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Quantum Simulation of Generalized Parton Distributions A Qudit-Based Approach

Florian Hechenberger
in collaboration with
Sebastian Grieninger, Tommaso Rainaldi,
Felix Ringer, and Ismail Zahed

Center for Nuclear Theory, Stony Brook University

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Outline

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Object: Boosted, Equal-Time Matrix Element

The quasi-GPD is an **equal-time** bilocal evaluated between **boosted** baryon states

$$\tilde{H}(x,\xi,\chi) = \int \frac{dz}{2\pi}\, e^{ixzP}\, \langle B(\chi_2)|\, \bar{\psi}(-\tfrac{z}{2})\, \gamma^z W\, \psi(\tfrac{z}{2})\, |B(\chi_1)\rangle,$$

prepared by a unitary boost of the rest baryon:

$$|B(\chi)\rangle = e^{-i\chi\mathbb{K}}\, |B(0)\rangle\,,$$

$$\langle\mathbb{P}\rangle = M \sinh\chi, \qquad \langle\mathbb{H}\rangle = M\cosh\chi\,.$$

- In/out rapidities $\chi_{1,2}$ fix the skewness ξ ($\xi = 0 \Rightarrow$ quasi-PDF)
- 2D $\Rightarrow$ ξ and t **kinematically locked**
- Polynomiality respected
- $P \to \infty$: matches the GPD

Advantages

- **Real-time**:
  no analytic continuation
  no sign problem
- **Direct matrix element**
  no deconvolution
  no shadow GPDs
- Same H for TN and QC

QCD_2: A Controlled Non-Perturbative Laboratory

QCD_2 with $N_c = 3$ keeps what matters:

- **Confinement**, non-Abelian **color**.
- **Baryons** as color-singlet 3-quark states.
- Analytic benchmarks:
  't Hooft ['t Hooft, 1974]
  Bars–Green [Bars and Green, 1978].

Drops transverse dimensions
$\Rightarrow$ no propagating, self-coupling gluons

Controlled testbed

- 1D gauge field non-dynamical
  $\Rightarrow$ eliminate by Gauss's law
- Reach the **continuum** cheaply
- TN, analytics, quantum hardware on *one model*

Meson predecessor in QED_2
[Grieninger et al., 2026, Grieninger et al., 2024].

Outline

Tensor Networks Primer — The Problem They Solve

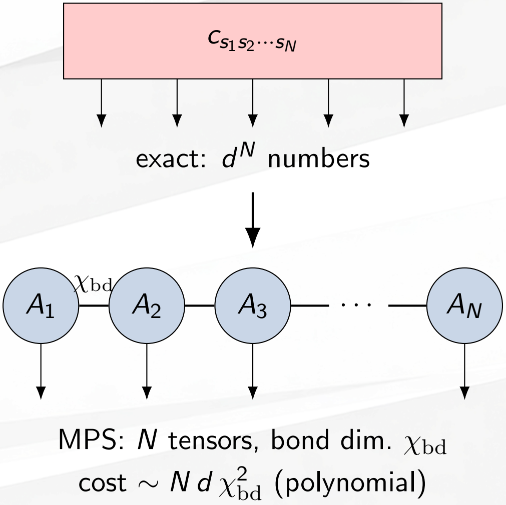
A state of N sites has **exponentially many** amplitudes:

$$|\Psi\rangle = \sum_{s_1 \ldots s_N} c_{s_1 \ldots s_N} |s_1 \ldots s_N\rangle , \quad d^N .$$

($2^{100} \approx 10^{30}$ — hopeless to store.)

But local, gapped ground states obey an **area law** — lightly entangled.

MPS: factorize into a *chain of small tensors* [Schollwoeck, 2011, Orus, 2014].



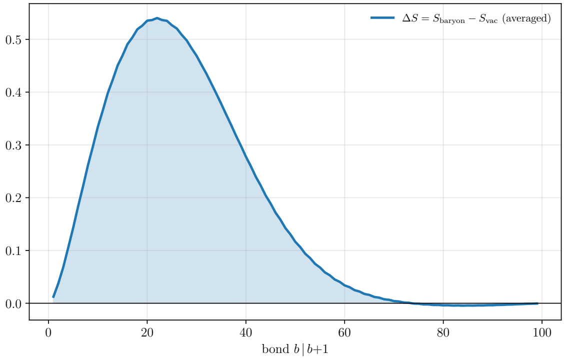
DMRG — static

Minimize $\langle\Psi|\,\mathbb{H}\,|\Psi\rangle$ over MPS [White, 1992]
$\Rightarrow$ vacuum, excited states, masses...

TDVP — real-time

Evolve on the MPS manifold [Haegeman et al., 2016] $\Rightarrow$ $e^{-i\mathbb{H}t}$ and the **boost** $e^{-i\chi\mathbb{K}}$.

Quantum numbers
($U(1)$ charge, color) kept explicit
$\Rightarrow$ block-sparse tensors, $B = 1$ & color singlet enforced *exactly*, no penalties.



Excess entanglement the baryon adds
$\Rightarrow$ modest bond dimension.
Software: `ITensor` [Fishman et al., 2022]
TN for QCD_2: [Silvi et al., 2020, Hayata et al., 2024, Bañuls et al., 2013]

Outline

$SU(3)$ QCD_2 on a Staggered Spin Chain

Kogut–Susskind [Kogut and Susskind, 1975]
staggered fermions, one flavor, $c \in \{r, g, b\}$.
Eliminate gauge field (axial gauge + Gauss):
$H_{\mathrm{QCD}_2} = H_{\mathrm{hop}} + H_m + H_C$

$$H_{\mathrm{hop}} = \frac{1}{2a} \sum_{i,c} \left(\psi^\dagger_{i,c}\psi_{i+1,c} + \mathrm{h.c.}\right)$$
$$H_m = m \sum_i (-1)^i \, n_i$$
$$H_C = \frac{g^2 a}{2} \sum_i \left(\sum_{j \le i} Q^A_j\right)^2, \quad Q^A_j = \psi^\dagger_{j,c}(T^A)_{cc'}\psi_{j,c'}$$

Non-Abelian Coulomb:
cumulative *color* charge, summed over all 8 generators A.

Local Hilbert space

Three color modes r, g, b per site, occupation $0/1$: $\dim \mathcal{H}_{\mathrm{site}} = 2^3 = 8$.

Baryon sector

Color singlet, $N_F = 3$:

$$|B_0\rangle \propto \sum_{i_1 i_2 i_3} \Phi \, \epsilon_{c_1 c_2 c_3} \psi^\dagger_{i_1 c_1} \psi^\dagger_{i_2 c_2} \psi^\dagger_{i_3 c_3} |0\rangle$$

Encoding the Color Site: Qubits or Qudits

Qubit encoding ($d = 2$)

Three qubits/site $\rightarrow$ triple physical sites
Mapped via Jordan-Wigner transform

$$\psi_j = \left(\prod_{k<j} Z_k\right)\sigma_j^-$$

repairs Fermi statistics $\rightarrow$ Pauli strings.

- Color charges: *short* strings.
- Hopping carries **non-local JW strings**

Qudit encoding ($d = 8$)

One $d = 8$ qudit per site [Illa et al., 2024]: $|n_r n_g n_b\rangle \rightarrow |s\rangle$.

- Gauss-law locality preserved
- Less JW overhead
- Compact $d = 8$ tensor (block-sparse!) Fits **multilevel hardware**
- Exact QN conservation ($B \equiv 1$)

Boost and Momentum Operators

From the stress tensor $T^{\mu\nu}$:

$$\mathbb{H} = \int dx\, T^{00}, \qquad \mathbb{K} = -\int dx\, x\, T^{00}$$

$$\mathbb{P} = \int dx\, T^{01}$$

Discretized $\mathbb{K} = K_{\rm hop} + K_m + K_C$
same three pieces, weighted by position x.

Poincaré algebra in 1+1D:

$$[\mathbb{P}, \mathbb{H}] = 0, \quad \underbrace{[\mathbb{K}, \mathbb{H}] = i\,\mathbb{P}}_{\text{closes on lattice}}, \quad \underbrace{[\mathbb{K}, \mathbb{P}] = i\,\mathbb{H}}_{\textbf{fails}\text{ at finite } a}.$$

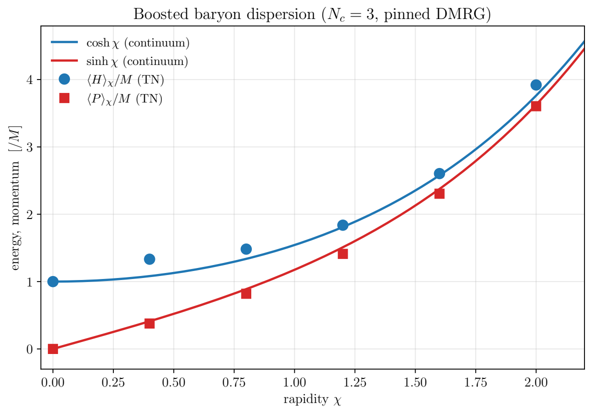
Boosting the Baryon

$$|B(\chi)\rangle = e^{-i\chi\mathbb{K}}|B(0)\rangle$$
$$\langle\mathbb{P}\rangle = M\sinh\chi, \; \langle\mathbb{H}\rangle = M\cosh\chi$$

- TDVP in "rapidity time" χ
- Pinned DMRG: min $\mathbb{H}$ at fixed $\langle\mathbb{P}\rangle$ — robust at large χ
- Convergence gate (energy only): $\dfrac{\langle\mathbb{H}\rangle - E_{\rm vac}}{M\cosh\chi} \to 1$

Outline

Relativistic Dispersion - Convergence Check



- Boost to χ, measure $\langle\mathbb{H}\rangle, \langle\mathbb{P}\rangle$.
- Track **continuum** $M\cosh\chi$, $M\sinh\chi$ to $\chi \sim 2$.
- Momentum is the pinned constraint (input); **energy is the free test** $\Rightarrow$ artifacts surface in $\langle\mathbb{H}\rangle$.
- Large-χ deviations $=$ lattice artifacts (cutoff, contraction).

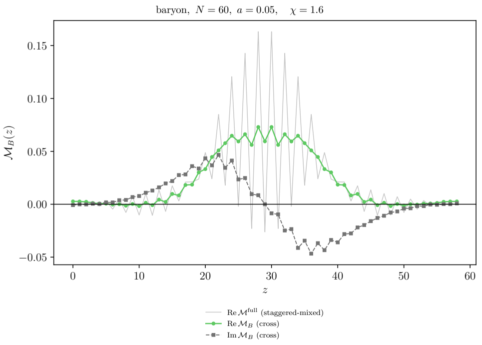
Preparation $+$ boost under control
$\Rightarrow$ Necessary for qGPD measurement

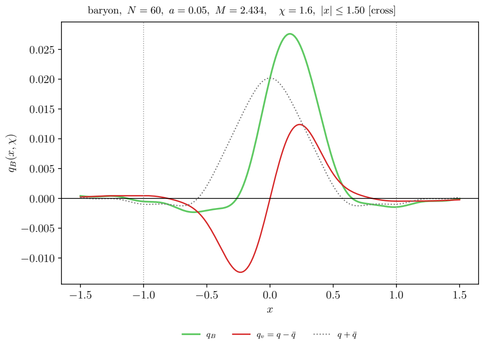
Forward limit: the baryon quasi-PDF ($\xi = 0$)

Forward ($\xi = 0$) case of the quasi-GPD — equal-time *bilocal* in the boosted baryon, and its Fourier transform:

$$\mathcal{M}_B(z,\chi) = \sum_c \langle B(\chi)|\, \psi_c^\dagger(-z)\, W\, \psi_c(z)\, |B(\chi)\rangle\,, \quad q_B(x,\chi) = \frac{aP}{\pi} \sum_z e^{-ixP(\chi)\, z}\, \mathcal{M}_B(z,\chi)$$

Staggered **point-split conserved vector current**



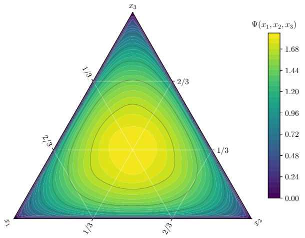


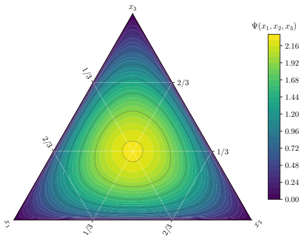
Benchmark: Large-N_c Light-Cone Wave Functions

A **large-N_c benchmark** for the TN numerics is obtained by projecting the **'t Hooft model** Hamiltonian onto the 3-quark Fock sector

$$M_B^2\Psi = \bar{m}^2\sum_i \frac{1}{x_i}\Psi - \frac{g^2}{\pi}\sum_{i<j}\mathrm{PP}\int dy\,\frac{\Psi(\dots)-\Psi}{(y-x_i)^2}$$

- 3-Fock truncation — 't Hooft-type
- Large-N_c
- Endpoint $\Psi \sim (x_1 x_2 x_3)^\beta$
- Solved by Rayleigh–Ritz
- Sets the qualitative shape the boosted quasi-PDF must reproduce.





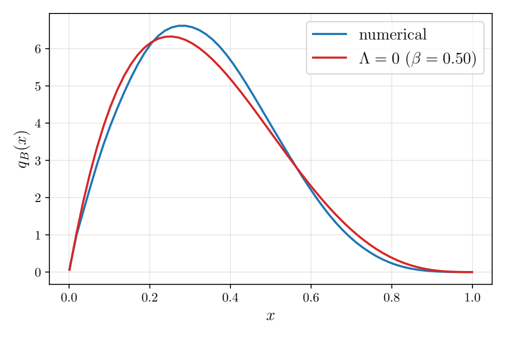
From Wave Function to the Valence PDF

Integrate out the two spectators:

$$q_B(x) = 3\int [dx]_2 \, |\Psi(x, x_2, x_3)|^2$$

$$q_B(x) = 3\,\frac{\Gamma(6\beta+3)}{\Gamma(2\beta+1)\Gamma(4\beta+2)}\, x^{2\beta}(1-x)^{4\beta+1}$$

- Normalizes to 3 quarks; $\langle x \rangle = \frac{1}{3}$.
- The **light-cone target** for qPDF



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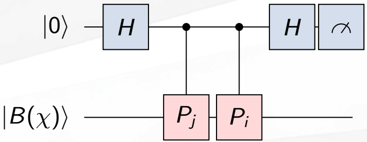
A Natively Quantum-Simulable Workflow

Reviews/Higher-dimensional:
[Bañuls et al., 2020, Zohar, 2021]:

1. **Prepare** $|B(0)\rangle$ (VQE / adiabatic) [Atas et al., 2021, Farrell et al., 2023].
2. **Boost** $e^{-i\chi\mathbb{K}}$ (two χ for $\xi \neq 0$).
3. **Measure** via **Hadamard test** [Grieninger et al., 2024].



$$P(0) - P(1) = \mathrm{Re}\,\langle B(\chi)|\,P_i P_j\,|B(\chi)\rangle$$

Resource counting

- N sites $\Rightarrow$ N $d = 8$ **qudits** (or $3N$ qubits)
- Local terms cheap; **bottleneck** = long-range Coulomb + JW strings
- $\Rightarrow$ Needs **all-to-all connectivity**
- Boost $e^{-i\chi\mathbb{K}}$: the costly unitary ($\mathbb{K}$ inherits the Coulomb)

Platforms and Resources: Near- vs. Long-Term

Best matched

- **Trapped ions**: native all-to-all, top fidelity; $\sim$30–50 qubits [Martinez et al., 2016].
- **Rydberg atoms**: 100s of qubits, **native qudits** for $d = 8$ [Bluvstein et al., 2024].
- Superconducting: fast but nearest-neighbor (SWAP overhead).

Near-term (NISQ)

$N \sim 8$–12: a *demonstration* [Preskill, 2018, Klco et al., 2018, Ciavarella et al., 2021]

Long-term

Continuum ($N \gtrsim 100$, $\chi \gtrsim 3$): needs fault tolerance

TN sets the **benchmarks** and tells us *when* quantum wins (large χ, real-time, higher-dimensionality ($\to \mathrm{QCD}_4$?).

Summary & Outlook

What we did

- $d = 8$ **qudit** $SU(3)$ QCD_2
  baryon & boost operators
- GPDs as **direct** matrix elements
  no deconvolution
- TN **validation**
  dispersion, baryon quasi-PDF
- Large-N_c LCWF benchmark

What's next

- Off-forward quasi-GPDs ($\xi \neq 0$);
  finer a, larger χ
- LCWFs
- Hadamard-test
  resource estimates
- Small-N **qudit-hardware** demo

A qudit-based, deconvolution-free route to GPDs validated by tensor networks.

Thank you!

florian.hechenberger@stonybrook.edu

References I

Atas, Y. Y., Zhang, J., Lewis, R., Jahanpour, A., Haase, J. F., and Muschik, C. A. (2021).
SU(2) hadrons on a quantum computer via a variational approach.
Nature Commun., 12:6499.

Bañuls, M. C., Cichy, K., Cirac, J. I., and Jansen, K. (2013).
The mass spectrum of the Schwinger model with Matrix Product States.
JHEP, 11:158.

Bañuls, M. C. et al. (2020).
Simulating Lattice Gauge Theories within Quantum Technologies.
Eur. Phys. J. D, 74(8):165.

Bars, I. and Green, M. B. (1978).
Poincare and Gauge Invariant Two-Dimensional QCD.
Phys. Rev. D, 17:537.

References II

Bluvstein, D. et al. (2024).
Logical quantum processor based on reconfigurable atom arrays.
Nature, 626(7997):58–65.

Ciavarella, A., Klco, N., and Savage, M. J. (2021).
Trailhead for quantum simulation of SU(3) Yang-Mills lattice gauge theory in the local multiplet basis.
Phys. Rev. D, 103(9):094501.

Farrell, R. C., Chernyshev, I. A., Powell, S. J. M., Zemlevskiy, N. A., Illa, M., and Savage, M. J. (2023).
Preparations for quantum simulations of quantum chromodynamics in 1+1 dimensions. I. Axial gauge.
Phys. Rev. D, 107(5):054512.

References III

- Fishman, M., White, S. R., and Stoudenmire, E. M. (2022).
  The ITensor Software Library for Tensor Network Calculations.
  SciPost Phys. Codeb., 2022:4.
- Grieninger, S., Ikeda, K., and Zahed, I. (2024).
  Quasiparton distributions in massive QED2: Toward quantum computation.
  Phys. Rev. D, 110(7):076008.
- Grieninger, S., Montgomery, J., Ringer, F., and Zahed, I. (2026).
  Tensor network simulations of quasi-GPDs in the massive Schwinger model.
  Phys. Rev. D, 113(7):074509.
- Haegeman, J., Lubich, C., Oseledets, I., Vandereycken, B., and Verstraete, F. (2016).
  Unifying time evolution and optimization with matrix product states.
  Phys. Rev. B, 94(16):165116.

References IV

Hayata, T., Hidaka, Y., and Nishimura, K. (2024).
Dense QCD_2 with matrix product states.
JHEP, 07:106.

Illa, M., Ciavarella, A. N., and Savage, M. J. (2024).
Qu8its for quantum simulations of lattice quantum chromodynamics.
Phys. Rev. D, 109(11):114507.

Klco, N., Dumitrescu, E. F., McCaskey, A. J., Morris, T. D., Pooser, R. C., Sanz, M., Solano, E., Lougovski, P., and Savage, M. J. (2018).
Quantum-classical computation of Schwinger model dynamics using quantum computers.
Phys. Rev. A, 98(3):032331.

References V

Kogut, J. B. and Susskind, L. (1975).
Hamiltonian Formulation of Wilson's Lattice Gauge Theories.
Phys. Rev. D, 11:395–408.

Martinez, E. A. et al. (2016).
Real-time dynamics of lattice gauge theories with a few-qubit quantum computer.
Nature, 534:516–519.

Orus, R. (2014).
A Practical Introduction to Tensor Networks: Matrix Product States and Projected Entangled Pair States.
Annals Phys., 349:117–158.

Preskill, J. (2018).
Quantum Computing in the NISQ era and beyond.
Quantum, 2:79.

References VI

Schollwoeck, U. (2011).
The density-matrix renormalization group in the age of matrix product states.
Annals Phys., 326:96–192.

Silvi, P., Sauer, Y., Tschirsich, F., and Montangero, S. (2020).
Tensor network simulation of an SU(3) lattice gauge theory in 1D.
Phys. Rev. D, 101(3):034508.

't Hooft, G. (1974).
A Two-Dimensional Model for Mesons.
Nucl. Phys. B, 75:461–470.

White, S. R. (1992).
Density matrix formulation for quantum renormalization groups.
Phys. Rev. Lett., 69:2863–2866.

Zohar, E. (2021).
Quantum simulation of lattice gauge theories in more than one space dimension—requirements, challenges and methods.
Phil. Trans. Roy. Soc. Lond. A, 380(2216):20210069.