Towards Distribution Amplitude and GPD Calculations on a Quantum Computer

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Parton Distributions---
a Great Challenge Both in
Exp. and Th.

LaMET (Ji, 2013) is the most popular theoretical approach so far.

This year's highlights

- Precision frontier: Coulomb-gauge, Renormalon/threshold resummation/long-distance extrapolation/higher loop matchings, enhanced Interpolators…
- Higher dimensional frontier: TMD/CS kernel (continuum/physical-point), GPD non-zero skewness, Λ-Baryon DA…
- Flavor frontier: gluonic helicity/PDF/CS, heavy flavor DAs…

Lattice QCD

- Feynman's Path Integral, Monte Carlo, sign problem and real time dependence challenging

An Alternative: Quantum Computation

- Studying how a state evolves under unitary transformation, including real time evolution
- Matrix elements of non-unitarity operators embedded into unitary transformations of larger (Hilbert) spaces with ancillary qubits added.
- Dealing with real time evolution and systems with the sign problem in classical computation in the same way with other problems

Why Quantum Computation?

- Memory:

  Classical: n bits can store an integer from 0 to $2^n - 1$. Quantum: n-qubits can store $2^n - 1$ complex numbers.

- However, if entanglement is lost (due to decoherence) such that the n-qubit state becomes a cross product state, then only n-1 complex numbers can be stored.
- Superposition allows the 2^n states/solutions to be studied at the same time.

Renormalon Ambiguity in LaMET

$$\tilde{q}(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP_z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P_z)^2}\right)$$

- OPE: separation of long and short distance physics not strict in MS-bar scheme
- Matching kernel: IR Borel resummation leads to $\Lambda_{\rm QCD}^2/P_z^2$ ambiguity to be canceled by power corrections
- Large momentum expansion fails when the parton momentum is small

Quantum PDF is IR Renormalon Ambiguity Free

- No IR renormalon ambiguity. Can access x near 0 and 1.

- No IR renormalon in GPD. What about TMD?

Other Possible Advantages of Quantum PDFs

- Accessing nonvalance partons with same expanses as valence ones (no disconnected diagram complications as in lattice QCD)

- 3+1 d QCD at end points of x--- a clear demonstration of quantum advantage on a scientifically important problem

Toy: PDF in QED_2

- Linear, confining potential. Positronium mimics meson in 3+1D QCD
- Working in Minkowski space, accessing lightcone correlators directly

$$\mathcal{O}(z_1, z_2) = \bar{\psi}(z_1 n) n \cdot \gamma W_n(z_1 n, z_2 n) \psi(z_2 n) \qquad n^\mu = (1, -1)$$

$$D(z_1 - z_2) = \frac{1}{2n \cdot P} \langle P | \mathcal{O}(z_1, z_2) | P \rangle$$

$$f(x) = \int_{-\infty}^{\infty} \frac{dz n \cdot P}{2\pi} e^{-izn \cdot P x} D(z)$$

Super renormalizable

$$f(x, a) = f(x) + \mathcal{O}(a)$$

Kogut Susskind Fermions in 1+1 d

$$\psi(x) = (\phi(n), \phi(n+1))^T$$

$$\{\phi(n), \phi^\dagger(k)\} = \delta_{n,k} \quad , \quad \{\phi(n), \phi(k)\} = 0 \quad .$$

(similar to 3+1d equal time $\{\psi_a(\mathbf{x}), \psi_b^\dagger(\mathbf{y})\} = \delta^{(3)}(\mathbf{x}-\mathbf{y})\, \delta_{ab}.$)

$$\hat{H} = \frac{i}{2a_n} \sum_n \left[\phi^\dagger(n)\phi(n+1) - \phi^\dagger(n+1)\phi(n) \right]$$

$$i\frac{d}{dt}\phi(n) = \left[\phi(n), \hat{H} \right] = \frac{i}{2a_n}\left(\phi(n+1) - \phi(n-1) \right)$$

$$\frac{d}{dt}\psi(x) = \alpha_x \frac{d}{dx}\psi(x) \quad , \quad \alpha_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad , \quad \gamma_x = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad , \quad \{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$$

Jordon Wigner Mapping

Occupied (unoccupied) fermion state at site n as spin up (down)

$$\phi(n) = \left(-i\hat{Z}_1\right)\left(-i\hat{Z}_2\right)\ldots\left(-i\hat{Z}_{n-1}\right)\sigma_n^-$$
$$\phi^\dagger(n) = \left(+i\hat{Z}_1\right)\left(+i\hat{Z}_2\right)\ldots\left(+i\hat{Z}_{n-1}\right)\sigma_n^+$$

$$\phi(0) = \sigma^- \otimes I \otimes I \otimes I \quad , \quad \phi^\dagger(0) = \sigma^+ \otimes I \otimes I \otimes I$$
$$\phi(1) = -i\hat{Z} \otimes \sigma^- \otimes I \otimes I \quad , \quad \phi^\dagger(1) = +i\hat{Z} \otimes \sigma^+ \otimes I \otimes I$$
$$\phi(2) = -\hat{Z} \otimes \hat{Z} \otimes \sigma^- \otimes I \quad , \quad \phi^\dagger(2) = -\hat{Z} \otimes \hat{Z} \otimes \sigma^+ \otimes I$$
$$\phi(3) = +i\hat{Z} \otimes \hat{Z} \otimes \hat{Z} \otimes \sigma^- \quad , \quad \phi^\dagger(3) = -i\hat{Z} \otimes \hat{Z} \otimes \hat{Z} \otimes \sigma^+$$

$$\{\phi(1), \phi^\dagger(1)\} = I \quad \{\phi(1), \phi(1)\} = 0 \quad \{\phi(0), \phi^\dagger(1)\} = 0$$

$$\hat{H} = -\frac{1}{2a}\sum_n \left[\, \sigma_n^+ \sigma_{n+1}^- + \sigma_{n+1}^+ \sigma_n^- \,\right] + \frac{m}{2}\sum_n (-)^n \hat{Z}_n$$

Hamiltonian of QED_2

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi, \qquad D_\mu = \partial_\mu + ieA_\mu$$

$$\Pi^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_0 A_\mu)} = F^{\mu 0}.$$ $\Pi^0 = 0$ implies A^0 is not dynamical.

$$\frac{\partial\mathcal{L}}{\partial A_0} = J^0 - \partial_\mu\Pi^\mu = 0$$ (Gauss's law, a quantum constraint)

A^1=0 (the axial gauge) (open boundary condition in x)

$$\begin{aligned}\mathcal{H} &= \pi\partial_0\psi - \mathcal{L} \\ &= \frac{1}{2}E^2 - i\bar{\psi}\gamma^1\partial_1\psi + m\bar{\psi}\psi,\end{aligned}$$

Wilson Line

$$W_n(z_1 n, z_2 n) = e^{-ie\int_{z_2}^{z_1} n\cdot A(z)dz}$$

Gauge invariant

$$O'(x,y) = \overline{\psi}(x) n\cdot\gamma Q_n(x)\overline{Q}_n(y)\psi(y)$$

$$\mathcal{L}' = \mathcal{L} + \overline{Q}_n(x) in\cdot D Q_n(x)$$

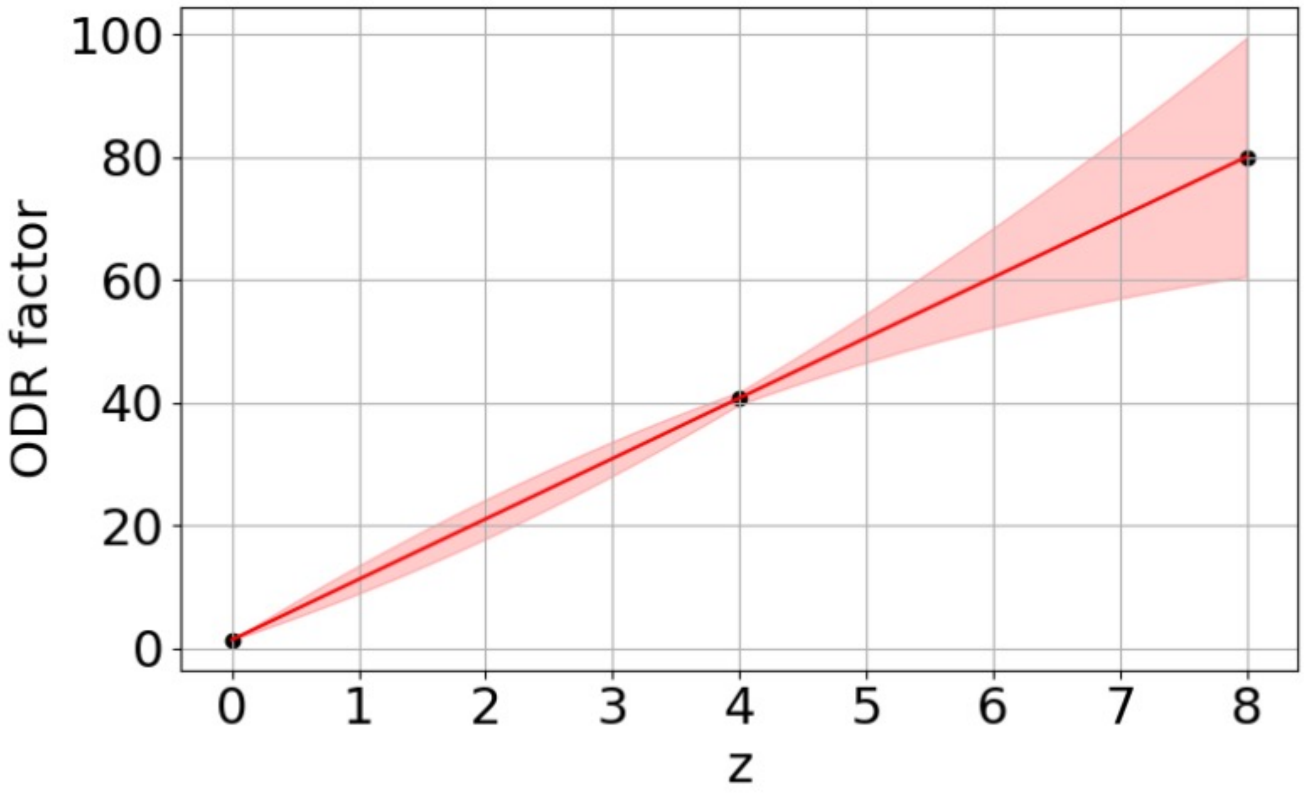
$$\delta J^\mu = -e\overline{Q}_n(x) n^\mu Q_n(x)$$

Wilson line carries the same charge as the fermion

Quantum Computation

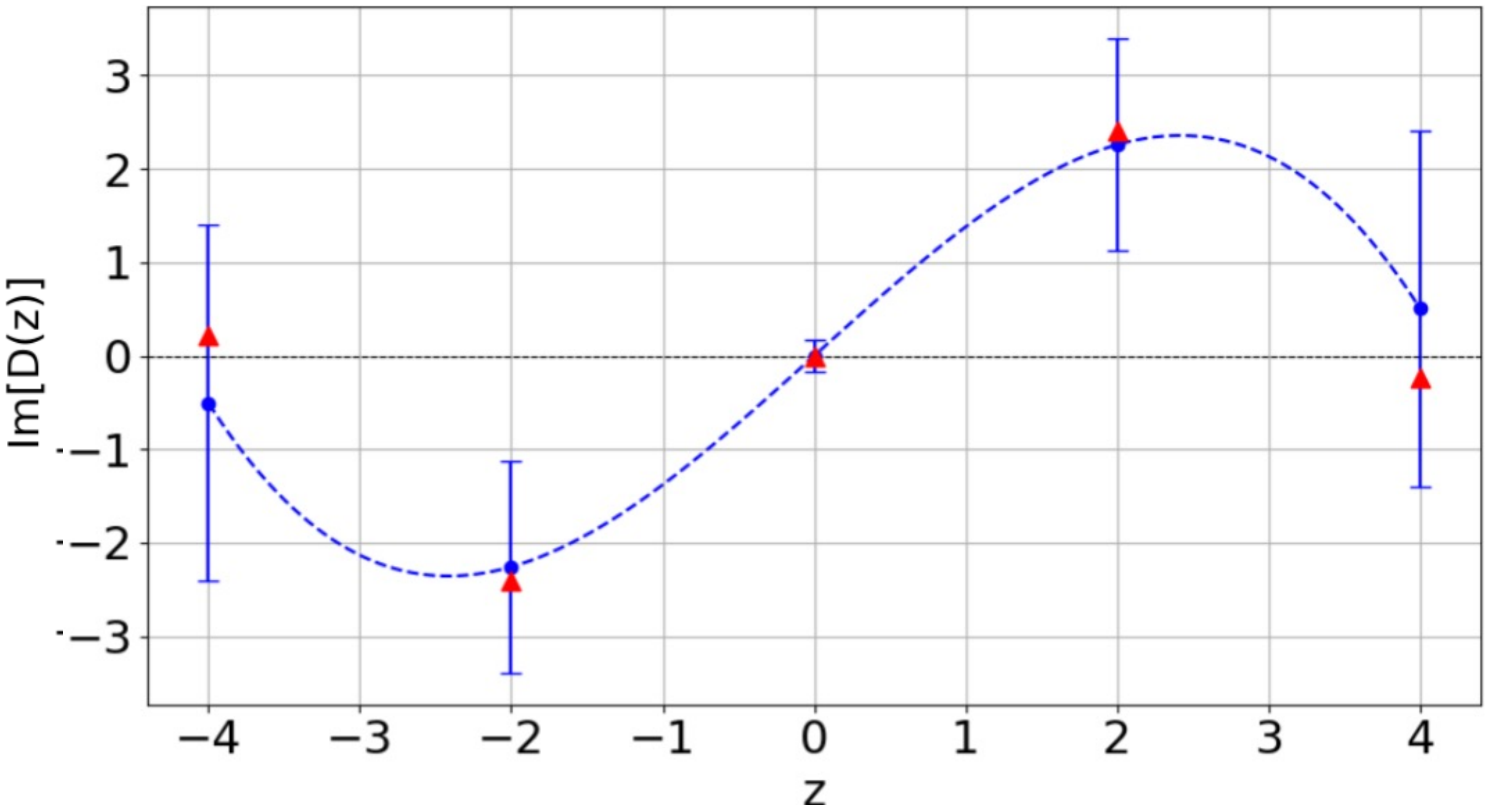
- IBM cloud quantum computers
- 11 qubits: 5 physical spatial sites, 10 staggered fermion sites, 1 ancillary qubit; Zigzag Wilson lines
- 2-qubit gate depth ~5K reduced to ~500 (very critical!). Mainly from changing state preparation, operator choice, and doubling the step size for time evolution and Wilson line. Each reduces the gate depth by half.

Error Mitigations



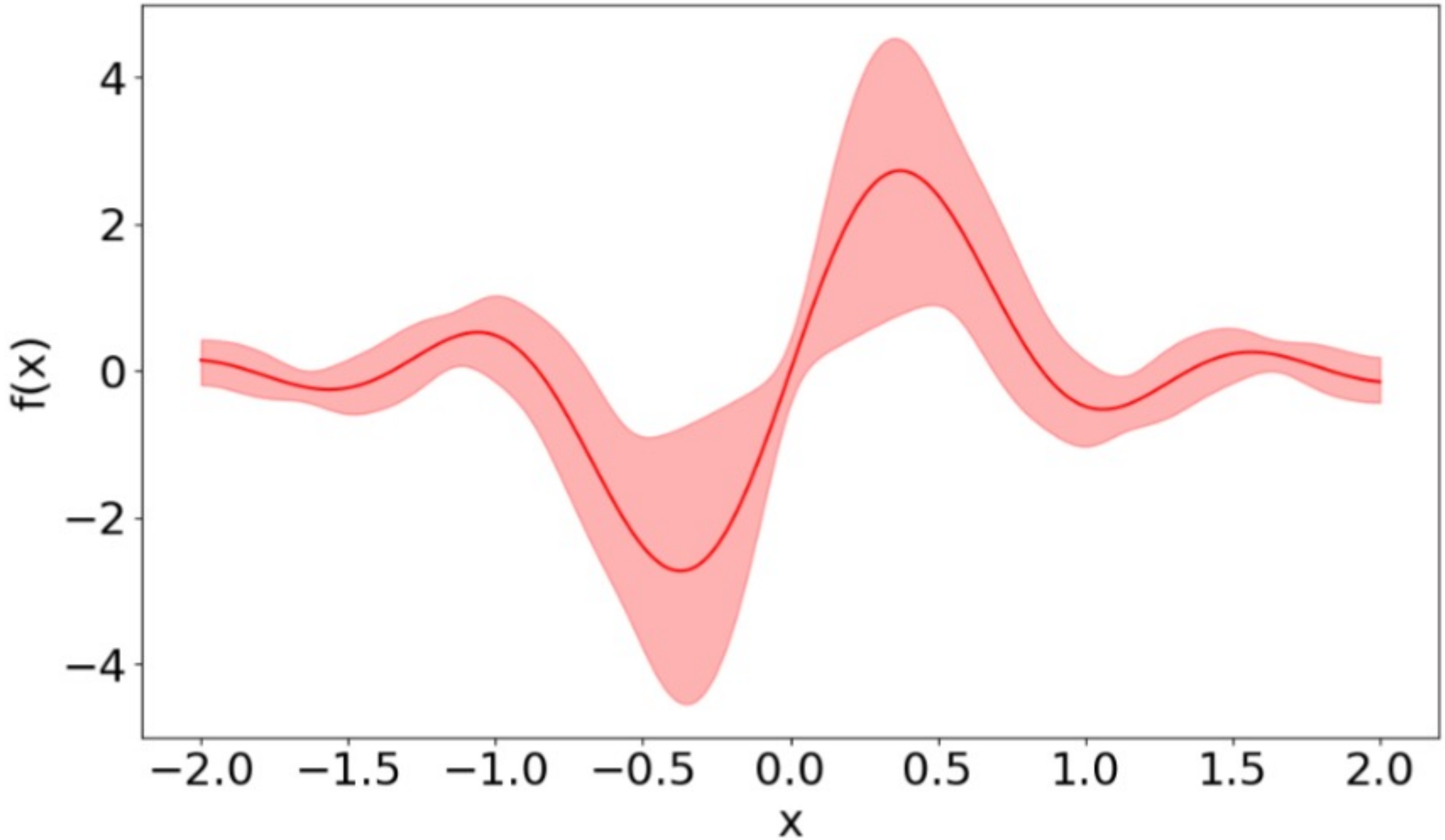
Dynamical decoupling (DD), Pauli twirling (PT) with randomization = 100, and Operator Decoherence Renormalization (ODR).

Result: Lightcone Correlator



8,000 shot result from IBM quantum computers.
Triangles from a classical simulator.

First “Quantum” PDF Result



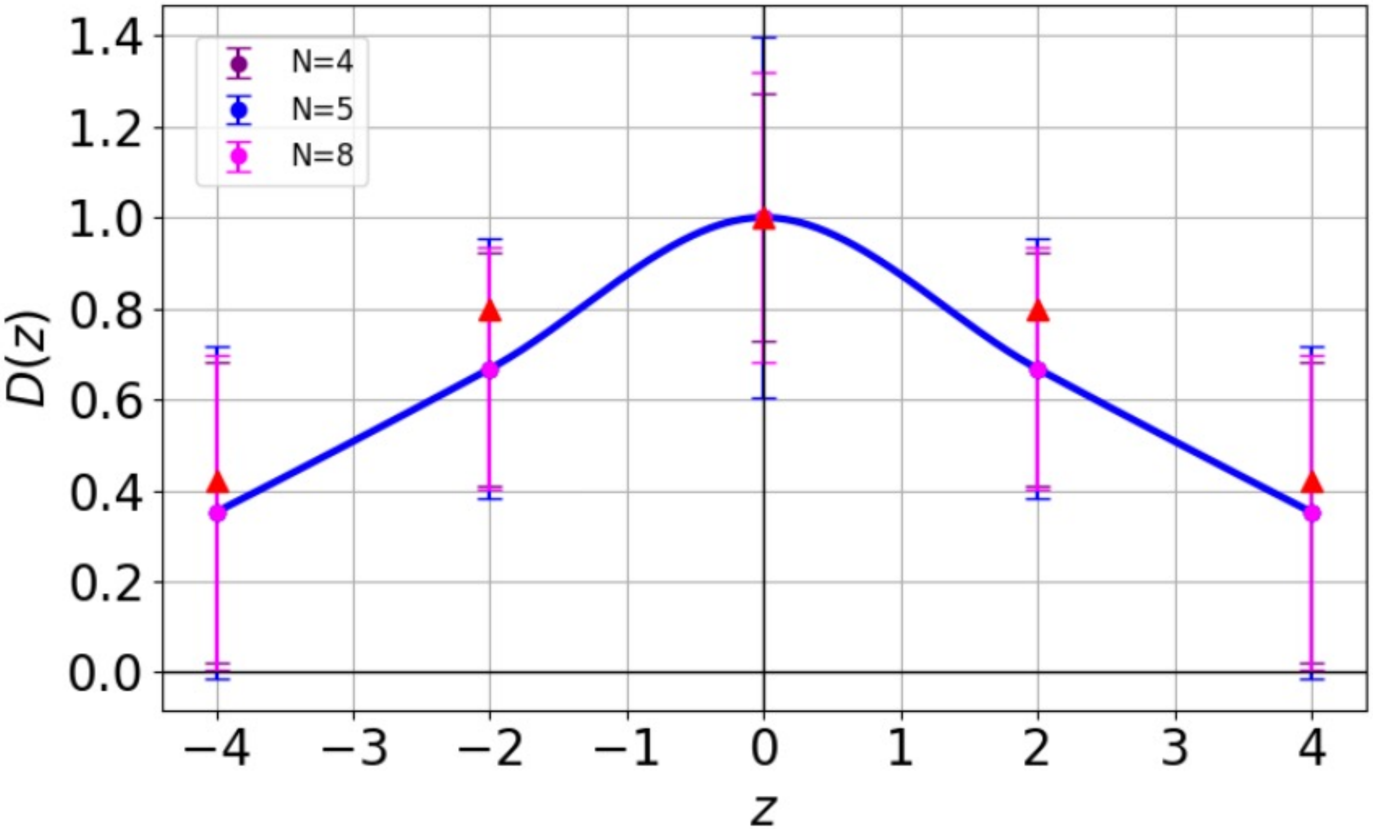
$$\int_{-1}^{1} dx\, x f(x) = 1.04^{+0.66}_{-0.76}, \qquad \int_{0}^{1} dx\, f(x) = 1.37^{+0.89}_{-1.02}$$

Scaling

- 2 qubit gate number scales as the cubic of the volume. This calculation is state of the art already.

- (1+1) d QED can be completely solved with machines available in 2029.

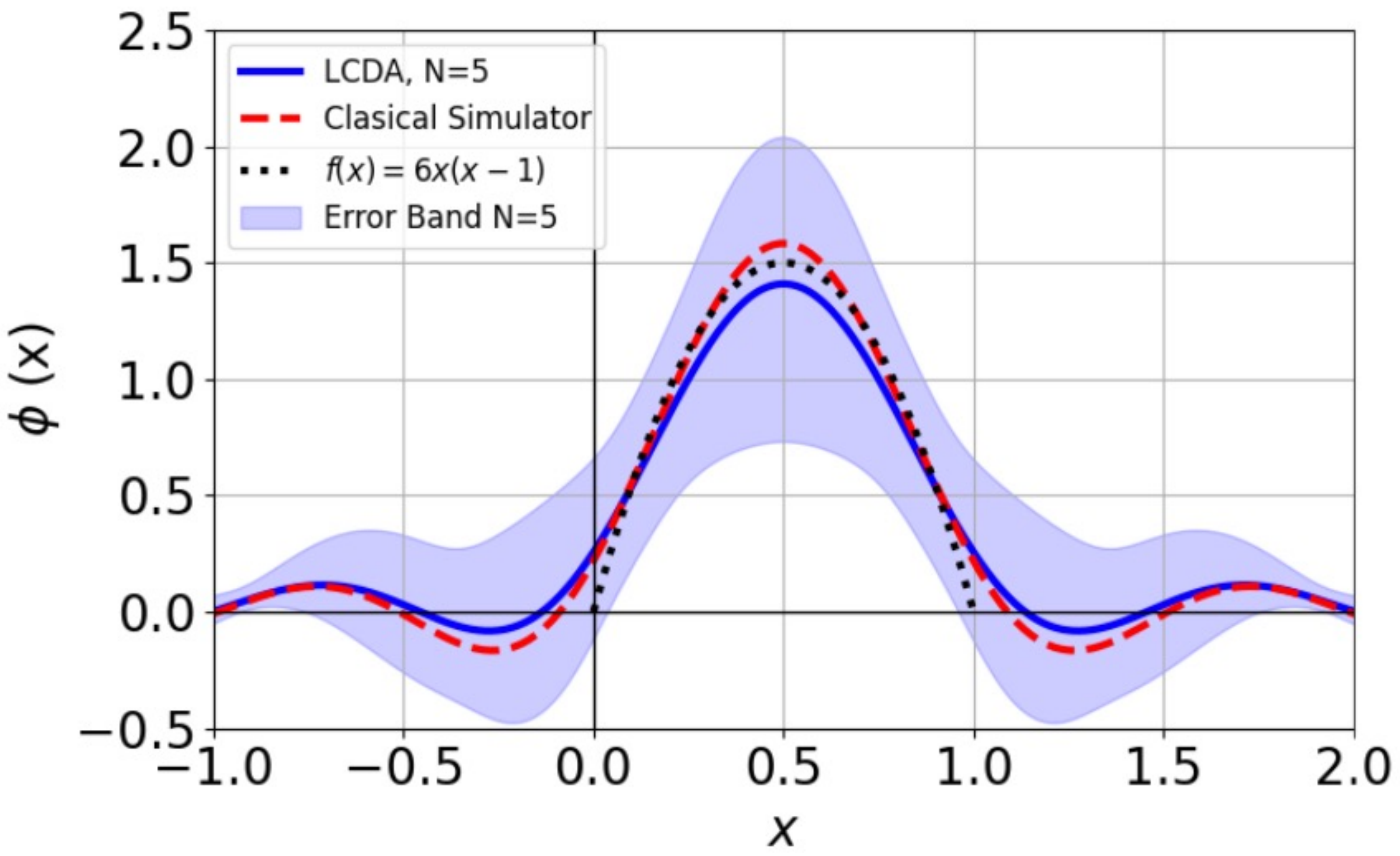
Meson DA



$\langle \Omega | \mathcal{O} | P \rangle$

Two qubit gate/depth ~1,000

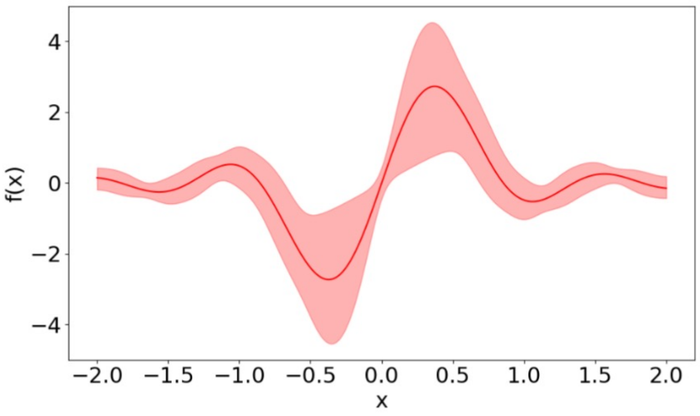
First “Quantum” DA Result



GPD?

$$H(x,\xi,t)=\frac{1}{2P^+}\int\frac{\mathrm{d}z^-}{2\pi}\,\mathrm{e}^{\mathrm{i}xP^+z^-}\,\langle M(q)|\bar{\psi}\left(-\frac{z^-}{2}\right)\gamma^+\mathcal{W}\psi\left(\frac{z^-}{2}\right)|M(0)\rangle$$

$$t=-\frac{4M^2\xi^2}{1-\xi^2}.$$



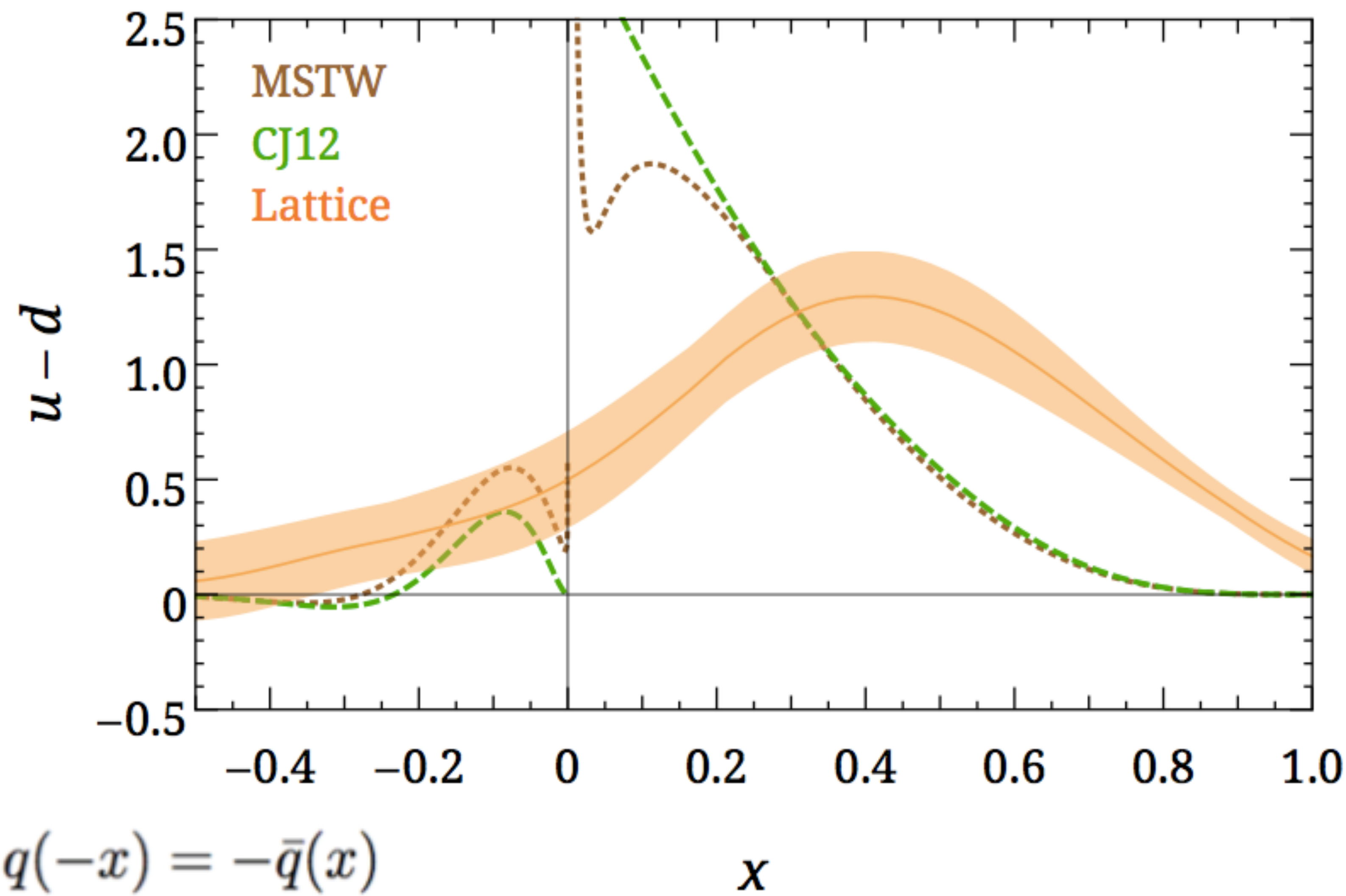
Two qubit gate/depth ~8,000

Cannot do it now.

Outlook

1st LaMET Calculation

Lin, JWC, Cohen, Ji (1402.1462)



$q(-x) = -\bar{q}(x)$

IBM Quantum Roadmap

Scaling faster than Moore's law.
200 qubits/0.1 B gates in 2029 (156/5K now)

IBM **Quantum**

2025	2026	2027	2028	2029	2033+
Enhancing quantum execution speed and parallelization with partitioning and quantum modularity	Improving quantum circuit quality to allow 7.5K gates	Improving quantum circuit quality to allow 10K gates	Improving quantum circuit quality to allow 15K gates	Improving quantum circuit quality to allow 100M gates	Beyond 2033, quantum-centric supercomputers will include 1000's of logical qubits unlocking the full power of quantum computing
Functions	Mapping Collection	Specific Libraries			General purpose QC libraries
Resource Management	Circuit Knitting x P	Intelligent Orchestration			Circuit libraries
Flamingo (5K) Error Mitigation 5k gates 156 qubits Quantum modular 156x7 = 1092 qubits	Flamingo (7.5K) Error Mitigation 7.5k gates 156 qubits Quantum modular 156x7 = 1092 qubits	Flamingo (10K) Error Mitigation 10k gates 156 qubits Quantum modular 156x7 = 1092 qubits	Flamingo (15K) Error Mitigation 15k gates 156 qubits Quantum modular 156x7 = 1092 qubits	Starling (100M) Error correction 100M gates 200 qubits Error corrected modularity	Blue Jay (1B) Error correction 1B gates 2000 qubits Error corrected modularity

See what will happen in 12 years!

Backup

Remaining Errors

- Statistical errors (standard errors): inversely proportional to the number of shots
- Remove finite volume effect and lattice spacing, need ~3 and ~2 times more sites
- 2-qubit gate depth scales with 3 powers of the spatial volume
- Machines available in 2029!