Brookhaven National Laboratory

U.S. Department of ENERGY

Hadronic Matrix Elements of the QCD EMT

—Lattice validation of hadron mass and trace-anomaly sum rules

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Based on arXiv: 2601.13070, accepted by PRL
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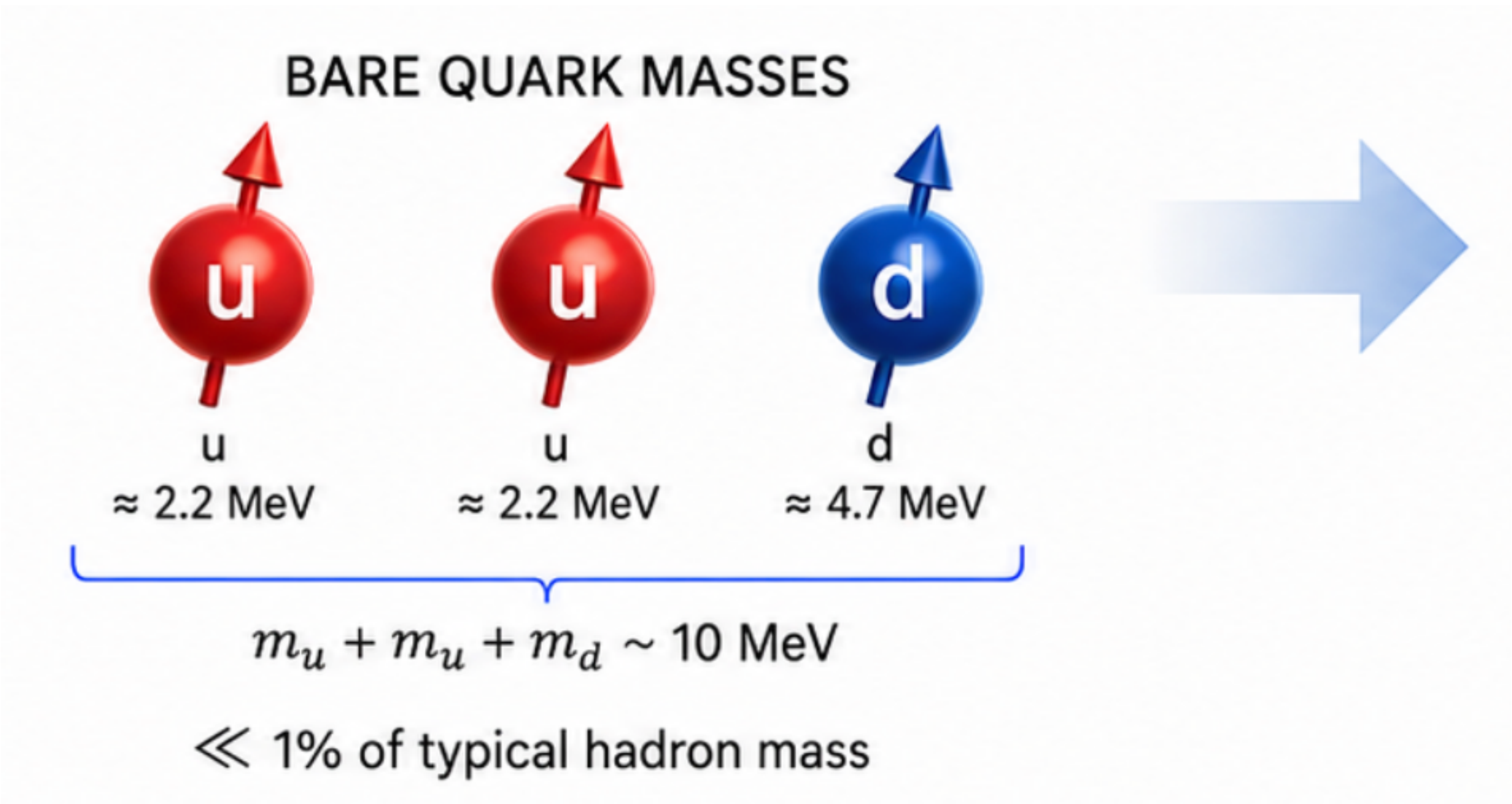
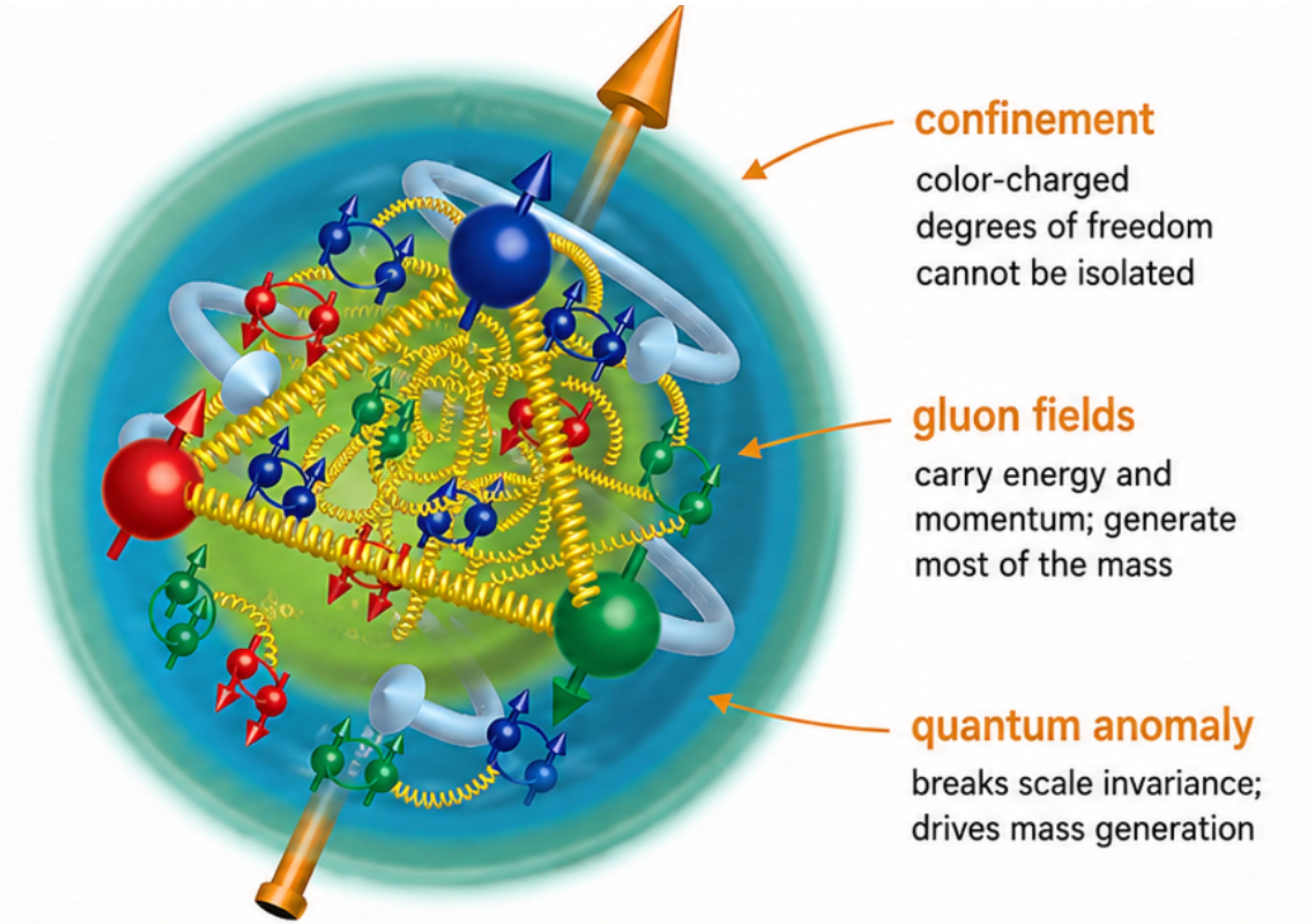
Hadron mass is a QCD question

Most visible mass is generated by QCD dynamics, not by bare quark masses.





How to understand the hadron mass in terms of quark and gluon fields?

The QCD EMT: from quark-gluon fields to hadron mass

To understand hadron mass in terms of quarks and gluons, we need the operator that measures energy and momentum in QCD.

Noether current of spacetime translations

$$T^{\mu\nu}_{\rm QCD} = T^{\mu\nu}_{q,R} + T^{\mu\nu}_{g,R}$$

$$P^\mu = \int d^3x\, T^{0\mu}$$

$$H_{\rm QCD} = \int d^3x\, T^{00}$$

Hadron matrix elements

$$\langle H \,|\, T^{\mu\nu}_{q,g} \,|\, H \rangle$$

$\langle p' \,|\, T^{\mu\nu} \,|\, p \rangle \longrightarrow$ *gravitational form factors*

$\langle p \,|\, T^{00} \,|\, p \rangle \longrightarrow$ *energy / rest-frame mass*

$\langle p \,|\, T^{\mu}_{\ \mu} \,|\, p \rangle \longrightarrow$ *scale breaking / trace anomaly*

The EMT connects quark-gluon dynamics to hadron mass, GFFs, and the trace anomaly.

Mass decomposition and the trace anomaly from the EMT

How do we weigh the hadron?

$$M_H = \frac{1}{2M_H}\langle H | T^{00} | H \rangle$$

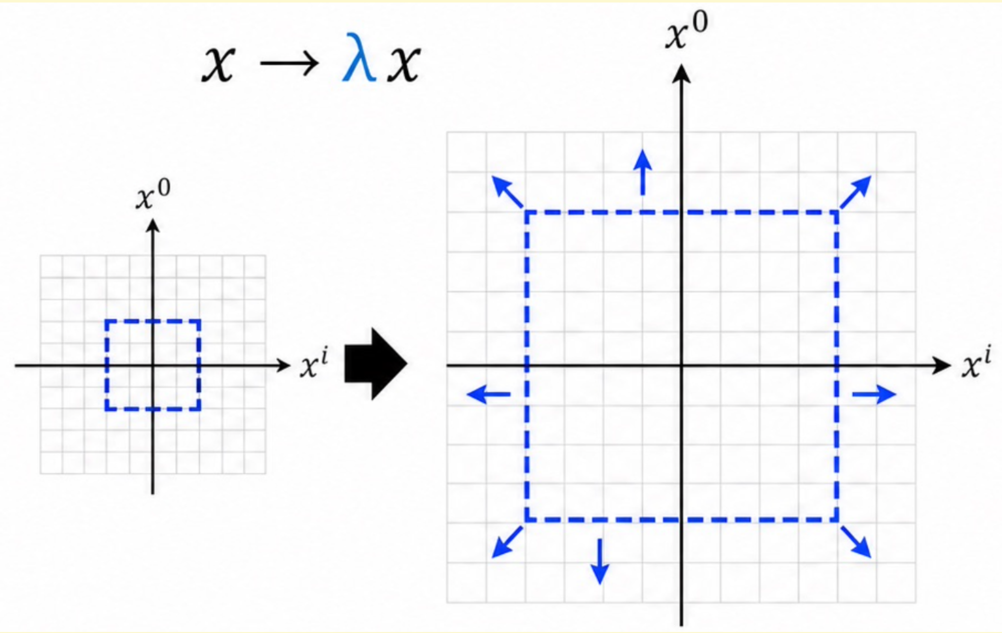
$$T^{00} = T^{00}_{q,R} + T^{00}_{g,R}$$

$$= \underbrace{\bar{T}^{00}_{q,R} + \bar{T}^{00}_{g,R}}_{\text{traceless}} + \frac{1}{4} T^{\mu}_{\ \mu}$$

T^{00} gives the Hamiltonian definition of mass

- $T^{\mu\nu}$ *is conserved current.*
- *q/g components are scale and scheme dependent.*

How does QCD acquire a scale?



$$D^{\mu} = x_{\nu} T^{\mu\nu}$$

$$\partial_{\mu} D^{\mu} = T^{\mu}_{\mu}$$

$$T^{\mu}_{\ \mu} = \frac{\beta(g)}{2g}\left(F^2\right)_R + (1+\gamma_m)\left(m_q \bar{\psi}\psi\right)$$

The trace measures the breaking of dilatation symmetry

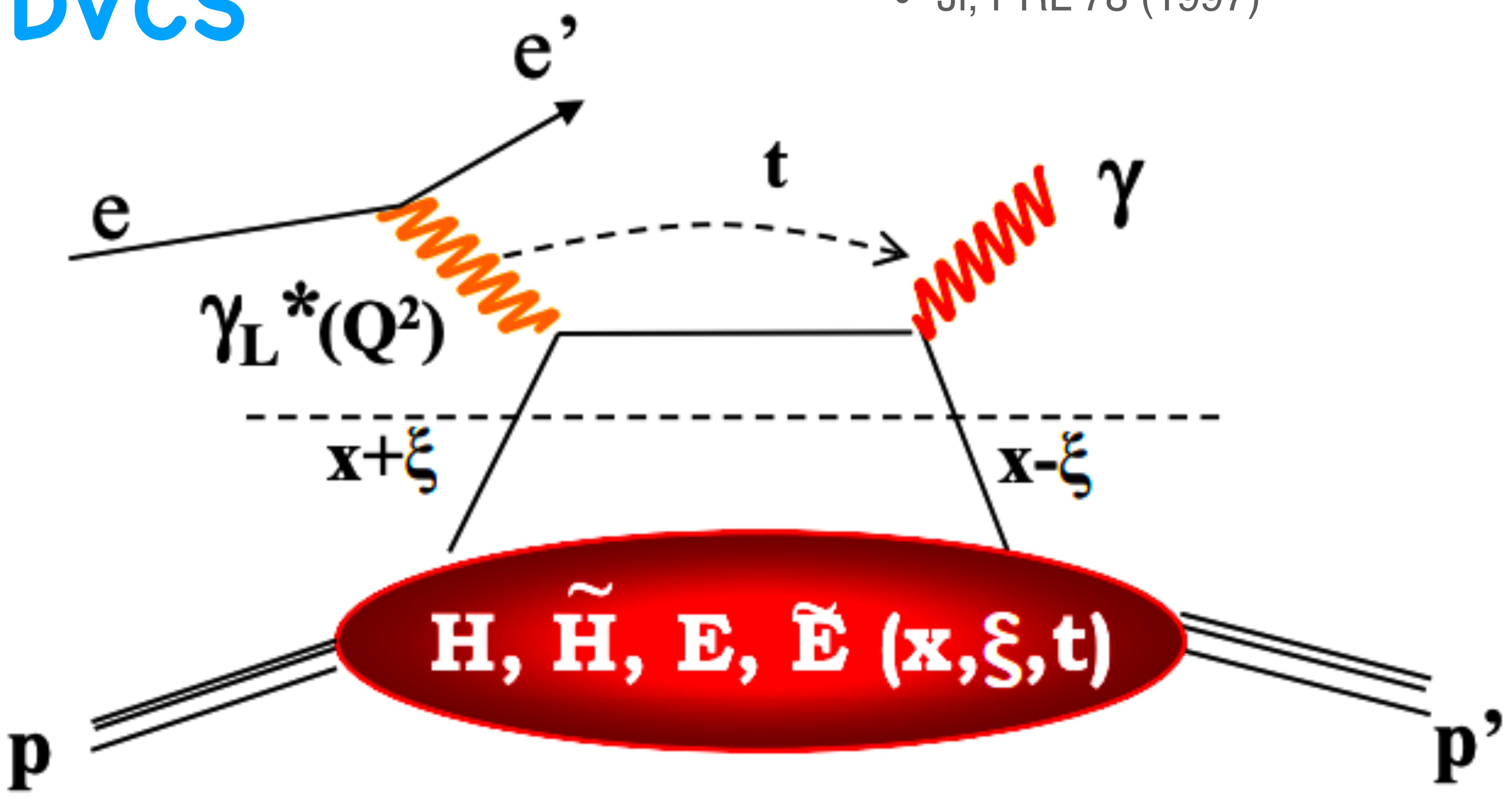
- *In classical limit* $\beta(g) = \gamma_m = 0$

$$T^{\mu}_{\ \mu} = m_q \bar{\psi}\psi \xrightarrow{m=0} 0$$

Experimental access: GFFs from exclusive processes

DVCS

- Ji, PRL 78 (1997)



E.g., moments of GPDs:

$$\int_{-1}^{1} dx\, x\, H^q(x,\xi,t) = A^q(t) + \xi^2 C^q(t)$$

$$\int_{-1}^{1} dx\, x\, E^q(x,\xi,t) = B^q(t) - \xi^2 C^q(t)$$

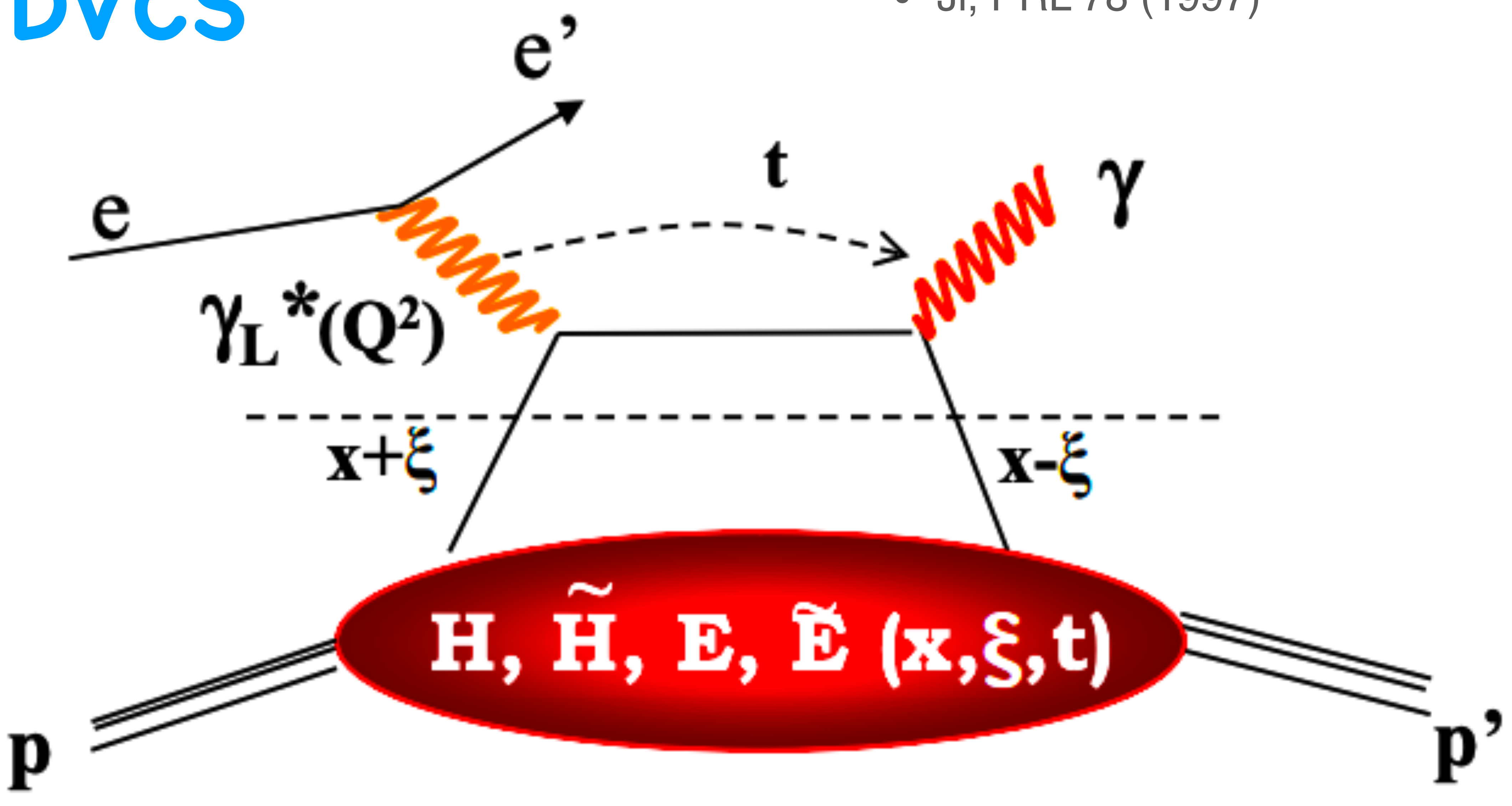
$$\langle p', s' | T^{\mu\nu}_{q,g} | p, s \rangle = \bar{u}(p', s') \left[A_{q,g}(t)\, \gamma^{(\mu} P^{\nu)} + B_{q,g}(t) \frac{P^{(\mu} i\sigma^{\nu)\rho} \Delta_\rho}{2M} + C_{q,g}(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{C}_{q,g}(t)\, M g^{\mu\nu} \right] u(p, s)\,.$$

$$P = \frac{p' + p}{2}, \qquad \Delta = p' - p, \qquad t = \Delta^2\,.$$

Experimental access: the missing trace channel

DVCS

- Ji, PRL 78 (1997)



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$$\bar{T}^{\mu\nu} = T^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^{\alpha}_{\ \alpha} :$$ *twist-2*

$T^{\alpha}_{\ \alpha}$: *scalar trace, twist-4*

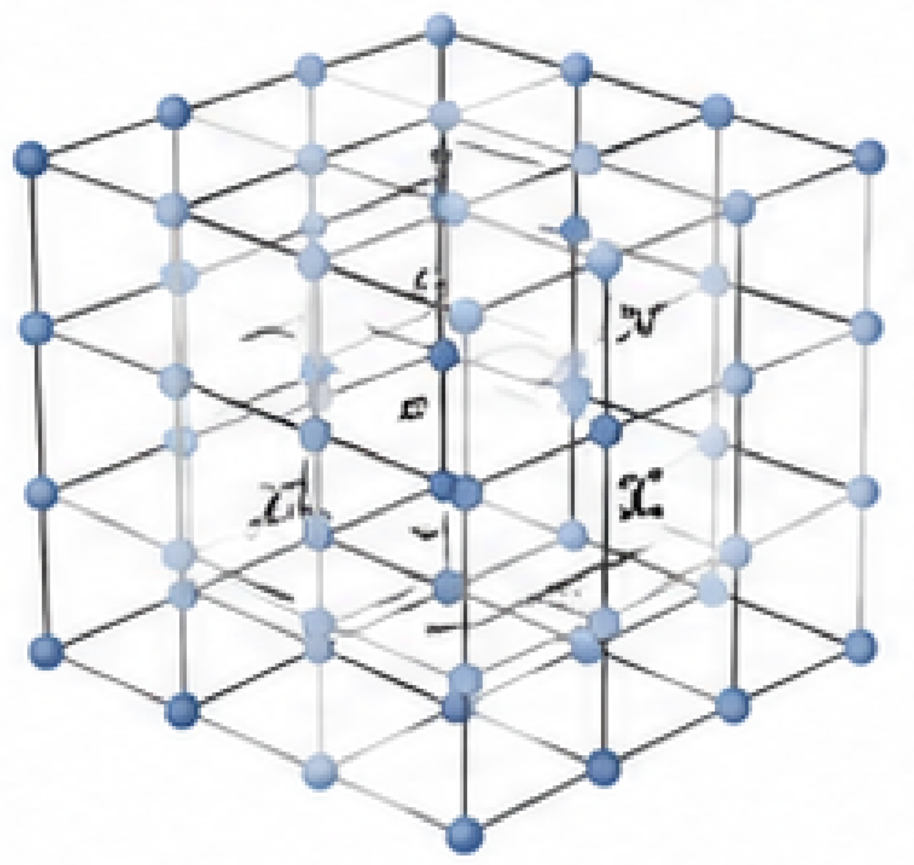
Twist expansion gives access to the traceless EMT: the trace sector is absent.

EMT matrix elements from lattice QCD: why hard?

$$T^{\mu\nu} = O_{1,\mu\nu} - \frac{1}{4}O_{2,\mu\nu} + \frac{1}{4}O_{3,\mu\nu} = \underbrace{-F_a^{\mu\alpha}F_{a,\alpha}^{\ \ \nu} + \frac{1}{4}g^{\mu\nu}F_a^{\alpha\beta}F_{a,\alpha\beta}}_{T_g^{\mu\nu}} + \underbrace{\sum_f i\bar{\psi}_f\gamma^{\{\mu}D^{\nu\}}\psi_f}_{T_q^{\mu\nu}}$$

Bare lattice EMT

$T_{\mu\nu}^{bare}(x)$



- **Lattice discretization breaks $O(4)$** to $H(4)$: only traceless $T^{\mu\nu}$ components in $H(4)$ irreps are safe.
- Trace mix with lower-dimensional operators with a^{-n} divergence, e.g.,

$$\langle\bar{\psi}\gamma_\mu\overleftrightarrow{D}^\mu\psi\rangle^R \sim \lim_{a\to 0}\left[\langle\bar{\psi}\gamma_\mu\overleftrightarrow{D}^\mu\psi\rangle + \frac{1}{a}\langle\bar{\psi}\Gamma\psi\rangle + \cdots\right]$$

No continuum limit!

- **No direct access to trace sector.**

Renormalized full EMT including trace

$T_{\mu\nu}^{R}(x)$

Gradient flow as heat-kernel smoothing

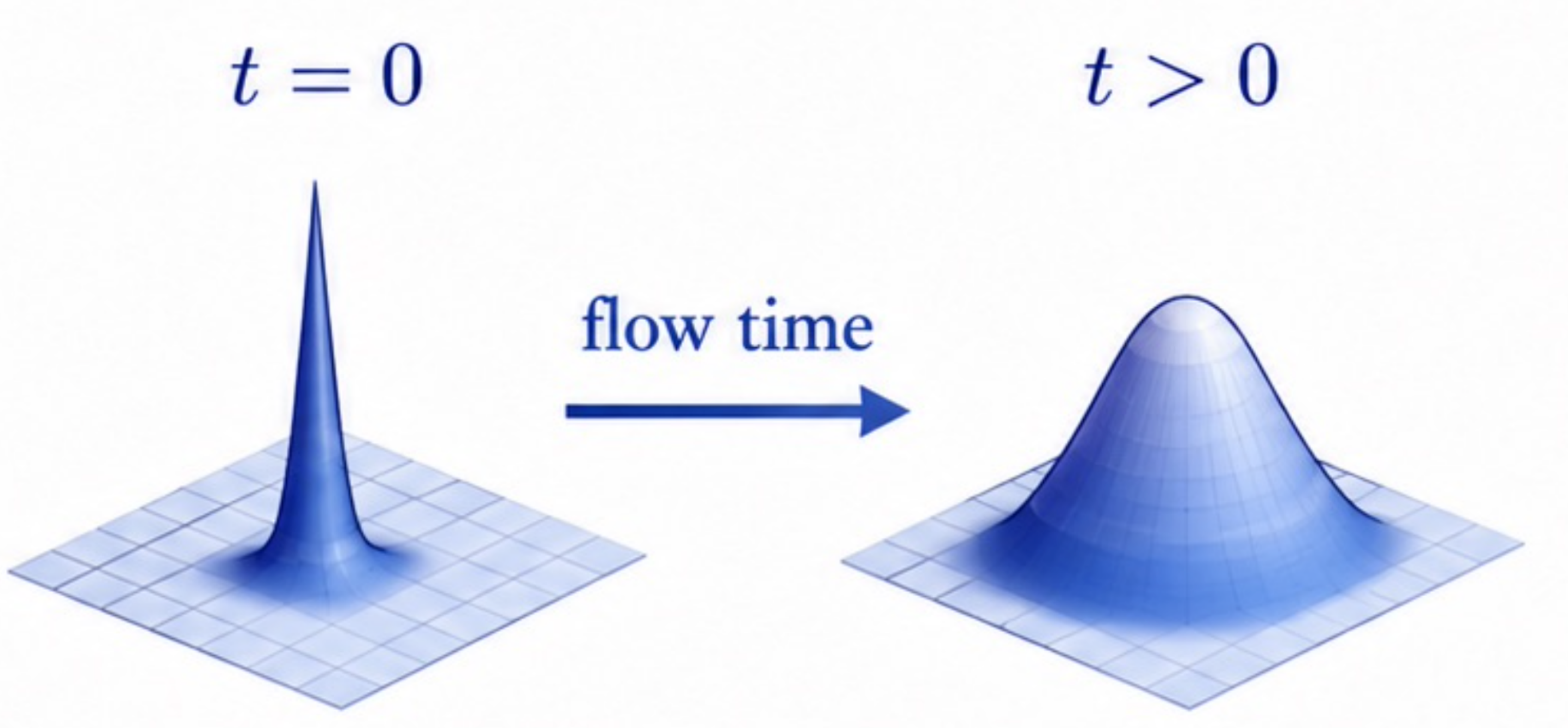
- M. Luscher, Commun.Math.Phys. 293 (2010)
- M. Luscher, JHEP 08 (2010) 071
- M. Luscher, P. Weisz, JHEP 02 (2011) 051

$$\partial_t B_\mu = D_\nu G_{\nu\mu}, \quad B_\mu(0,x) = A_\mu(x)$$

$$B_\mu(t,p) \simeq e^{-tp^2} A_\mu(p)$$

$$r_{\text{flow}} \simeq \sqrt{8t}, \qquad \mu_t \sim \frac{1}{\sqrt{8t}}$$

- UV modes are exponentially suppressed.
- At finite t, the operator has a physical resolution scale.
- The flowed operator is a smeared probe, not a local QCD operator.



Local operator pointlike

$\frac{1}{4} G^a_{\mu\nu}(0,x) G^a_{\mu\nu}(0,x)$

Smeared over $r_{\text{flow}} \simeq \sqrt{8t}$

$\frac{1}{4} G^a_{\mu\nu}(t,x) G^a_{\mu\nu}(t,x)$

Gradient flow smooths the operator over a radius $r_{\text{flow}} \simeq \sqrt{8t}$

UV finiteness and well-defined continuum limit

- **Flowed composite operator**

$$\tilde{O}(t_f, x) = O\left[B_\mu(t_f, x), \chi(t_f, x), \bar{\chi}(t_f, x)\right]$$

- **Continuum limit at fixed physical flow time**

$$\tilde{O}(t_f, a) = \tilde{O}(t_f, 0) + c_1 \frac{a^2}{t_f} + c_2 a^2 + \cdots$$

Continuum limit (finite)

Cut-off effects

- For $t_\mathrm{f} > 0$, correlation functions of flowed fields are UV finite.
- Regulator independent: holds for any underlying regulator (lattice, dimensional regularization, …).
- Safe to take continuum limit $a \to 0$ at finite t_f.

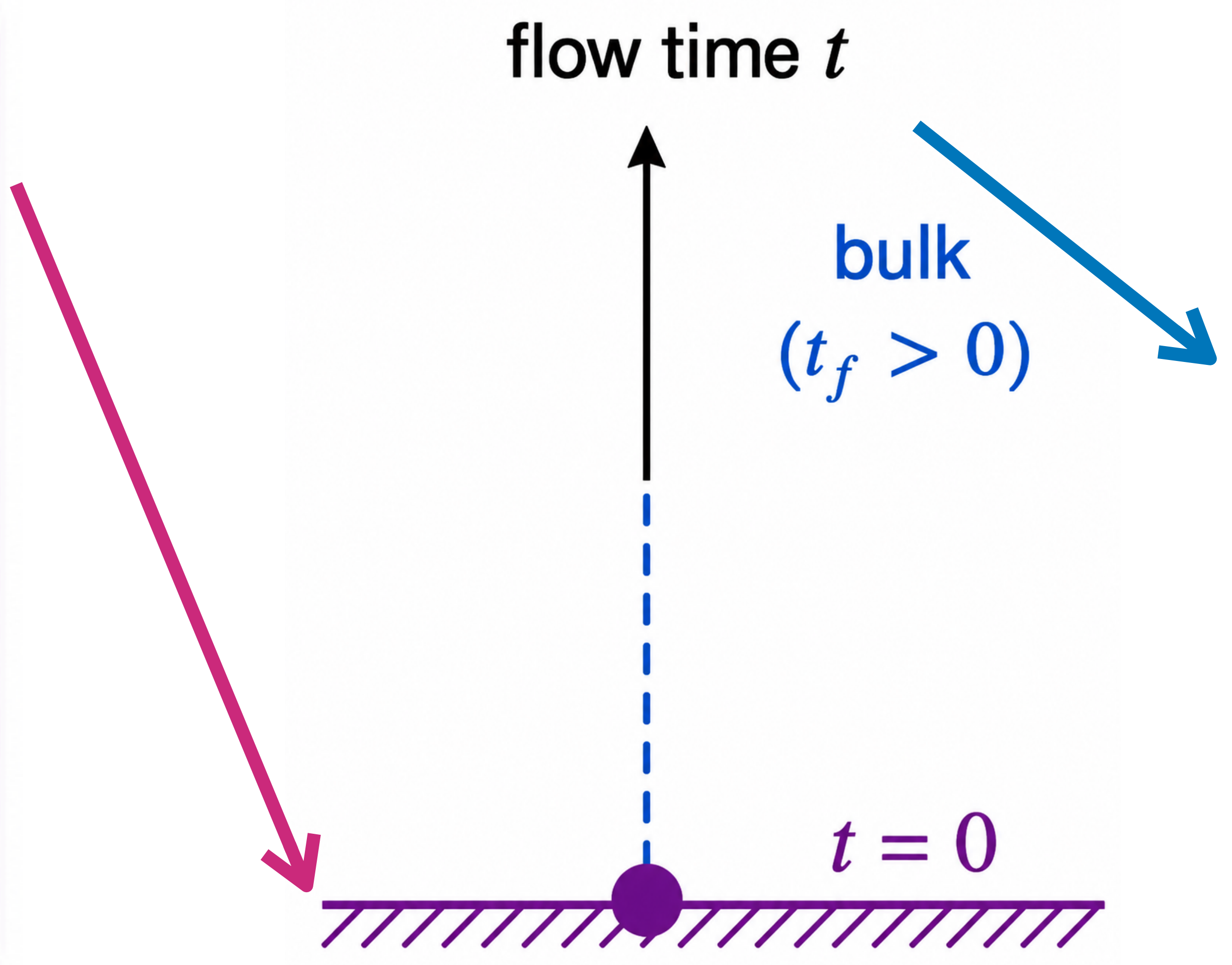
Back to the local EMT at the $t = 0$ boundary

$t = 0$ (boundary):

local 4D QCD EMT and scale Ward identity

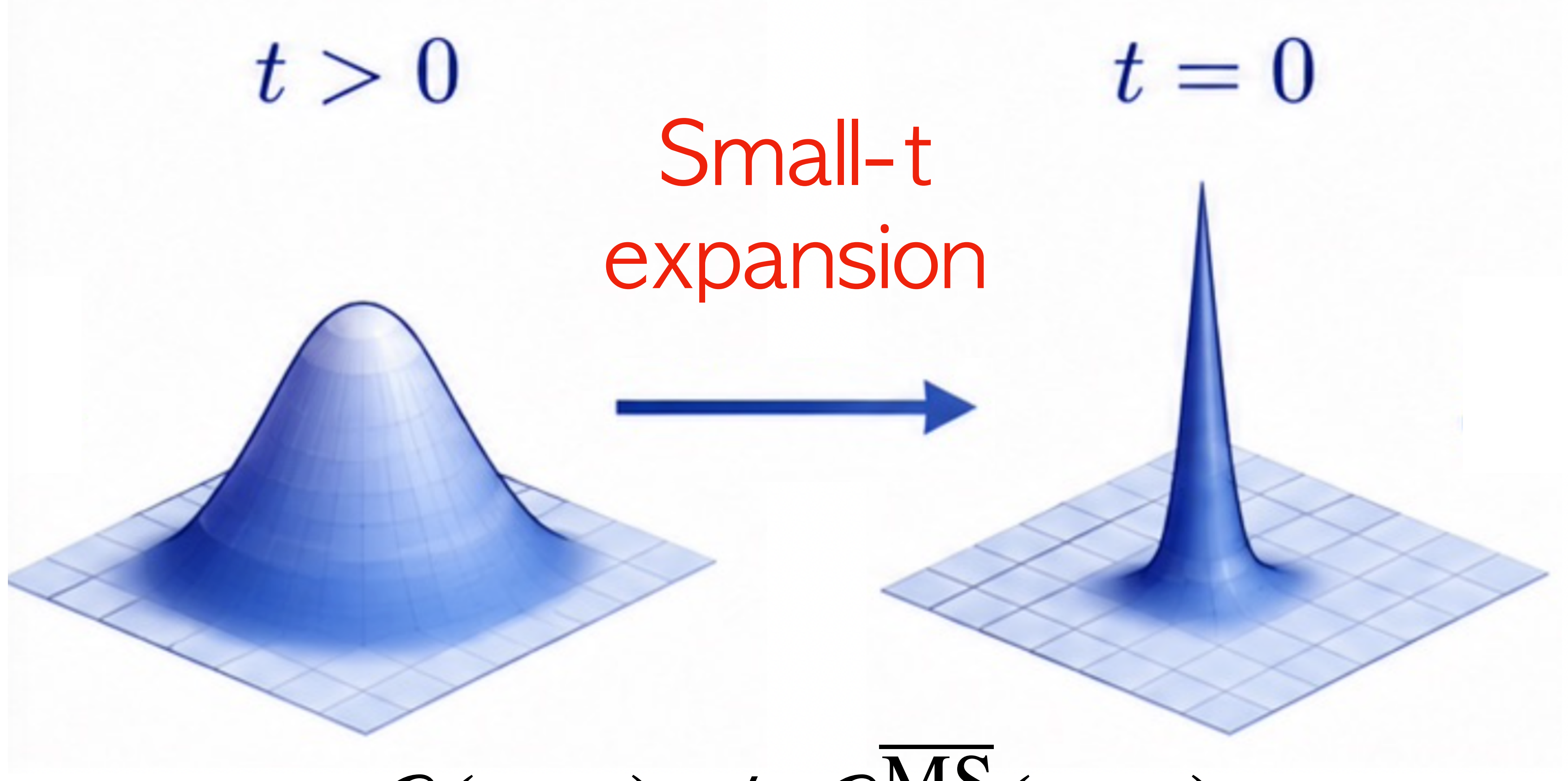
$$T^{\mu}_{\mu} = \frac{\beta(g)}{2g}(F^2)_R + (1 + \gamma_m)(m\bar{\psi}\psi)_R$$

trace anomaly of 4D QCD



$t > 0$

UV finite smeared composite operator: no trace anomaly.

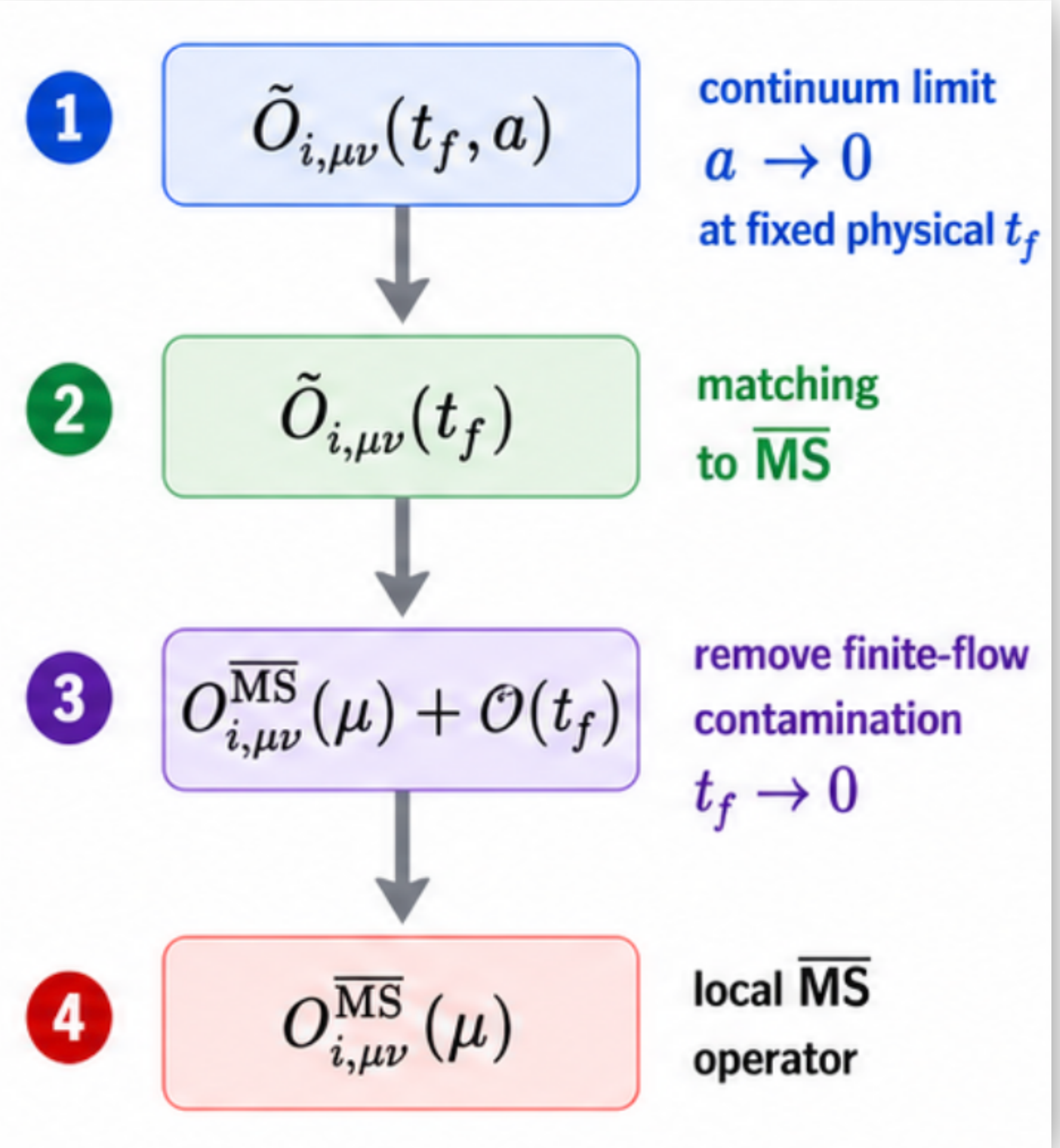


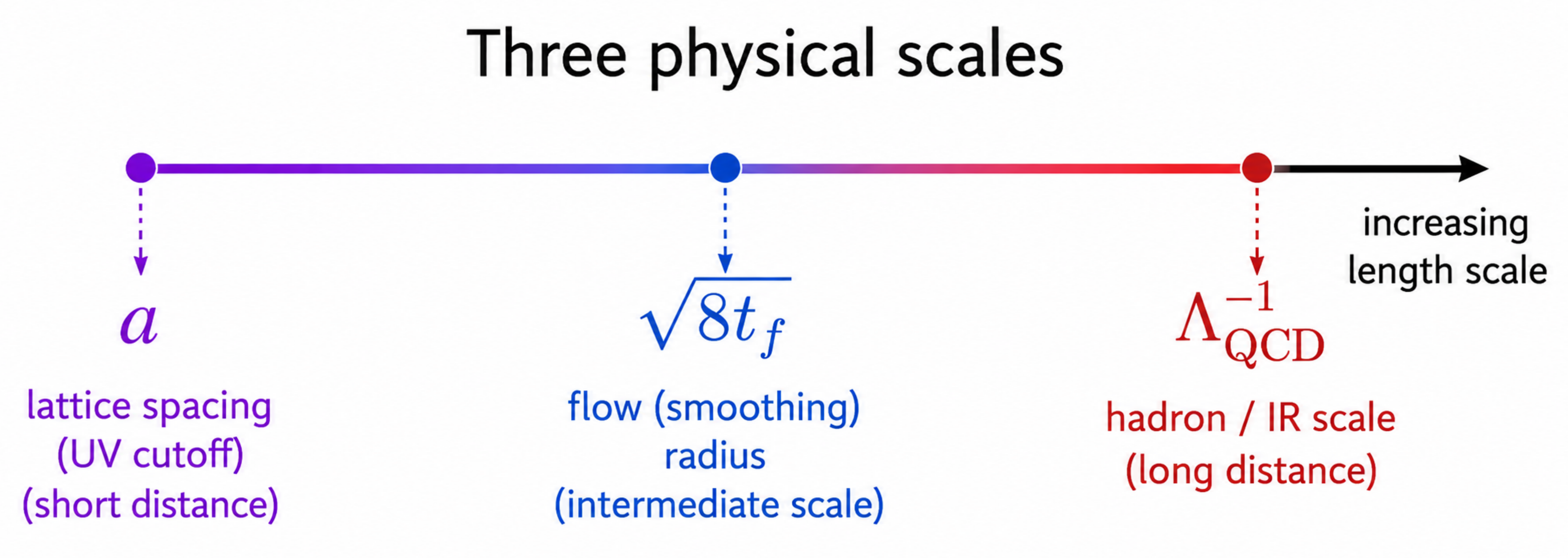
$$\mathcal{O}(t_f, x) \neq \mathcal{O}^{\overline{\text{MS}}}(\mu, x)$$

$$\mathcal{O}_i(t_f, x) = \sum_j C_{ij}(t_f, \mu)\, \mathcal{O}_j^{\overline{\text{MS}}}(\mu, x) + \mathcal{O}(t_f)$$

- The local EMT is recovered after matching and $t_f \to 0$ limit.
- The anomaly enters the flowed representation through the coefficients $C_{ij}(t_f, \mu)$ in the $t_f \to 0$ limit.

A controlled path to the full local EMT





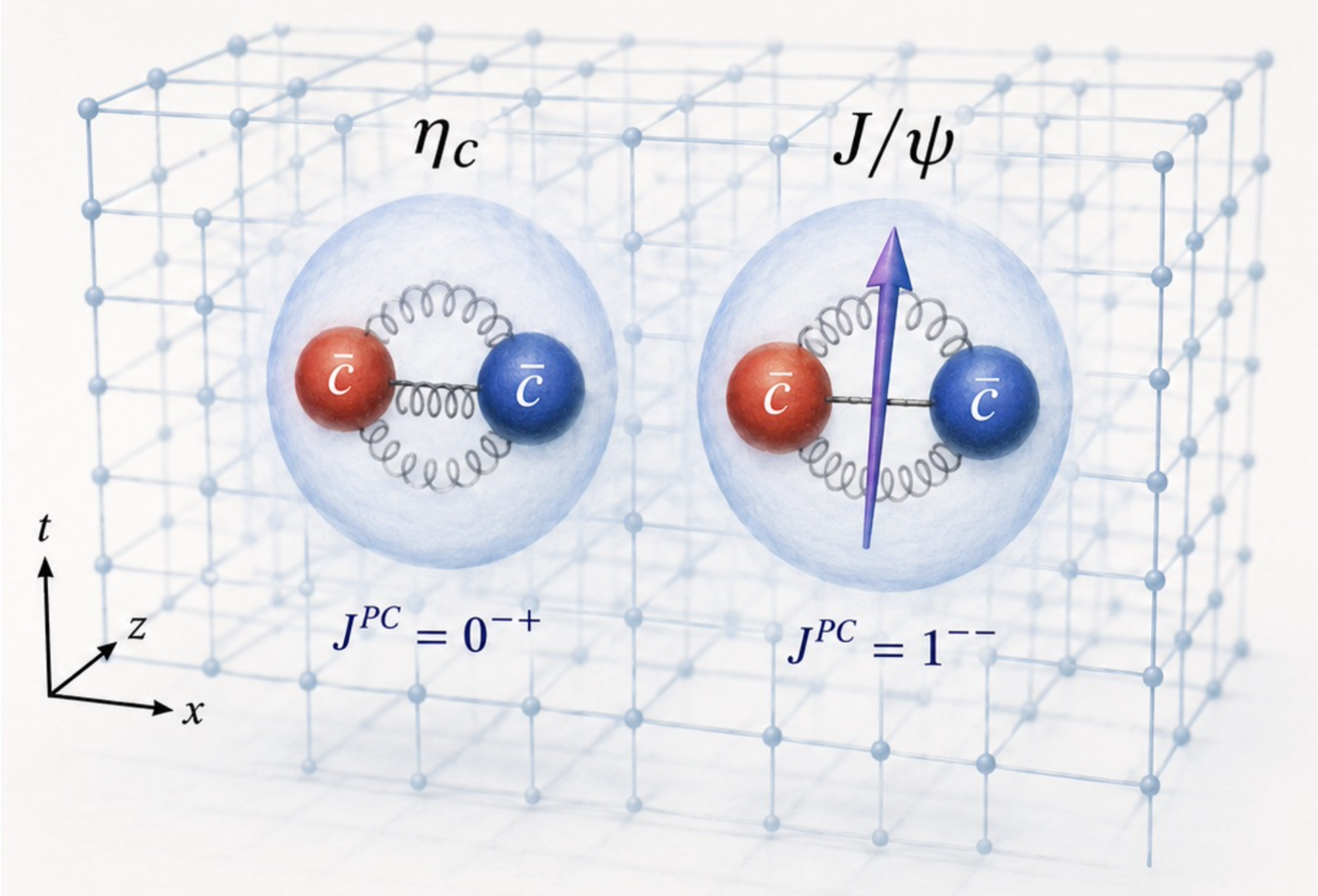
$$a \ll \sqrt{8t_f} \ll \Lambda^{-1}_{\text{QCD}}$$

- t_f too small: cutoff artifacts dominate ($\sim a^2/t_f, a^2, \ldots$).
- t_f too large: oversmearing, loss of short-distance information.

Lattice calculation: charmonium states as a demonstration

- η_c and J/ψ states: three ensembles with physical η_c and J/ψ meson masses; Light quark contribution ignored.

a [fm]	N_s	N_t	$m_s^{\text{sea}}/m_l^{\text{sea}}$	M_π^{sea} [MeV]	c_{sw}	am_c^{val}
0.06	48	64	20	160	1.0336	0.306
0.05	64				1.030934	0.236
0.04	64				1.02868	0.167



- Start with the rest frame: $P=0$:

$$\frac{1}{2M}\langle P=0|T^{\mu\nu}_{q/g}|P=0\rangle = M\begin{pmatrix} A_{q/g}(0)+\bar{C}_{q/g}(0) & 0 & 0 & 0 \\ 0 & -\bar{C}_{q/g}(0) & 0 & 0 \\ 0 & 0 & -\bar{C}_{q/g}(0) & 0 \\ 0 & 0 & 0 & -\bar{C}_{q/g}(0) \end{pmatrix}$$

Bare EMT matrix elements from flowed operators

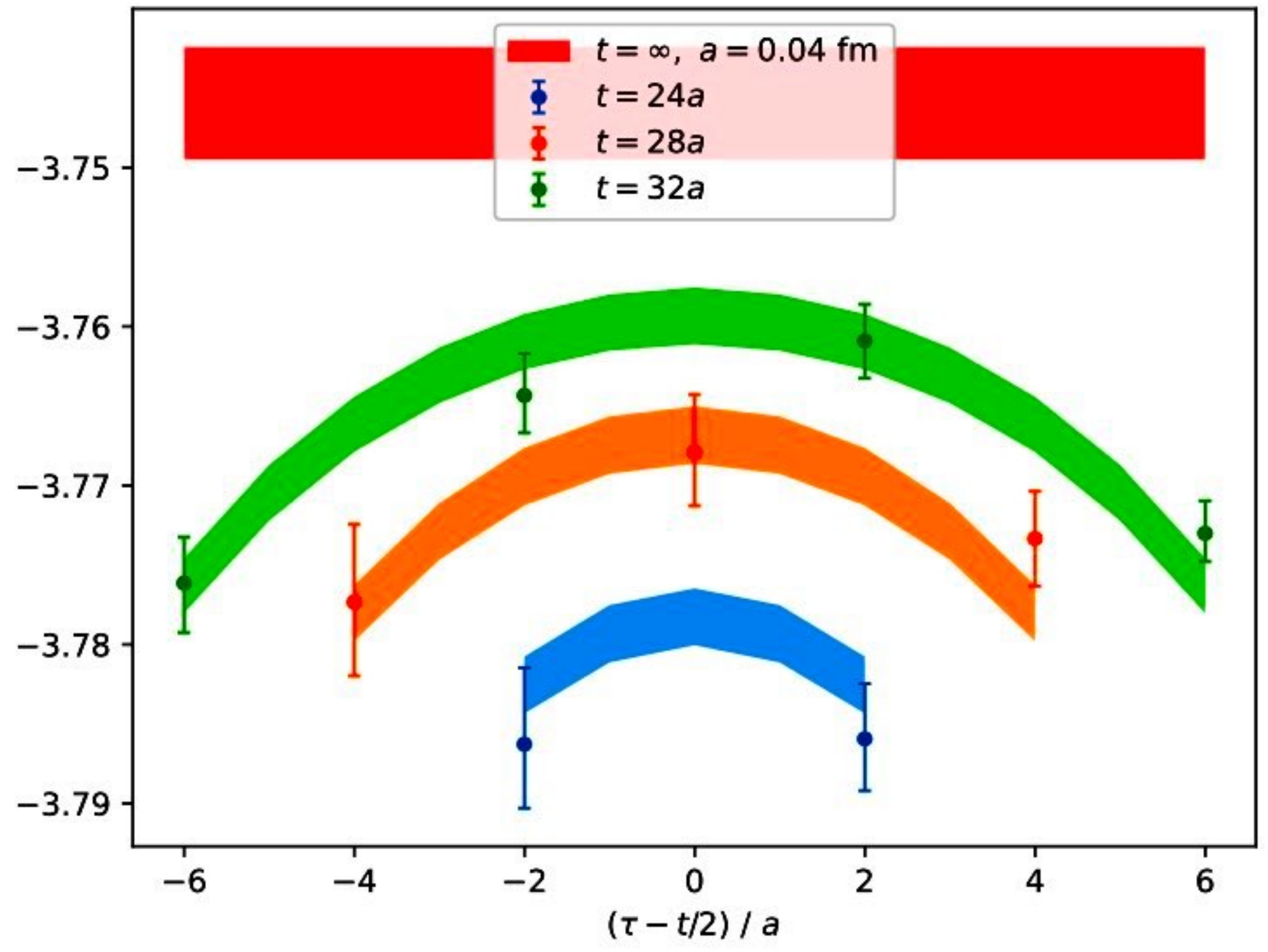
- Three-point function of flowed operator:

$$C^{3\mathrm{pt}}_{i,\mu\nu}(t_\mathrm{f}; t_s, \tau) = \sum_{x,y} \Big\langle \chi(y, t_s)\, \tilde{O}_{i,\mu\nu}(t_\mathrm{f}; x, \tau)\, \chi^\dagger(0,0) \Big\rangle - \Big\langle C^{2\mathrm{pt}}_\Gamma(t_s) \Big\rangle \sum_x \Big\langle \tilde{O}_{1,\mu\nu}(t_\mathrm{f}; x, \tau) \Big\rangle$$

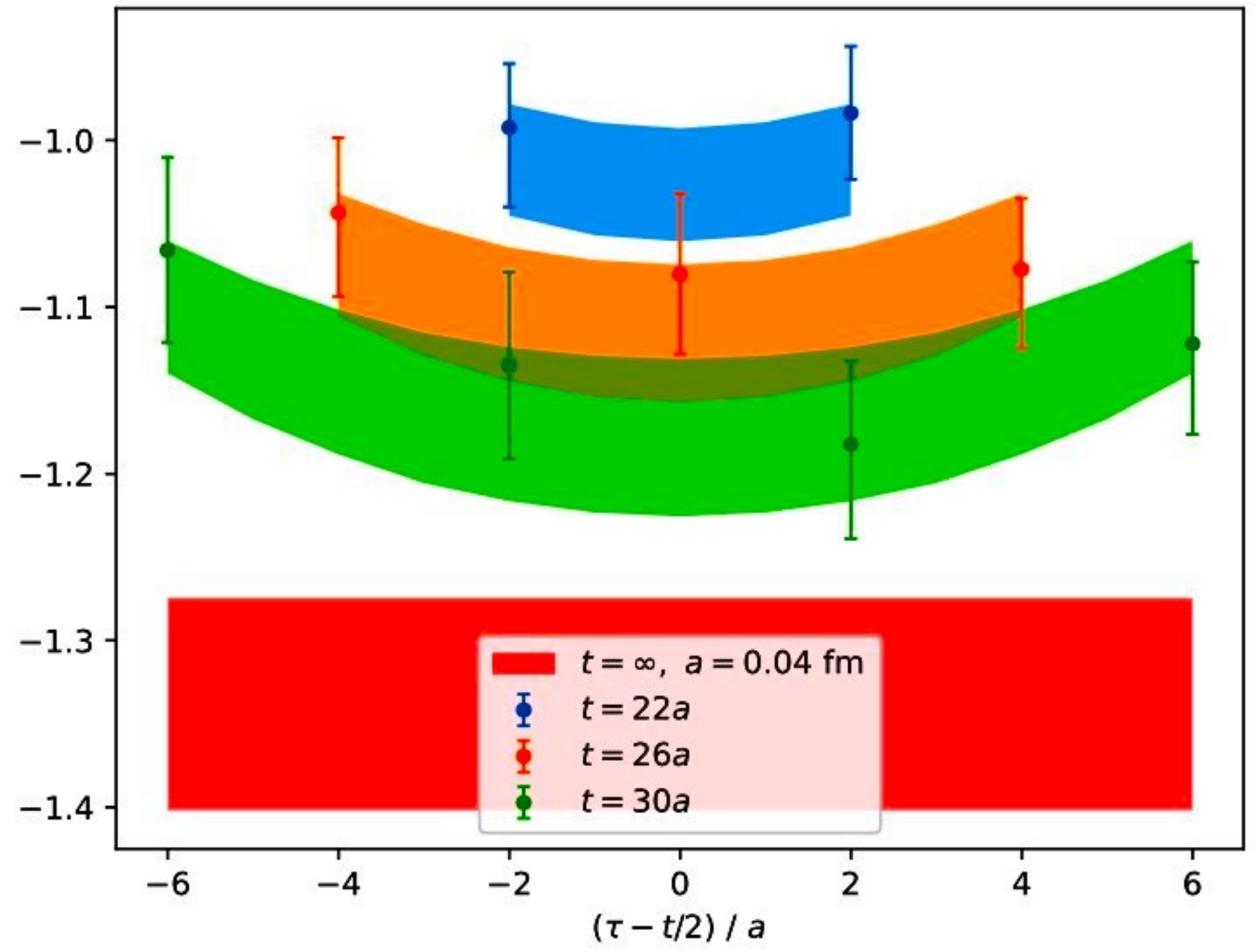
- Bare matrix elements from two-state fit:

$$C^{3\mathrm{pt}}_{i,\mu\nu}(t_\mathrm{f}; t_s, \tau) = \sum_{m,n} Z^*_m Z_n \frac{\langle m \,|\, \tilde{O}_{i,\mu\nu}(t_\mathrm{f}) \,|\, n \rangle}{2M} e^{-\tau E_n} e^{-(t-\tau) E_m}$$

quark EMT $\langle \eta_c \,|\, \tilde{O}_{3,\mu\nu}(t_\mathrm{f}) \,|\, \eta_c \rangle$



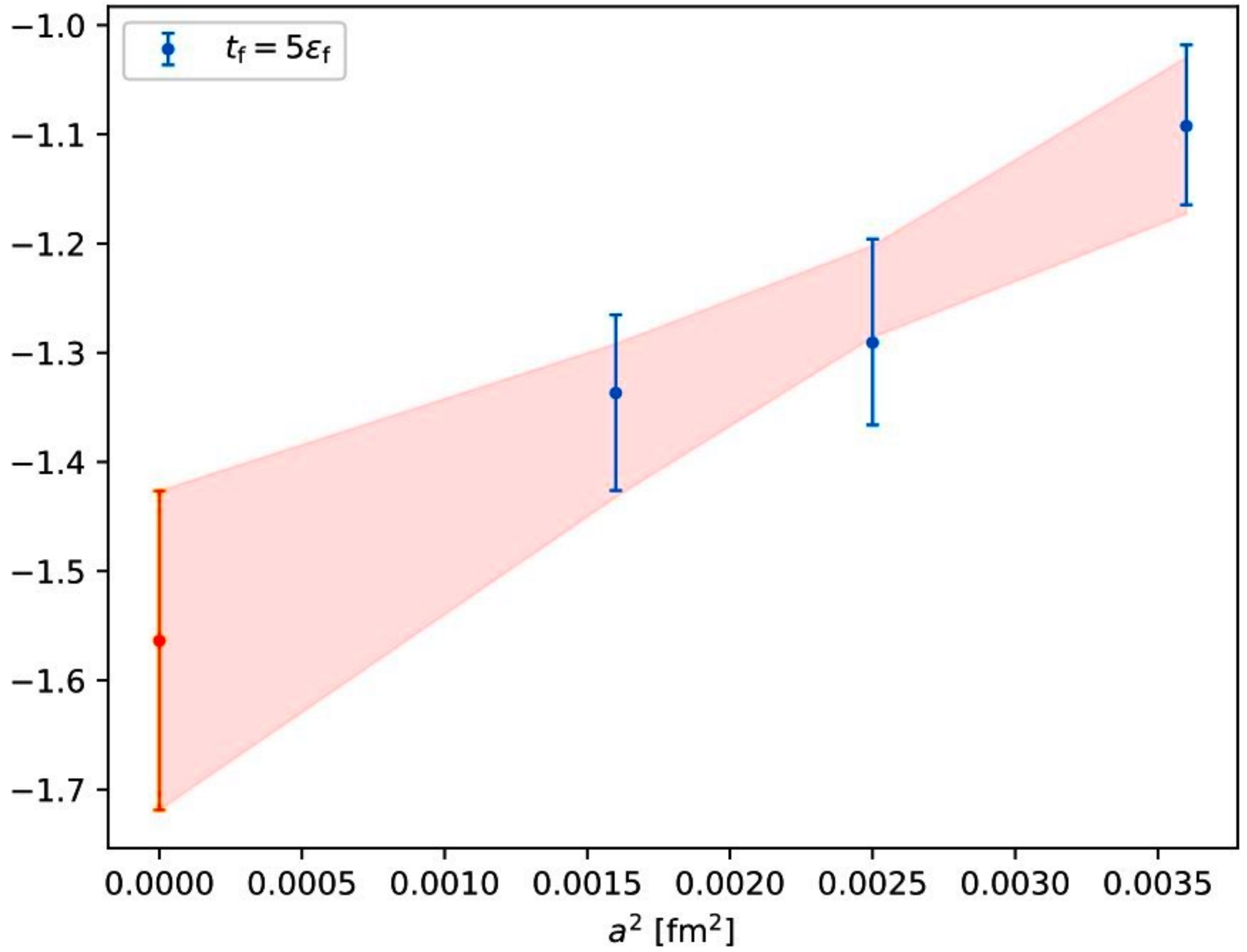
gluon EMT $\langle \eta_c \,|\, \tilde{O}_{1,\mu\nu}(t_\mathrm{f}) \,|\, \eta_c \rangle$



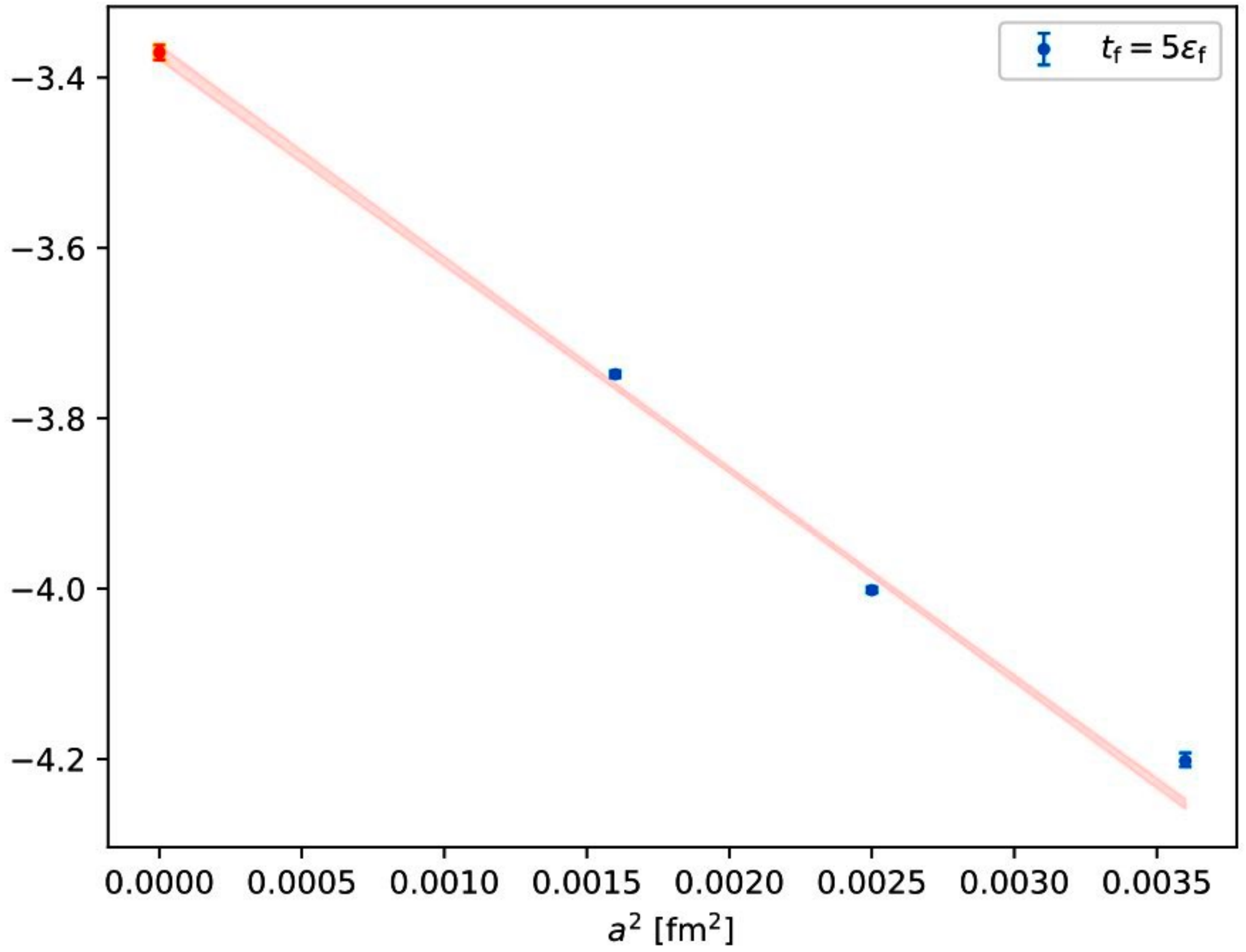
Continuum extrapolation at fixed flow time

$$\langle \eta_c | \tilde{O}_{i,\mu\nu}(t_f, a) | \eta_c \rangle = \langle \eta_c | \tilde{O}_{i,\mu\nu}(t_f) | \eta_c \rangle + a^2 X_{i,\mu\nu}(t_f)$$

gluon EMT $\langle \eta_c | \tilde{O}_{1,\mu\nu}(t_f) | \eta_c \rangle$



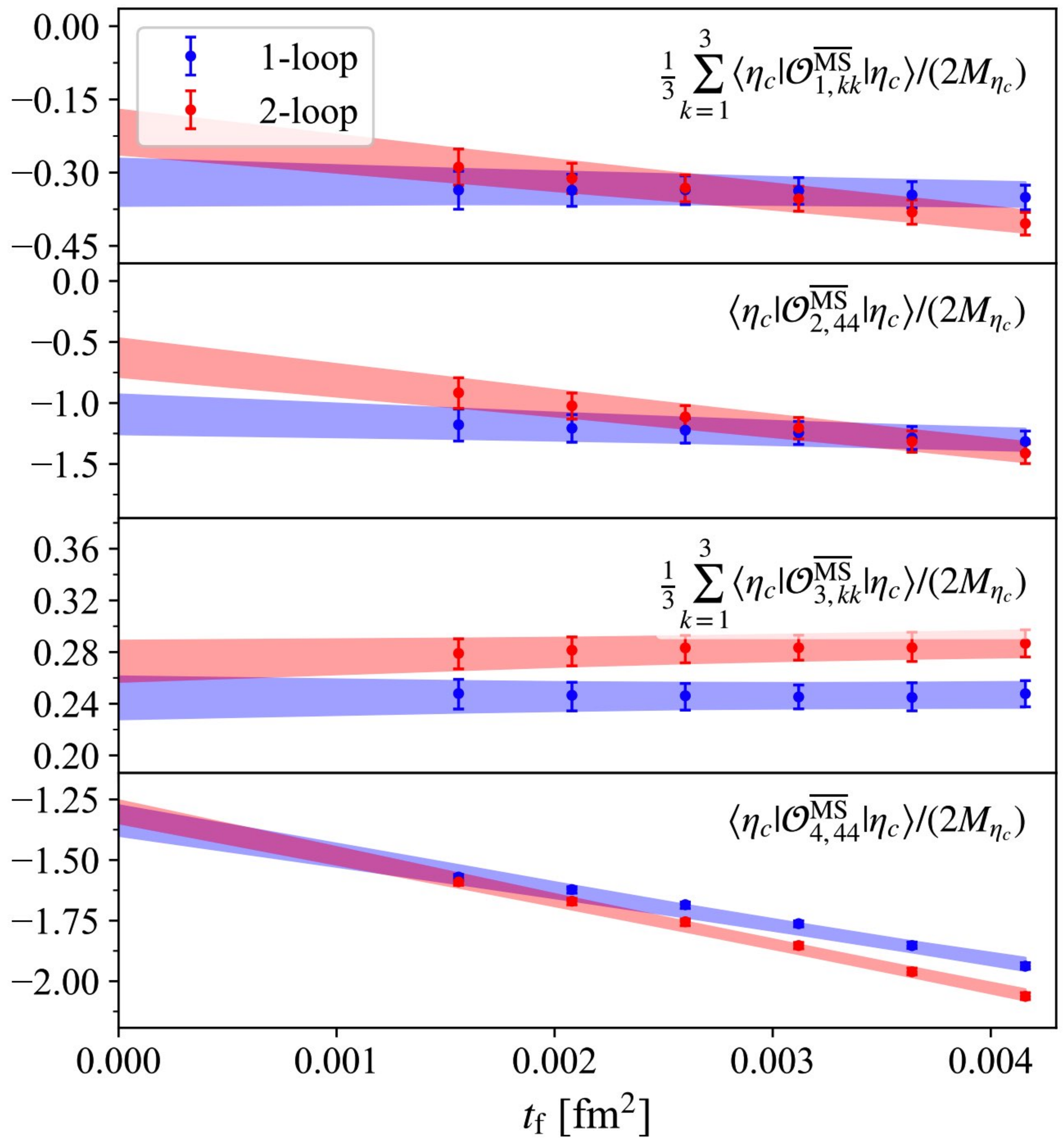
quark EMT $\langle \eta_c | \tilde{O}_{3,\mu\nu}(t_f) | \eta_c \rangle$



At fixed physical t_f, the continuum extrapolation is well controlled due to UV finiteness

Matching to $\overline{\mathrm{MS}}$ and extrapolation to zero flow time

$$O^{\overline{\mathrm{MS}}}_{i,\mu\nu}(\mu) = M^{\overline{\mathrm{MS}}\leftarrow\mathrm{Flow}}_{ij}(\mu, t_{\mathrm{f}})\tilde{O}_{j,\mu\nu}(t_{\mathrm{f}}) + \mathcal{O}(t_{\mathrm{f}}\Lambda^2)$$



- H. Suzuki, PTEP 2013 (2013) 083B03
- H. Makino, H. Suzuki, PTEP 2014 (2014) 063B02
- R. Harlander, Y. Kluth, F. Lange, Eur.Phys.J.C 78 (2018) 11, 944

- One- and two-loop matching are close.
- small-flow-time window is under control.
- the final result is the local MS EMT matrix element.

GFFs from EMT matrix elements (rest frame P=0)

- η_c (spin-0)

$$\langle P | T_i^{\mu\nu} | P \rangle = 2P^\mu P^\nu A_i(0) + 2M^2 g^{\mu\nu} \bar{C}_i(0)$$

- J/ψ (spin-1): denote the polarization by γ_λ

$$\langle P, \gamma_\lambda | T_i^{\mu\nu} | P, \gamma_\lambda \rangle = 2P^\mu P^\nu A_{0,i}(0) + \left(2\delta^{\mu\lambda}\delta^{\nu\lambda} + \frac{1}{2}g^{\mu\nu}\right) M^2 \bar{f}_i(0) - g^{\mu\nu} M^2 \bar{C}_{0,i}(0) \,.$$

	η_c		J/ψ		
	$A(0)$	$\bar{C}(0)$	$A(0)$	$\bar{C}(0)$	$\bar{f}(0)$
gluon(g)	0.084(45)	0.058(18)	0.077(35)	-0.146(29)	-0.012(32)
charm(c)	0.918(17)	-0.068(4)	0.926(34)	0.176(87)	0.023(5)
total	1.000(46)	−0.010(18)	0.997(43)	0.030(30)	0.010(32)

- $A_{q/g}$ probes the momentum fraction $\langle x \rangle_{q/g}$.
- $\bar{C}_{q/g}$ probes the trace-related channel.
- The quark and gluon sectors combine into the conserved total EMT: $A_g(0) + A_q(0) \approx 1, \bar{C}_g(0) + \bar{C}_q(0) \approx 0$

The sigma term from the QCD equation of motion

- σ term: the contribution to the hadron mass due to explicit chiral-symmetry breaking; scale and scheme invariant.

$$\sigma \equiv \frac{1}{2M}\langle P|\sum_f m_f \bar{\psi}_f(x)\psi_f(x)|P\rangle$$

- Use QCD EOM we have,

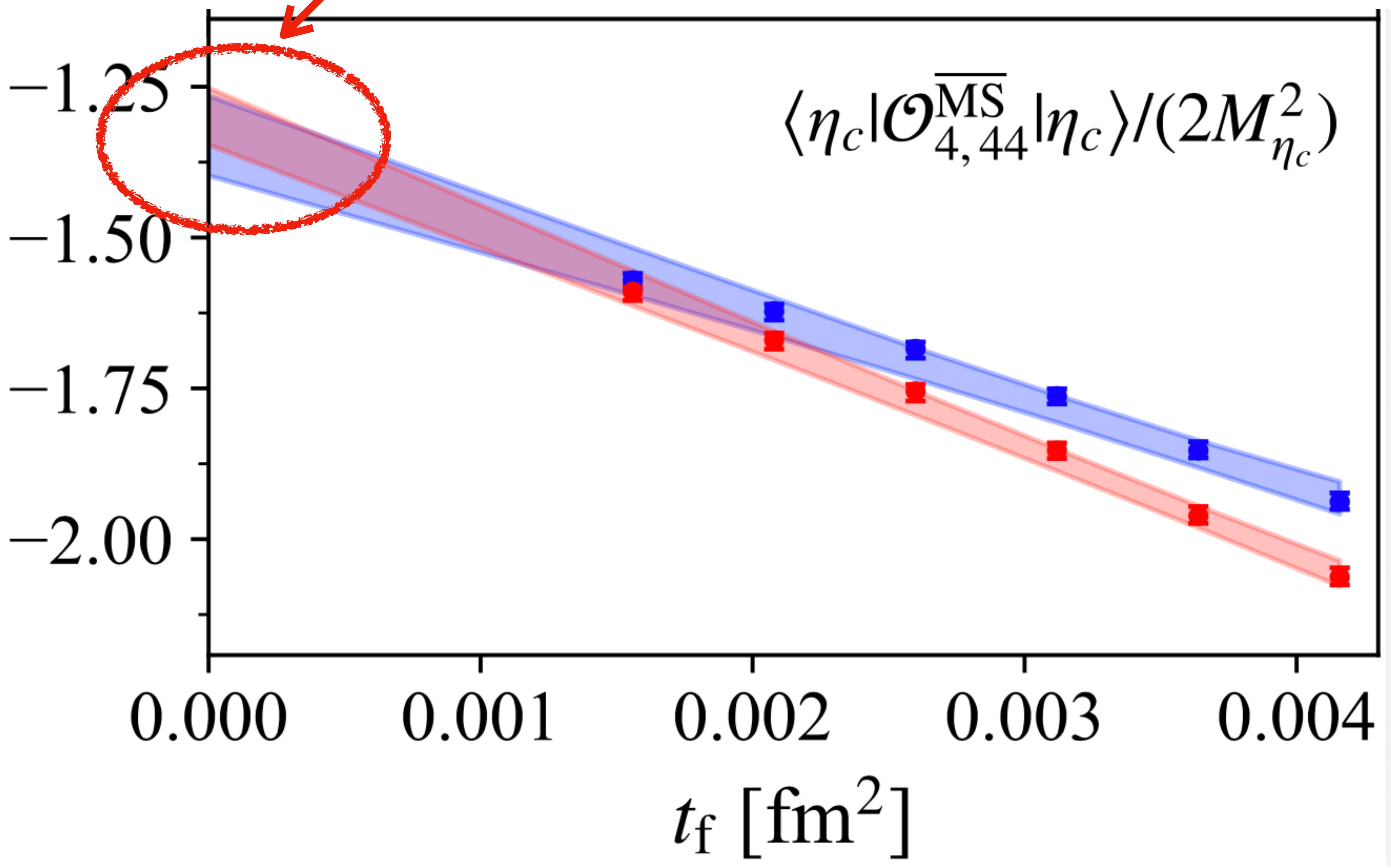
$$\sum_f \bar{\psi}_f(x)\overleftrightarrow{\not{D}}\psi_f(x) = -2m_f\bar{\psi}_f(x)\psi_f(x)$$

$$\sigma = -\frac{1}{4M}\langle P|\sum_f \bar{\psi}_f(x)\overleftrightarrow{\not{D}}\psi_f(x)|P\rangle$$

$$= -\frac{1}{4M}\langle P|O^{\overline{\text{MS}}}_{4,\mu\nu}|P\rangle$$

Trace operater: power divergent mixing avoided through GF

Matching remove the flow effects: scale and scheme invariant



Quark-gluon decomposition of mass and trace anomaly

$$M \equiv \frac{1}{2M}\langle H|T^{00}|H\rangle$$

$$\langle P|T^{\mu}_{\ \mu}|P\rangle = 2P^{\mu}P_{\mu} = 2M^2$$

- Ji' four-term decomposition

$$T^{00} = \bar{T}^{00}_{q,R} + \bar{T}^{00}_{g,R} + \frac{1}{4}T^{\mu}_{\ \mu}$$

$$M = M_q + M_g + M_a + M_m$$

- Two-term/three-term decomposition

$$T^{00} = T^{00}_{q,R} + T^{00}_{g,R}$$

$$M = U_q + U_g$$
$$= (U_q - \sigma) + U_g + U_m$$

- Quark--gluon separation of the trace

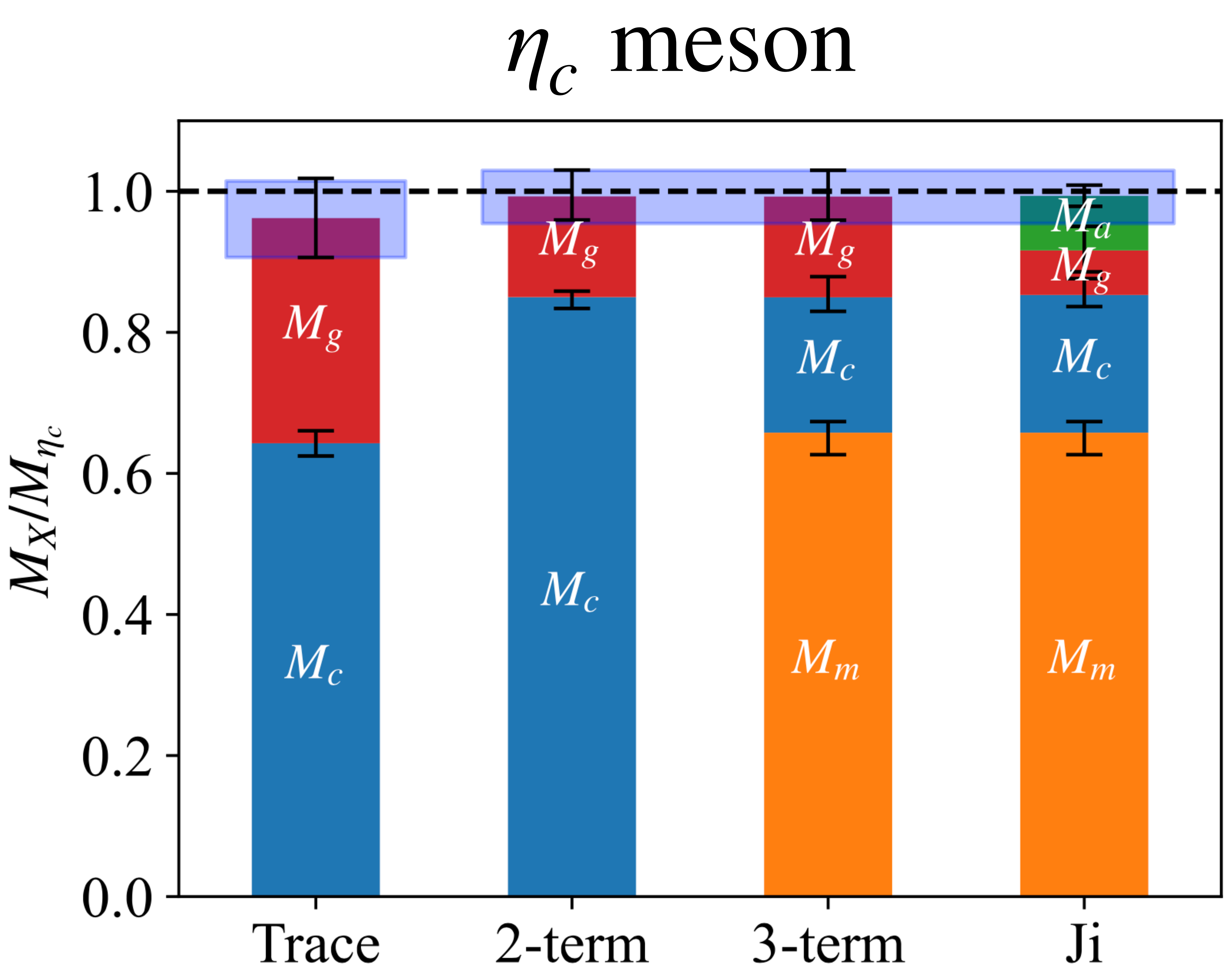
$$T^{\mu}_{\ \mu} = \sum_f \left(1+\gamma_m\right)\, m_f \bar{\psi}_f\psi_f + \frac{\beta(g)}{2g} F^a_{\mu\nu}F^a_{\mu\nu}$$

$$M = \frac{1}{2M}\left(\langle (T_{q,R})^{\mu}_{\mu}\rangle + \langle (T_{g,R})^{\mu}_{\mu}\rangle\right)$$

$$M_m = U_m = \sigma$$

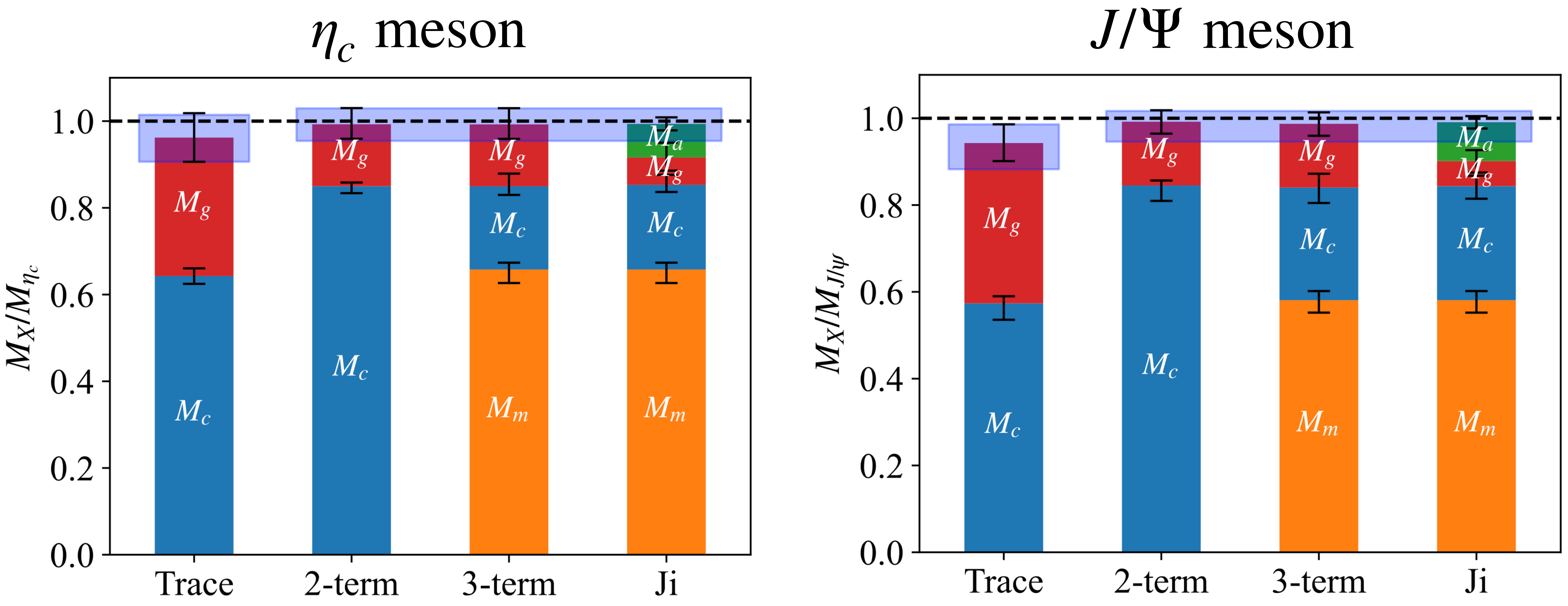
- X.-D. Ji, PRL 74 (1995) 1071-1074
- C. Lorcé, EPJC 78 (2018) 2, 120
- A. Metz, B. Pasquini and S. Rodini, PRD 102 (2020) 114042
- Y. Hatta, A. Rajan and K. Tanaka, JHEP 12 (2018) 008

Quark-gluon decomposition of mass and trace anomaly



- All mass/trace-anomaly sum rules are simultaneously satisfied in a common $\overline{\text{MS}}$ framework (first direct lattice validation).
- Even deeply bound $c\bar{c}$ states contain sizable gluon energy and a non-negligible trace-anomaly contribution.

Quark-gluon decomposition of mass and trace anomaly



- Gluon contributions are remarkably similar between η_c and J/ψ。
- Differences are dominated by the charm contributions, indicating that the spin change mainly redistributes charm energy/sigma components.

Summary

- Gradient flow provides a UV-finite and regulator-independent definition of flowed EMT operators, which admits a controlled continuum limit and avoids power-divergent mixing. Combined with perturbative matching to $\overline{\mathrm{MS}}$ and a $t_\mathrm{f} \to 0$ extrapolation, this gives us access to the full EMT including the trace.
- In charmonium, we extract quark and gluon EMT matrix elements and determine forward-limit GFFs, including the trace-related $\bar{C}_i$.
- We directly validate the momentum, trace-anomaly, and mass/energy decomposition sum rules simultaneously in a common scheme.
- The framework is general and can be straightforwardly adopted for lattice-QCD calculations of mass and spin decompositions as well as gravitational form factors of other hadrons and nuclei.