MASSACHUSETTS INSTITUTE OF TECHNOLOGY

arxiv:2606.16877

Direct calculation of parton distributions in momentum space from lattice QCD

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Factorization of QCD processes

Collins, et.al, ASDHEP(1989)

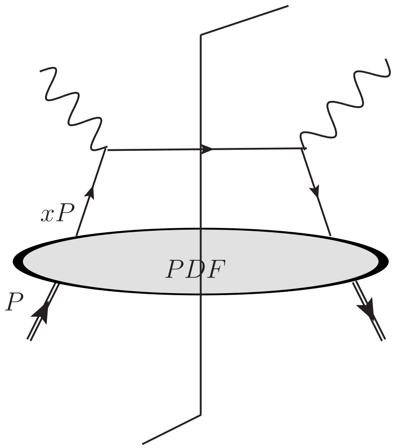
Factorization is the foundation of studying hadron collider physics

Pert calculable hard kernel — Universal NP parameters

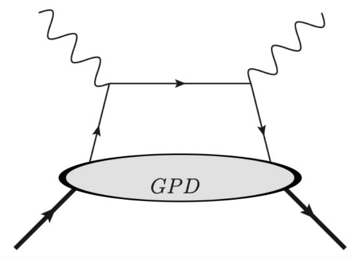
$$\mathrm{OB} = \mathrm{UV} \otimes \mathrm{IR} + O\left(\frac{\Lambda_{\mathrm{QCD}}^{n}}{Q^{n}}\right)$$

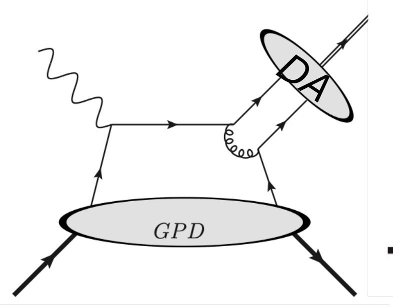
Power corrections

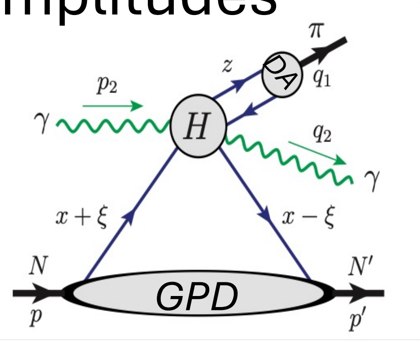
- Inclusive process
  - Factorization of cross section
  - Parton Distribution Function



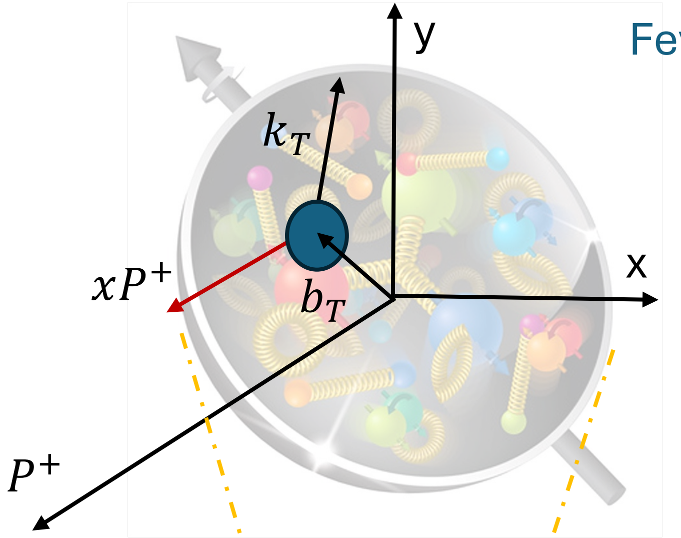
- Exclusive process
  - Factorization of amplitude
  - Generalized parton distribution
  - Meson distribution amplitudes



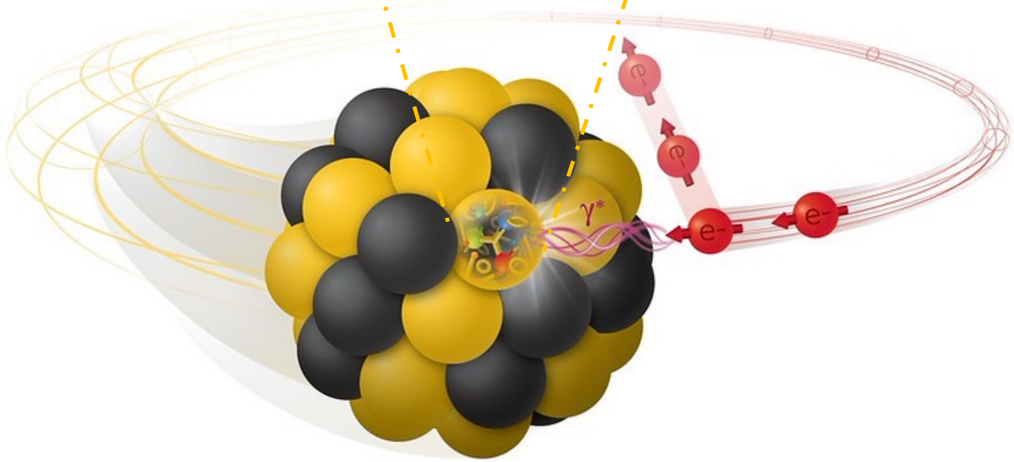




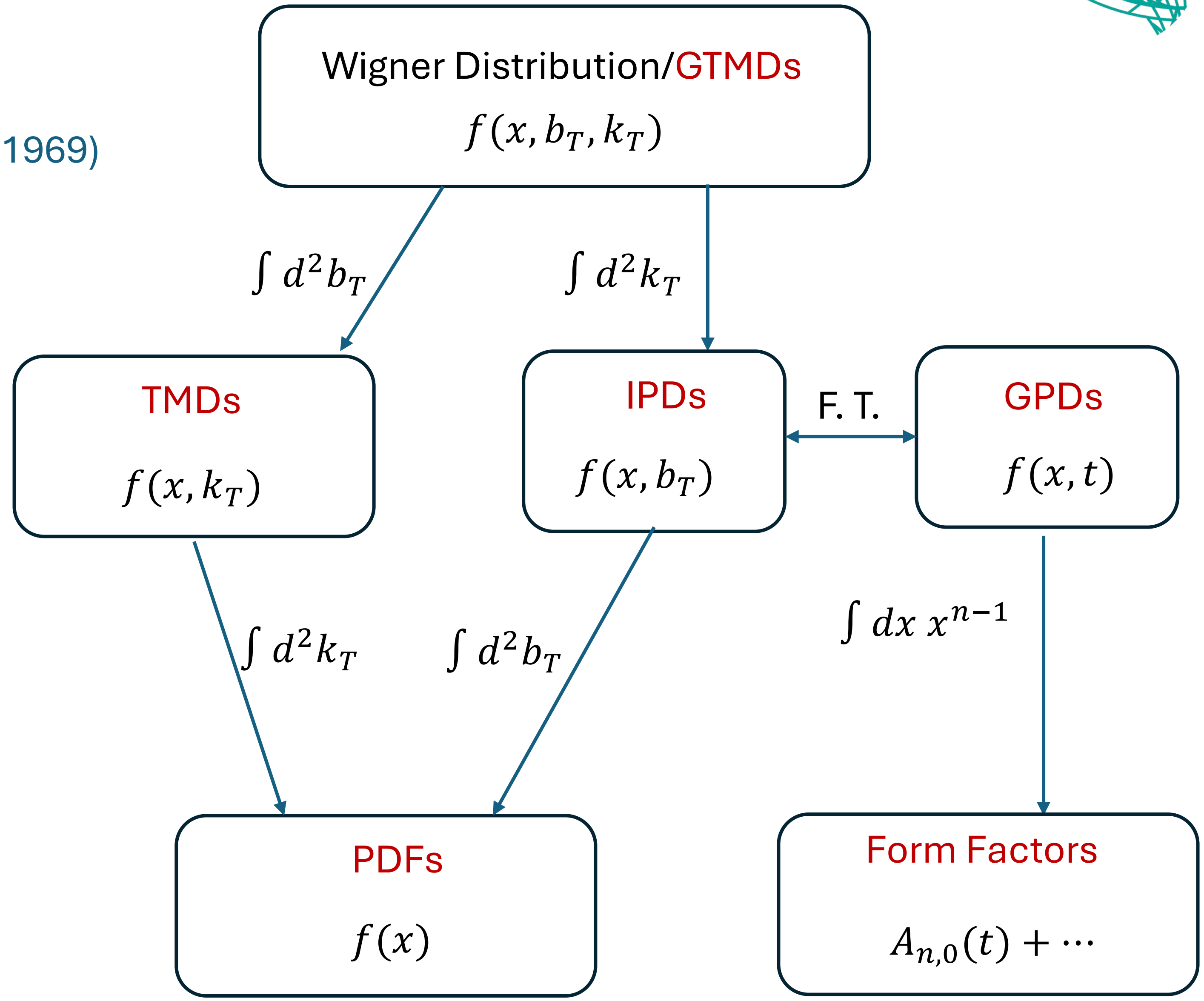
Parton Distributions



Feynman, PRL(1969)



credit: BNL

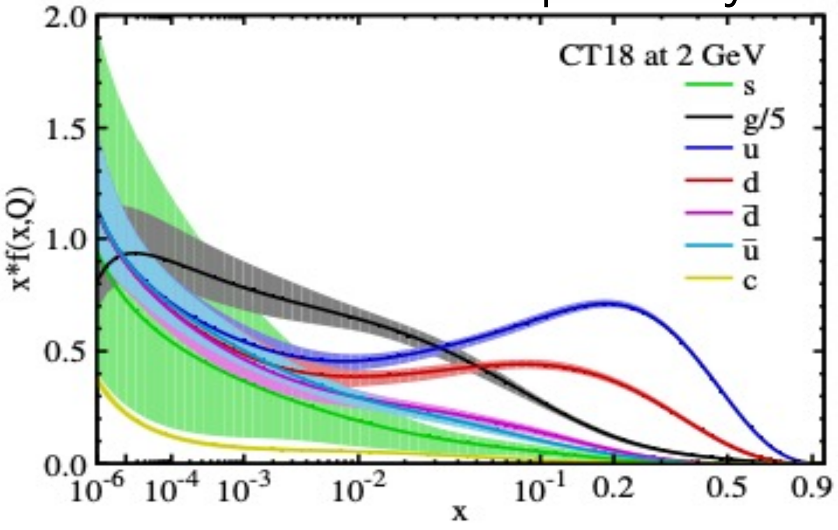


Extracting Parton Physics from Exp Data

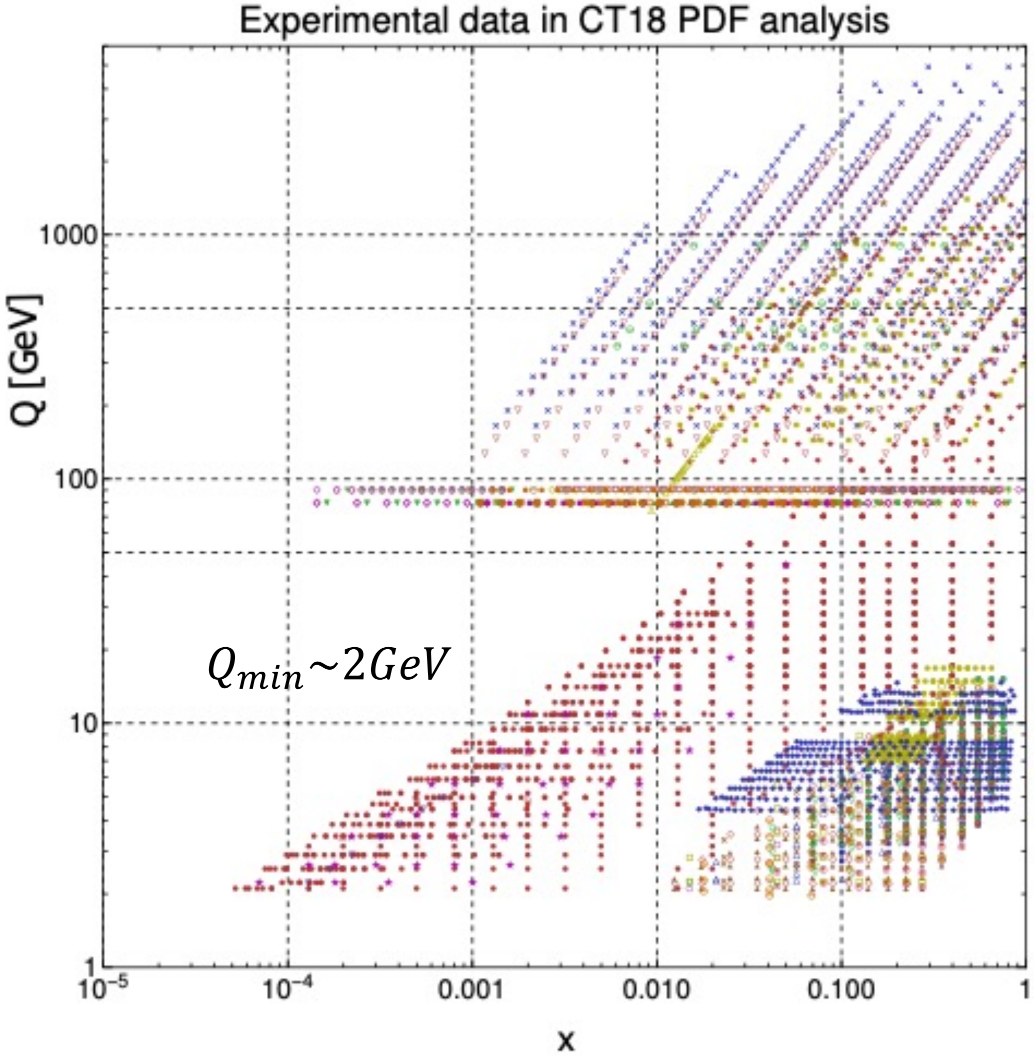
Global Fitting is solving an inverse problem (IP) with large enough data points:

More data, less model assumptions

- Unpolarized PDF has been precisely determined.



- Many other observables are poorly constrained.



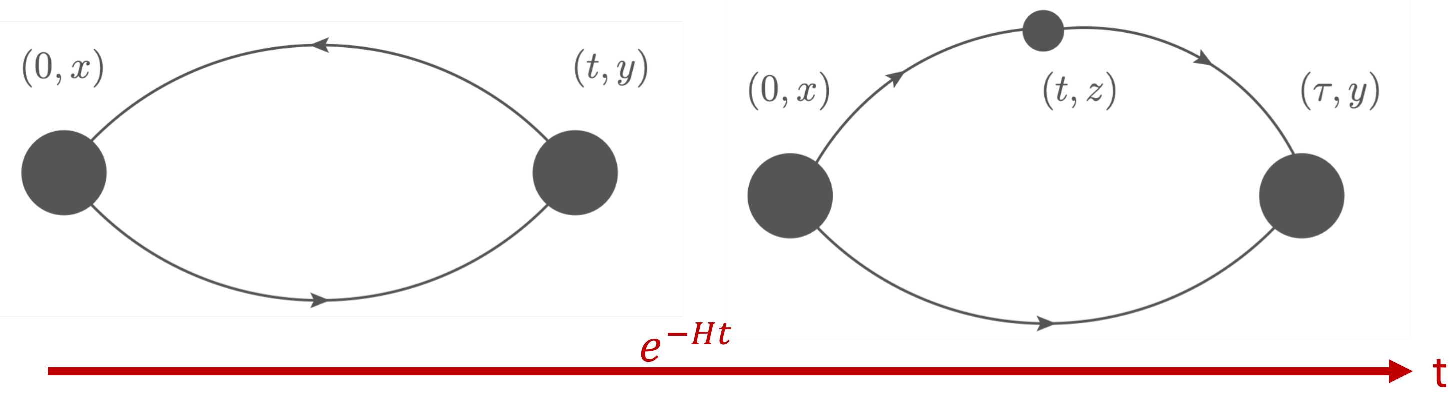
Hou et al. PRD (2019)

Lattice QCD

Wilson, PRD(1974)

- Discretization of QCD action
- Construction of correlators:

$C_{2pt}(t) = \langle \chi_{snk}(t) | \chi_{src}(0) \rangle$ $\quad$ $C_{3pt}(t) = \langle \chi_{snk}(t) | O(t) | \chi_{src}(0) \rangle$



χ overlaps with all physical states carrying the same quantum number

Euclidean 4D spacetime

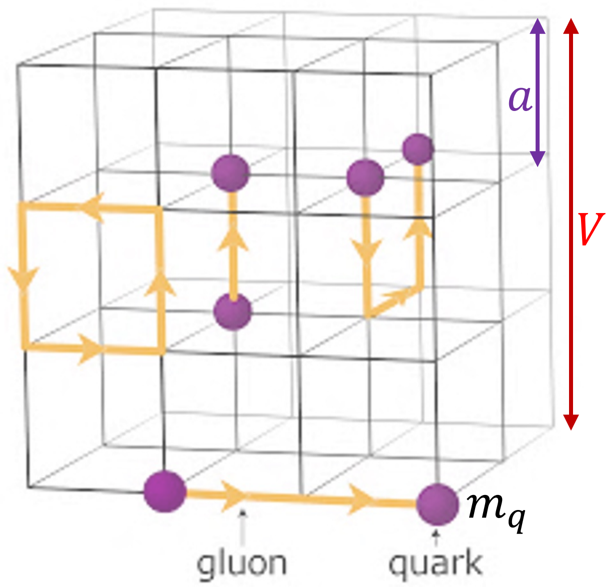


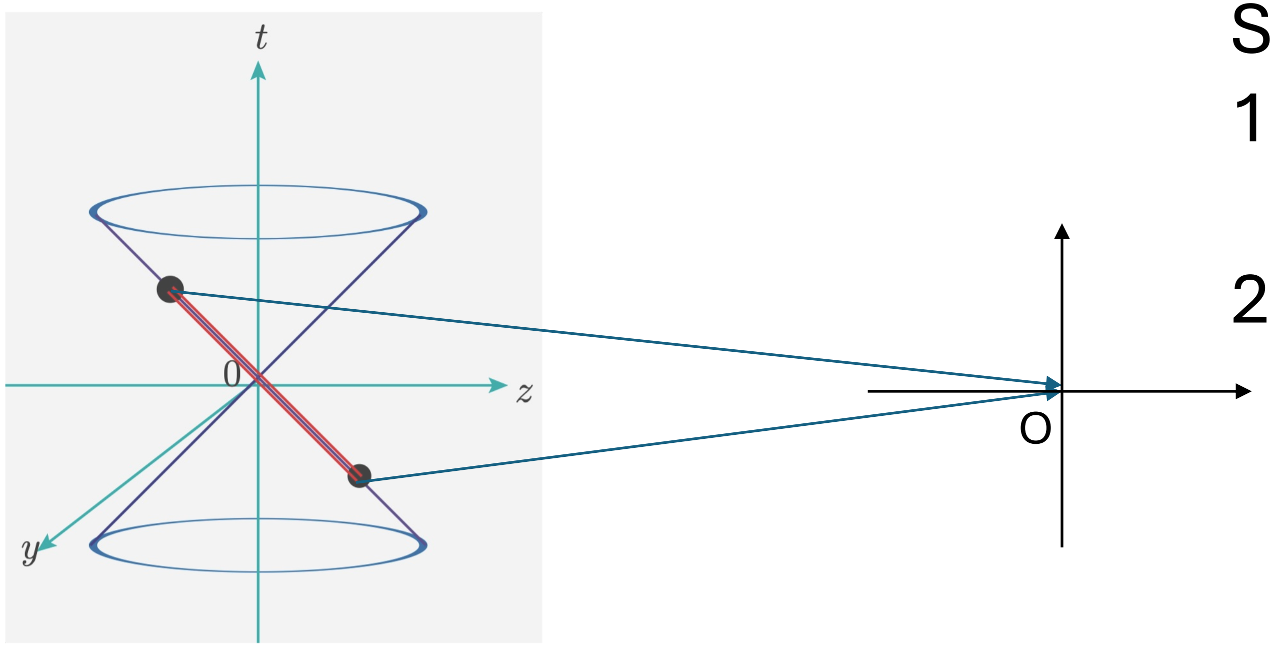
Image credit: M. J. Savage

Spectrum expansion:

$C_{2pt}(t) = \sum |c_n|^2 e^{-E_n t}$

$C_{3pt}(t, \tau) = \sum c_m^* c_n \langle m | O | n \rangle e^{-E_m(\tau - t)} e^{-E_n t}$

But for parton physics on lattice ...



Minkowski Spacetime

Euclidean Lattice

Solutions:

1. Local operators from OPE
   - Measure moments
2. Indirect probe
   - “Lattice cross sections”
   - HOPE
   - Ioffe time distribution
   - Pseudo-PDF
3. Effective theory
   - LaMET

Not directly measurable?

Detmold, et.al, PRD(2006)
Braun, EPJC(2008)
Radyushkin, PRD(2017)
Ma & Qiu, PRL(2018)
Chambers, et.al, PRL(2017)

Indirect probe of parton physics

Lattice: $h(z,\omega) = UV \otimes Parton\ Physics + O\left(z^n \Lambda_{\mathrm{QCD}}^n\right)$

($z \to q^{-1}$ in HOPE and Compton amplitude methods)

Mellin space: $h(z,\omega) = \sum \omega^n C(z,\mu)\langle x^n \rangle + O\left(z^n \Lambda_{\mathrm{QCD}}^n\right)$

Hard scale: $Q \sim z^{-1} \gg \Lambda_{\mathrm{QCD}}$

1. Extracting Lowest few Mellin Moments, or
2. Solve the inverse problem to obtain x-dependence

Be careful!
Don't use small scales:

FIG. 18. Matching vs pure evolution of a toy model from $z = 0.56$ fm to 2 GeV, for $\xi = 0$ (top) and $\xi = 0.5$ (bottom). $z = 0.56$ fm corresponds to an initial scale of 0.35 GeV if $\mu_0 = 1/\sqrt{-z^2}$ (dotted and dashed lines), which we vary between 0.25 and 0.49 GeV to give an account of scale fixing uncertainty (error bands).

Large Momentum Effective Theory (LaMET)

Ji, PRL (2013)
Ji, SCPMA(2014)
Ji, NPB (2024)

K. Wilson:
$a \to 0$ is a critical point.

Physics on finite lattices tells us about continuum.

Ji (Inspired by Wilson):
$P_z \to \infty$ limit is a critical point.

Physics at large P_z tells us about lightcone physics.

Symanzik's Effective lattice Lagrangian:

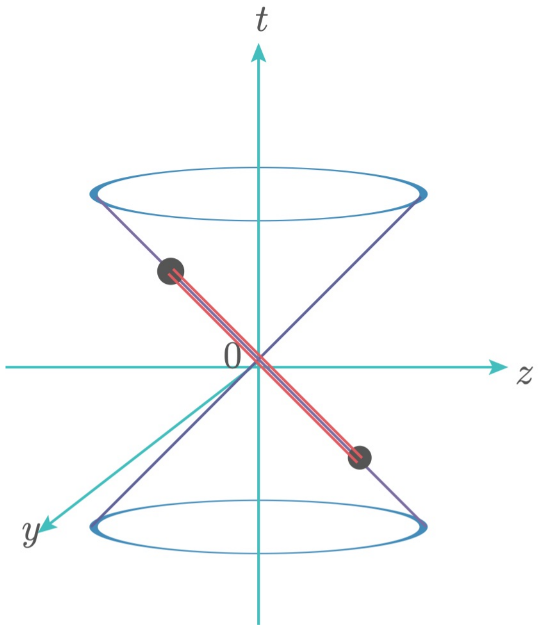
$$\mathcal{L}_{eff}^{QCD} = C_0(\alpha(a))\,\mathcal{L}_a^{(0)} + \sum a^i C_i(\alpha(a))\,\mathcal{L}_a^{(i)}$$

LaMET Expansion:

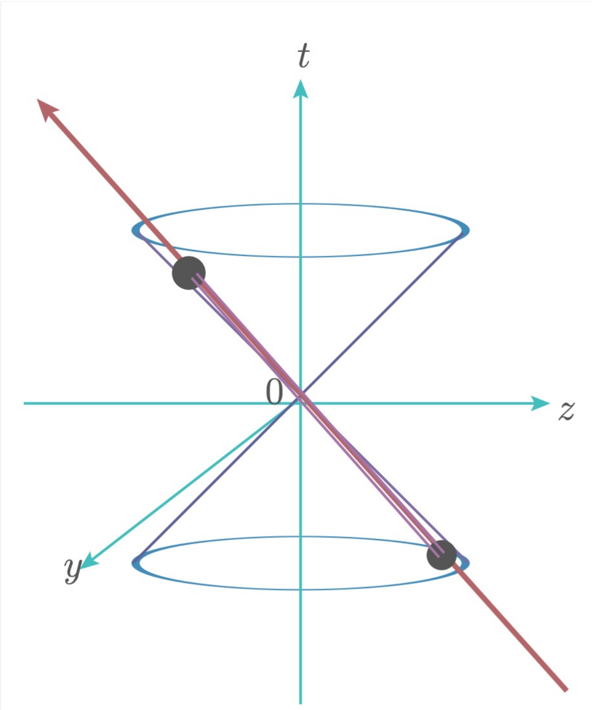
$$\mathcal{L}_{eff}^{Parton} = C_0(\alpha)\,\mathcal{L}_v^{(0)} + \sum \gamma^{-i} C_i(\alpha)\,\mathcal{L}_v^{(i)}$$

Large Momentum Effective Theory (LaMET)

Ji, PRL (2013)
Ji, SCPMA(2014)



Large P_z
Expansion



Infinite Power Corrections

$$+\mathcal{O}\left(\frac{1}{P_z^n}\right)$$

$$F(x,\mu) = C\,(x,y,\mu,P_z) \otimes \tilde{F}(y,P_z)$$

Lightcone Distribution = Quasi-Distribution

$$+\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{\mathrm{x}^2 P_z^2}, \frac{\Lambda_{QCD}^2}{\bar{\mathrm{x}}^2 P_z^2}\right)$$

LaMET universality class (example for PDFs)

- $\tilde{F}(y, P_z)$ can be anything within a universality class
  - Quasi-PDF: $\int e^{ixzP_z}\langle P|\bar{\psi}(z)W(z,0)\Gamma\psi(0)|P\rangle$
  - Coulomb gauge quasi-PDF: $\int e^{ixzP_z}\langle P|\bar{\psi}_C(z)\Gamma\psi_C(0)|P\rangle$
  - Current-Current distribution: $\int e^{ixzP_z}\, z^3\langle P|\bar{\psi}(z)\Gamma^a\psi(z)\bar{\psi}(0)\Gamma^b\psi(0)|P\rangle$
  - …

Zhao, PRL (2024)
Gao, et.al., PRD(2023)

Zhang, et.al., (2026)

- Same IR physics as lightcone parton distributions
  - Lightcone parton distributions / physical cross sections can be factorized into quasi distributions and perturbative matching, if $\tilde{F}(y, P_z)$ is fully known.

Forward Matching!

Systematics for x-dependence in LaMET

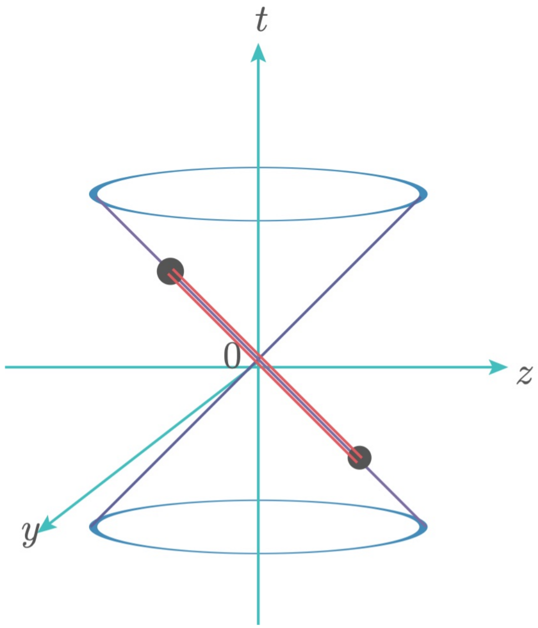
Chen, et.al., PRD (2026)

- What systematics?
  - Lattice data become extremely noisy at large z, one will have to truncate the data and model the longtail (formally an IP if without constraint)
- Solution: Physics correlations decay exponentially
  - Asymptotic form derived through dispersive analysis in arxiv:2601.12189
  - Finite degrees of freedom for a system with exponential decay
- Prerequisite: Data are precise enough until $z \sim 0.8(0.5)$ fm
- Conclusion: large z contribution to x-space results is below a systematically improvable bound

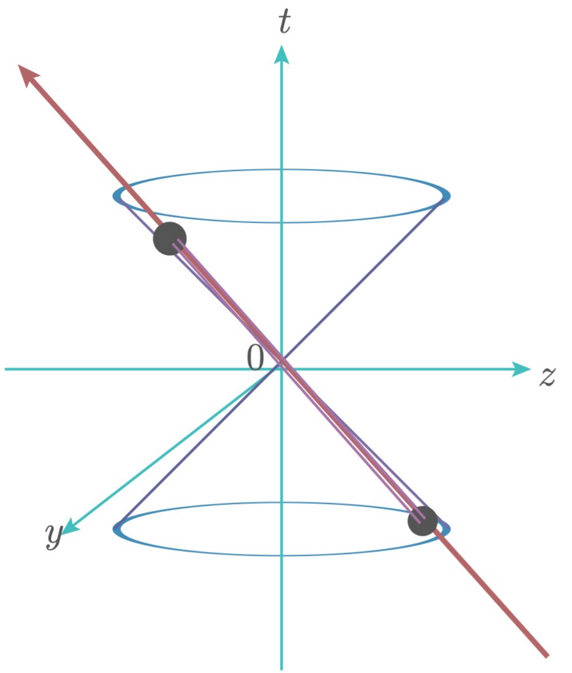
$$\delta f(x, P, \lambda_L) < \frac{4N_x |h(z, P; \lambda_L)|_{\max}}{\pi x}$$

Directly start from momentum space?

LaMET really just needs an x-space $\tilde{F}(y, P_z)$:



Large P_z Expansion



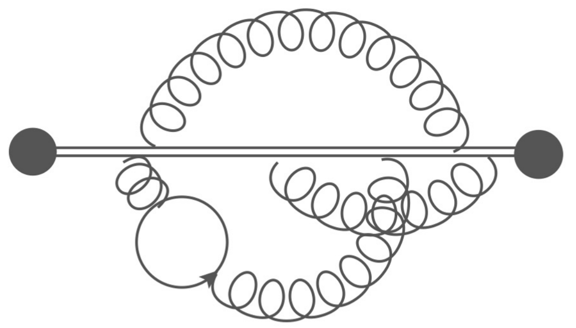
Infinite Power Corrections

$$+\mathcal{O}\left(\frac{1}{P_z^n}\right)$$

$$F(x, \mu) = C\,(x, y, \mu, P_z) \otimes \tilde{F}(y, P_z) \quad +\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{\bar{x}^2 P_z^2}\right)$$

Lightcone Distribution = Quasi-Distribution

Why we had to start from z-space?



- Renormalization of operator $O(z) = \bar{\psi}(z)W(z,0)\psi(0)$
- Linearly divergent self-energy $\delta m(a) \sim \frac{1}{a}$
  - $h^B(\mathrm{z}) \sim e^{-\delta m(a)\cdot|z|}$
  - Renormalization constant $Z(a,z,\mu) \sim e^{-\delta m(a)\cdot|z|}$ is z-dependent

$$\tilde{q}^R(x, P_z, \mu) = \int \frac{dzP_z}{2\pi} e^{ixzP_z} \frac{\tilde{h}^B(z, P_z, a)}{Z(z, a, \mu)}.$$

- Renormalization must be done before F.T.
- Renormalized data have to be truncated due to exponentially growing noise → asymptotic analysis in LaMET

Quasi-PDF without Wilson line

Gao, et.al., PRD (2023)
Zhao, PRL (2024)

- Coulomb gauge $O_C(z) = \bar{\psi}_C(z)\Gamma\psi_C(0), \quad \psi_C(x) = e^{-ig\frac{\nabla\cdot A}{\nabla^2}}\psi(x)$
- In the $P_z \to \infty$ limit, $\nabla\cdot A = 0 \to A^+ = 0$
- No Linear divergence
- Renormalization of $O_C(z)$ is independent of z

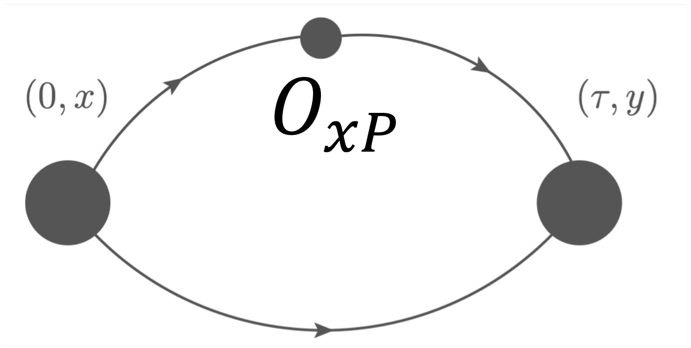
$$O_C^R(z,\mu) = O_C^B(z,a)/Z_C(a,\mu)$$

- Directly define momentum density operator:

$$\tilde{O}_C(k_z) \equiv \sum_{z=-L/2}^{L/2-a} e^{izk_z}\bar{\psi}(0)\gamma^t\psi(z)$$

$$\tilde{q}_C^R(x,P_z,\mu) = \frac{\tilde{q}_C^B(x,P_z,a)}{Z_C(a,\mu)} = P_z\langle P|O_C^R(xP,\mu)|P\rangle$$

x-space Recipe



- Lattice correlator
- ME: multi-state fit

$$\frac{C_{3pt}(t,\tau)}{C_{2pt}(\tau)} = \tilde{q}^B(x, P_z, a) + \mathcal{O}(e^{-\Delta E t}, e^{-\Delta E(\tau - t)})$$

- Renormalization

$$\tilde{q}^R(x, P_z, \mu) = \tilde{q}^B(x, P_z, a)/Z^R(a, \mu)$$

- Matching to lightcone PDF

$$q(x, \mu) = \int C(x, y, \mu, P_z)\tilde{q}(y, P_z, \mu)$$

Where does the systematics go?

$z-$ space method

Uncertainty control in FT from $z-$space

Bounded, improvable model dependence of asymptotic analysis

$x-$ space method

Finite volume effects for off-grid momenta

$$xP_z \neq n\times\frac{2\pi}{L}$$

$$\frac{\Delta\tilde{q}(\mathrm{x})}{\tilde{q}(\mathrm{x})} \sim e^{-\Lambda L}|\sin(xLP_z/2)|, \quad \Lambda \geq m_\pi$$

A type of common lattice artifacts. Automatically removed during infinite volume extrapolation.

Lattice test for pion PDF

- The ensemble is generated by MILC collaboration

Lattice Spacing	m_π	Lattice Volume	$m_\pi L$	Fermion Action (sea)	Fermion Action (Valence)
0.06 fm	310 MeV	$48^3 \times 144$	3.7	2+1+1f HISQ	Clover-Wilson

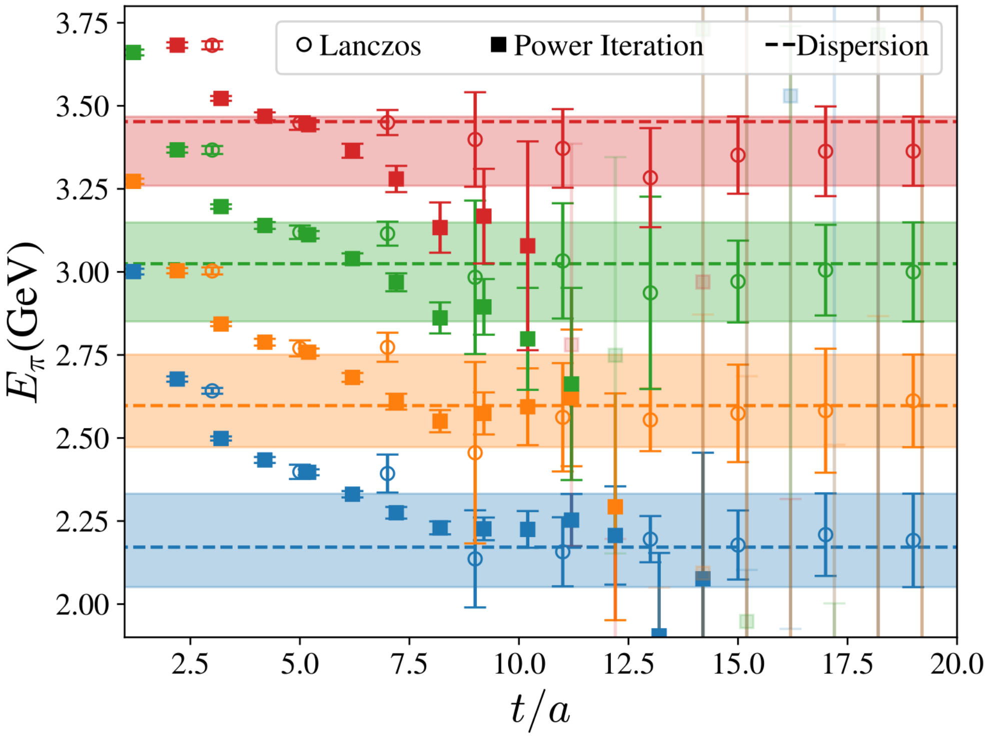
Momentum Smearing	P_z	Gauge Smear	Samples	Sources/cfg
$k = 1.7$ GeV	{2.15,2.6}GeV	1-HYP	100	16

Extraction of ground-state matrix elements

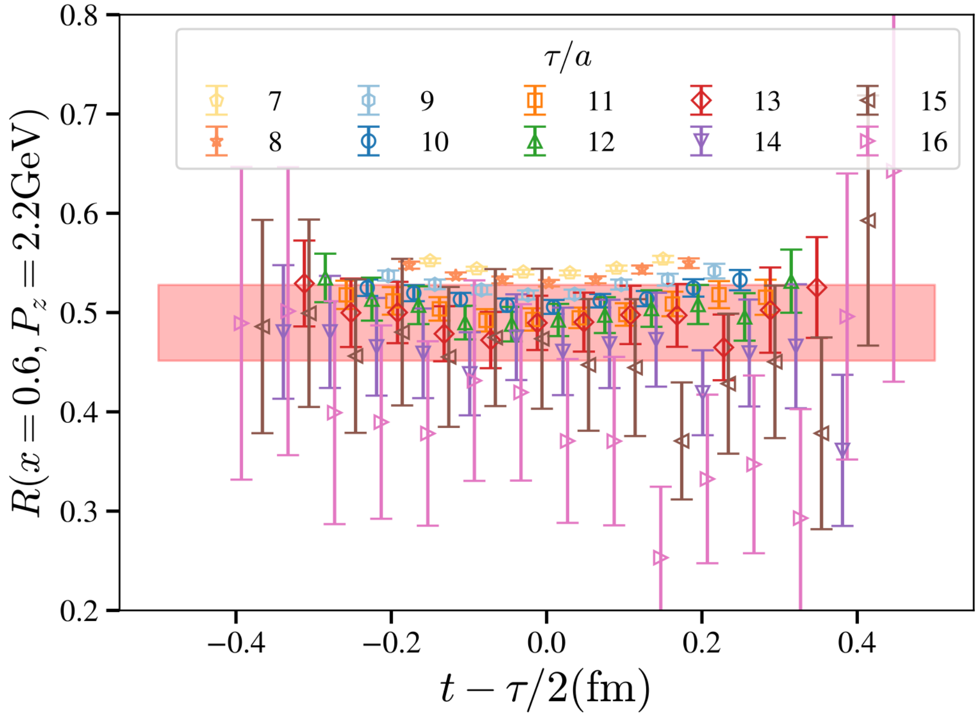
Wagman, PRL (2025)
Hackett, et.al., PRD(2025)

We use Lanczos analysis to extract ground state

Grounds-state energy for different momenta

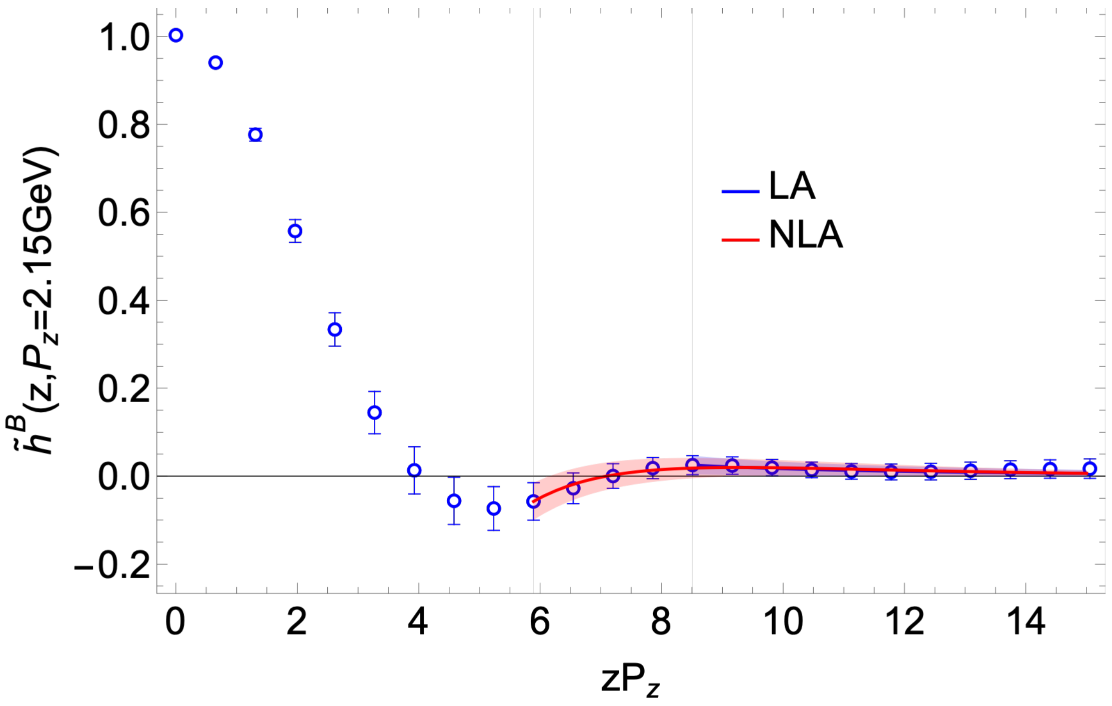


Ground-state matrix elements in x-space

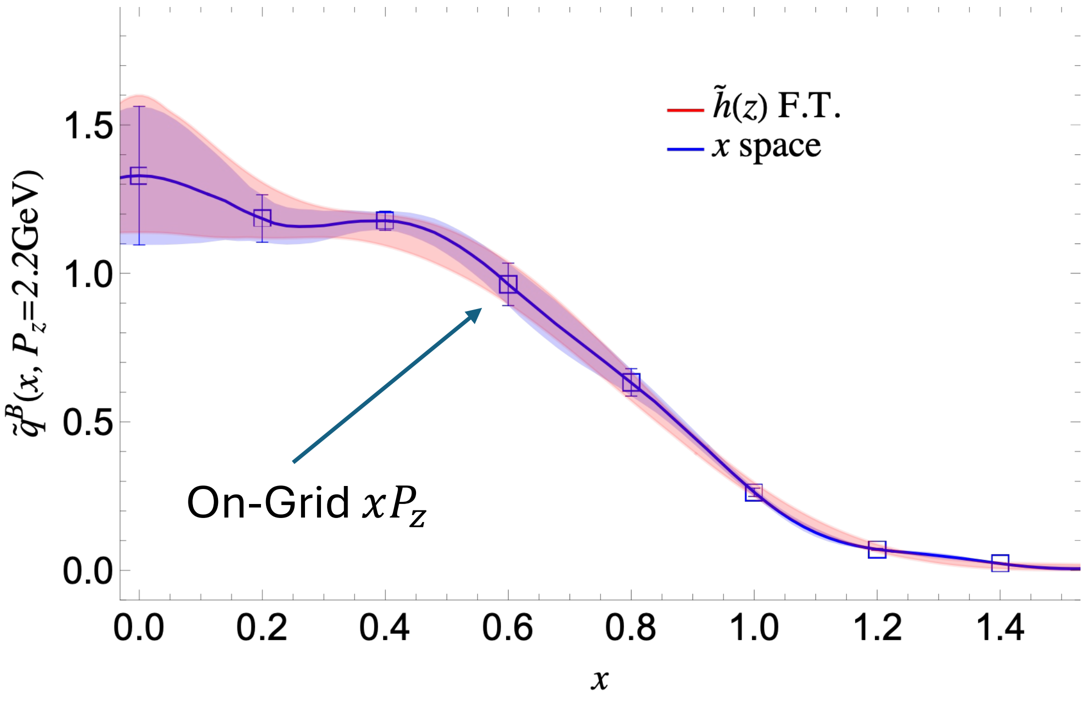


Compare with z-space analysis

Asymptotic analysis in z-space



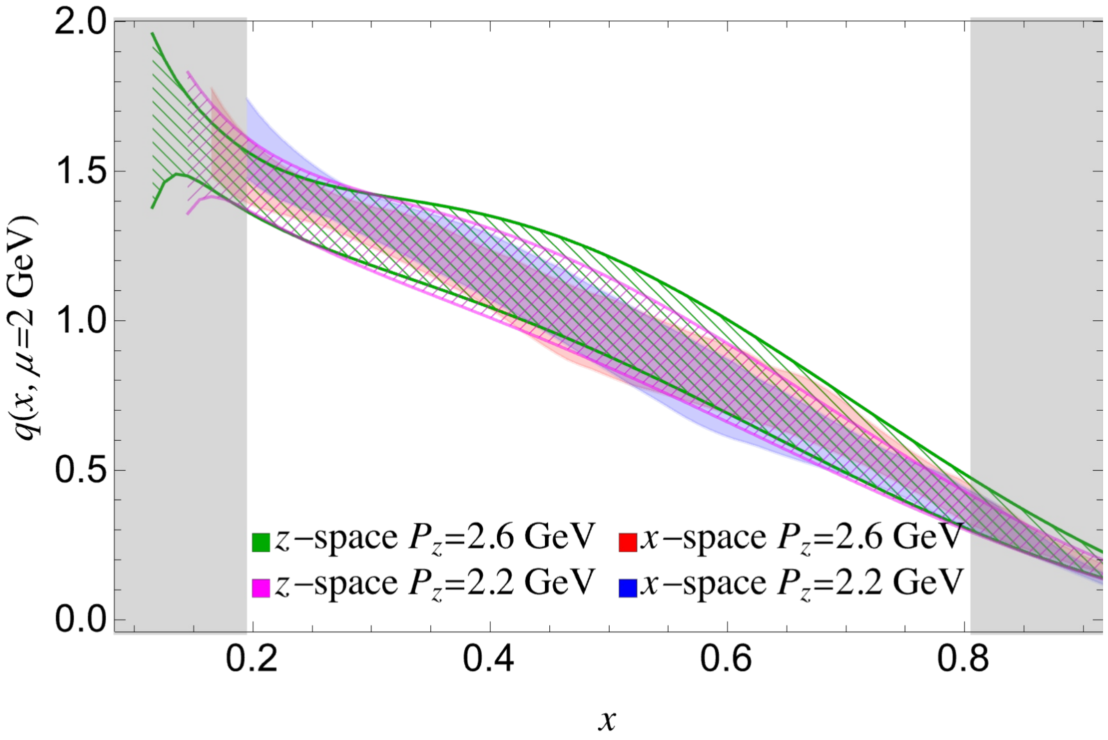
Compare two approaches



Given precise enough data in the asymptotic z region, the systematics from the FT is well controlled

Comparison with traditional LaMET approach

- Traditional approach starts from another renormalization scheme in z-space.
- Same raw lattice data, independent renormalization and matching are consistent
- Uncertainties include systematics from different asymptotic extrapolation, and scale variation in perturbative matching.



More challenging observable: TMDPDF

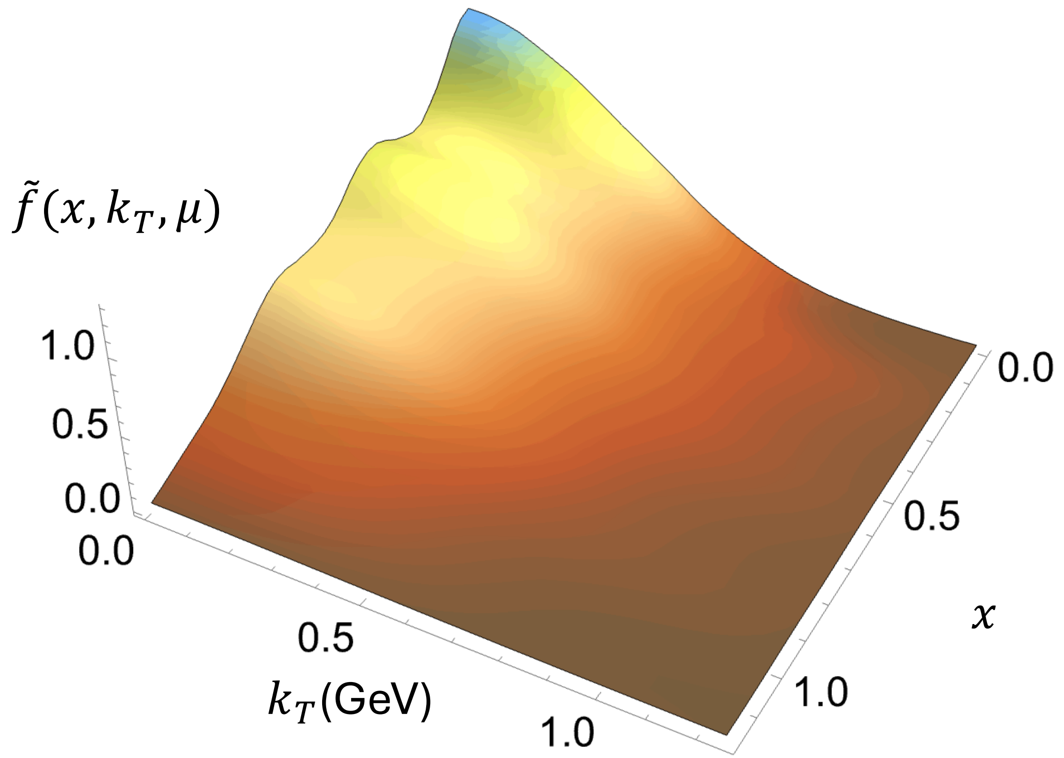
- Previous studies found slow decay in b_T space for the quasi-beam functions
  - Gaussian modeling of large b_T has been required for F.T. to k_T space

- Now: directly define 3D momentum density distribution operator

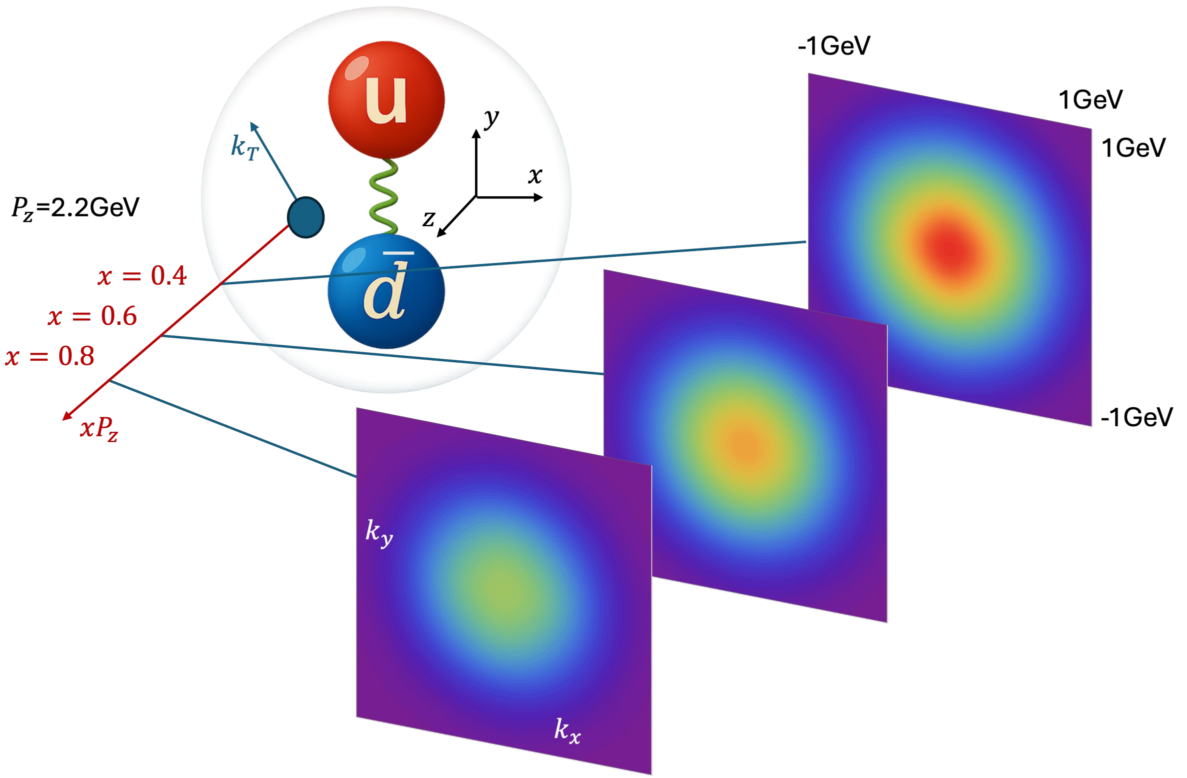
$$\tilde{O}_C(\vec{k}) \equiv \sum_{\vec{b}\in V} e^{i\vec{k}\cdot\vec{b}} \bar{\psi}(0)\gamma^t \psi(\vec{b})$$

Quasi-TMD beam functions in (x, k_T) space

- 3D Momentum Distribution



- k_T distribution at different x



Summary

- We propose an approach based on Coulomb gauge operators to directly calculate parton distributions in momentum space.
- The FT uncertainty is encoded in finite volume effects thus can be automatically removed during the infinite-volume extrapolation.
- Consistent with traditional LaMET approach, testify both approaches on the well control of FT uncertainty. Can be used to cross-check with traditional approach with almost no extra cost.
- The method is straightforwardly generalized to multi-dimensional parton distributions. A first direct lattice 3D imaging of a boosted pion is presented.

Thank you for listening!