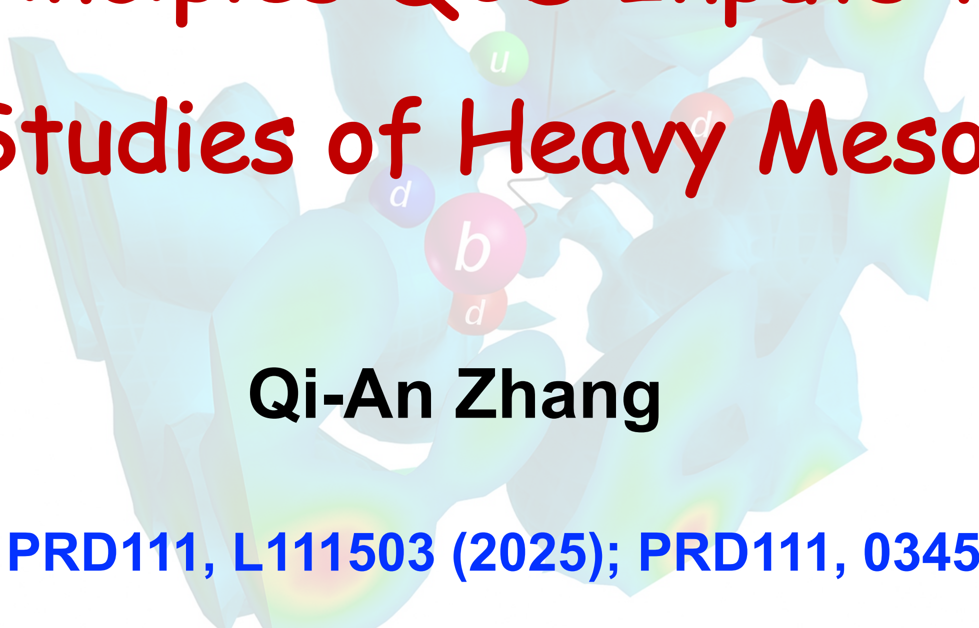
Lattice Parton Collaboration

北京航空航天大學
BEIHANG UNIVERSITY

First-principles QCD Inputs toward Precision Studies of Heavy Meson Decays



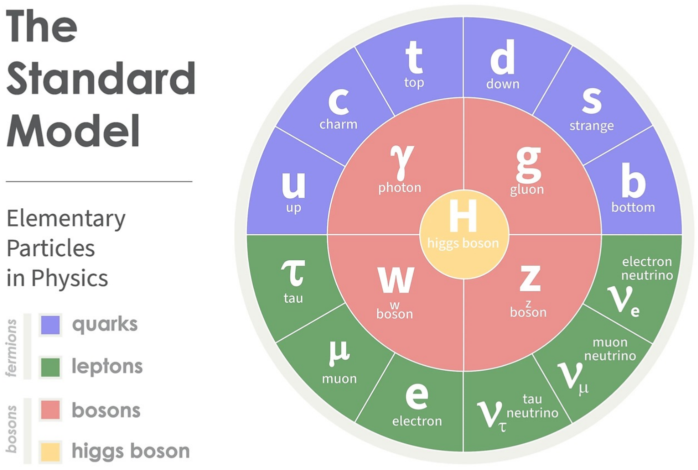
Qi-An Zhang

Based on: PRD111, L111503 (2025); PRD111, 034503 (2025);
arXiv:2604.25802; 2605.10946

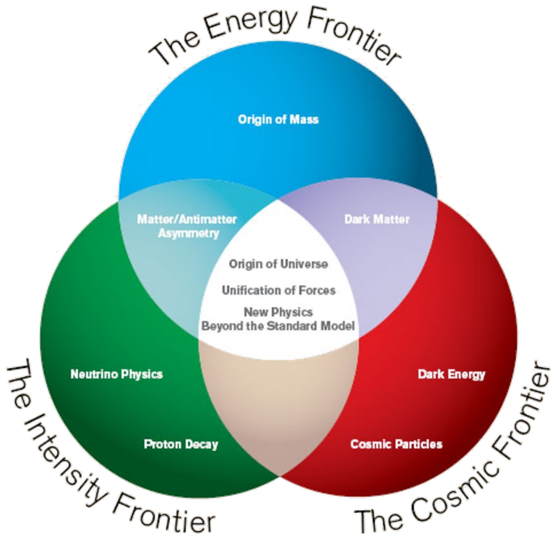
Jul. 07, 2026, **LaMET2026 @ Cracow, Poland**

Motivation for Heavy Hadron Decays

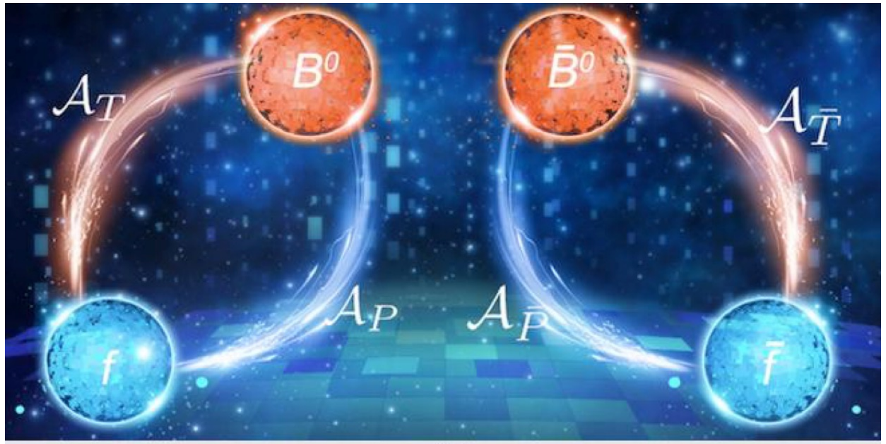
- **Precision tests of the Standard Model**



- **Indirect searches for new physics**



- **Study on CP violation**

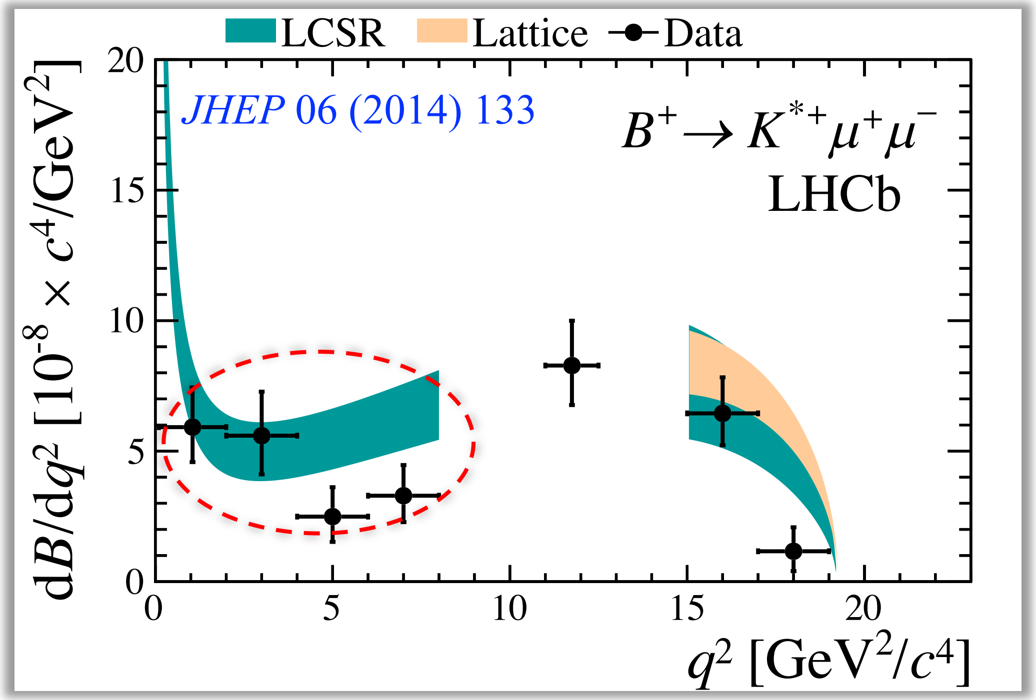


$$\mathcal{A} = \mathcal{A}_T e^{i(\delta_1+\phi_1)} + \mathcal{A}_P e^{i(\delta_2+\phi_2)} \overset{\text{CP}}{\leftrightarrow} \bar{\mathcal{A}} = \bar{\mathcal{A}}_T e^{i(\delta_1-\phi_1)} + \bar{\mathcal{A}}_P e^{i(\delta_2-\phi_2)}$$

Heavy Meson LCDA in the Precision Calculation

- e.g. FCNC processes $b \to s\ell\ell$

Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035



$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}$$
$${}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

Dominant systematic uncertainties:

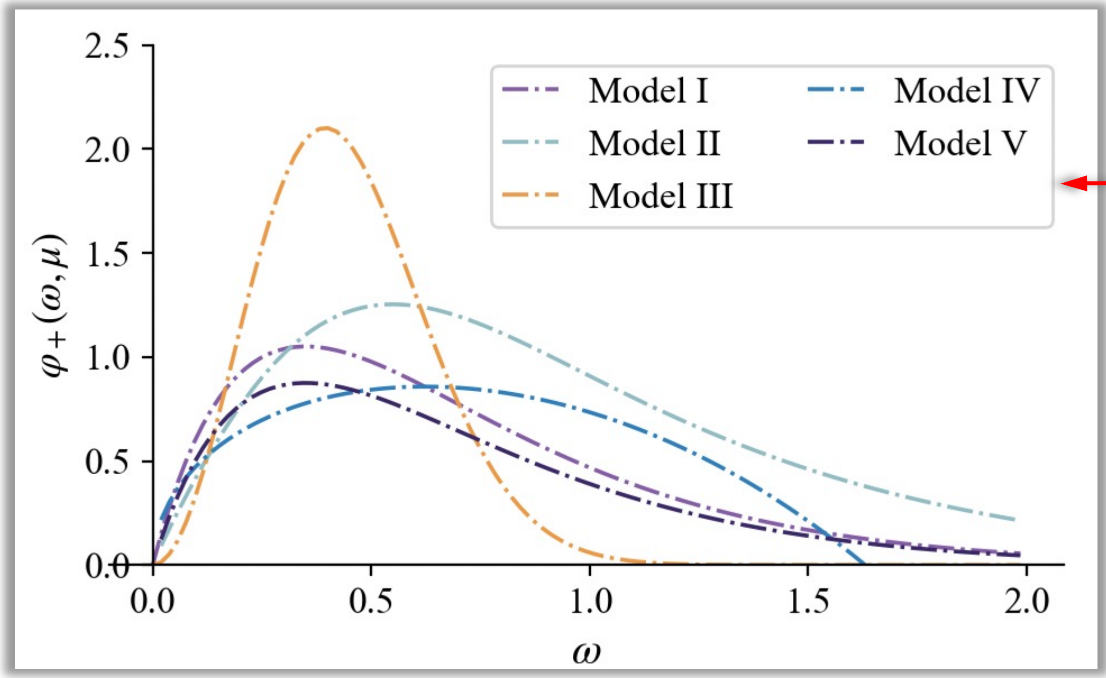
- Uncertainties of inverse moment;
- Model-dependence of LCDA.

Model Parametrization of the Heavy Meson LCDA

- Models for leading twist LCDA of B meson:

Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$



Dominant systematic uncertainties:

- Uncertainties of inverse moment;
- Model-dependence of LCDA.

Challenges for Heavy Meson Distributions from Lattice QCD

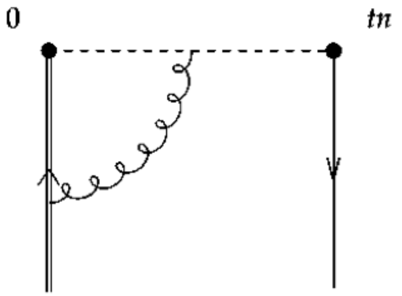
- **Scheme I: Operator Product Expansion (OPE) for QCD quark bilinears:**

$$\bar q(z_2 n)\not n\gamma_5 W_c(z_2 n, z_1 n)\, q'(z_1 n) = \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k!l!} n^\rho n^{\mu_1}\dots n^{k+l}\mathcal{M}^{(k,l)}_{\rho\mu_1\dots\mu_{k+l}} + \cdots,$$

with

$$\mathcal{M}^{(k,l)}_{\rho\mu_1\dots\mu_{k+l}} = \bar q(0)\overleftarrow{D}_{(\mu_1}\dots\overleftarrow{D}_{\mu_k}\overrightarrow{D}_{\mu_{k+1}}\dots\overrightarrow{D}_{\mu_{k+l}}\gamma_{\rho)}\gamma_5 q'(0).$$

While for HQET fields, the OPE break down……



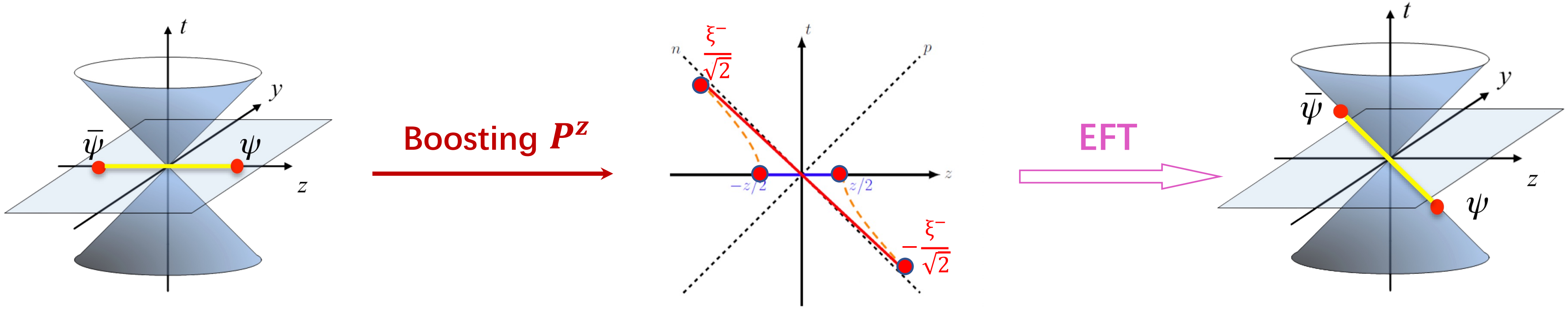
$$O_v^{\text{ren}}(t,\mu) = \frac{4}{\hat\epsilon}\underline{\ln(it\mu)}\, O_v^{\text{bare}}(t)+\dots \rightarrow \log 0! \Rightarrow \text{No Local Limit}$$

Cusp Divergence!

Braun, Ivanov, Korchemsky, PRD69, 034014 (2004)

Challenges for Heavy Meson Distributions from Lattice QCD

- **Scheme II: Large-momentum Effective Theory (LaMET):** connecting the Euclidean and Minkowski correlators in the large momentum limit



Quasi distributions:
Directly calculable on the lattice

Ji,PRL110 (2013), 262002
Ji, et al., Rev.Mod.Phys. 93 (2021) 3, 035005

Light-cone distributions

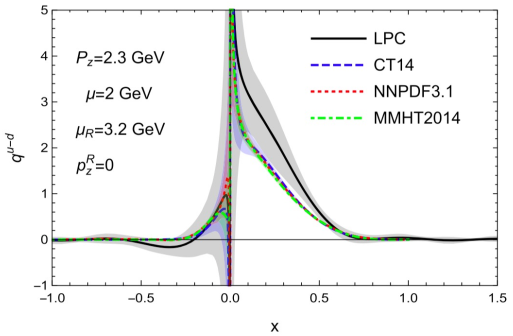
Effective field theory: one can perform an expansion for large but finite P^z

$$f(y,\mu) = \int \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \tilde{f}(x, P^z, \mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{(yP^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{((1-y)P^z)^2}, \frac{M^2}{(P^z)^2}\right)$$

Challenges for Heavy Meson Distributions from Lattice QCD

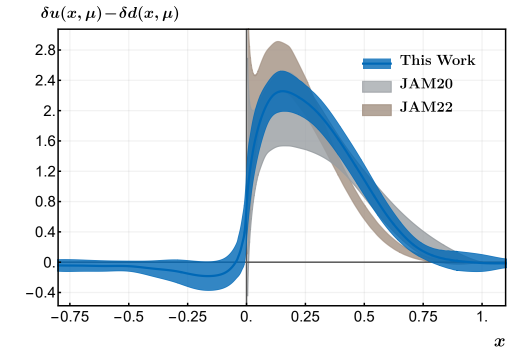
- **Significant progress has been made on PDFs, LCDAs, TMDs, GPDs, *et al.*:**

Unpolarized PDF



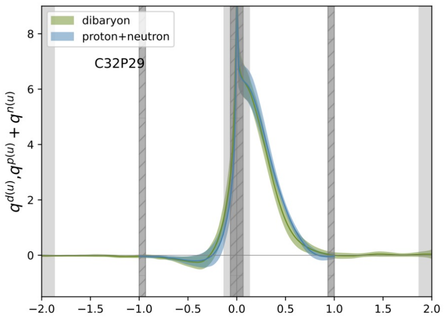
LPC, 2020

Transversity PDF



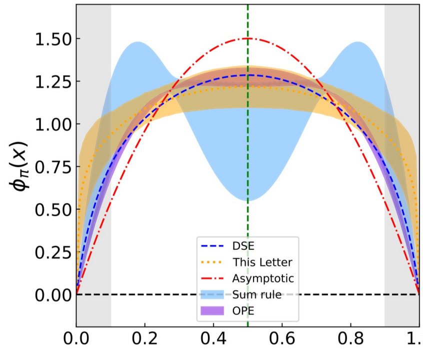
LPC, 2024

Deuteron PDF



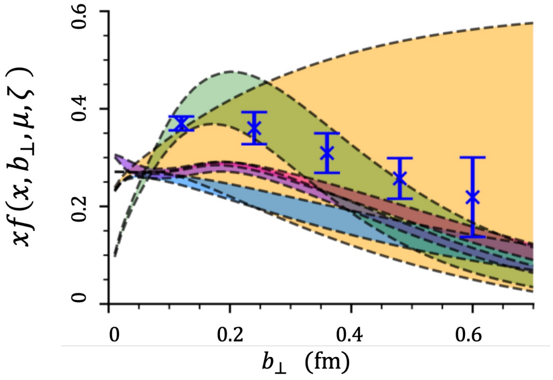
LPC, 2025

Pion LCDA



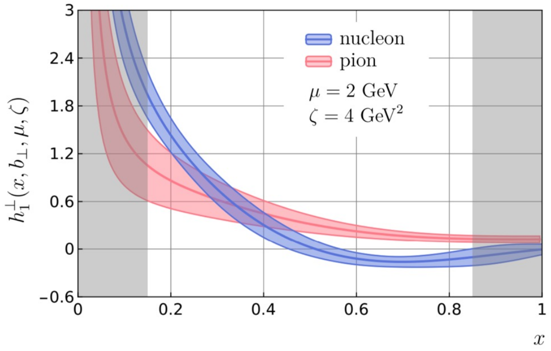
LPC, 2022

Unpolarized TMDPDF



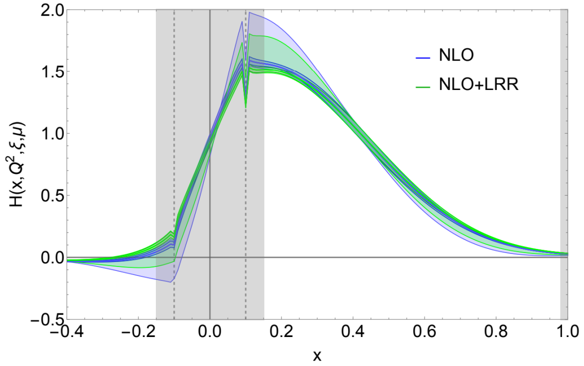
LPC, 2024

Boer-Mulders function



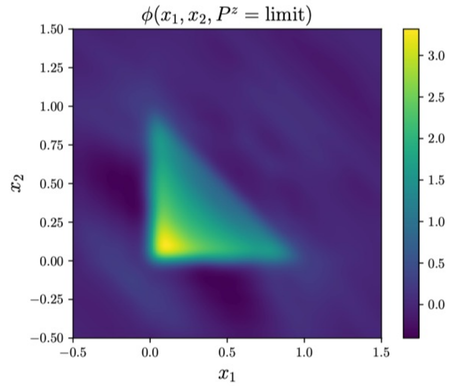
LPC, 2025

GPD



Holligan, Lin, 2024

Λ-baryon LCDA



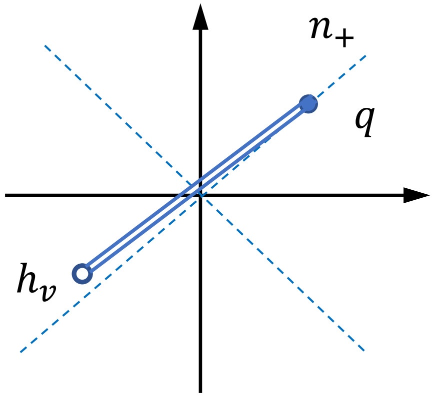
LPC, 2025

Challenges for Heavy Meson Distributions from Lattice QCD

- **Turns to correlators with HQET fields:**
  - An intuitive approach: adopt **off light-cone Wilson line** to avoid cusp divergence

$$\langle \boldsymbol{H_Q(P_{H_Q})}\,|\,\bar{q}(z)\not{n}_z\gamma_5\, W_c(z\,,0)h_v(0)\,|\,0\rangle$$

Wang, Wang, Xu, Zhao, PRD 102, 011502 (2020);
Xu, Zhang, PRD 106, 114019 (2022);
Hu, Xu, Zhao, EPJC 84, 502 (2024); ……



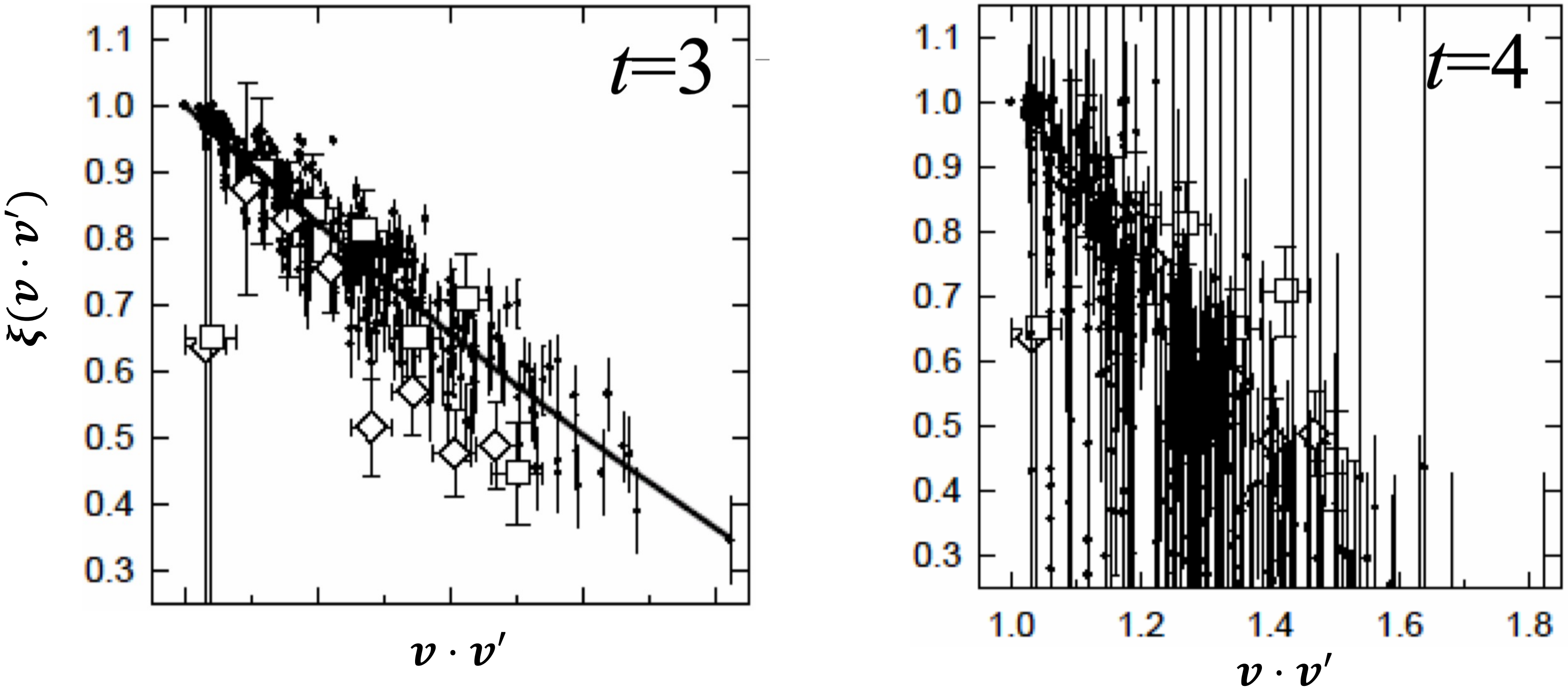
Need to realize the **boosted HQET fields on lattice**.

Challenges for Heavy Meson Distributions from Lattice QCD

- Boosted HQET on Lattice suffers significant signal-to-noise (StN) problem

Isgur-Wise function:

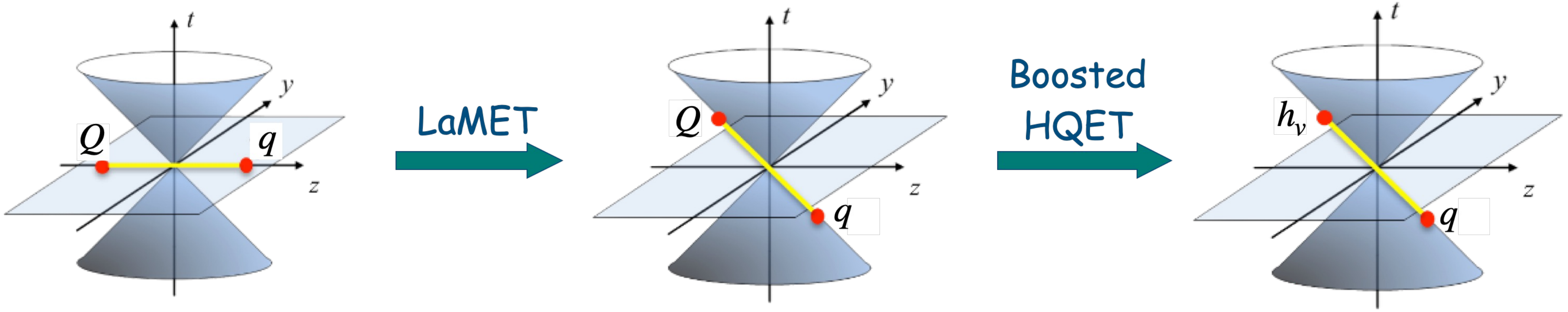
Mandula, Ogilvie, PRD 45, 2183-2187 (1992); NPB 34, 480-482 (1994)



Recipes for Heavy Meson Distributions from LQCD

Heavy Quark LaMET

LPC, Phys.Rev.D 111 (2025), L111503; PRD111 (2025), 034503



P^z

m_H/m_Q

Λ_{QCD}

- Quasi-DA
- LQCD calculable
- $\Lambda_{\text{QCD}} \ll m_H \ll P^z$

m_H/m_Q

Λ_{QCD}

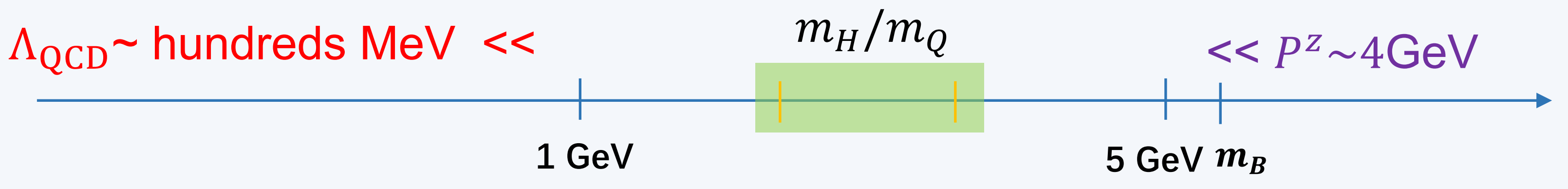
- LaMET
- Integrate out P^z
- QCD LCDA

Λ_{QCD}

- Boosted HQET
- Integrate out m_Q
- HQET LCDA

Recipes for Heavy Meson Distributions from LQCD

- **A two-step effective theory on the lattice: $\boldsymbol{\Lambda_{\mathrm{QCD}} \ll m_H/m_Q \ll P^z}$**



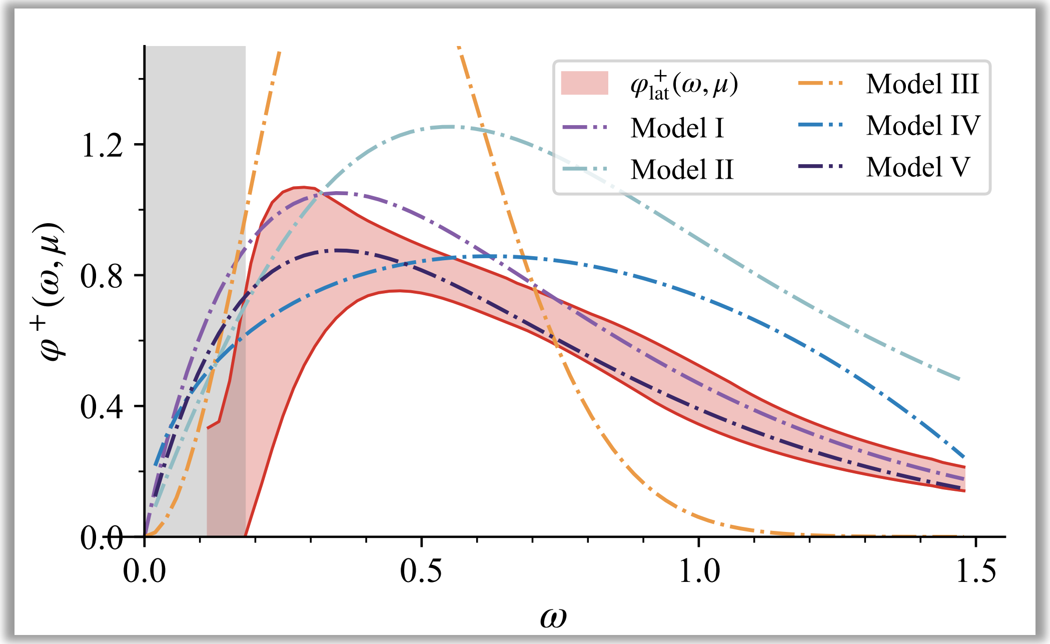
Window for heavy meson mass

- ✓ Heavy quark flavor symmetry ensures that the HQET measurement is independent of heavy quark mass;
- ✓ m_H/m_Q only contributes to the power corrections.

Power corrections: $\frac{\Lambda_{\mathrm{QCD}}^2}{(yP^z)^2}, \frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\mathrm{QCD}}}{m_Q}$

Recipes for Heavy Meson Distributions from LQCD

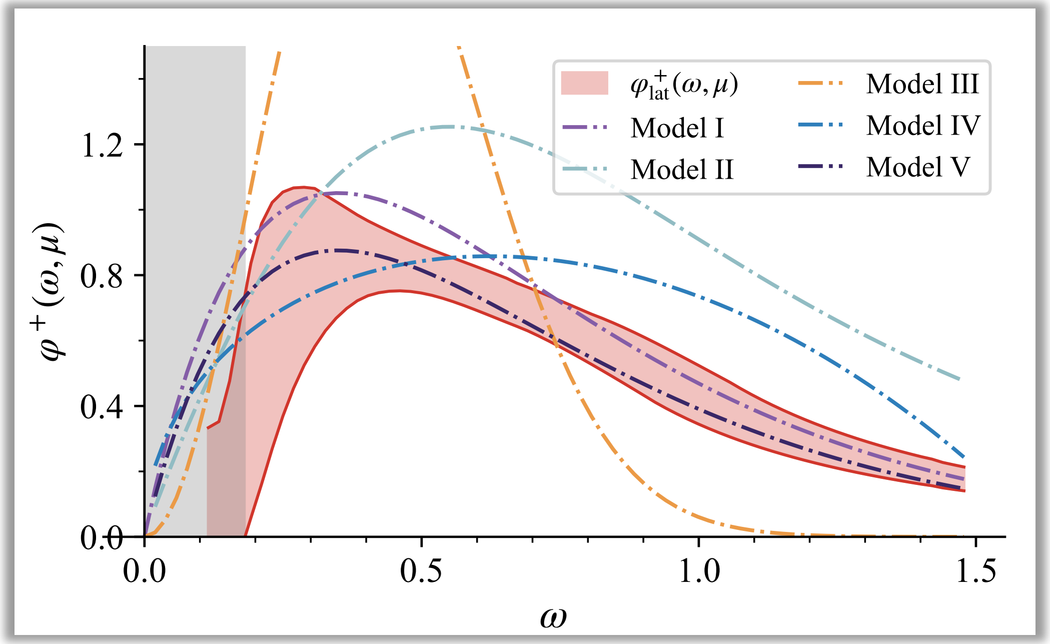
LQCD result, 2025



LPC, Phys.Rev.D 111 (2025), L111503; PRD111 (2025), 034503

Recipes for Heavy Meson Distributions from LQCD

LQCD result, 2025



LPC, Phys.Rev.D 111 (2025), L111503; PRD111 (2025), 034503

- Only single lattice spacing ⇒ Continuum limit
- Unphysical mass ⇒ Chiral extrapolation
- Leading power ⇒ Power corrections?

LaMET:

$$\frac{\Lambda_{\rm QCD}^2}{(yP^z)^2} \sim O\left(\frac{0.4^2}{(4y)^2}\right) \sim O(1\%)$$

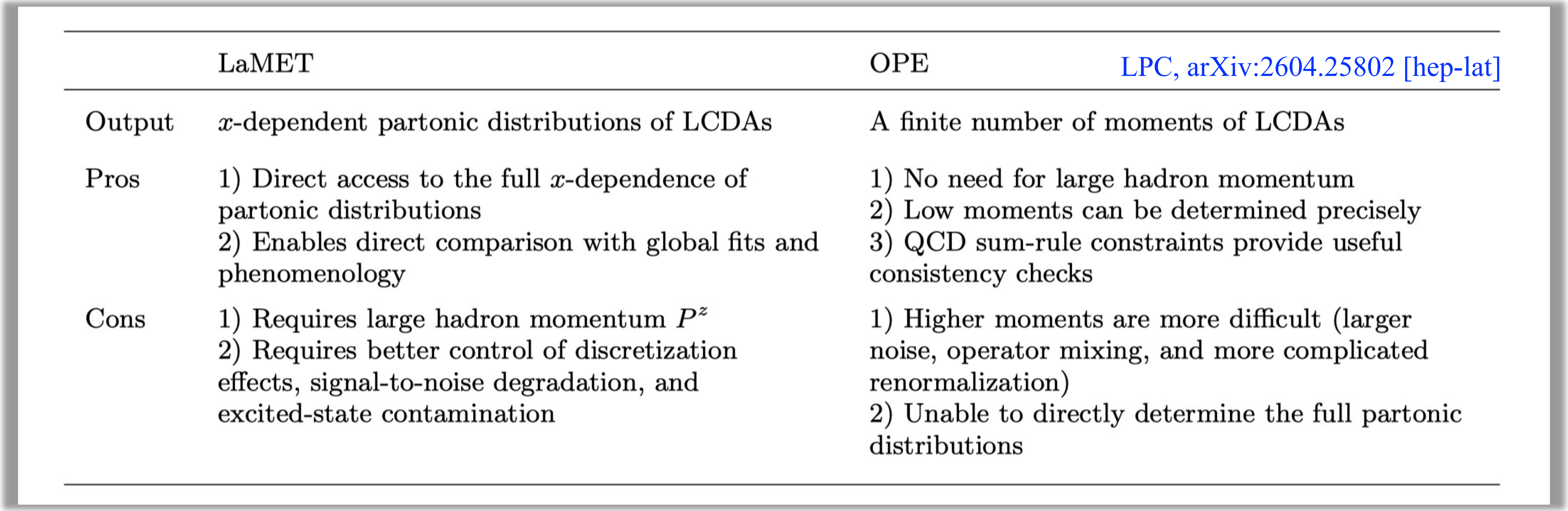
$$\frac{m_D^2}{(P^z)^2} \sim O\left(\frac{2^2}{4^2}\right) \simeq O(25\%)$$

bHQET:

$$\frac{\Lambda_{\rm QCD}}{m_c} \sim O\left(\frac{0.4}{1.2}\right) \simeq O(33\%)$$

Recap: LaMET vs. Lattice OPE

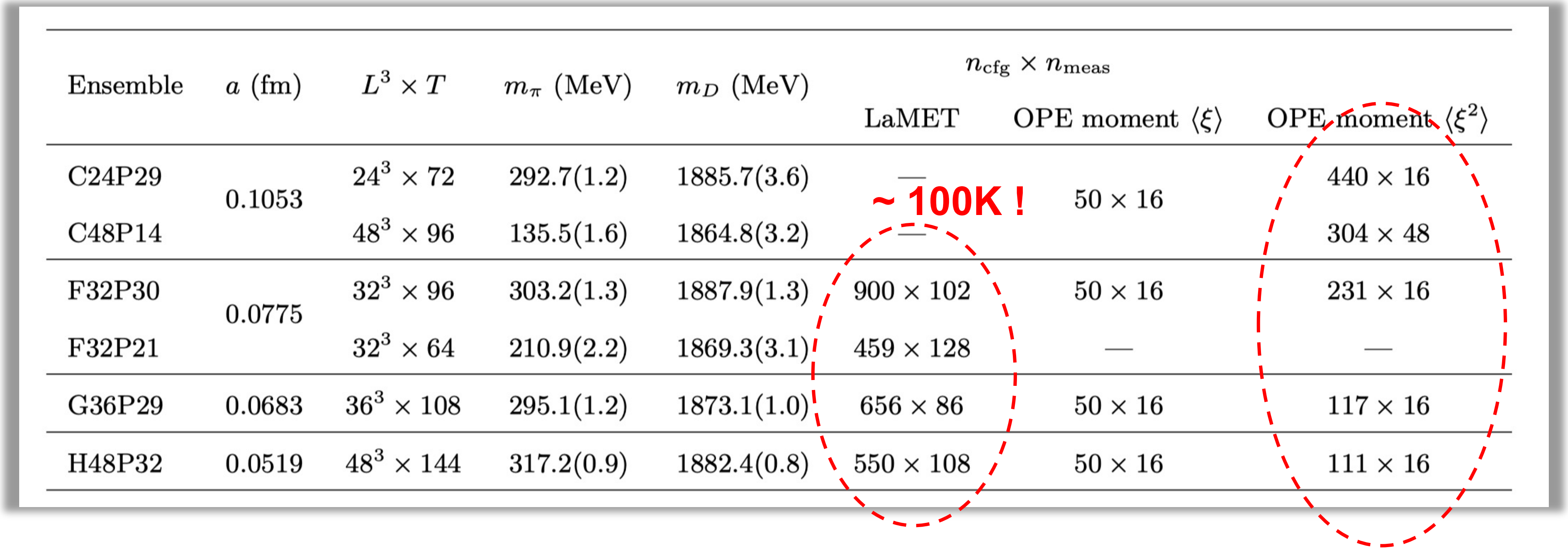
- In the lattice OPE calculation of heavy meson LCDAs, the largest scale is m_H
- But only the lowest moments can be accessed
- The power correction $\mathcal{O}(m_D^2/(P^z)^2)$ can be benchmarked by the OPE moments



	LaMET	OPE
Output	x-dependent partonic distributions of LCDAs	A finite number of moments of LCDAs
Pros	1) Direct access to the full x-dependence of partonic distributions 2) Enables direct comparison with global fits and phenomenology	1) No need for large hadron momentum 2) Low moments can be determined precisely 3) QCD sum-rule constraints provide useful consistency checks
Cons	1) Requires large hadron momentum P^z 2) Requires better control of discretization effects, signal-to-noise degradation, and excited-state contamination	1) Higher moments are more difficult (larger noise, operator mixing, and more complicated renormalization) 2) Unable to directly determine the full partonic distributions

LPC, arXiv:2604.25802 [hep-lat]

Precision Calculation of Heavy Meson LCDAs



Ensemble	a (fm)	$L^3 \times T$	m_π (MeV)	m_D (MeV)	$n_{\mathrm{cfg}} \times n_{\mathrm{meas}}$		
					LaMET	OPE moment $\langle \xi \rangle$	OPE moment $\langle \xi^2 \rangle$
C24P29	0.1053	$24^3 \times 72$	292.7(1.2)	1885.7(3.6)	—	50×16	440×16
C48P14		$48^3 \times 96$	135.5(1.6)	1864.8(3.2)	—		304×48
F32P30	0.0775	$32^3 \times 96$	303.2(1.3)	1887.9(1.3)	900×102	50×16	231×16
F32P21		$32^3 \times 64$	210.9(2.2)	1869.3(3.1)	459×128	—	—
G36P29	0.0683	$36^3 \times 108$	295.1(1.2)	1873.1(1.0)	656×86	50×16	117×16
H48P32	0.0519	$48^3 \times 144$	317.2(0.9)	1882.4(0.8)	550×108	50×16	111×16

- ✓ Multi lattice spacings: Continuum extrapolation.
- ✓ Multi pion masses: Chiral extrapolation.
- ✓ Large statistic.
- ✓ LaMET + Lattice OPE for QCD LCDA.

Matching from Quasi-DA to QCD LCDA of D meson

$$\phi(y,\mu)=\int_{-\infty}^{+\infty}\frac{dx}{|x|}\,\mathcal{C}\left(\frac{y}{x},\frac{\mu}{xP^z}\right)\tilde\phi(x,P^z)+\mathcal{O}\left(\frac{m_H^2}{(P^z)^2},\frac{\Lambda_{\rm QCD}^2}{(yP^z,(1-y)P^z)^2}\right).$$

Matching kernel @ 1-loop:

Liu, Wang, Xu, QAZ, Zhao, PRD99, 094036 (2019)

LPC, PRD111, 034503 (2025)

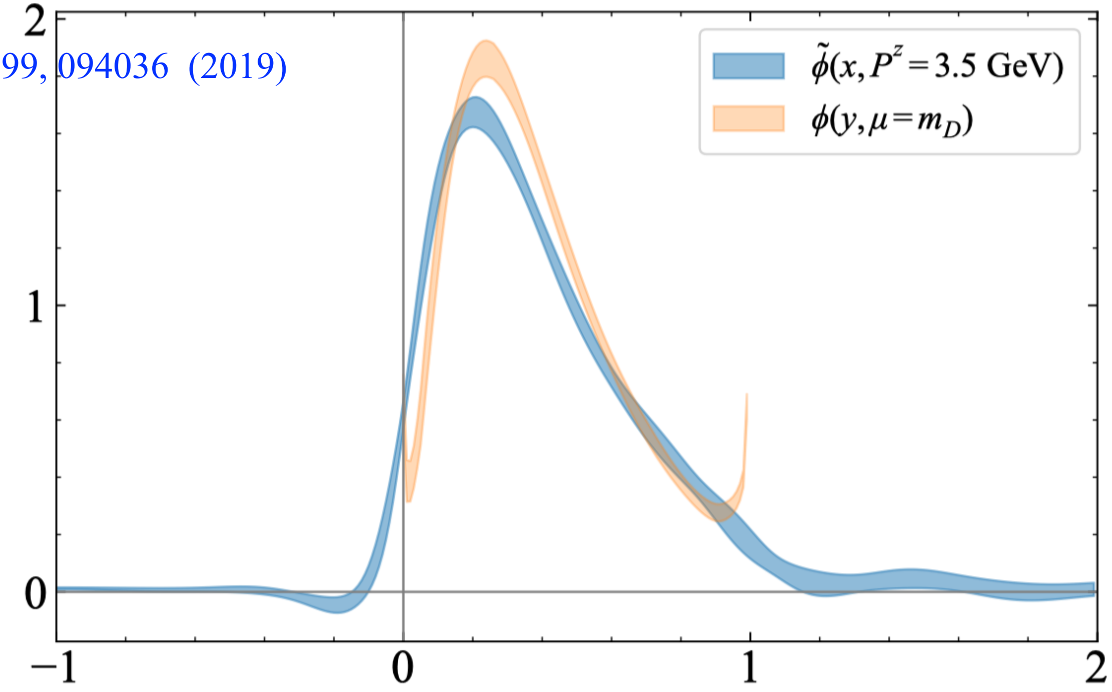
$$C_B^{(1)}\left(x,y,\frac{\mu}{P^z}\right)$$

$$=\frac{\alpha_sC_F}{2\pi}\begin{cases}[H_1(x,y)]_+ & x<0<y\\ [H_2(x,y,P^z/\mu)]_+ & 0<x<y\\ \left[H_2\left(1-x,1-y,\frac{P^z}{\mu}\right)\right]_+ & y<x<1\\ [H_1(1-x,1-y)]_+ & y<1<x\end{cases},$$

where

$$H_1(x,y)=\frac{1+x-y}{y-x}\frac{1-x}{1-y}\ln\frac{y-x}{1-x}+\frac{1+y-x}{y-x}\frac{x}{y}\ln\frac{y-x}{-x},$$

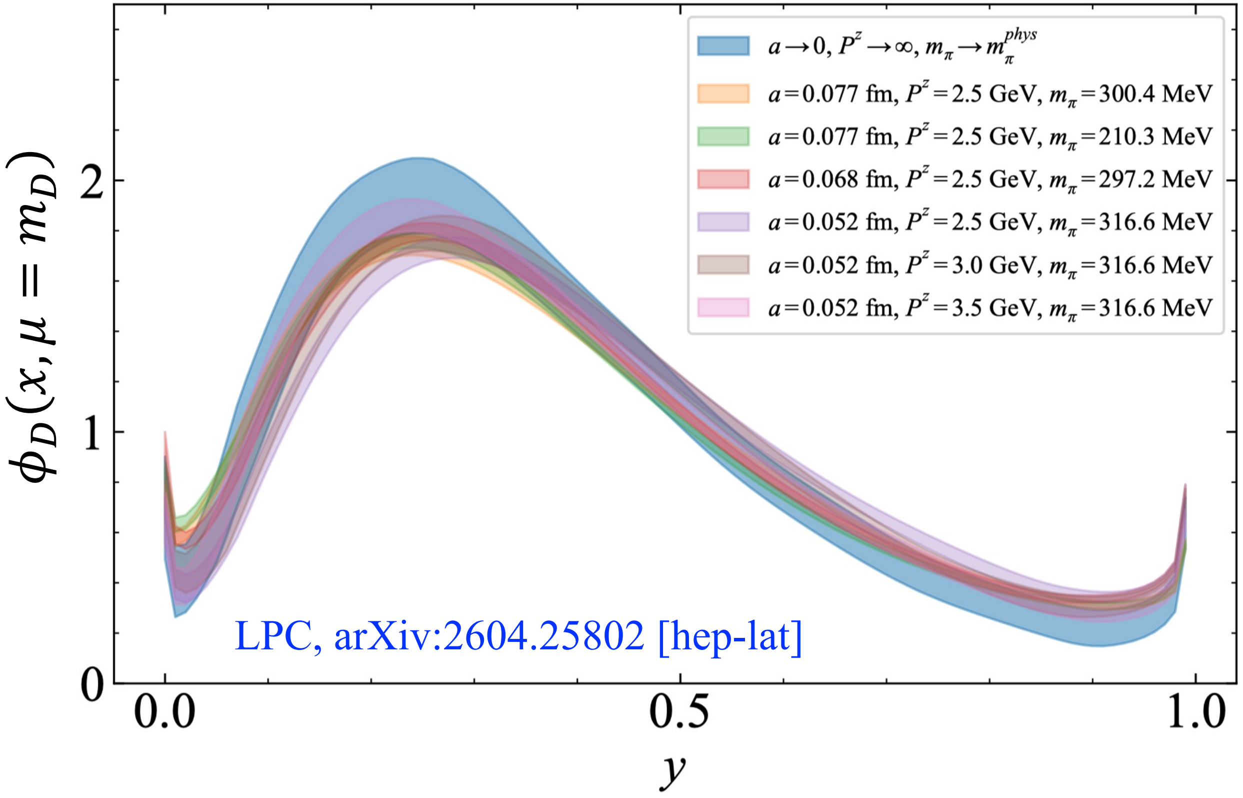
$$H_2(x,y,P^z/\mu)=\frac{1+y-x}{y-x}\frac{x}{y}\ln\frac{4x(y-x)(P^z)^2}{\mu^2}+\frac{1+x-y}{y-x}\left(\frac{1-x}{1-y}\ln\frac{y-x}{1-x}-\frac{x}{y}\right).$$



$$C_{CT}^{(1)}=-\frac{3\alpha_sC_F}{4\pi}\left[\frac{2\,\mathrm{Si}[(x-y)z_sP^z]}{\pi(x-y)}\right]_+$$

QCD LCDA of D Meson from LaMET

- ➢ Full distribution of D meson QCD LCDA from LaMET:



LPC, arXiv:2604.25802 [hep-lat]

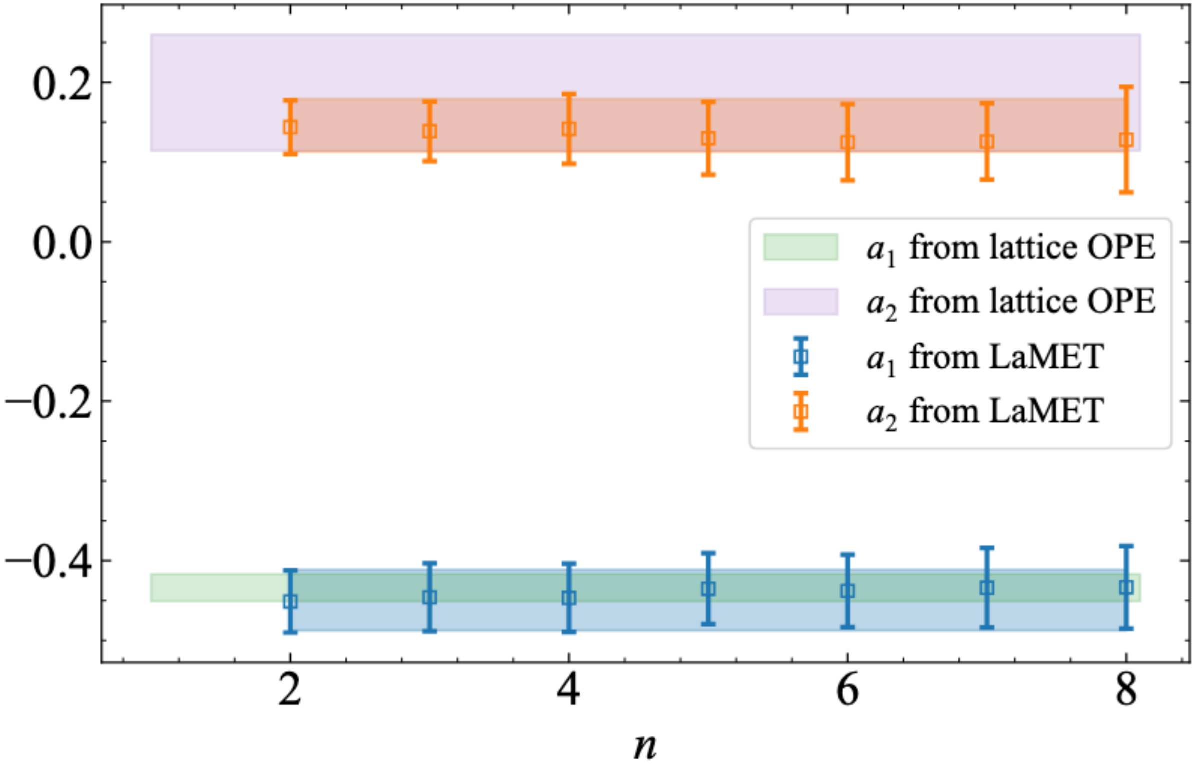
- Peak region: $y \sim \mathcal{O}(\Lambda_{\text{QCD}}/m_H)$
- Tail region: $y \sim \mathcal{O}(1)$
- End-point region: $\left(yP^z \lesssim \Lambda_{\text{QCD}}\right)$

- Gegenbauer moments can be extracted from the full distribution

Benchmark the Full Distributions from moments

- Gegenbauer moments a_1 and a_2 from LaMET and Lattice OPE:

$$\phi(x,\mu) = 6x(1-x)\left[1 + \sum_{n=1}^{\infty} a_n(\mu) C_n^{(3/2)}(2x-1)\right]$$



LaMET

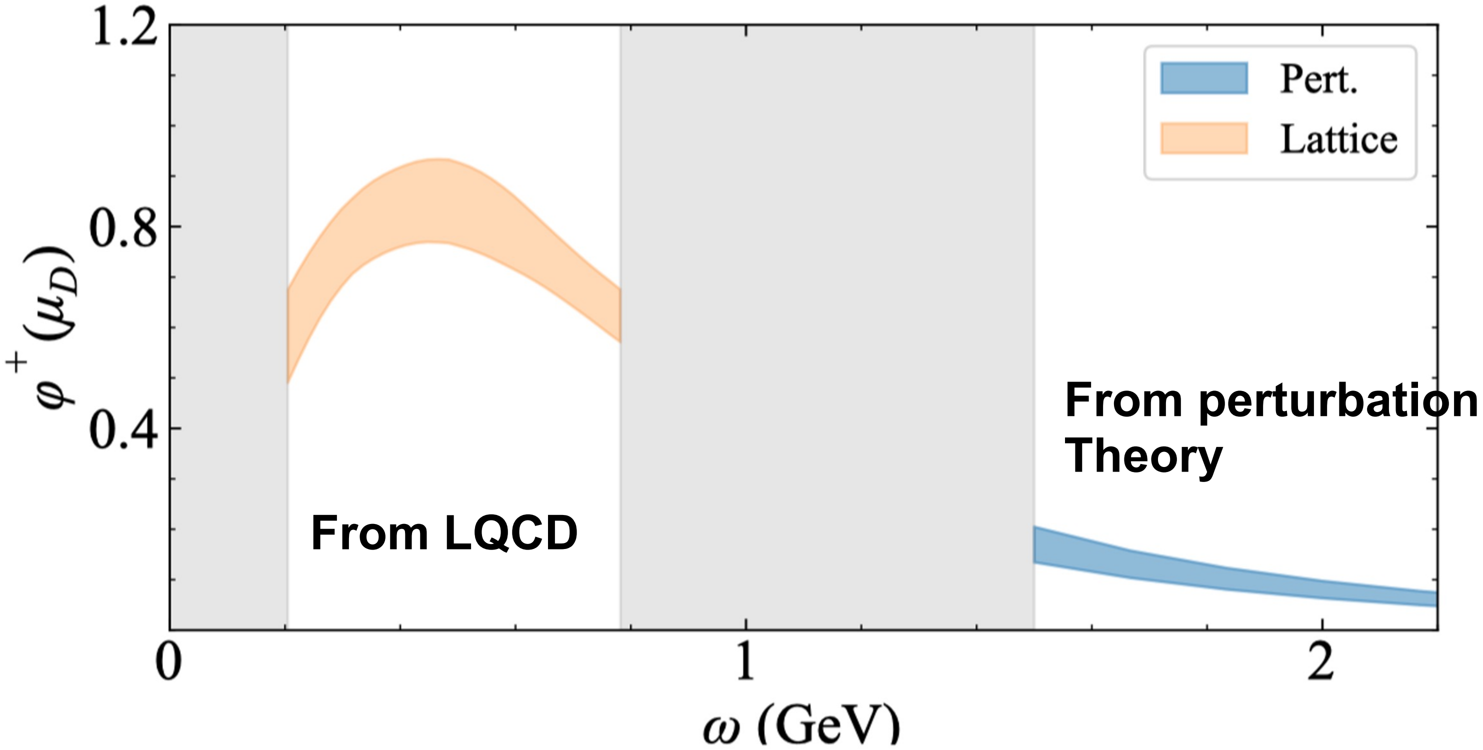
$$a_1^{\mathrm{LaMET}}(\mu) = -0.450(38),$$
$$a_2^{\mathrm{LaMET}}(\mu) = 0.146(33),$$

OPE

$$a_1^{\mathrm{OPE}}(\mu) = -0.434(17),$$
$$a_2^{\mathrm{OPE}}(\mu) = 0.183(73),$$

Prediction for the HQET LCDA

$$\varphi^+(\omega,\mu) = \begin{cases} \frac{1}{m_H}\left[1-\frac{\alpha_s C_F}{4\pi}\left(\frac{1}{2}\ln^2\frac{\mu^2}{m_H^2}+\frac{1}{2}\ln\frac{\mu^2}{m_H^2}+\frac{3}{2}\ln\frac{\mu^2}{m_Q^2}+\frac{\pi^2}{12}+4\right)+\mathcal{O}\left(\alpha_s^2\right)\right]\phi\left(\frac{\omega}{m_H},\mu\right), & \omega\sim\Lambda_{\rm QCD} \\ \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2}-\ln\frac{\omega}{\mu}\right)+\frac{4\bar{\Lambda}}{3\omega}\left(2-\ln\frac{\omega}{\mu}\right)\right]+\mathcal{O}\left(\alpha_s^2\right), & \omega\sim m_H \end{cases}$$



Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

Lee, Neubert, PRD 72, 094028 (2005)

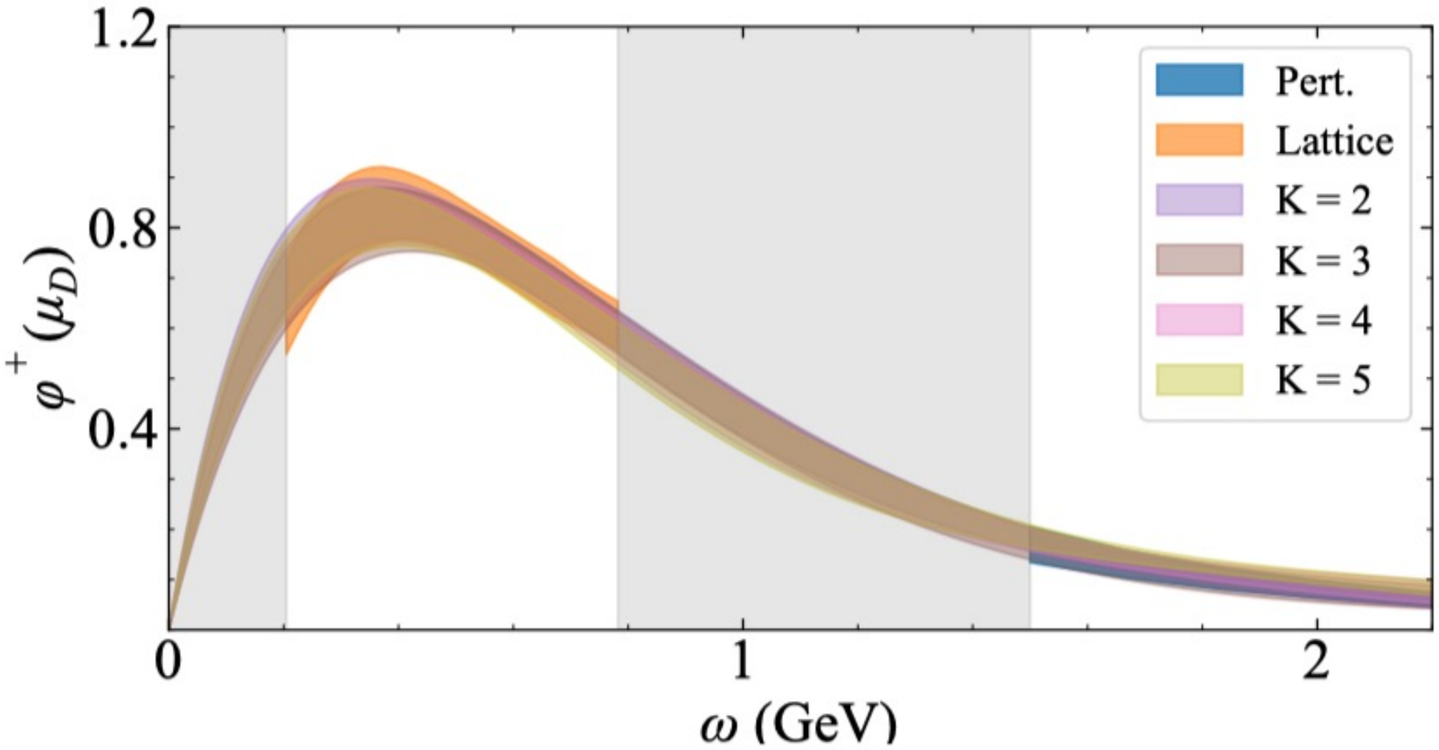
Full distribution is still not accessible!

Prediction for the HQET LCDA

- Rebuild the full distribution of the HQET LCDA from a **model-independent parametrization**:

$$\varphi^+(\omega,\mu) = \frac{\omega e^{-\omega/\omega_0}}{\omega_0^2}\sum_{k=0}^{K}\frac{a_k(\mu)}{1+k}L_k^{(1)}(2\omega/\omega_0),$$

Feldmann, Lughausen, Dyk, JHEP10, 162 (2020)

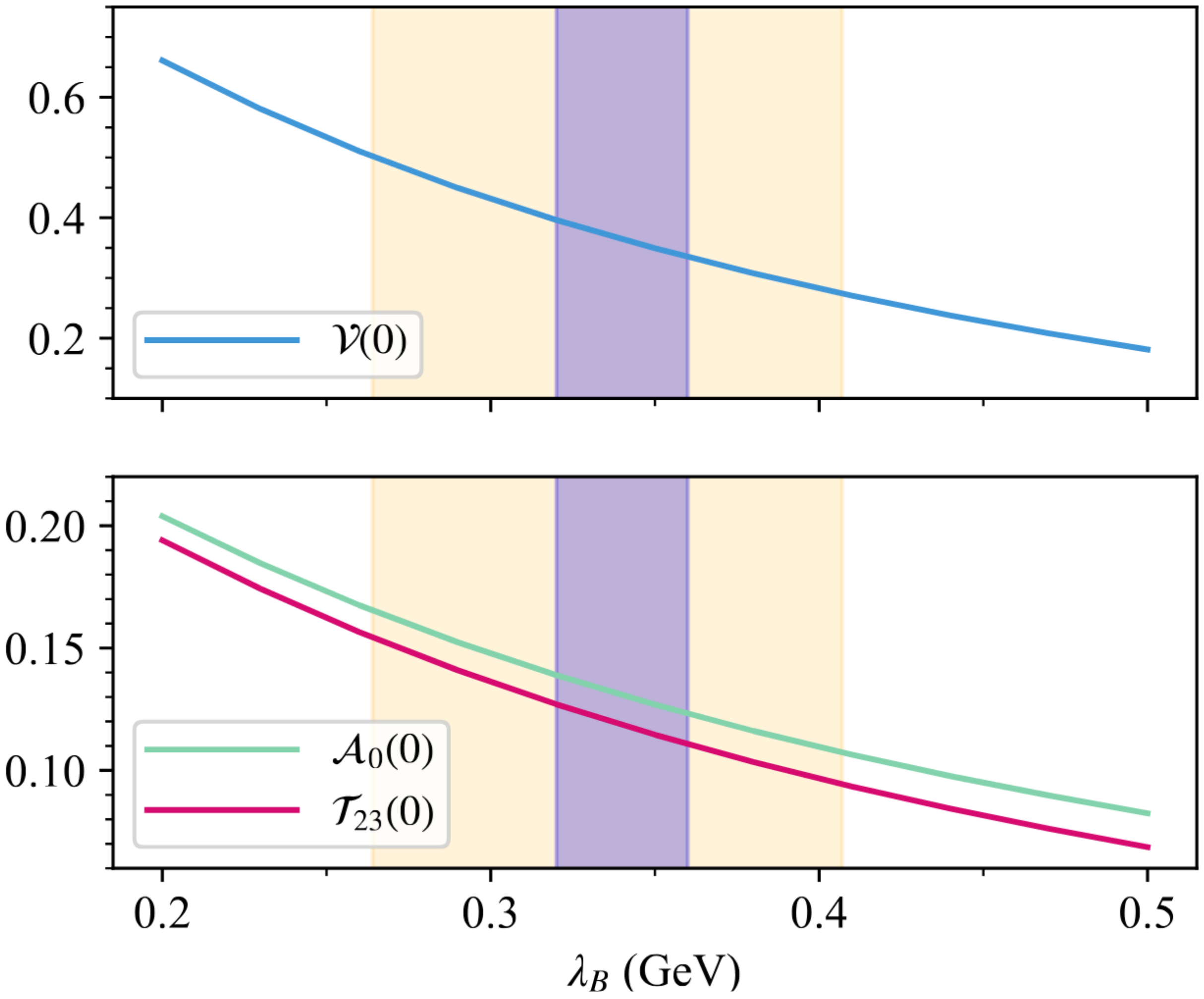


- Agree well with the data.
- With full distributions, further phenomenological discussions can be preformed.

Inverse and inverse-logarithmic moments

μ	Reference (Method)	λ_B (GeV)	$\sigma_B^{(1)}$	
1 GeV	This work	0.340(20)	1.685(63)	
	Ref. [39] (LQCD)	0.376(63)	1.66(13)	PRD111 (2025), 034503
	Ref. [19] (Experiment)	> 0.24	–	PRD98 (2018), 112016
	Ref. [13] (QCD sum rule)	$0.343^{+0.064}_{-0.079}$	1.4(4)	PRD101 (2020), 074035
	Ref. [23] (QCD sum rule)	0.46(11)	1.4(4)	PRD69 (2004), 034014
	Ref. [21] (QCD sum rule)	0.383(153)	–	JHEP10 (2020), 043
	Ref. [22] (OPE)	0.48(11)	1.6(2)	PRD72 (2005), 094028
	Ref. [18] (Asymptotic behavior)	0.35(15)	–	PRD55 (1997), 272290
	Ref. [83] (Global Fit)	0.338(68)	–	PLB848 (2024), 138345

B→K* Form Factors in LCSR



- In PRD101 (2020), 074035: (orange band)

$$V(0) = 0.359^{+0.141}_{-0.085}, \quad A_0(0) = 0.129^{+0.035}_{-0.021},$$
$$T_{23}(0) = 0.116^{+0.036}_{-0.022},$$

- With our updated λ_B: (purple band)

$$V(0) = 0.362^{+0.034}_{-0.029}, \quad A_0(0) = 0.131^{+0.008}_{-0.008},$$
$$T_{23}(0) = 0.119^{+0.008}_{-0.008}.$$

We thank Yu-Ming Wang for his code to calculate the B → V form factors in LCSRs @ *PRD 101 (2020) 074035*

Boosted-HQET beyond Leading Power: Work in Progress

- Factorization formula up to NLP:

$$\phi(u,\mu) = \boxed{\left\{ m_H \frac{\tilde{f}_H(\mu)}{f_H} \mathcal{J}_{\mathrm{peak}}\varphi_+(\omega,\mu) + \mathcal{O}(\alpha_s^2, 1/m_Q)\right\}}$$

NLO+LP

$$\boxed{+ \left\{ m_H \frac{\tilde{f}_H(\mu)}{f_H} \left[\varphi_+^1(\omega,\mu) + \varphi_+^{\mathrm{kin}}(\omega,\mu)\right] + \mathcal{O}(\alpha_s, 1/m_Q^2)\right\},}$$

LO+NLP correction

Long-Shun Lu, Yan-Bing Wei, QAZ, *et.al.*, in preparation.

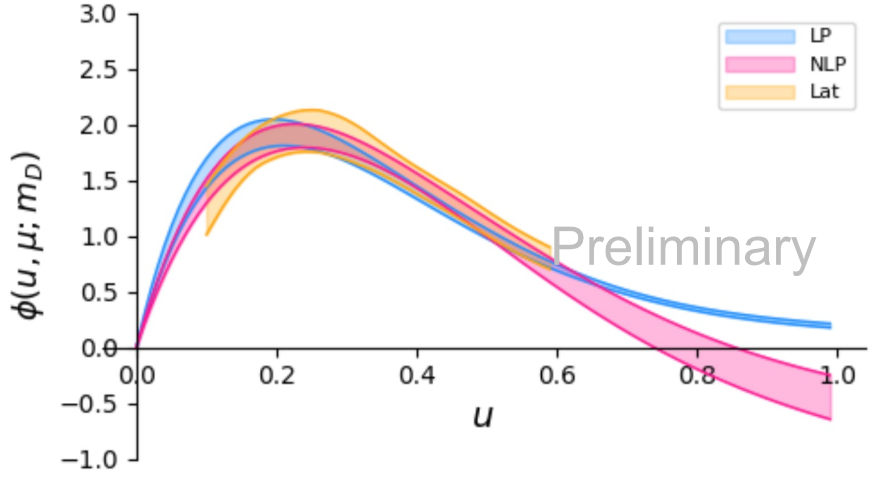
- Predictions for the inverse moment (@ $\mu = m_D$):

$$\lambda_B = 0.457 \pm 0.039 \pm (\sim 40\%)|_{\mathrm{NLP,NNLO}},$$

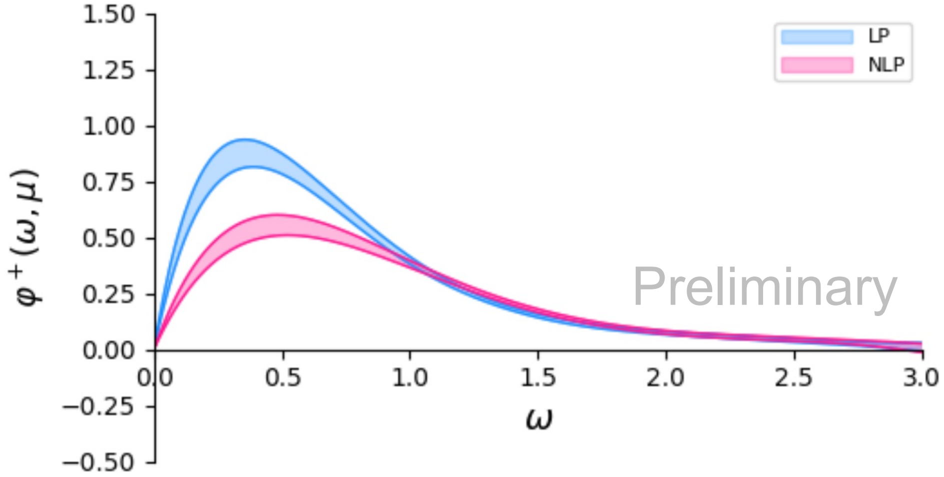
$$\lambda_B = 0.457\, {}^{+0.283}_{-0.039} \pm (\sim 4\%)|_{\mathrm{NNLP,NNLO}}$$

- Further analyses are in progress.

- Constrained by lattice data in peak region



- HQET LCDA constrained by lattice data



Summary and Outlook

- Heavy-meson HQET LCDAs are essential nonperturbative inputs for precision studies of exclusive weak B-meson decays, but have long suffered from model dependence.
- We provide a precision lattice QCD determination of the heavy-meson HQET LCDA within HQLaMET:

  Continuum limit ✓, Physical-mass ✓, Systematic uncertainties ✓

  Precision: Leading power, Error <10%

  Reliably control of Power corrections 🕓

- A new era of lattice QCD calculations of heavy meson distributions, with the prospect of providing more precise nonperturbative inputs for both theory and experiment.

Thanks for your attention