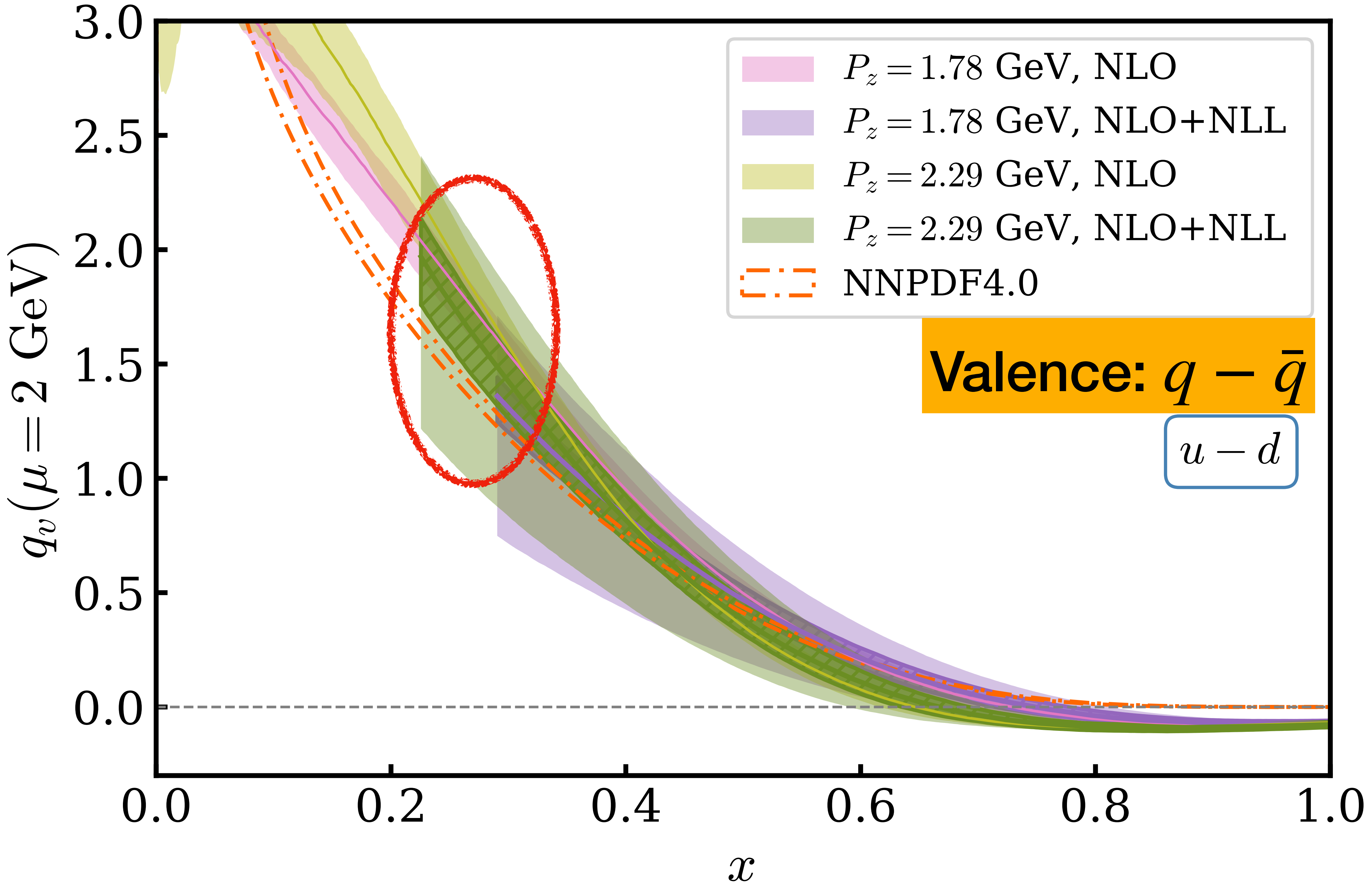
$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \left[\delta\left(1 - \frac{x}{y}\right) - \frac{\alpha_s(\mu) C_F}{2\pi} C^{(1)}_{\text{hyb.}}\left(\frac{x}{y}, \frac{\mu}{p_z}, z_s p_z\right) \right] \tilde{q}(y, P_z, z_s), \quad p_z = xP_z$$

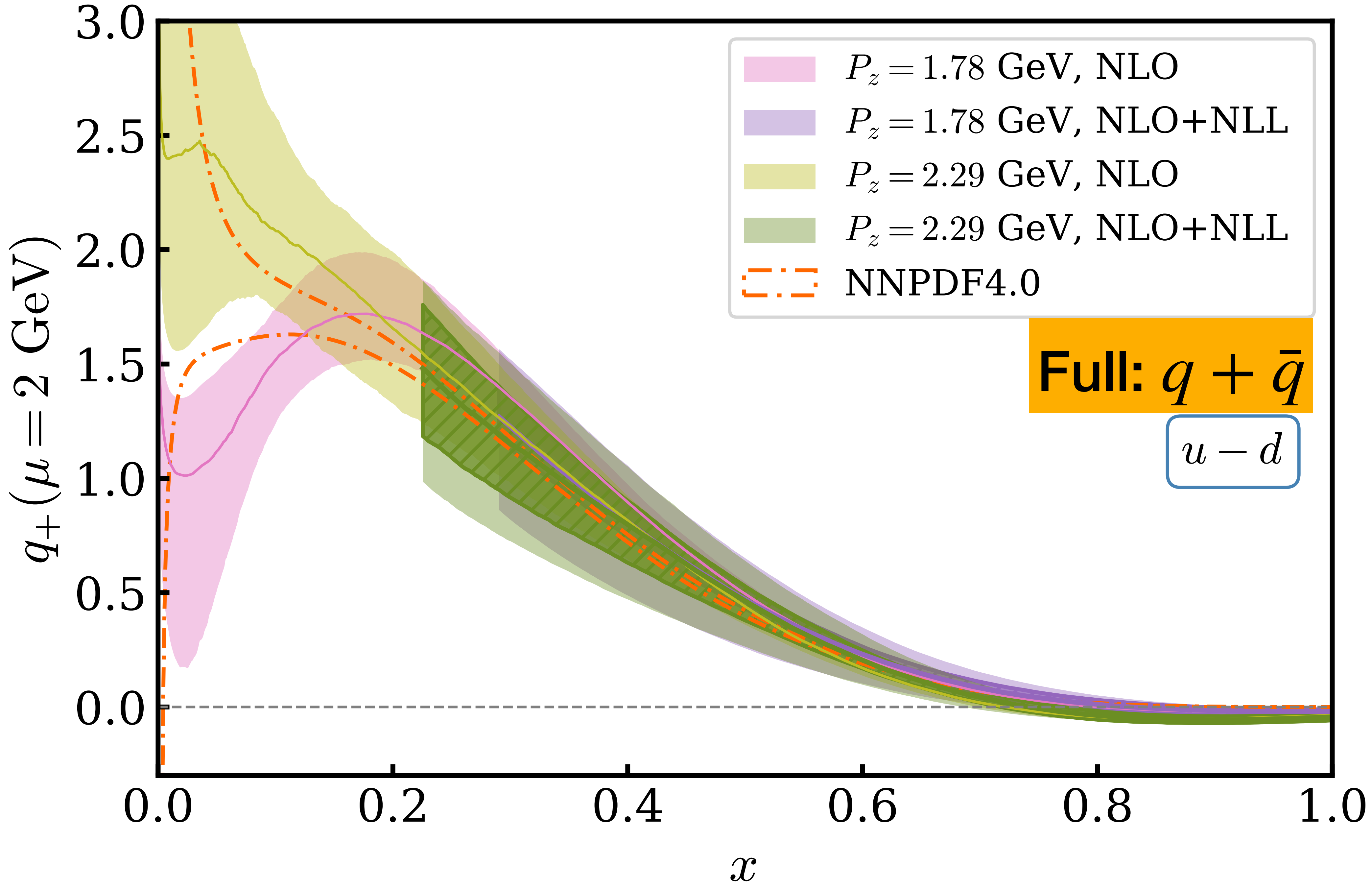
☆ NLO+NLL RGR: $\mu_0 = 2\,\kappa\, zP_z$, $\ln\left(\mu_0^2 / (2yP_z)^2\right) \Rightarrow \mu = 2$ GeV   NPB 991 (2023) 116201

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \int_{-\infty}^{\infty} \frac{dz}{|z|} C^{-1}_{\text{evo}}\left(\alpha_s(\mu_0), \frac{x}{z}, \frac{\mu}{\mu_0}\right)$$

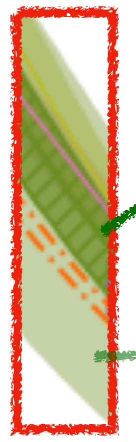
$$\times \left[\delta\left(1 - \frac{z}{y}\right) - \frac{\alpha_s(\mu_0) C_F}{2\pi} C^{(1)}_{\text{hyb.}}\left(\frac{z}{y}, \frac{\mu_0}{p_z}, z_s p_z\right) \right] \tilde{q}(y, P_z, z_s)$$

Light-cone PDF: $u-d$





① Bands:



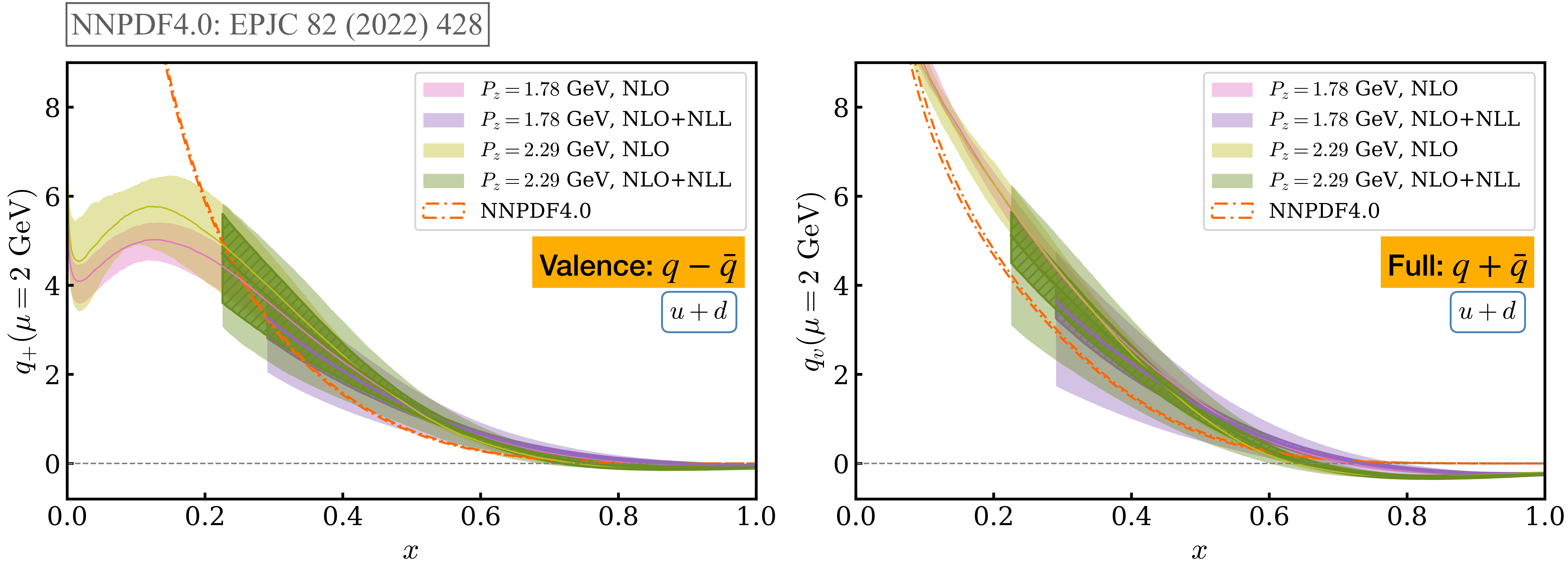
Central scale $\kappa = 1$

Scale variation $\kappa \in \{0.5, 1, 2\}$

② Cutoff: $\mu_0 = 2\kappa x P_z$, $\alpha_s(\mu_0) < 1 \to x_{\min}$

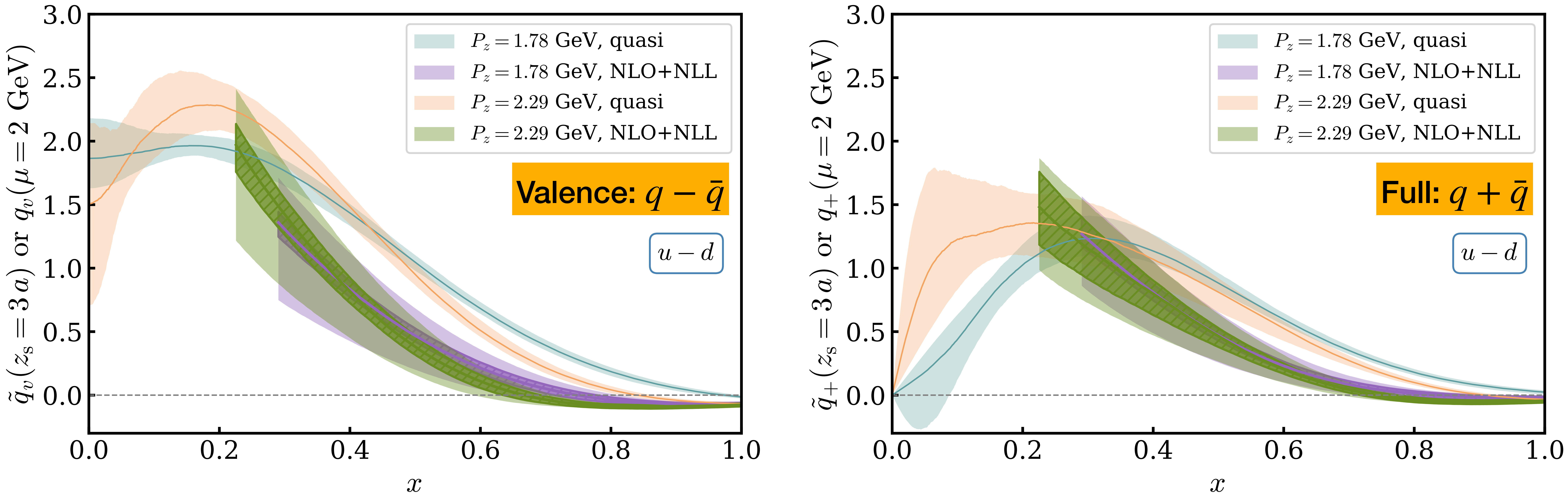
- $x \downarrow$ — scale dependece $\uparrow$
- $x \lesssim 0.5$: $\mu_0/\mu < 1$, RGR correction
- $0.25 \lesssim x \lesssim 0.75$: Consistent with global fits
- $x \to 1$: threshold resummation in the future

Light-cone PDF: connected $u + d$



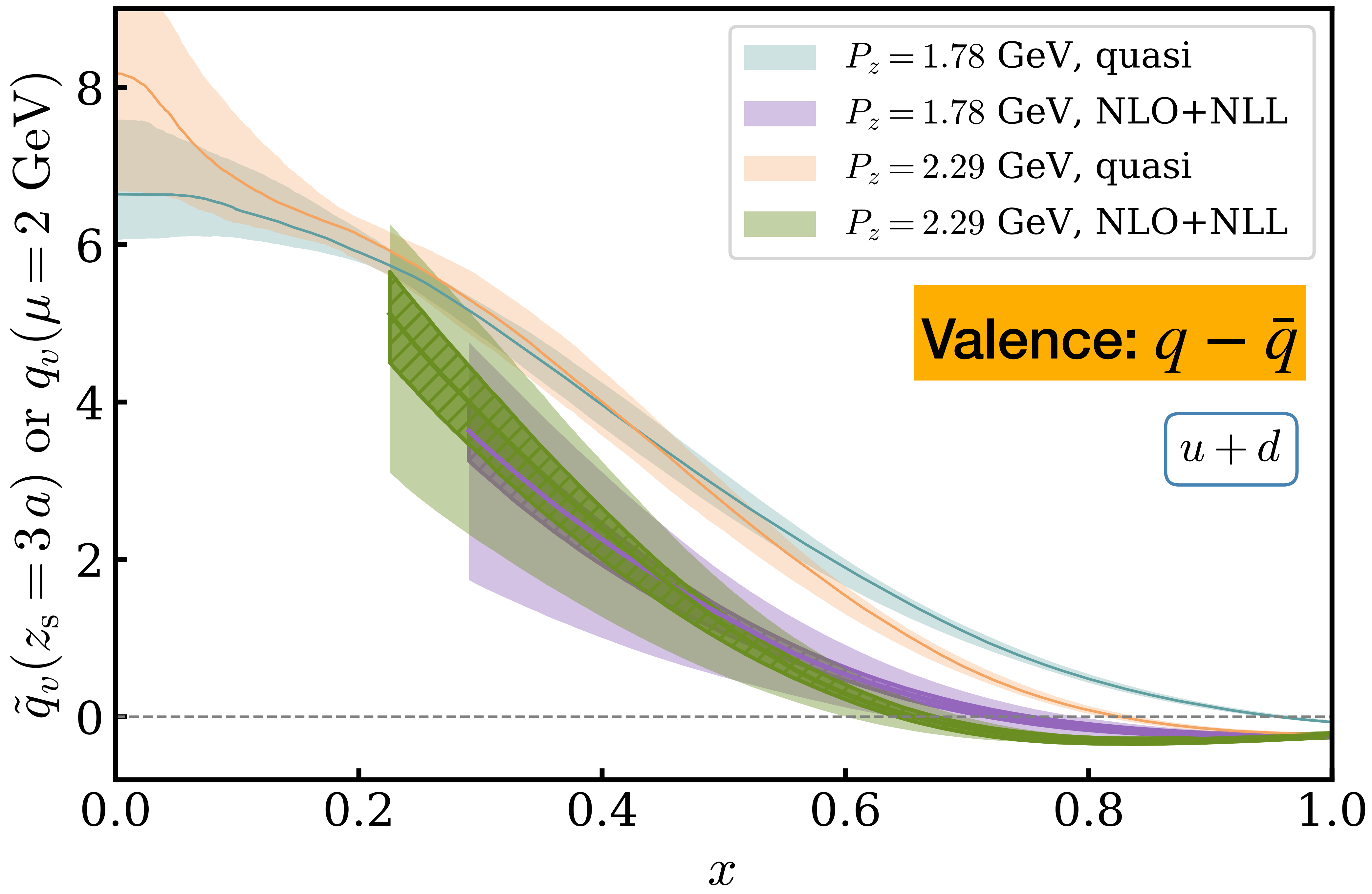
- Consistent within the scale variation
- Omit the disconnect diagrams → Offset in the central curves

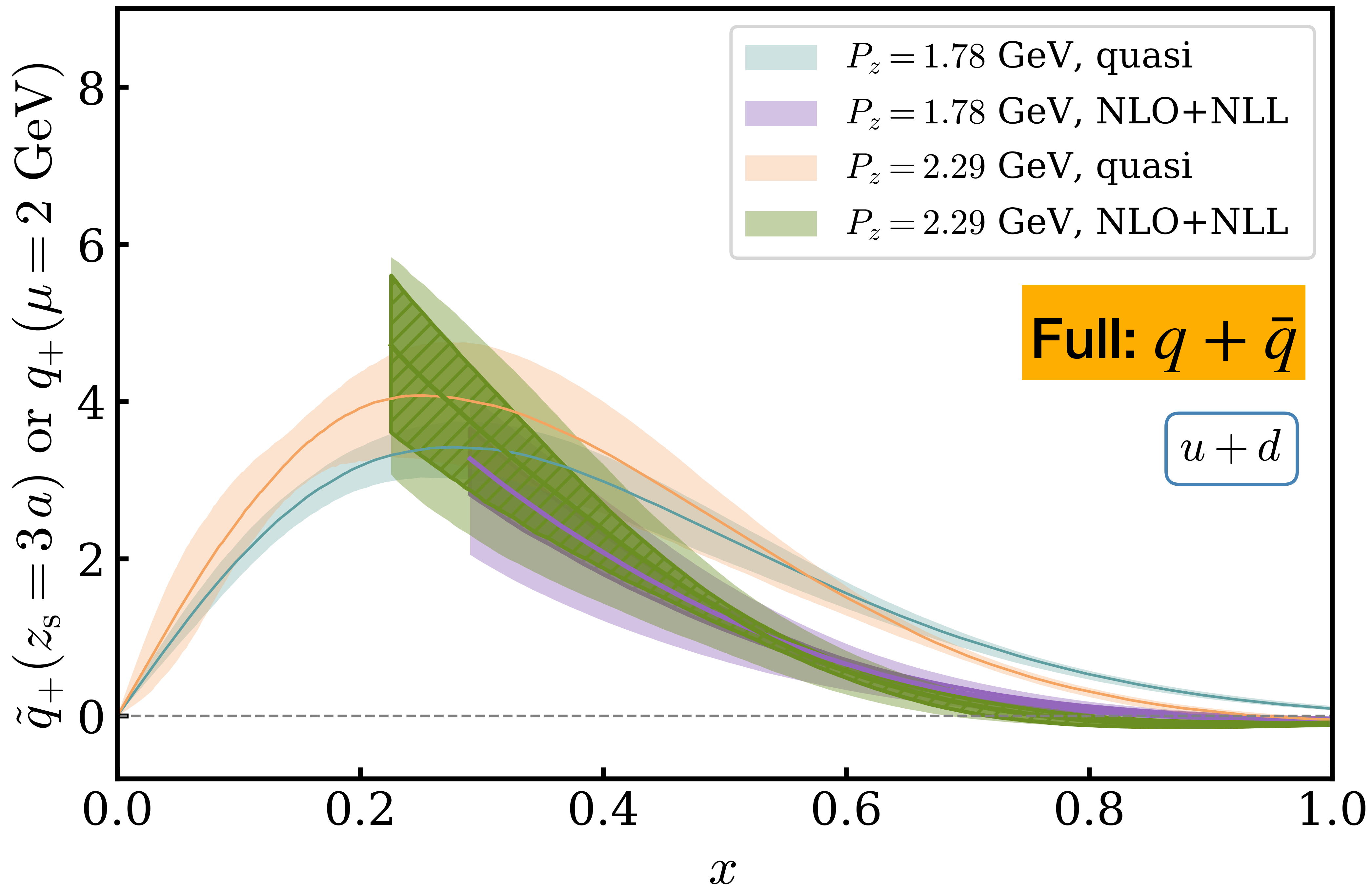
P_z-dependence: $u - d$



- Perturbative matching improves the consistency between diff. momenta
- LaMER expectation: momentum-independent light-cone results

P_z-dependence: connected $u+d$





Valence: $q - \bar{q}$

Full: $q + \bar{q}$

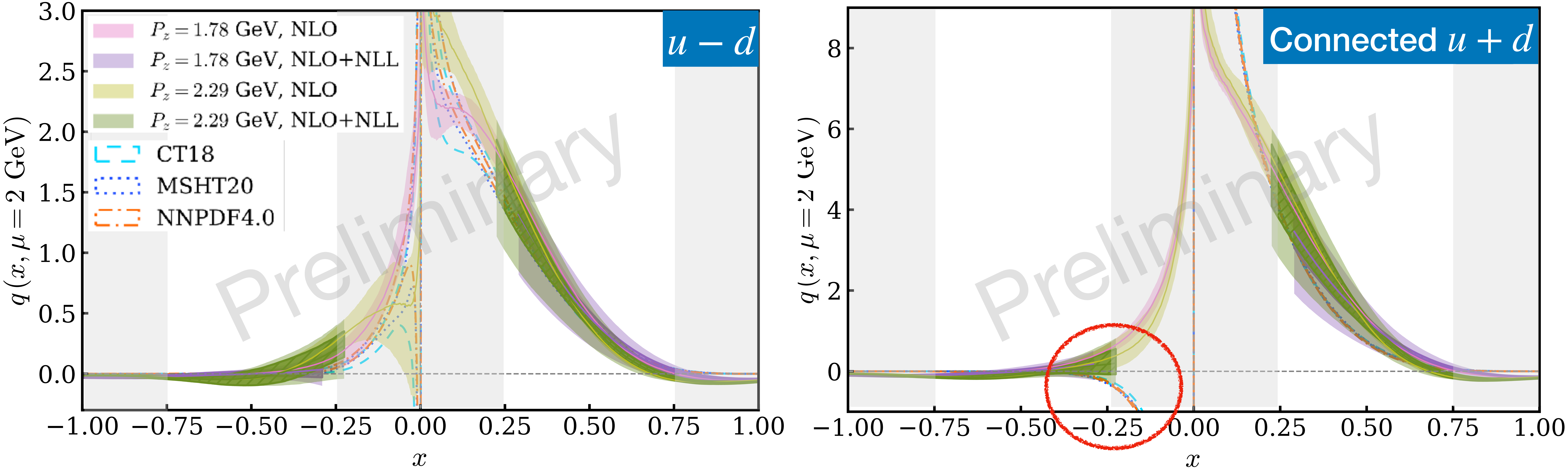
$\Rightarrow$

Positive x: q

$x \in [-1, 1]$

Negative x: $\bar{q}$

Entire-x PDF: Compare w. Global fits



CT18:
PRD 103 (2021) 014013
MSHT20:
EPJC 81 (2021) 341
NNPDF4.0:
EPJC 82 (2022) 428

- RGR-improved results are needed
- Isovector: good agreement with global analyses; antiquark: $\bar{d}(x) > \bar{u}(x)$ for $x > 0$
- Connected isoscalar:
  quark:
  antiquark: $-[\bar{u}(x) + \bar{d}(x)] \leq 0$ for $x > 0$, overlook the disconnected diagrams

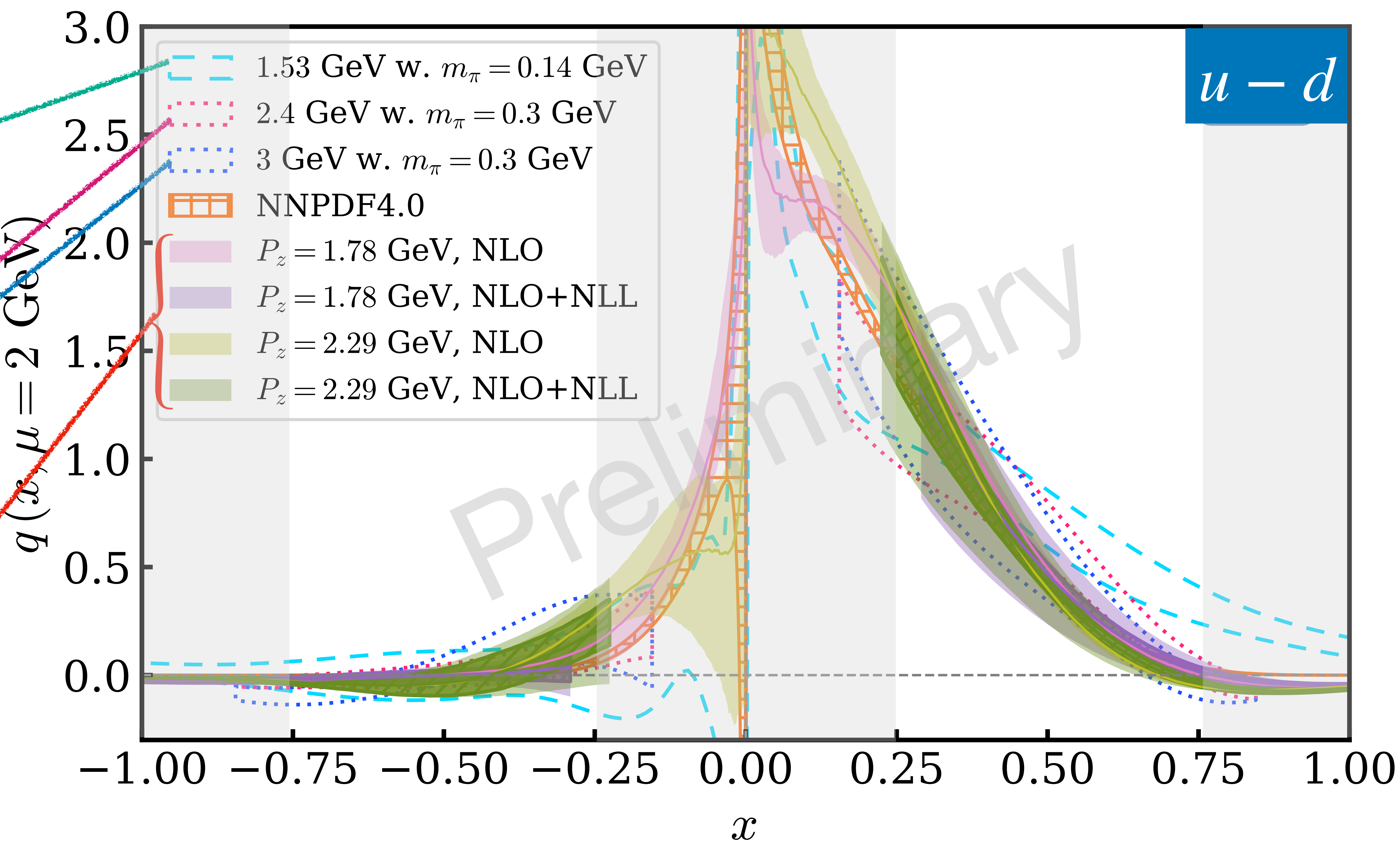
Entire-x PDF: Compare w. Other lattice results

Dominance of P_z ✔
Accidental coincidence ✘

PRD 107 (2023) 074509
Physical, low P_z

Heavier, high P_z
arXiv: 2602.11283

Physical, high P_z



The universality of unpolarized PDF from first principle QCD

Summary & Outlook

- ☑ Nucleon unpolarized PDF
  - physical point
  - large, discretization-controlled P_z
  - isovector and connected isoscalar
- ☑ Consistent with global fits
  - NLO & NLL RGR
  - $0.25 \lesssim x \lesssim 0.75$
  - $\bar{u} + \bar{d}$ is underestimated
- ☑ Universality of Lattice QCD determination
  - combining previous works

- ☐ NNLO + NNLL RGR
  - improve the perturbative accuracy
  - study the perturbative convergence
- ☐ Threshold resummation
  - control the large logarithms near $x \to 1$
- ☐ Disconnected diagrams in $u + d$
- ☐ Extend to multiple a & continuum limit
  - remove the residual discretization artifacts
  - more definitive test of the universality

Thanks for your attention!