

# 3D Tomography of the Nucleon: Lattice QCD Calculations of the TMDs

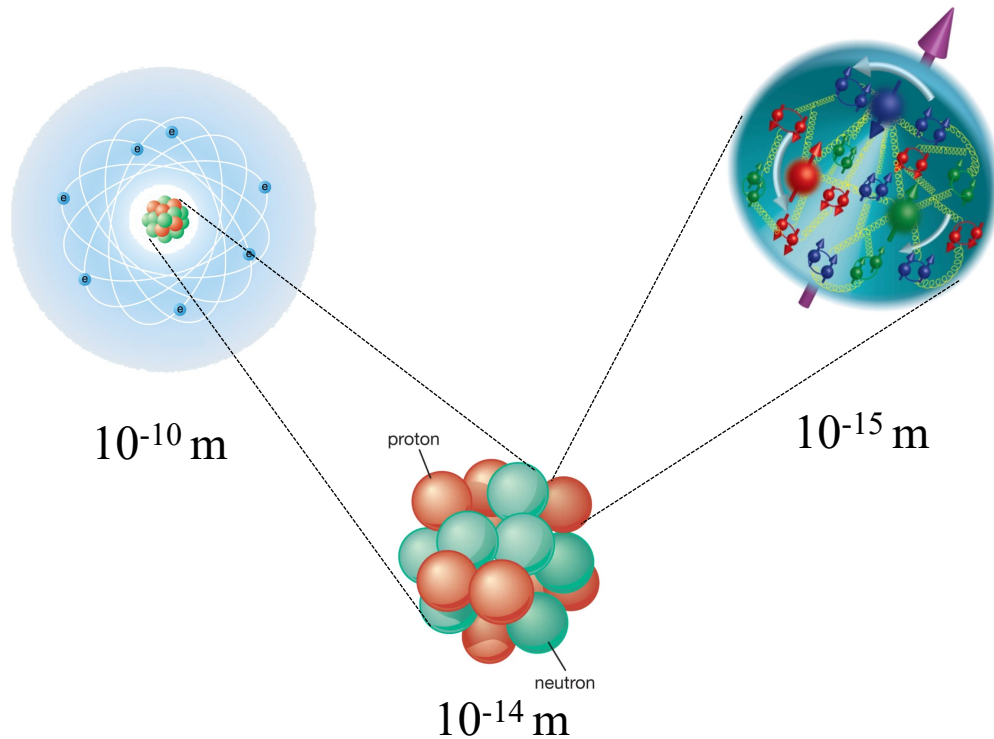
► Jin-Xin Tan (Shanghai Jiao Tong University)

► Based on: PRD 113 (2026) 5, 054505

Jul.08,2026, LaMET 2026 @ Krakow, Poland

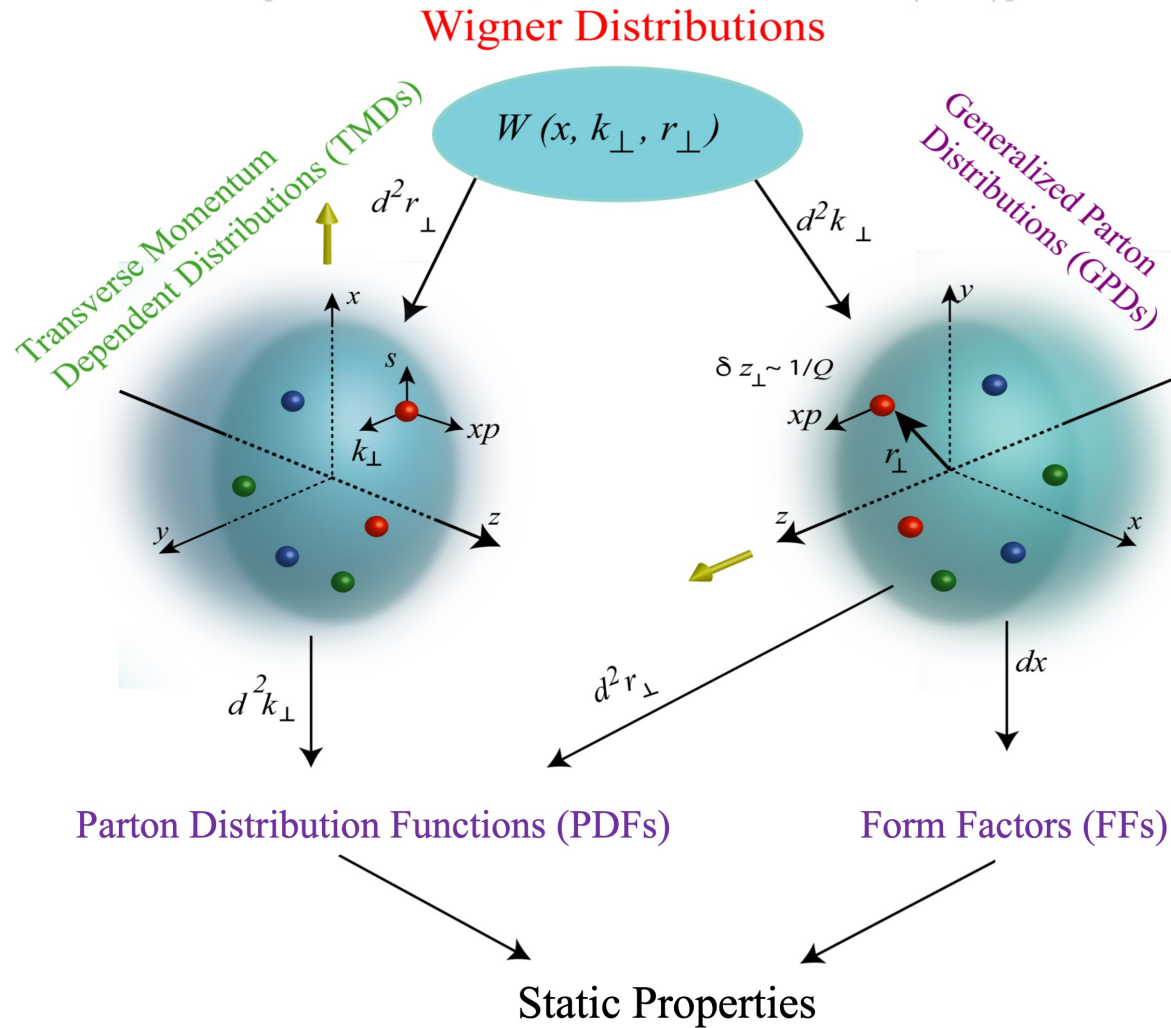
# Nucleons: Building Blocks of the Visible Universe

Despite more than half a century of exploration, the nucleon is still a very mysterious object, and the most abundant piece of matter in the visible Universe



- How do quarks & gluons form a nucleon, how do they distribute?
- How does nucleon mass & spin emerge from strong interactions?

# The Landscape of Nucleon Structure

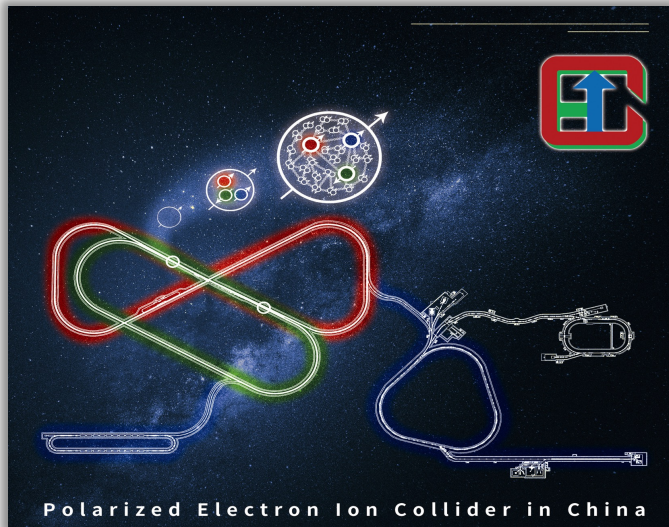


- Wigner distribution (5-D)  
Not directly accessible in experiments
- TMDs and GPDs (3-D)  
Still limited in experiments, with relatively low precision
- PDFs and FFs (1-D)  
Precisely measured over a broad kinematic range
- Spins, charge, magnetic moment, charge radius, ... (0-D)  
Have been determined with high precision

# The Landscape of Nucleon Structure

## The Next Experiment Frontier

EicC (China)



EIC (US)



- Wigner distribution (5-D)

Not directly accessible in experiments

- TMDs and GPDs (3-D)

Still limited in experiments, with relatively low precision

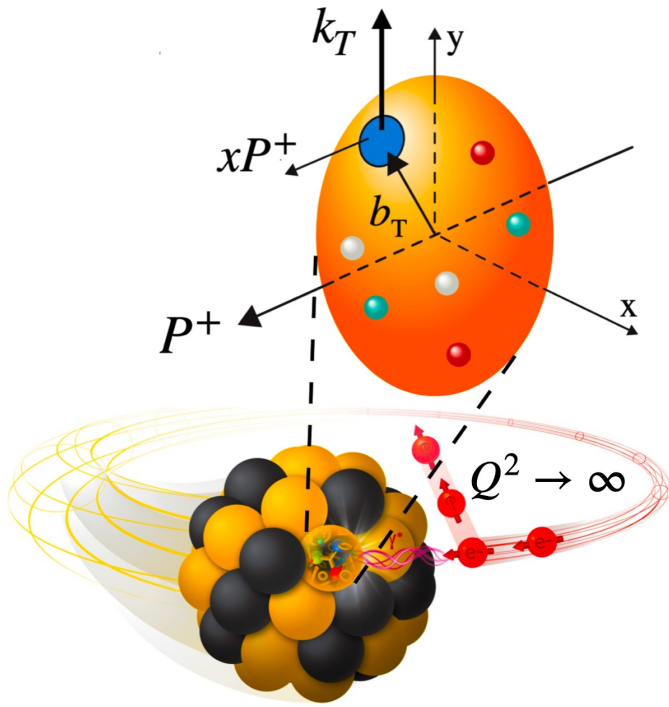
and FFs (1-D)

precisely measured over a broad kinematic range

charge, magnetic moment, charge radius, ...

Have been determined with high precision

# TMDs: 3D Momentum Tomography of Nucleon



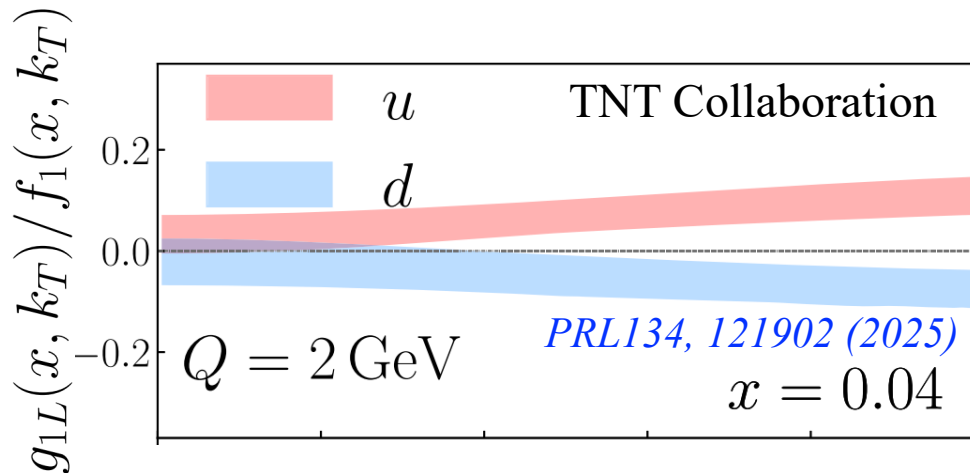
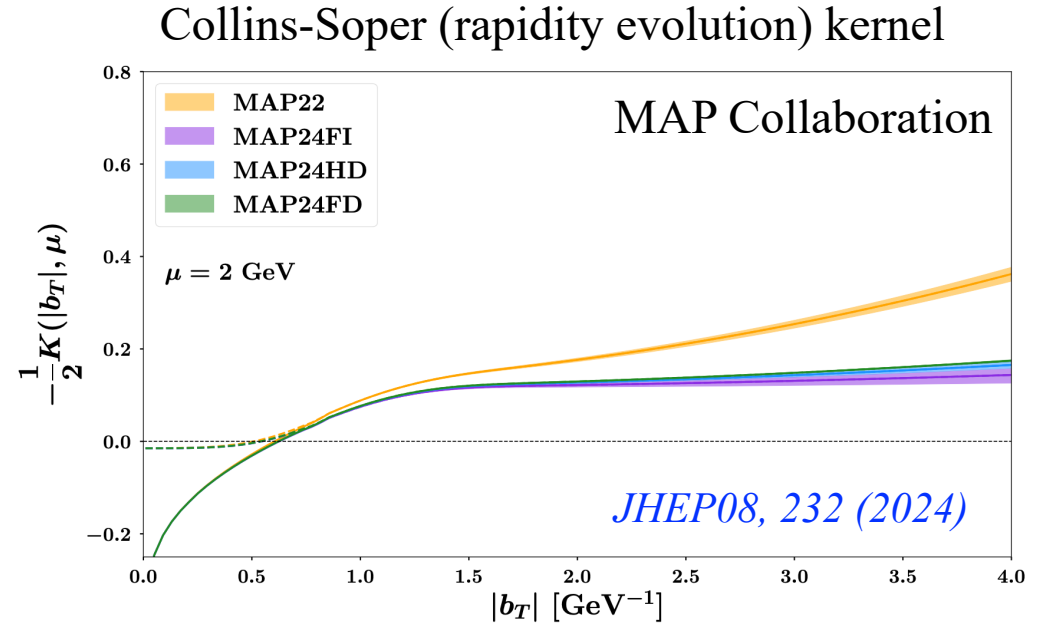
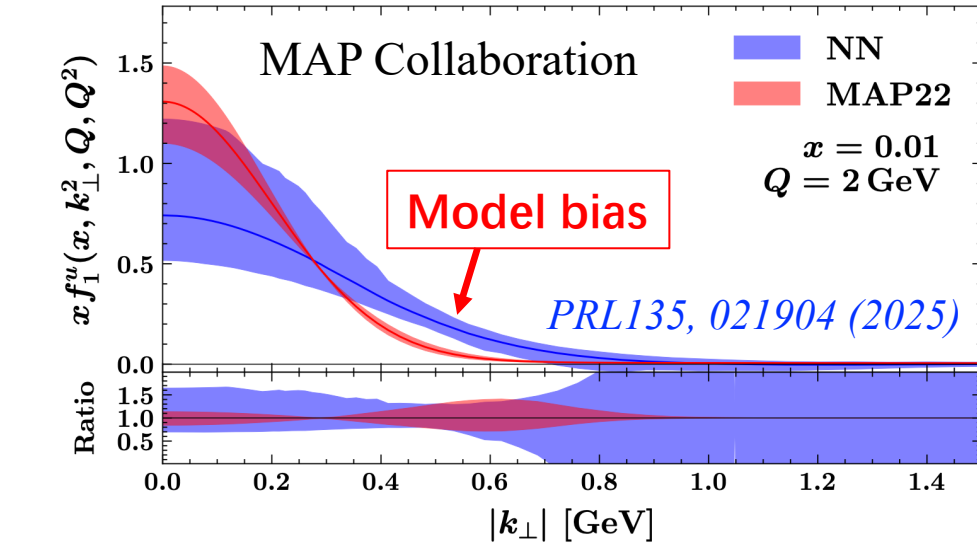
**TMDs:** Spin-dependent 3D momentum space images from semi-inclusive scattering

Leading Quark TMDPDFs



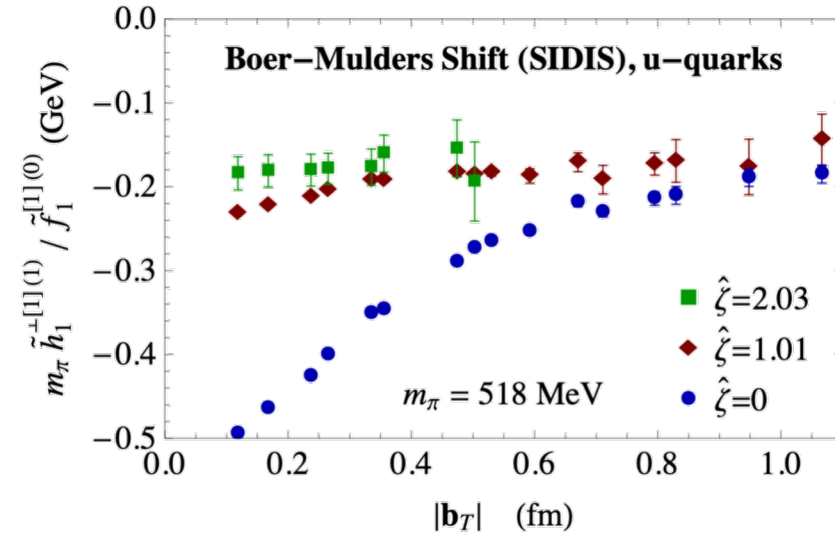
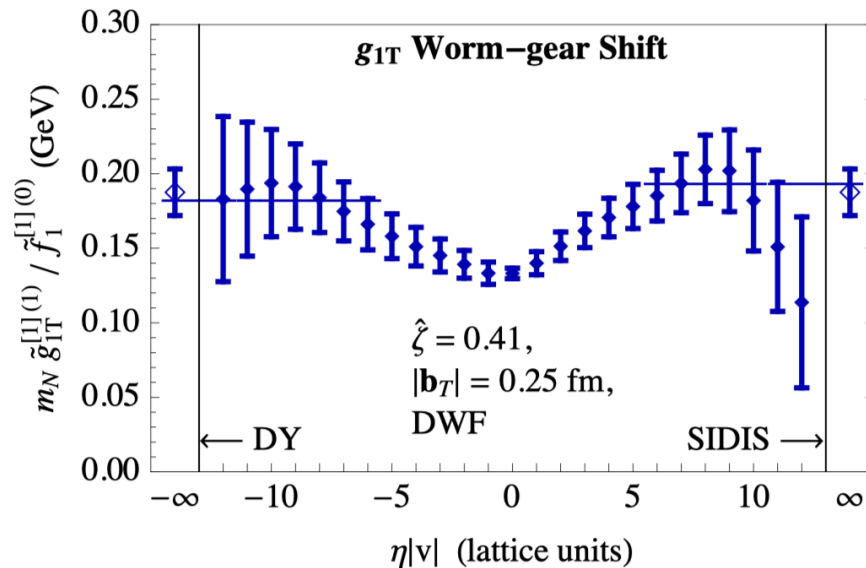
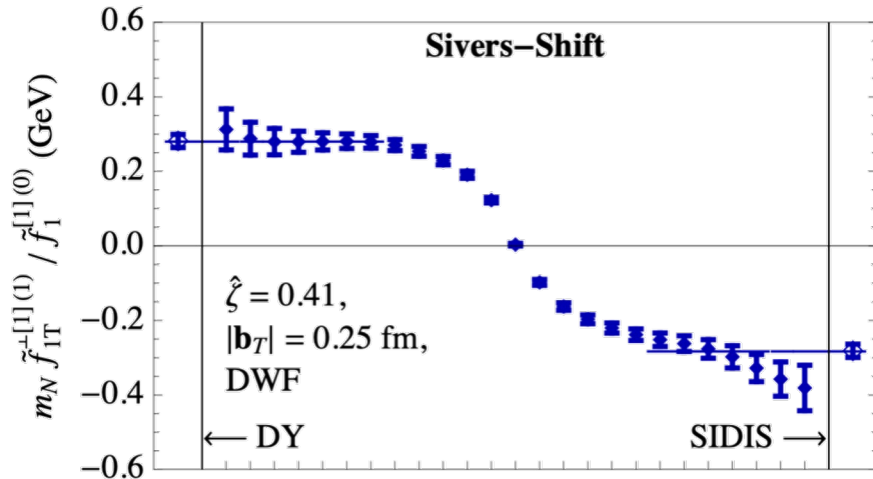
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \text{○} \cdot$ Unpolarized		$h_1^\perp = \text{○} \downarrow - \text{○} \uparrow$ Boer-Mulders
	L		$g_1 = \text{○} \rightarrow - \text{○} \rightarrow$ Helicity	$h_{1L}^\perp = \text{○} \nearrow - \text{○} \searrow$ Worm-gear
	T	$f_{1T}^\perp = \text{○} \uparrow - \text{○} \downarrow$ Sivers	$g_{1T}^\perp = \text{○} \uparrow \rightarrow - \text{○} \uparrow \leftarrow$ Worm-gear	$h_1 = \text{○} \uparrow \downarrow - \text{○} \downarrow \uparrow$ Transversity $h_{1T}^\perp = \text{○} \nearrow \uparrow - \text{○} \searrow \uparrow$ Pretzelosity

# TMDs from Global Analysis of Experimental Data



- Global fit is limited by **sparse data**.
- Rely on **model assumptions**, and produce noticeable inconsistencies.

# TMDs from Lattice: OPE Moments

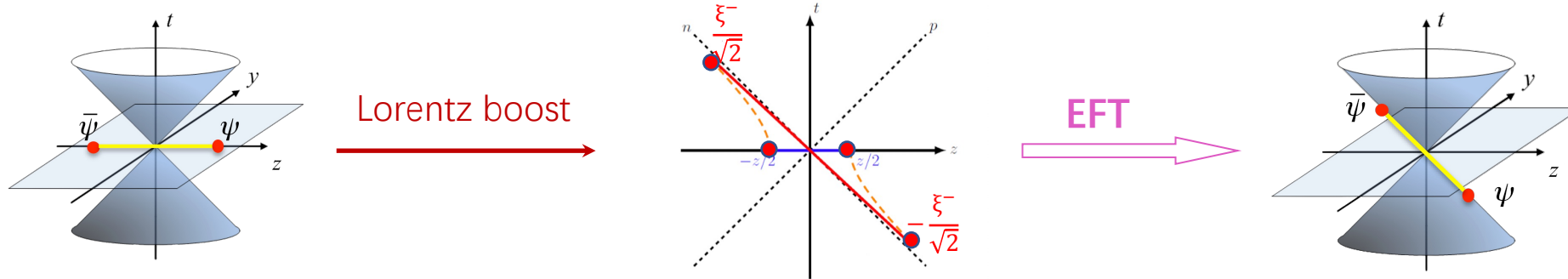


- Without soft function, **only ratios** can be accessed.
- Only the ratio of **lowest order moments**.

*PRD93, 054501 (2016), PRD96, 094508 (2017)*

# LaMET: A New Horizon for Nucleon Structure from Lattice

Large-momentum effective theory (LaMET): connecting Euclidean lattice and physical observables



*Ji, PRL110, 262002 (2013), Rev.Mod.Phys.93, 035005 (2021)*

- Equal-time distributions can be directly computed in lattice QCD:

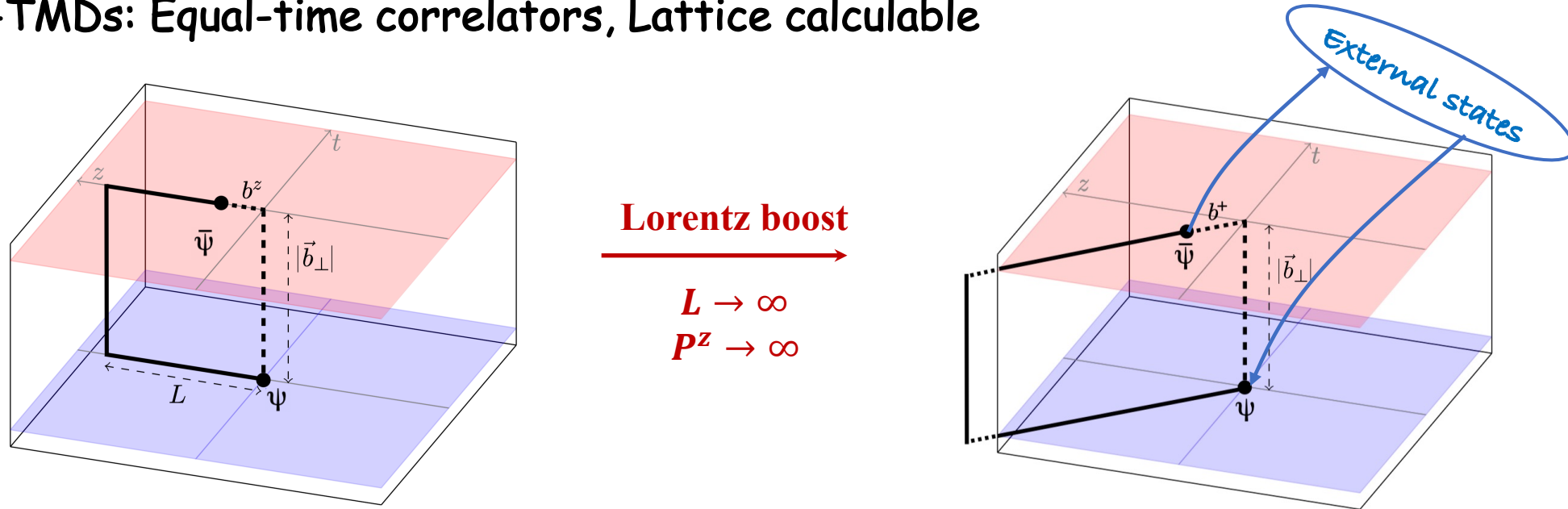
$$\tilde{q}(x, P^z) = \int \frac{dz}{2\pi} e^{-izP^z} \langle P^z | \bar{\psi}(z) \Gamma \mathcal{U}(z, 0) \psi(0) | P^z \rangle$$

- Recovering light-cone distributions in the large-momentum limit:

$$q(y, \mu) = \int_{-\infty}^{+\infty} \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \tilde{q}(x, P^z) + \mathcal{O}\left(\frac{M^2}{(P^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{(yP^z, (1-y)P^z)^2}\right)$$

# TMDs from Lattice QCD within LaMET

Quasi-TMDs: Equal-time correlators, Lattice calculable



Factorization in LaMET:

*PLB811, 135946 (2020); JHEP04, 178, (2022)*

Perturbative matching kernel

$$\tilde{f}_\Gamma(x, b_\perp, \mu, \zeta^z) \sqrt{S_I(b_\perp, \mu)} = H_\Gamma(\zeta^z, \mu) \exp\left[\frac{1}{2} K(b_\perp, \mu) \ln \frac{\zeta^z}{\mu}\right] f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{\zeta^z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta^z}\right)$$

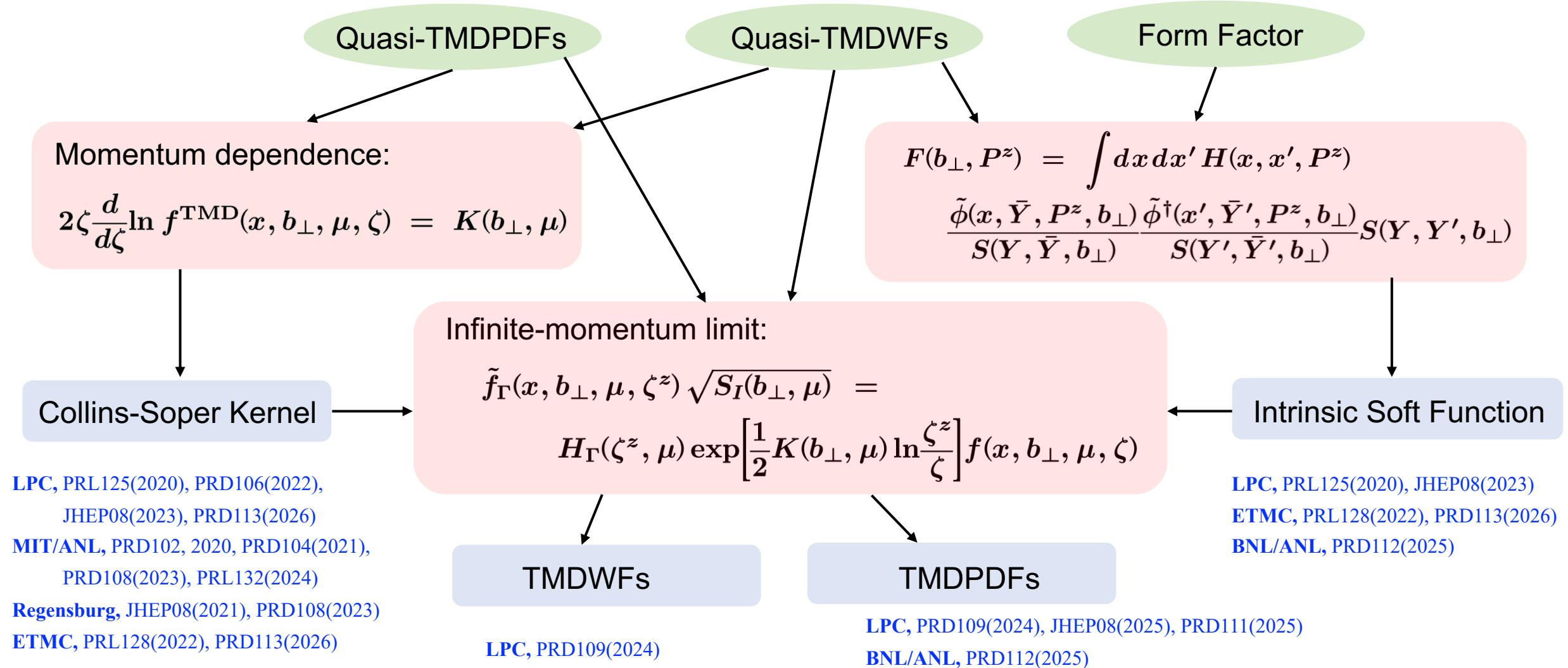
Quasi TMDs

Intrinsic Soft Function

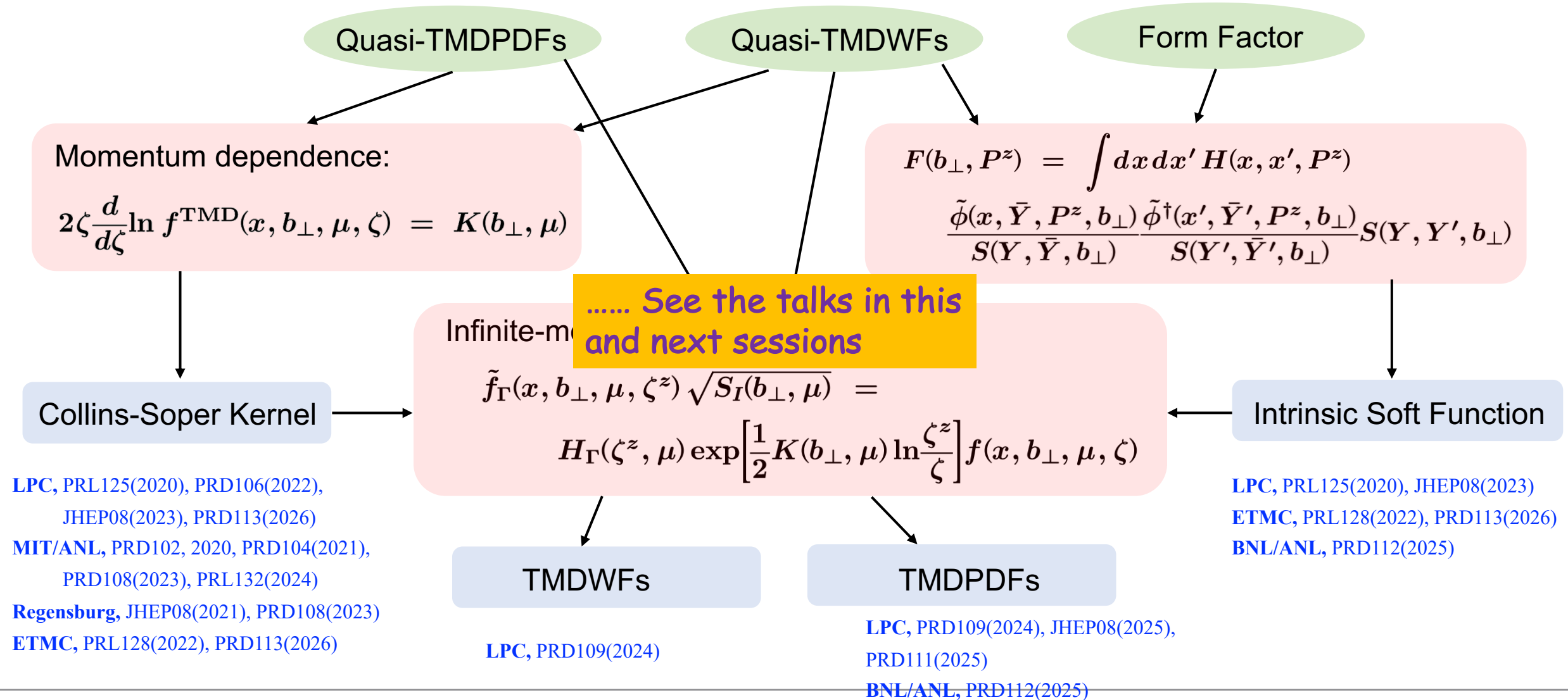
Collins-Soper kernel

Light-cone TMDs

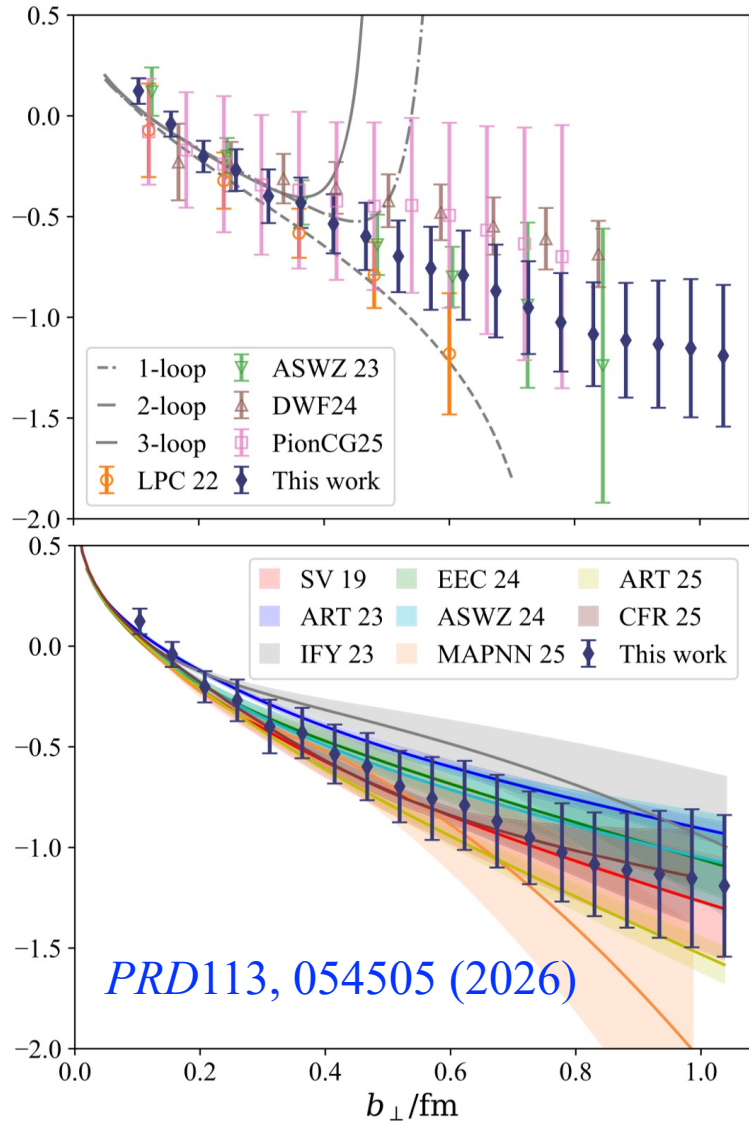
# LQCD Workflow for TMD Calculations and Factorization



# LQCD Workflow for TMD Calculations and Factorization



# Collins-Soper Kernel from Lattice QCD

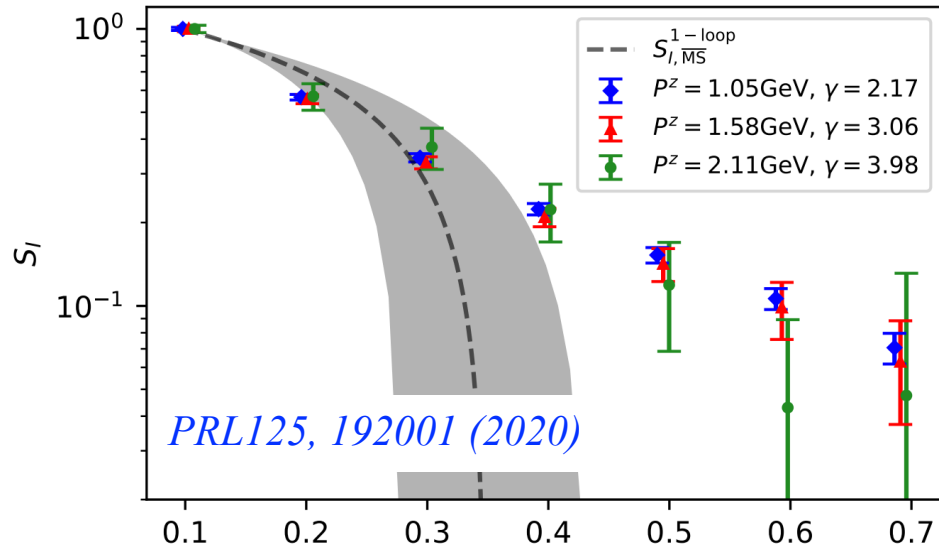


- A benchmark quantity for lattice TMD studies
  - ✓ Continuum, physical mass, and infinite-momentum limits are now being addressed
  - ✓ Lattice QCD provides first-principles constraints up to  $b_{\perp} \sim 1\text{fm}$
  - ✓ Agreement with global fits provides mutual validation
- A precise CS kernel is a key step toward first-principles predictions of TMD observables

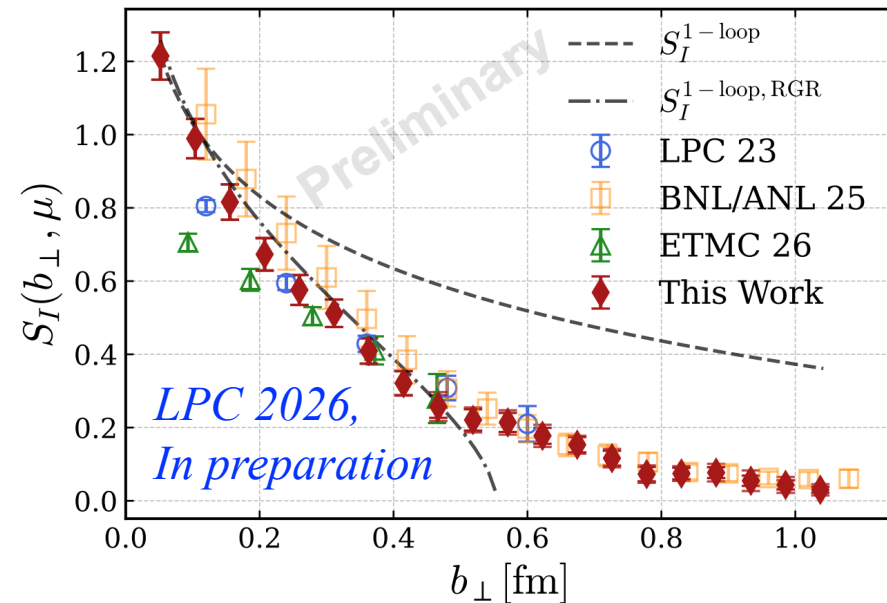
# Intrinsic Soft Function from Lattice QCD

See Zhi-Chao's talk for details.

LPC, 2020



LPC, 2026



- ✓ Continuum, physical mass and infinite-momentum limits
- ✓ Precise first-principles prediction up to  $b_T \sim 1\text{fm}$
- ✓ Essential input for the lattice calculation of TMDPDFs

- Quasi-TMDPDFs
- Collins-Soper Kernel
- Intrinsic Soft Function

Matching in LaMET:

$$\tilde{f}_\Gamma(x, b_\perp, \mu, \zeta^z) \sqrt{S_I(b_\perp, \mu)} = H_\Gamma(\zeta^z, \mu) \exp\left[\frac{1}{2}K(b_\perp, \mu) \ln\frac{\zeta^z}{\zeta}\right] f(x, b_\perp, \mu, \zeta)$$

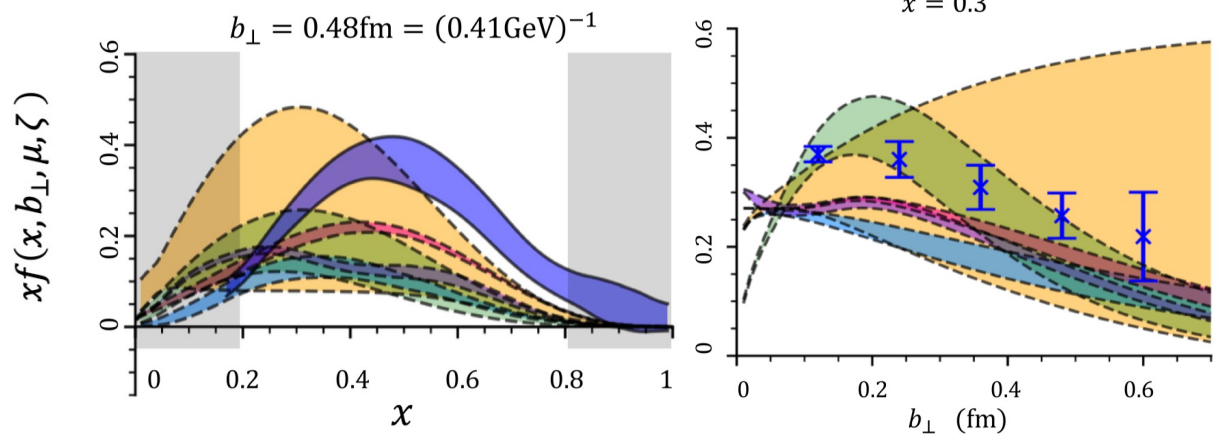
- Single lattice spacing
- Limited statistics
- Incomplete systematic uncertainties

Unpolarized TMDPDF

Quark Spin-transverse-momentum Correlation

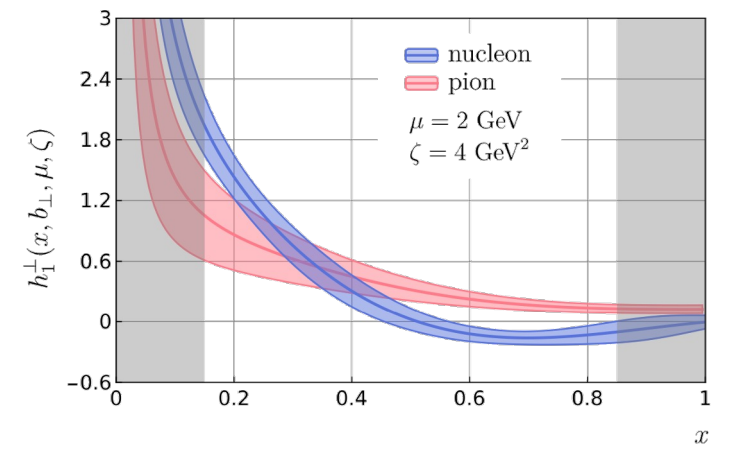
*LPC, PRD111, 094507 (2025); JHEP 08, 086 (2025)*

*LPC, PRD109, 114513 (2024)*



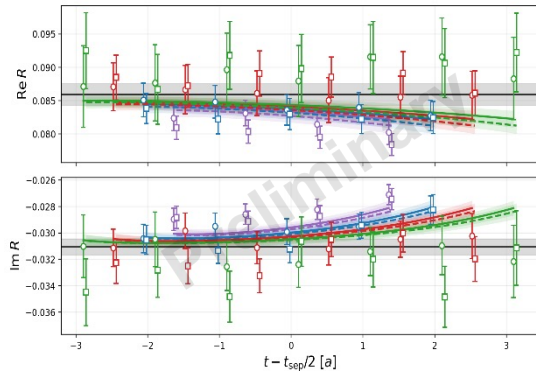
- This work
- ART23
- BHLSVZ22
- MAPTMD22
- SV19
- PV17

$P^z = 1.84 \text{ GeV}, b_\perp = 0.196 \text{ fm}$

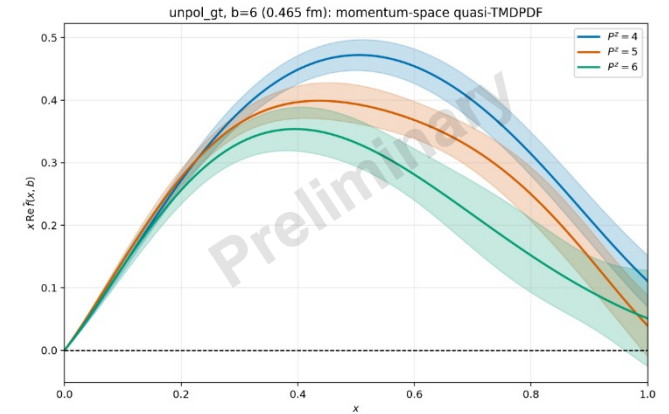
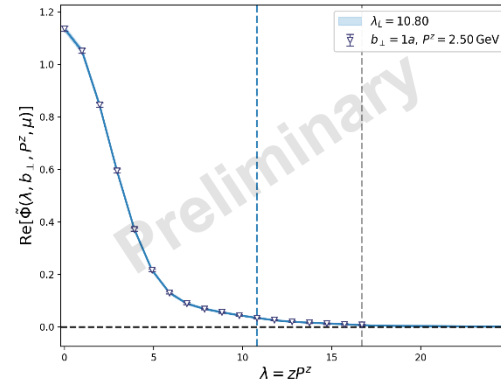
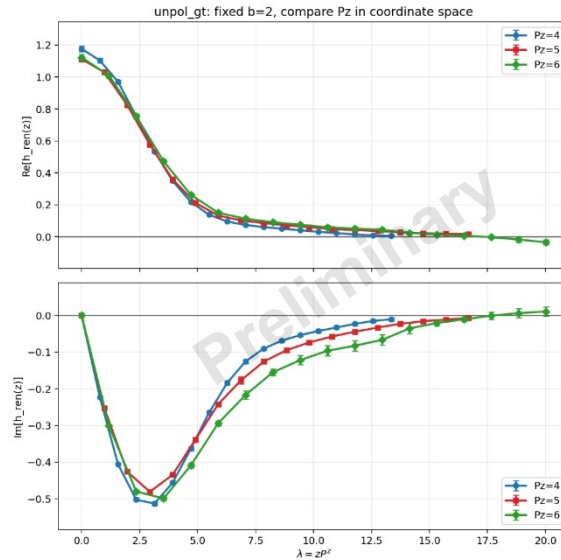


- E.g.: Unpolarized TMDPDF

- ✓ Multi lattice spacings  $\Rightarrow$  Continuum Limit
- ✓ Multi pion masses  $\Rightarrow$  Chiral Limit
- ✓ Large statistics



LPC (2026), in preparation



Ground-state fit  
the ratio of 3pt/2pt

Nonlocal renormalization  
to get coordinate-space  
distribution

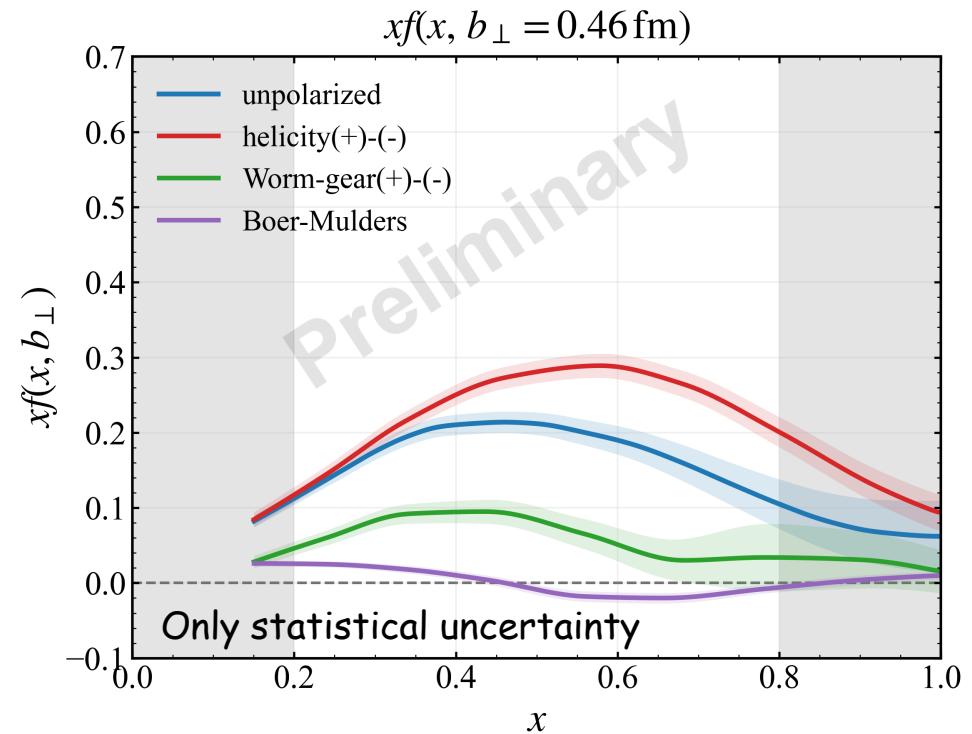
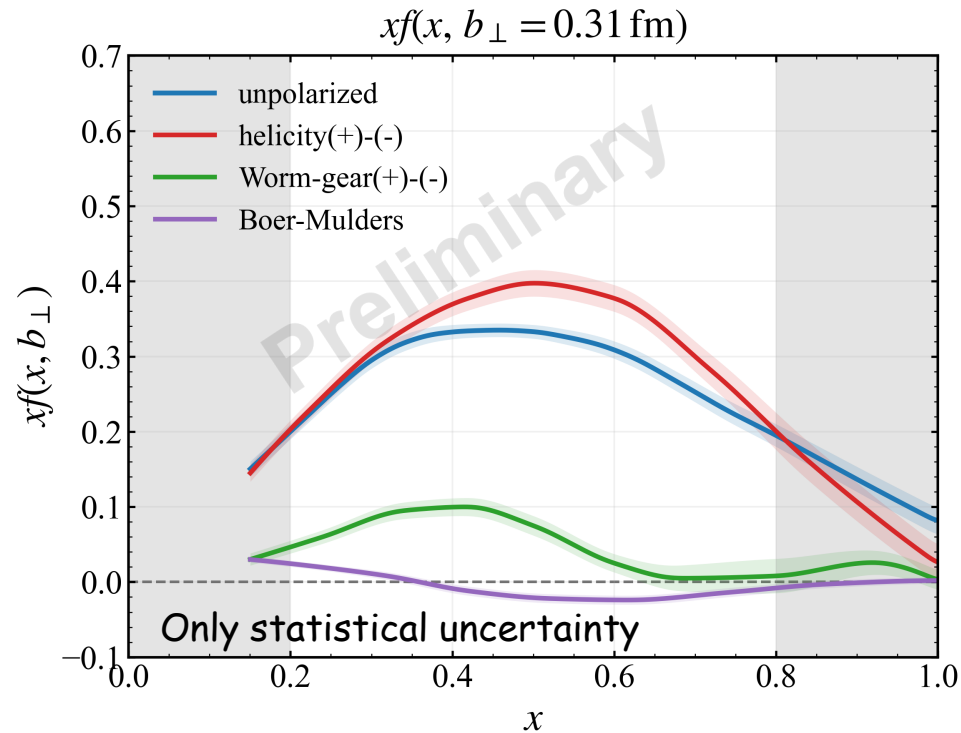
Large- $\lambda$   
extrapolation

Fourier transform to get  
momentum-space  
quasi-TMDPDF

# Precise Calculation for Polarized TMDPDFs

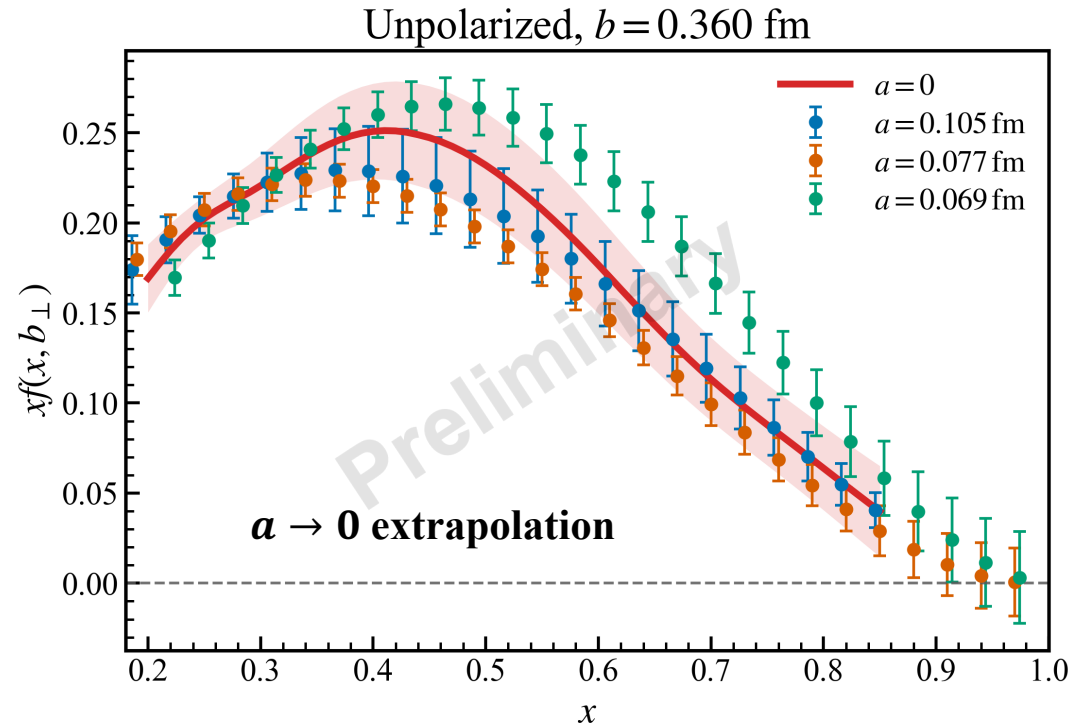
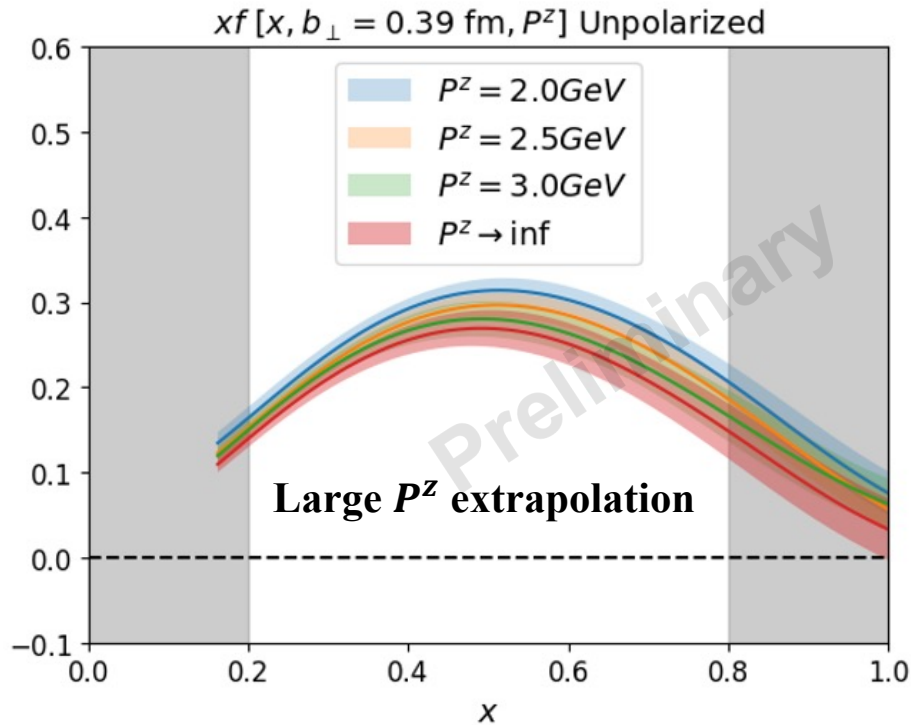
- Preliminary results of TMDPDFs for unpolarized and longitudinal polarized nucleon ( $a=0.077\text{fm}$ ,  $m_\pi=300\text{MeV}$ )

*LPC (2026), in preparation*



# Unpolarized TMDPDFs @ Physical Limit

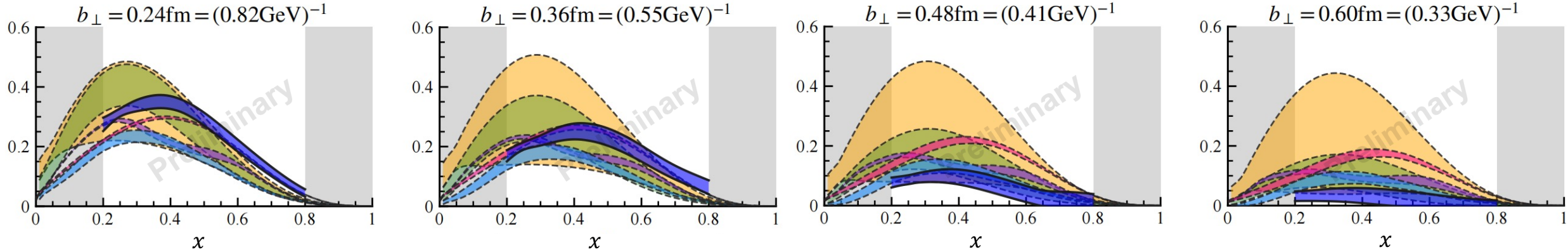
- Unpolarized TMDPDFs at ✓ Infinite-momentum limit and 🕒 Chiral limit  
✓ Continuum limit



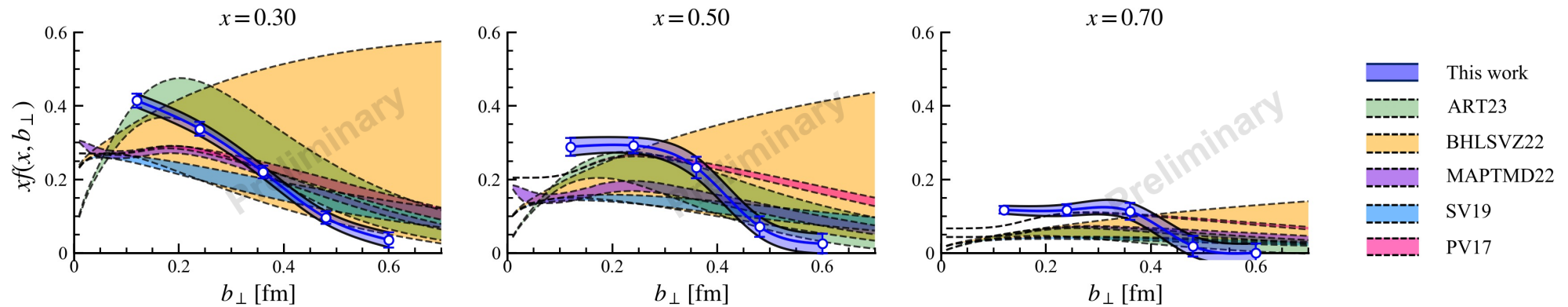
# Unpolarized TMDPDFs @ Physical Limit

- x-dependence

*LPC (2026), in preparation*



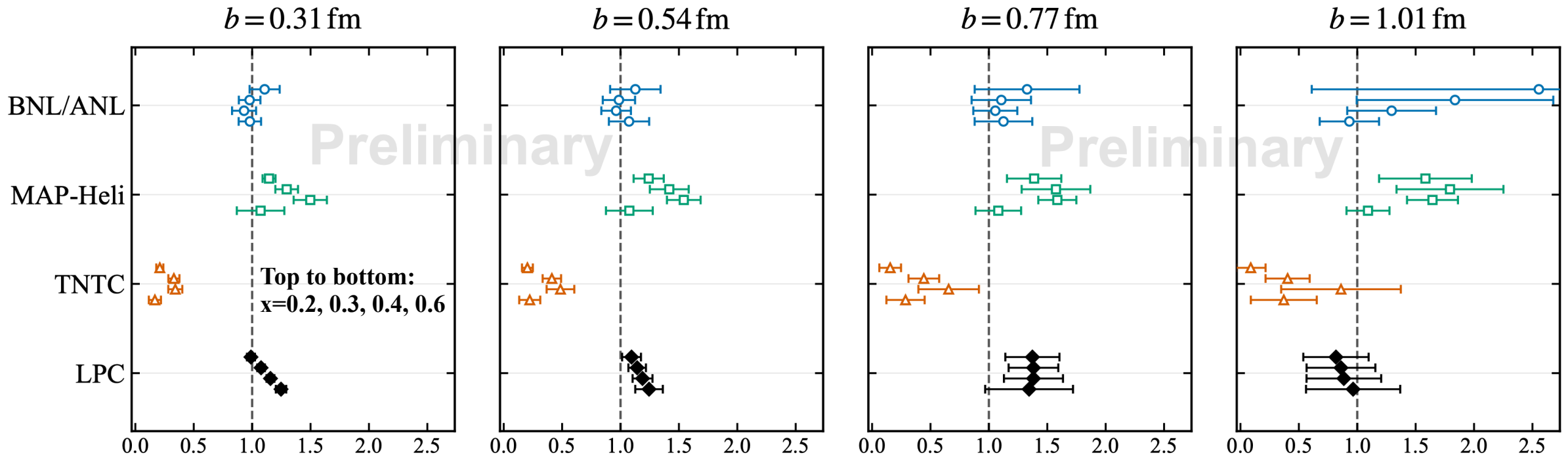
- $b_{\perp}$ -dependence



# Ratios between nucleon helicity and unpolarized TMDPDF

- Preliminary results on the ensemble with  $a=0.077\text{fm}$ ,  $m_\pi=300\text{ MeV}$

*LPC (2026), in preparation*



More systematic analyses are in progress .....

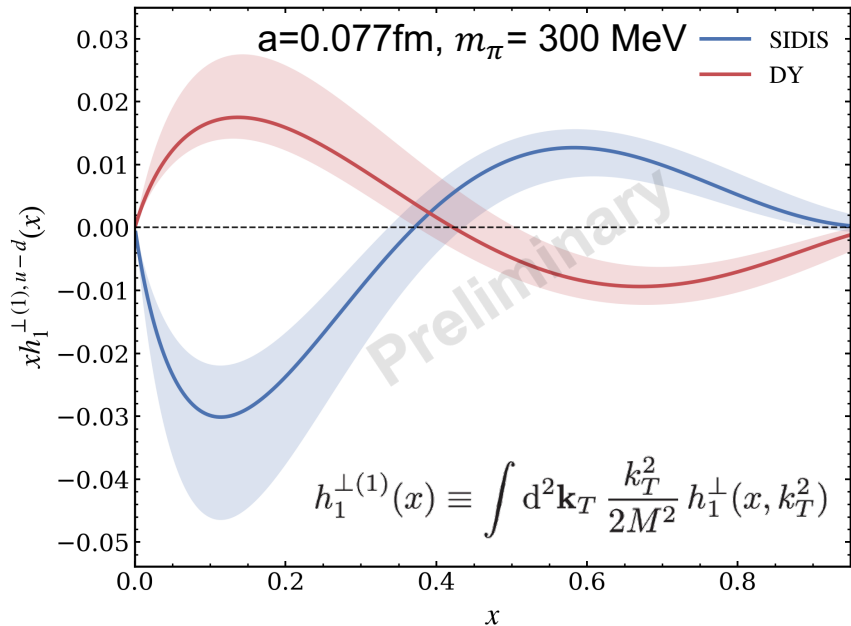
BNL/ANL (LQCD): PRL135, 20 (2025)  
 MAP (Global fit): PRL134, 121901 (2025)  
 TNTC (Global fit): PRL134, 121902 (2025)

# Quark Spin-transverse-momentum Correlation

- First moment of quasi Boer-Mulders function and its matching

$$\tilde{h}_1^{\perp(1)}(x, b_{\perp}^2, P^z, \mu) = \frac{1}{iM\epsilon_{\perp}^{ij}b_{\perp j}} \frac{E}{P^z} \int \frac{dz}{2\pi} e^{ixP^z z} \frac{\sum_S \text{Tr} \left[ P_+ u(P, S) \langle N(P, S) | O^{i\sigma^{\perp z} \gamma_5}(z, b_{\perp}) | N(P, S) \rangle \bar{u}(P, S) \right]_R}{\sum_S \text{Tr} [P_+ u(P, S) \bar{u}(P, S)]_R}$$

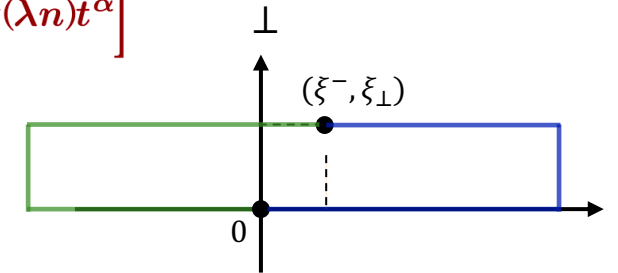
$$\tilde{h}_1^{\perp(1)}(x, b_{\perp}^2, P^z, \mu) \sqrt{S_I(b_{\perp}, \mu)} = \exp \left[ \frac{1}{2} \ln \frac{\zeta_z}{\zeta} K(b_{\perp}, \mu) \right] H \left( \frac{x}{y}, \frac{\mu^2}{\zeta_z} \right) h_1^{\perp(1)}(y, b_{\perp}^2, \mu, \zeta) + \mathcal{O} \left( \frac{\Lambda_{\text{QCD}}^2}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_{\perp}^2 \zeta_z} \right)$$



- Gauge links generate initial and final state interactions and distinguish the SIDIS & Drell-Yan processes

$$\mathcal{U}_n(a, b) = \exp \left[ -ig \int_a^b d\lambda n \cdot A^\alpha(\lambda n) t^\alpha \right]$$

LPC (2026), in preparation



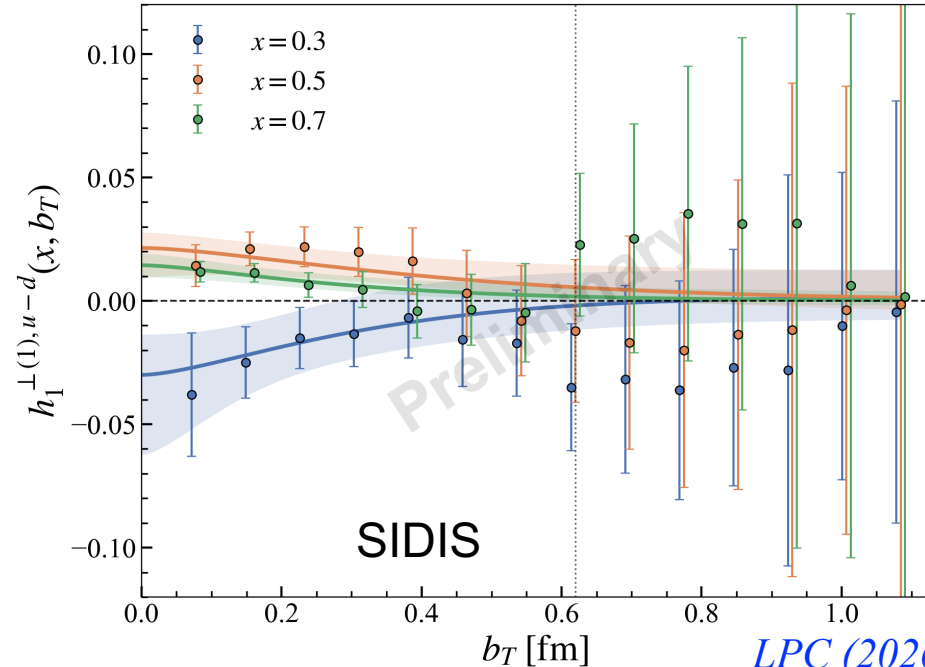
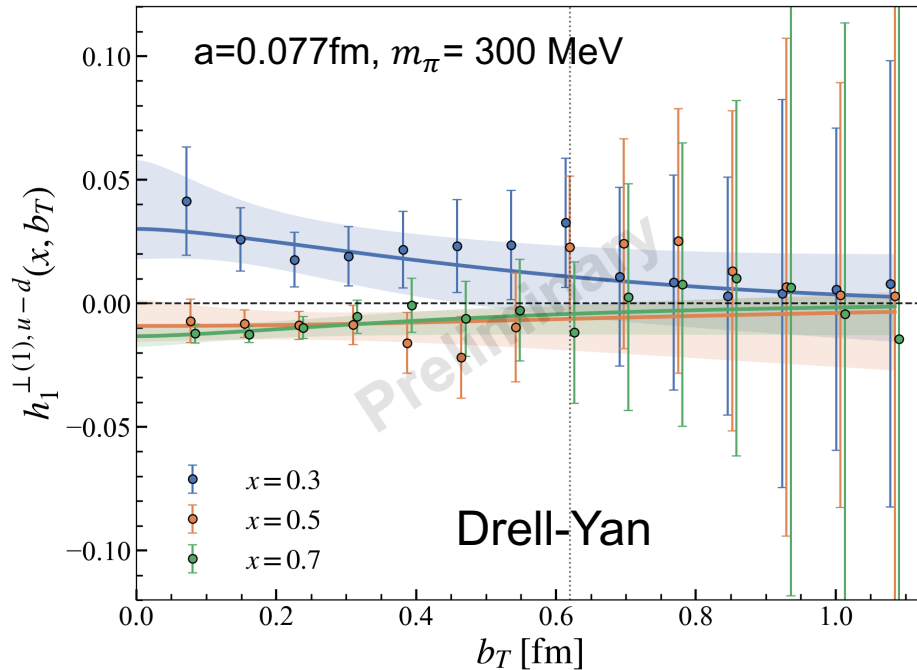
# Quark Spin-transverse-momentum Correlation

- Rebuild the full distribution of  $h_1^\perp(x, k_\perp^2)$  from model parametrization

*Bacchetta, Conti, Radici, PRD78, 074010 (2008)*

$$h_1^{\perp, u-d}(x, k_\perp^2) = \frac{M_N^2}{2\pi k_\perp} \int db_\perp b_\perp^2 J_1(k_\perp b_\perp) h_1^{\perp(1)}(x, b_\perp^2)$$

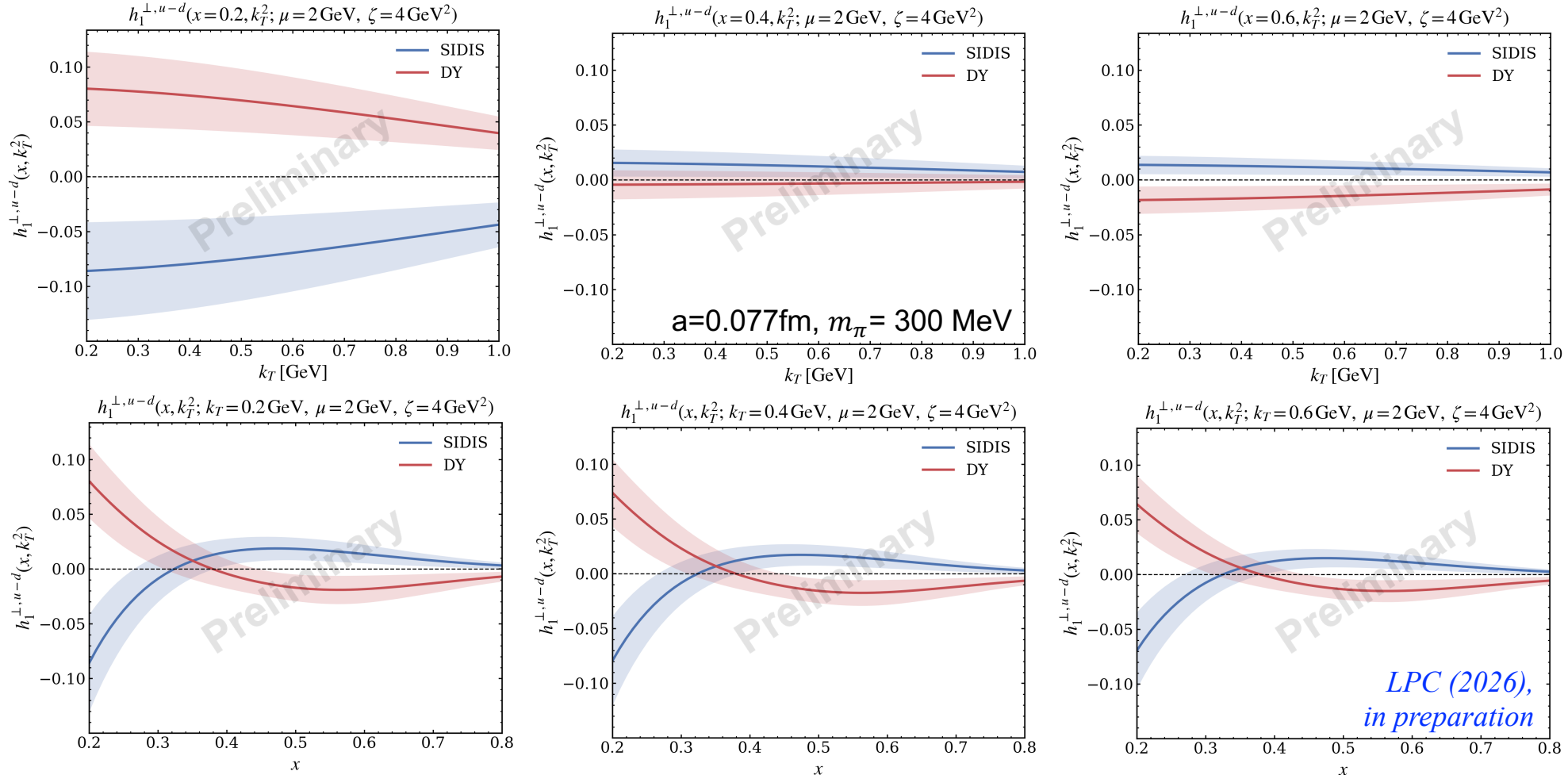
$$\equiv H_S(x) \frac{4M^2}{\pi L_S^4(x)} \left(1 + \frac{k_\perp^2}{L_S^2(x)}\right)^{-3} - H_A \frac{4M^2}{\pi L_A^4(x)} \left(1 + \frac{k_\perp^2}{L_A^2(x)}\right)^{-3}$$



*LPC (2026), in preparation*

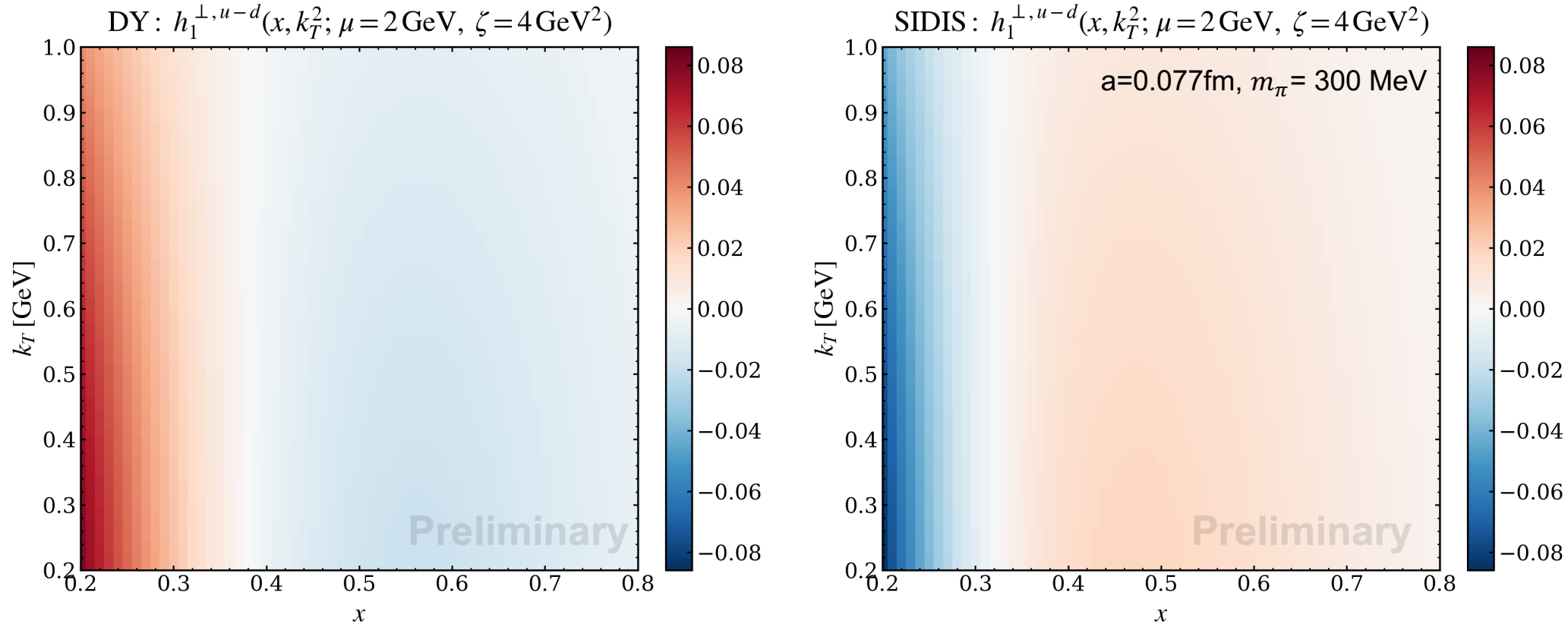
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# Quark Spin-transverse-momentum Correlation

- Rebuild the full distribution of  $h_1^\perp(x, k_\perp^2)$  from model parametrization



Further analyses and discussions are underway...

# Summary and Outlook

TMDs of the nucleon are part of the future frontier of nuclear science, especially in the Electron-Ion Collider and first-principles QCD calculations.

Lattice QCD + LaMET provides a new perspective on exploring the 3-D structure of hadrons.

With controlled continuum, physical-mass, and large-momentum limits, lattice calculations will deliver essential nonperturbative inputs for precision TMD phenomenology.



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- J. X. Tan, *et al.* (LPC), in preparation.

Backup

# Lattice Setup

	a/fm	$n_s^3 \times n_t$	$m_\pi/\text{MeV}$	$n_{cfg} \times n_{meas}$		
				CS kernel	Intrinsic Soft Function	TMDPDFs
C32P29		$32^3 \times 64$	292.4	$984 \times 4$	$984 \times 2$	$1000 \times 2$
C32P23	0.10530	$32^3 \times 64$	228.0	$448 \times 10$	$450 \times 24$	$1000 \times 4$
C48P14		$48^3 \times 96$	135.5	$204 \times 24$	$448 \times 48$	$300 \times 12$
F32P30	0.07746	$32^3 \times 96$	303.2	$1153 \times 6$	$1100 \times 6$	$1000 \times 4$
G36P30	0.06889	$36^3 \times 108$	297	—	—	$500 \times 4$
H48P32	0.05187	$48^3 \times 144$	317.2	$550 \times 8$	$550 \times 8$	—