Towards Gluon PDFs in the Coulomb Gauge

Bill Good
07.05.2026
In Collaboration with:
Joshua Lin, Yong Zhao, Huey-Wen Lin

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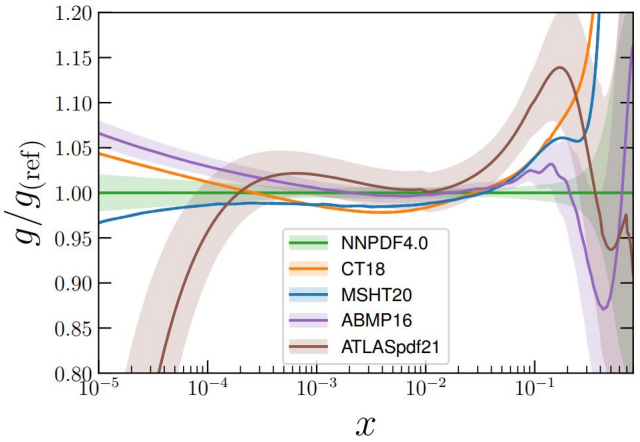

Amaroso, *et al.* (Snowmass) Acta Physica Polonica. B, 53, 12 (2023)

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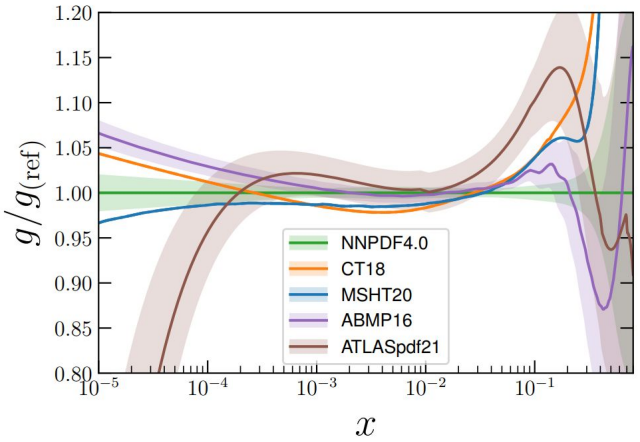

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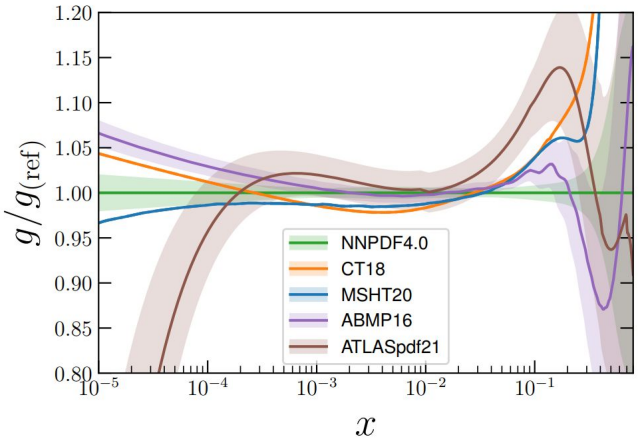

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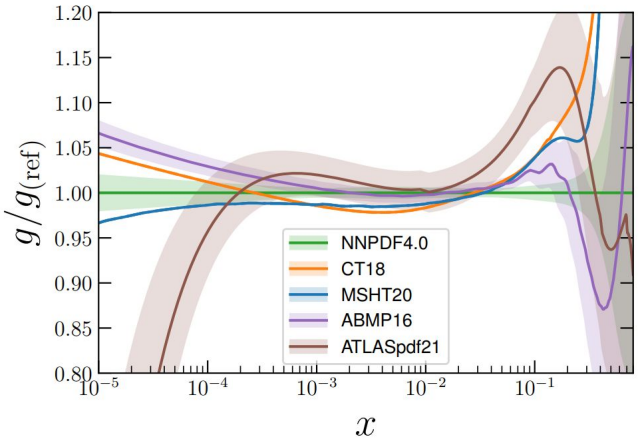
- Coulomb gauge PDFs promise to reduce statistical errors and in turn allow us to go to higher momenta, lower pion mass, etc.



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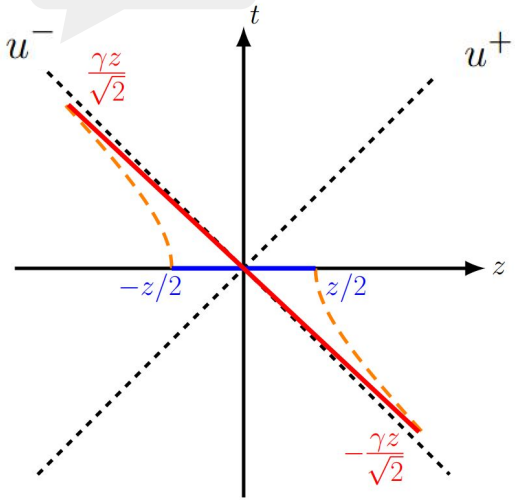
Gao, *et al.* PRD 109(0):094506 (2024)

Light Cone Gluon PDFs

Ji, PRL 110:262002 (2013)
Ji, Sci. China Phys. Mech. Astron. 57:1407 (2014)

$$u^{\pm} = \frac{u^0 \pm u^3}{\sqrt{2}}$$



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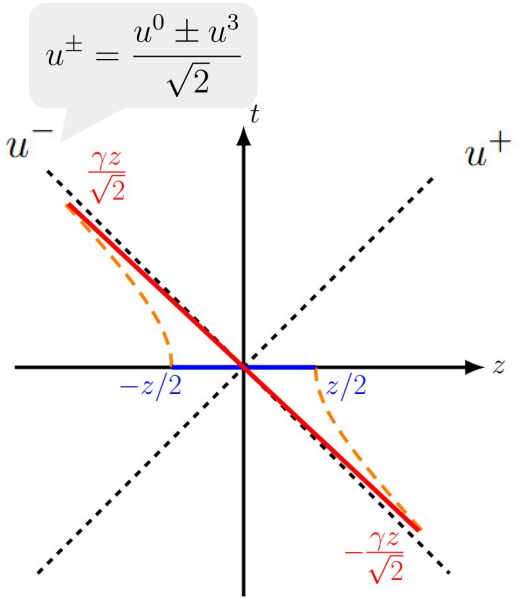
$$g(x) = \frac{1}{xP^+} \int dz^- e^{ixP^+z^-} \left\langle P \left| F^{+\mu}\left(z^-\right) W\left(0, z^-\right) F^+_{\ \mu}(0) \right| P \right\rangle$$



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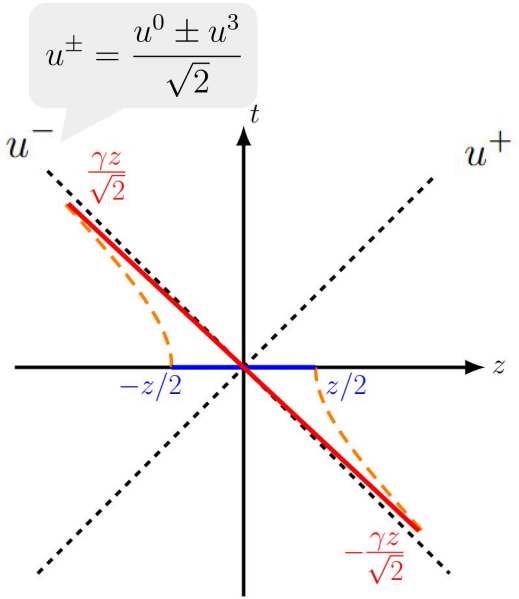
With wilson line: $$W(z,0) = \mathcal{P}\exp\left[-ig\int_0^z dz' A^z(z')\right]$$



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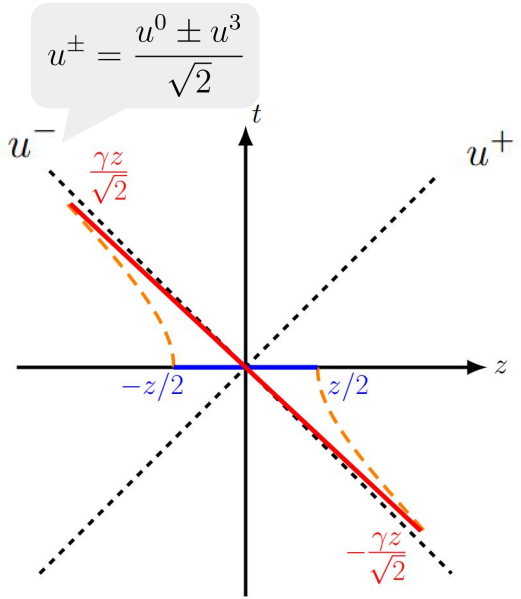

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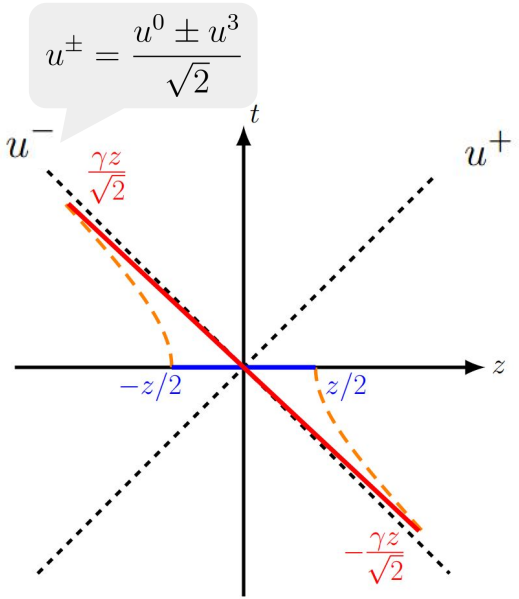

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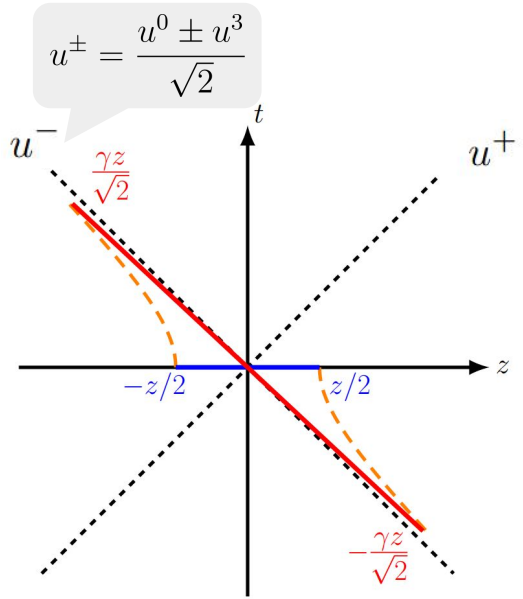

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$$xg(x,\mu^2) = \int dy\, C_{gg}(\frac{x}{y},\frac{\mu^2}{P_z^2})y\tilde{g}(y,P_z)+C_{gq}(\frac{x}{y},\frac{\mu^2}{P_z^2})\tilde{q}(y,P_z)+\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(xP_z)^2},\frac{\Lambda^2_{\rm QCD}}{((1-x)P_z)^2}\right)$$

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[MSULat] First physical continuum lattice nucleon gluon PDFs

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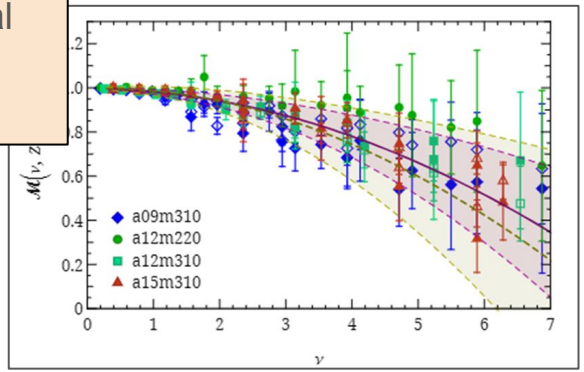

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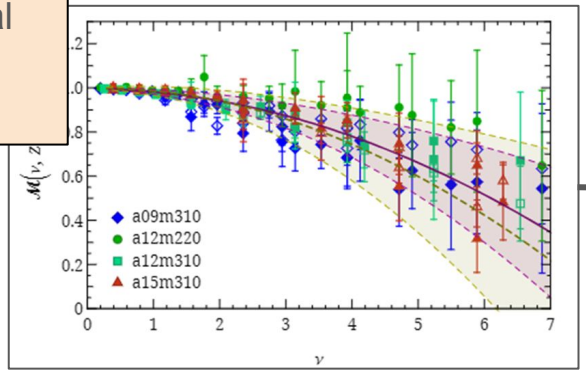

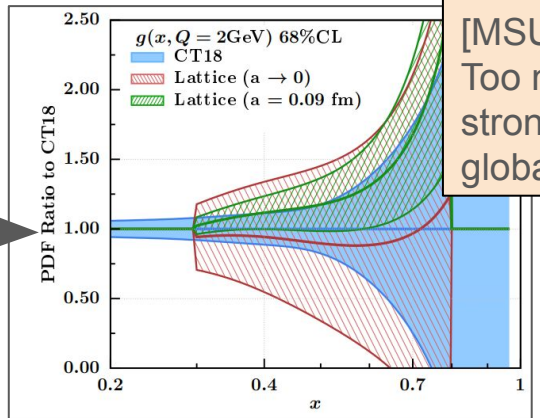

[MSULat+CTEQ] Too noisy to strongly impact global fit arXiv:2502.10630

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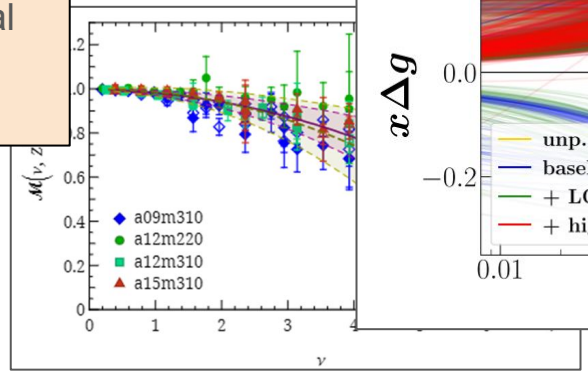

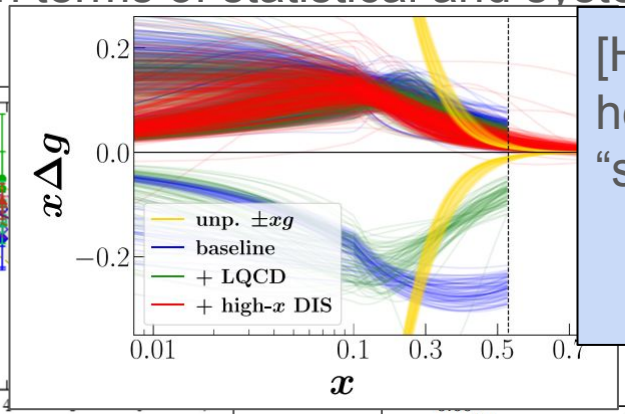

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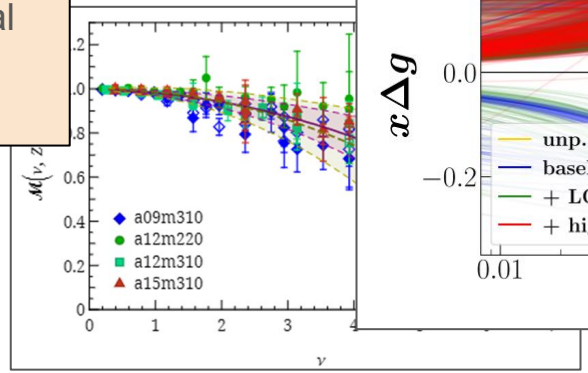

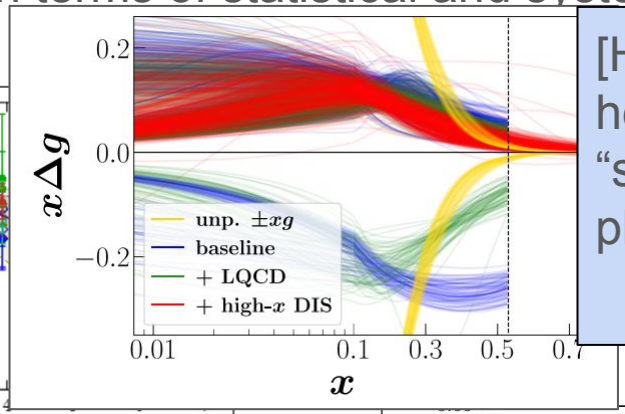

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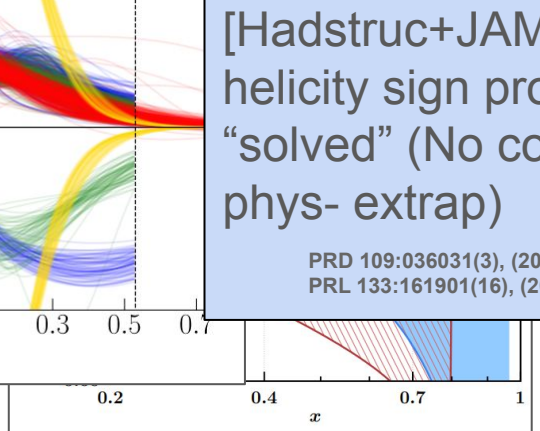

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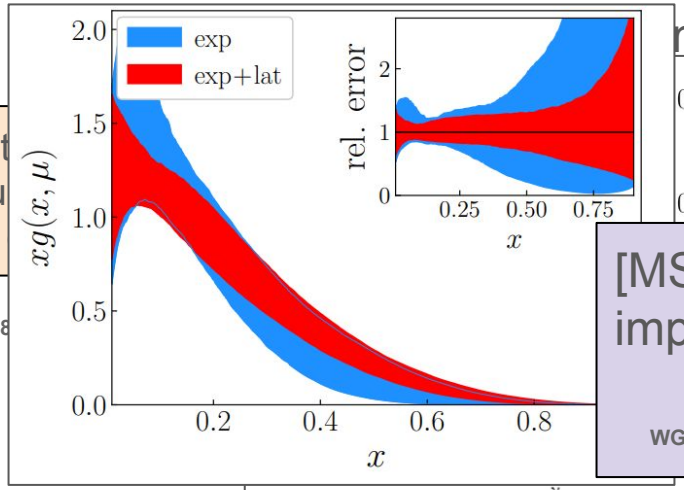

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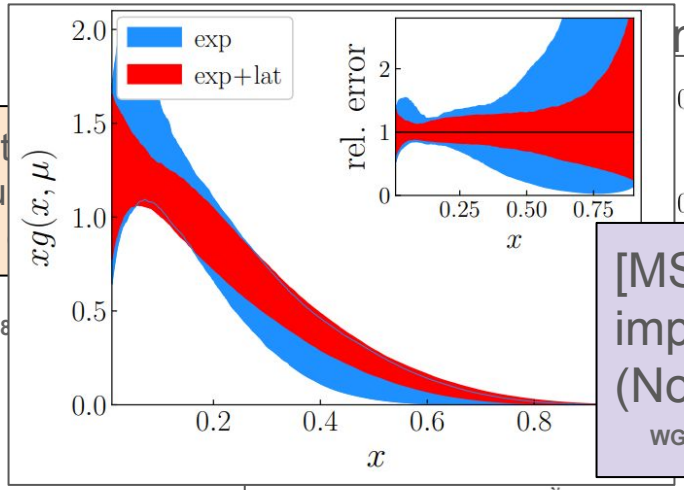

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NieMiera, *et al.* arXiv 2510.17758 Chen, *et al.* arXiv 2510.26425

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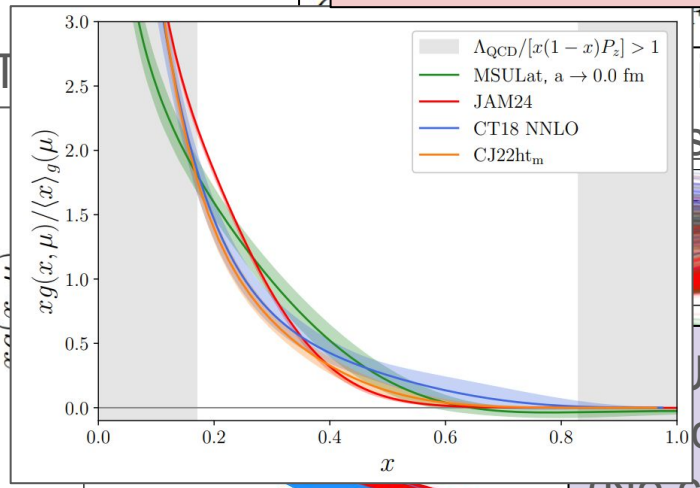

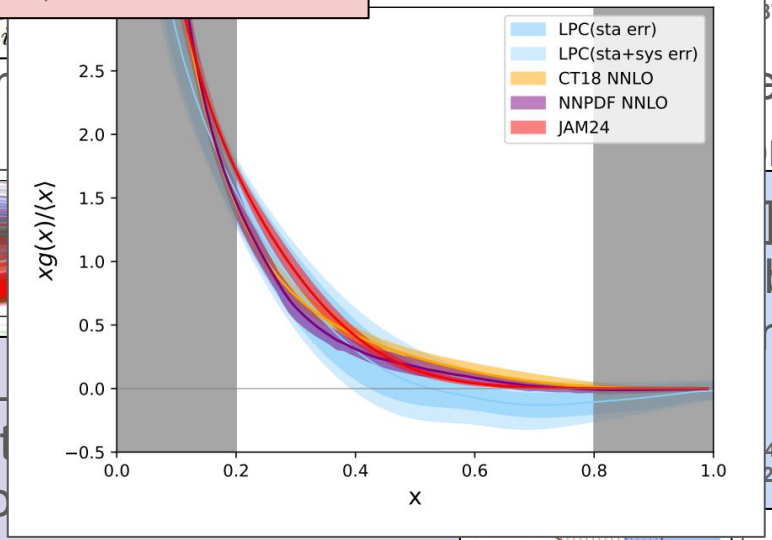

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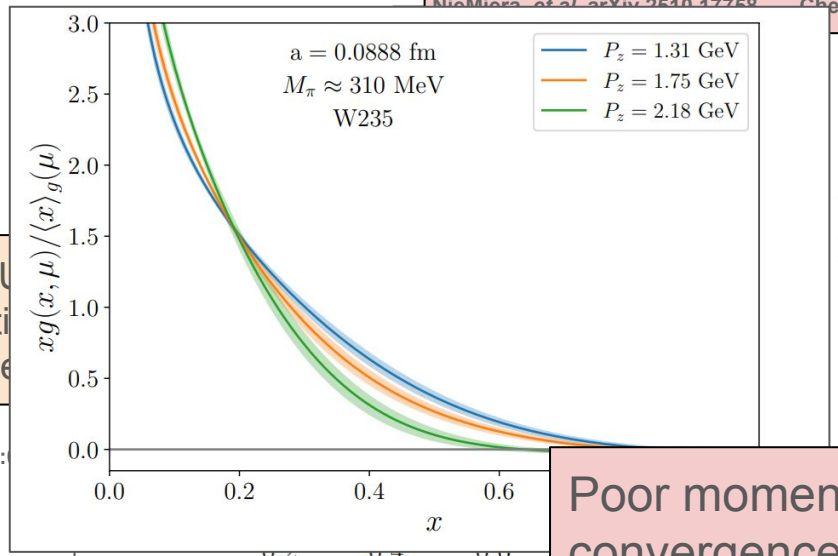

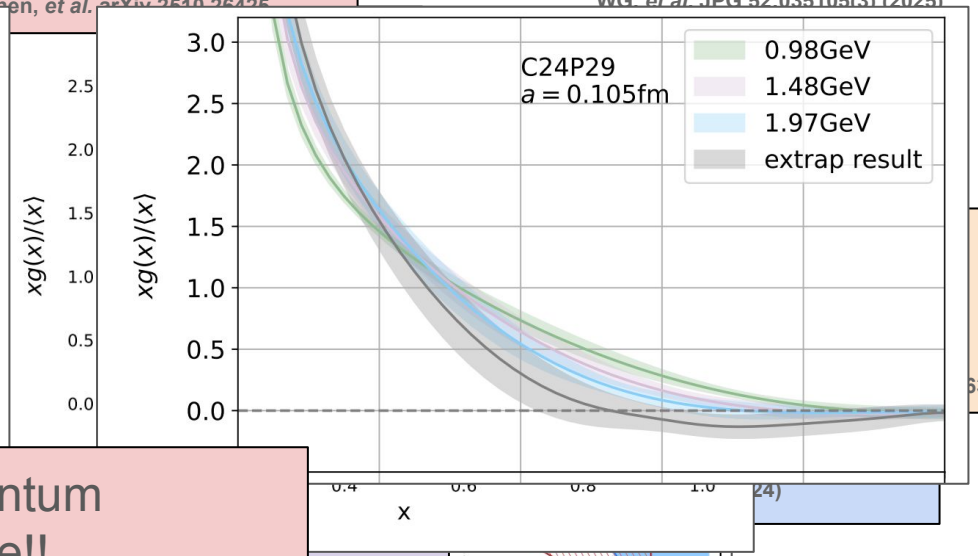

[MSU… conti… nucle…

PRD 108:…

Poor momentum convergence!!

Coulomb Gauge to the Rescue

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- The self-energy of the Wilson line in the gauge invariant quasi-PDF operators causes the signal to noise ratio to decay exponentially at long distances

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Gao, *et al*. PRD 109(0):094506 (2024)

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- With this method is that matching must be recalculated for these new CG operators, which is a major challenge in the gluon case. I'll share our progress

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- Matching to relevant lattice renormalization schemes is straightforward

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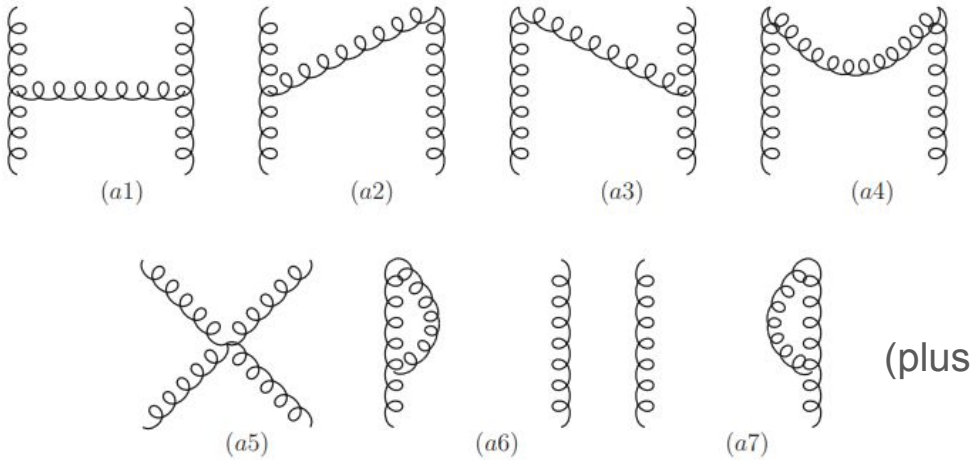

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Wang, *et al*. PRD 100:074509 (2019)

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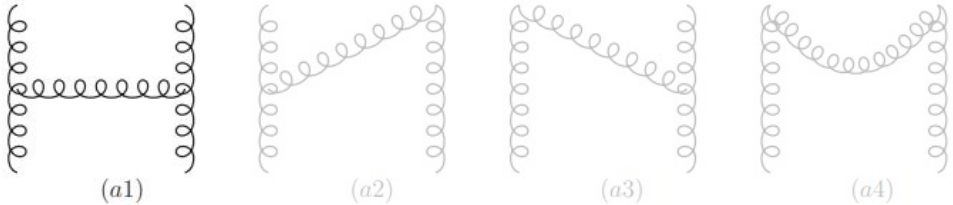

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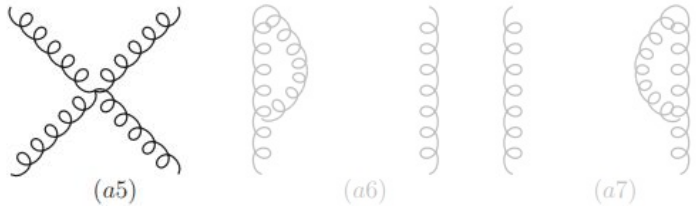

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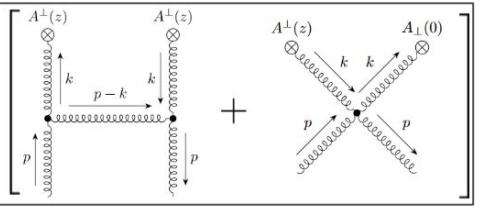

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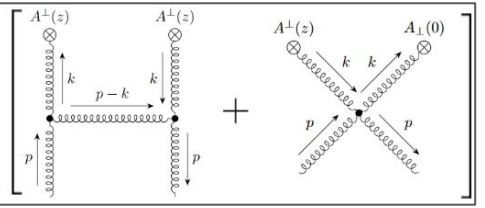

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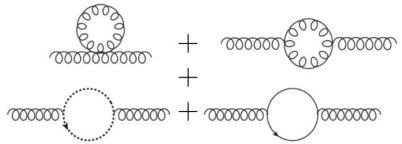

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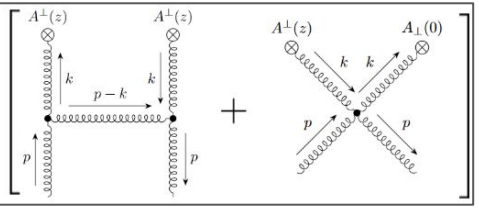

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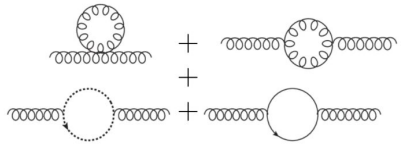

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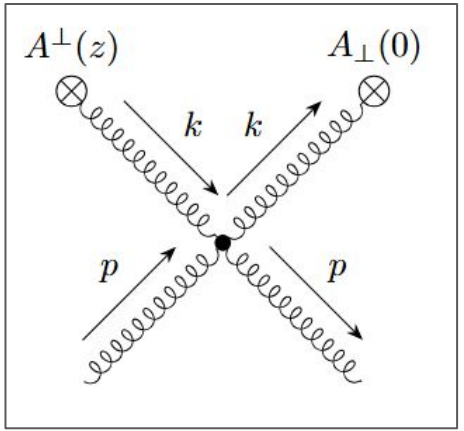

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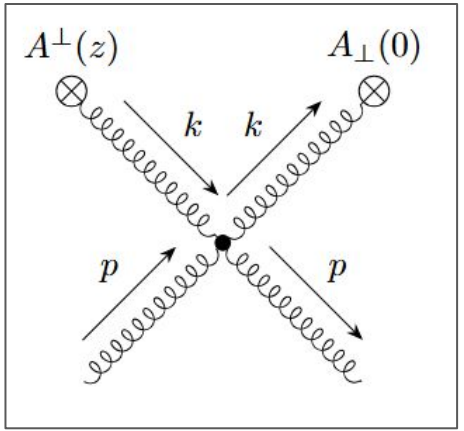

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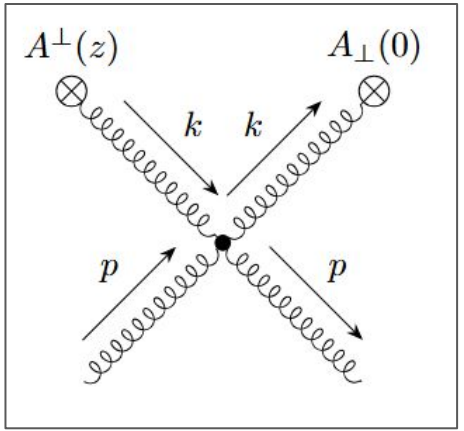

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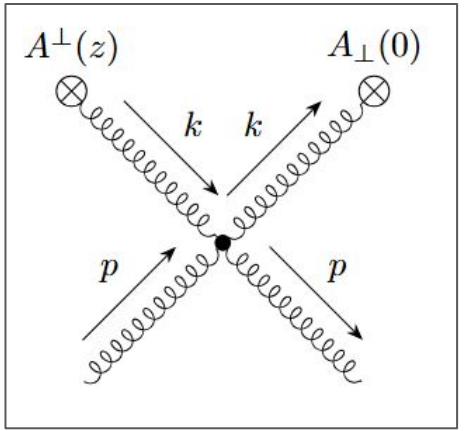

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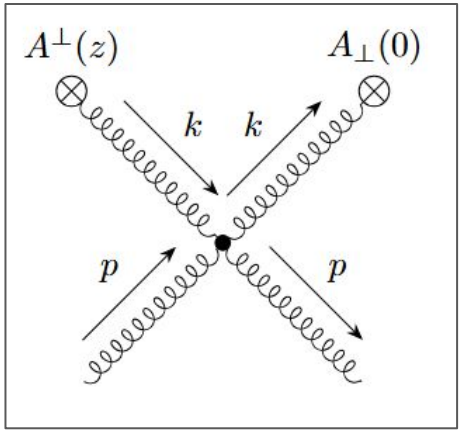

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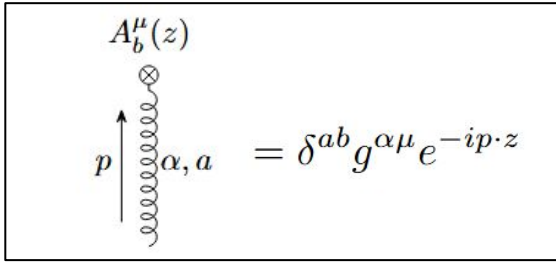

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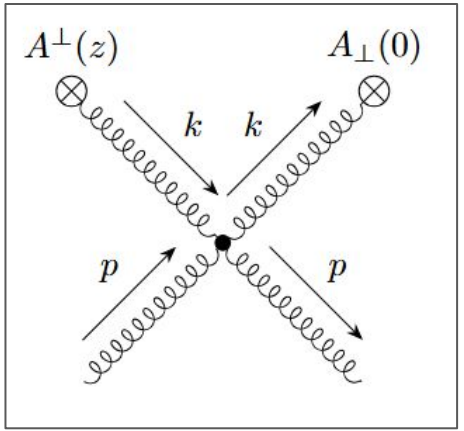

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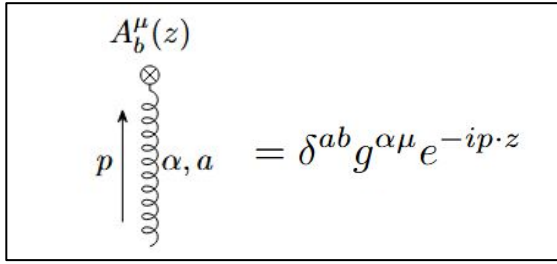
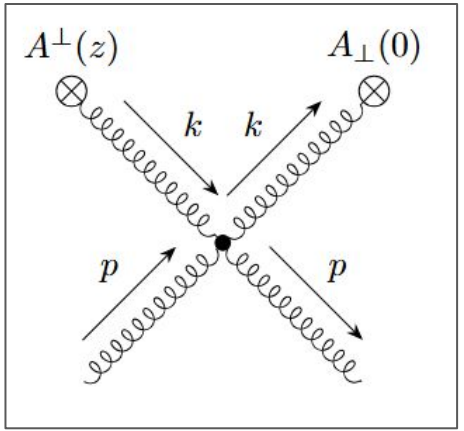


Easy Part: Four Gluon Vertex Diagram



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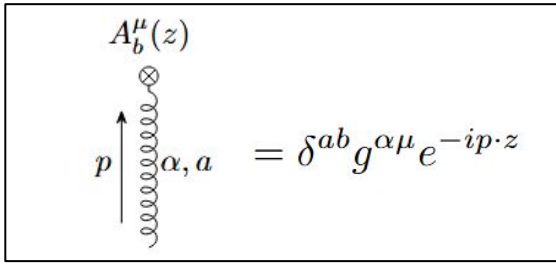

- We treat the temporal momentum integral as d = 1

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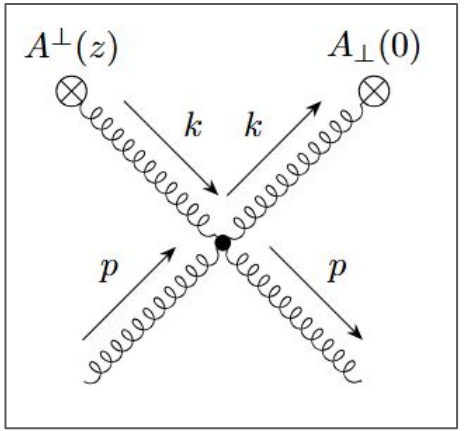

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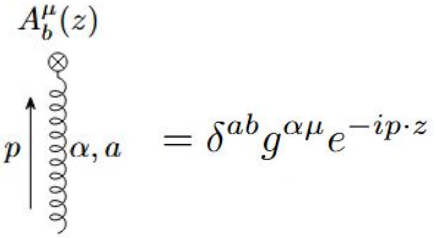

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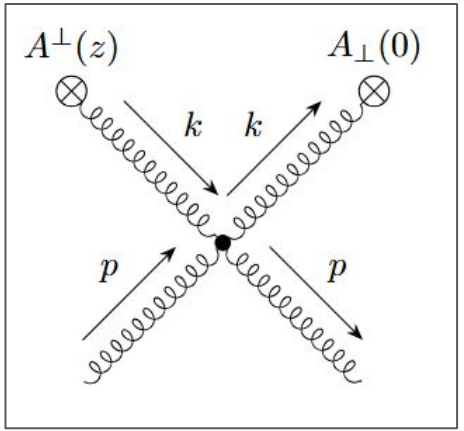

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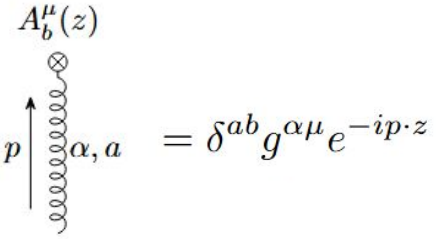

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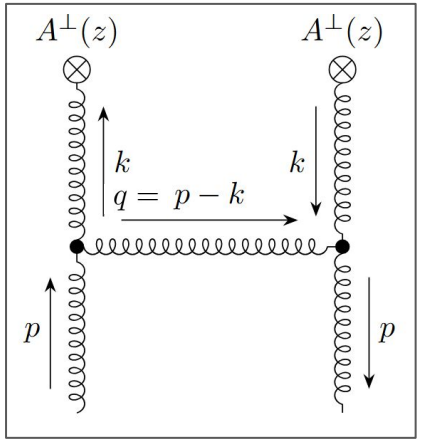
$$A \times \mathrm{sgn}(x)$$

Box Diagram I



Box Diagram I

- This diagram becomes a lot harder with 3 propagators

$$Ag_\perp^{\alpha\beta}g_\perp^{ij}\Pi^{\gamma i}(k)\Pi^{\sigma j}(k)\Pi^{\mu\nu}(p-k)\left(2k^\alpha g^{\gamma\mu}+(p^\mu-k^\mu)g^{\alpha\gamma}+2p^\gamma g^{\alpha\mu}-2p^\mu g^{\alpha\gamma}\right)$$
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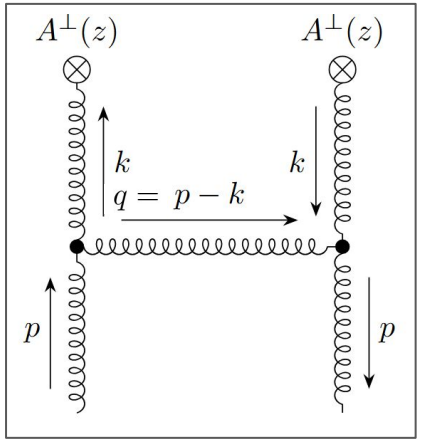

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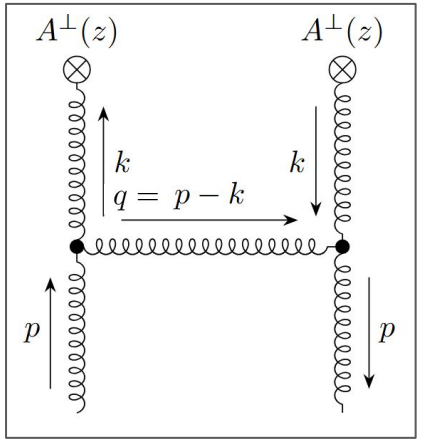

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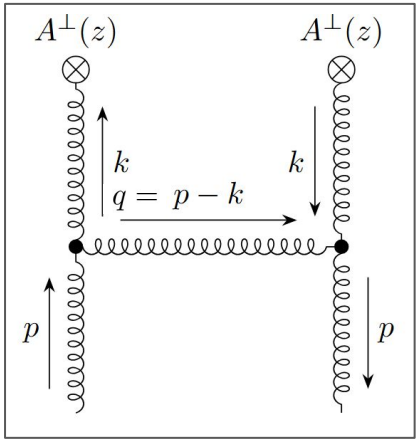

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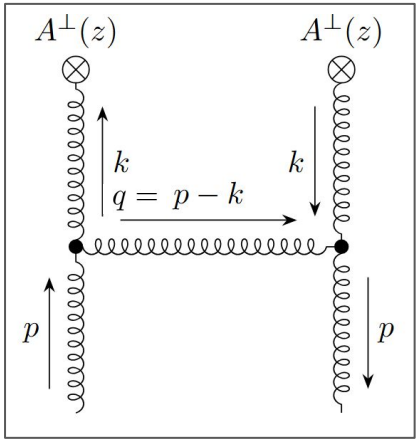

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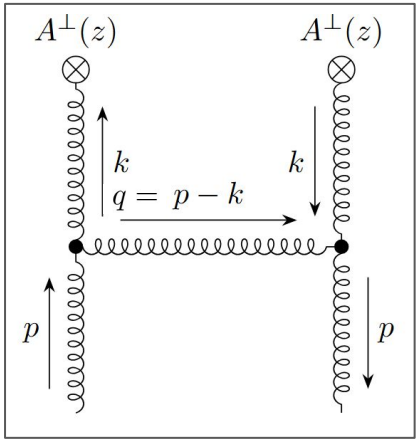

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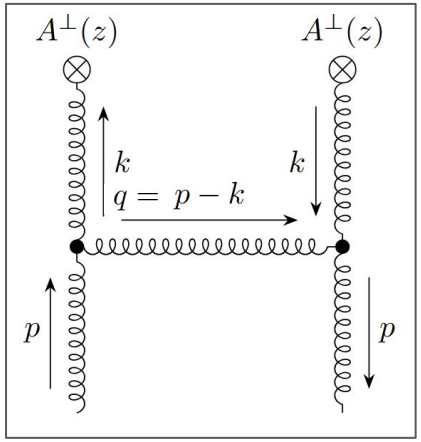
- Or we can use Feyncalc or the likes to handle these and solve the integrals algorithmically

Box Diagram II

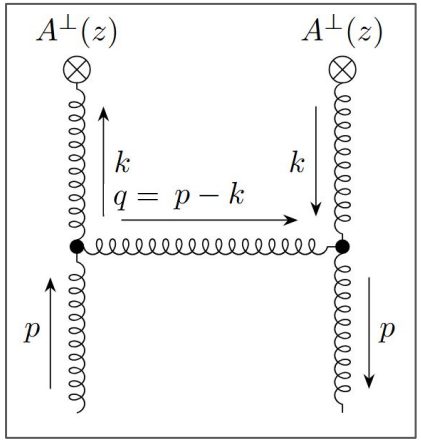


Box Diagram II

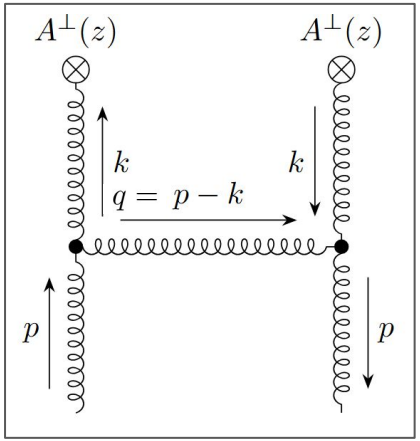
- The general form of this diagram looks like the following:

$$\int \frac{d^{D-1}k}{(2\pi)^{D-1}} \sum_i \frac{A_i(x)\vec{k}_\perp^{2n_i}}{(\vec{q}^2)^{a_i}(\vec{k}^2)^{b_i}(q^2)^{c_i}(k^2)^{d_i}}$$

$$k_z = xp_z$$



Box Diagram II



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- If we do the temporal integration carefully, we can reduce everything to a nice set of 2-2ϵ integrals, which have closed forms

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(if a or b = 0, use textbook formula)

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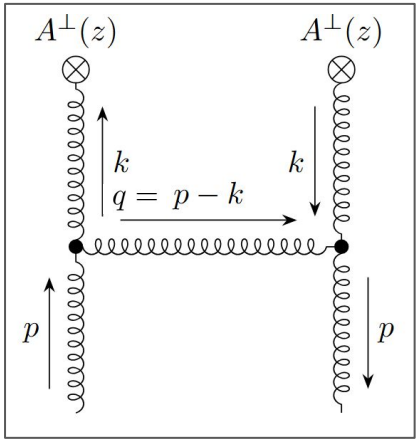

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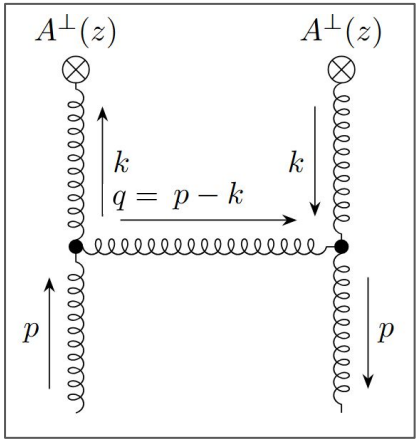

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$${}_2F_1(a,b-\epsilon;c;\frac{2x-1}{x^2}) \xrightarrow{x\to 1} (1-x)^{\min[0,c-b-a+\epsilon]}$$

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$$\widetilde{g}^{(1)}(x) = \frac{F(x,\epsilon)}{|1-x|^{1+2\epsilon}}$$

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- Expanding like this carefully and adding the contribution from the gluon wavefunction renormalization, we recover exactly the DGLAP kernel for our IR divergence, which nearly proves NLO factorization!

$$-\frac{P_{gg}(x)}{\epsilon_{\rm IR}} = \frac{1}{\epsilon_{\rm IR}}\left(-2C_A\left(\frac{x}{(1-x)_+} + x(1-x) + \frac{1-x}{x}\right) + \frac{\beta_0}{2}\delta(1-x)\right)$$

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- For x ≠ 1, the results look something as follows (we haven't simplified the expression with full handling of the plus prescription yet)

$$\mathrm{sgn}(1-x)\left(\log(|1-x|)\,P_{gg}(x)+\frac{11x^6-34x^5+71x^4-84x^3+60x^2-24x+4}{4(x-1)x(2x-1)^2}\right)+$$

$$\mathrm{sgn}(x)\left(\log(|x|)\,P_{gg}(x)+\frac{(x^4+4x^2-4x+1)x^2\tanh^{-1}\left(\frac{\sqrt{2x-1}}{x}\right)}{(x-1)(2x-1)^{5/2}}+\frac{90x^5-210x^4+434x^3-590x^2+357x-74}{30(2x-1)^2x}-\right.$$

$$\left.\frac{(5x^6-32x^5+78x^4-120x^3+108x^2-48x+8)\sin^{-1}\left(\frac{\sqrt{2x-1}}{x}\right)}{4(2x-1)^{5/2}x}\right)+\theta(1-x)\,\theta(x)\,P_{gg}(x)\log\left(\frac{4\,pz^2}{\mu^2}\right)$$

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- This suggests that the matching, convolution does not produce convergent results for any gluon PDF that does not approach 0 as y → 0

$$\widetilde{g}(x)=\int_{-1}^{1}\frac{dy}{|y|}C_{gg}\left(\frac{x}{y}\right)g(y)$$

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This observation and the explicit expression given by Eq. (7.28) may be used to check the gluon-gluon matching kernels in Refs. [17,18]. Our check shows that $Z_1^{gg}(y,1,p_3)|_{y>1}$ corresponding to Eq. (64) of Ref. [18] does not satisfy the constraint (7.27). The difference is by a constant term (-2/3) that leads to a linear divergence in the integral of $Z_1^{gg}(y,1,p_3)|_{y>1}$ over y. The same difference (with the opposite sign) appears in the $Z_1^{gg}(y,1,p_3)|_{y<0}$ term in Eq. (64) of Ref. [18]. Apparently, these differences result from the use of the off-shell external gluon fields in the calculations of Ref. [18], but a discussion of this topic is outside of the scope of our paper.

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- We hope to track down some prescription to remove these singularities, we're open to suggestions

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- We are actively measuring Coulomb gauge operators and plan to get numerical results before Lattice2026