Lattice Parton Collaboration

SHANGHAI JIAO TONG UNIVERSITY

Complete Access to Leading-twist Baryon Light-cone DAs from Lattice QCD

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(Lattice Parton Collaboration)

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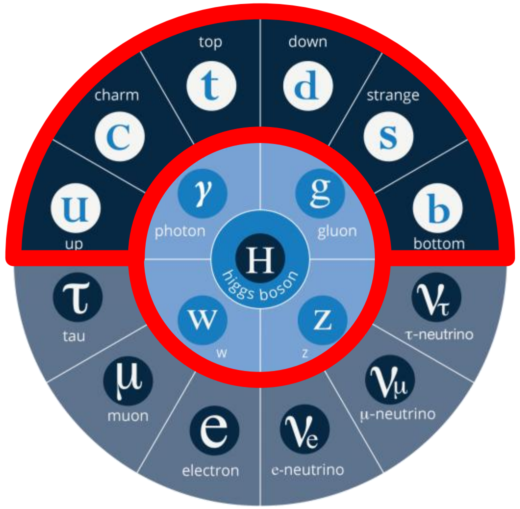
Why baryons ?

Baryon — cornerstone particles

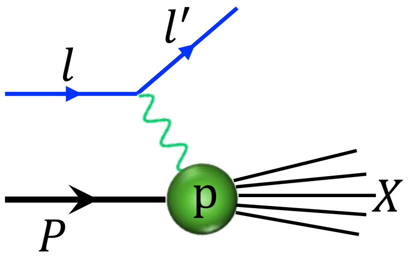


- Matter genesis in the Universe
- Tools to probe micro world
- Astronomy & Cosmology
- …………

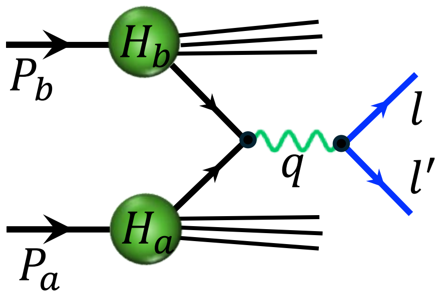
Why LCDAs ?

- **PDFs**: internal structure of hadrons in **Inclusive processes**

DIS

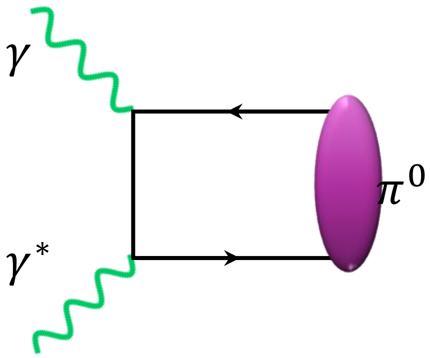


Drell-Yan

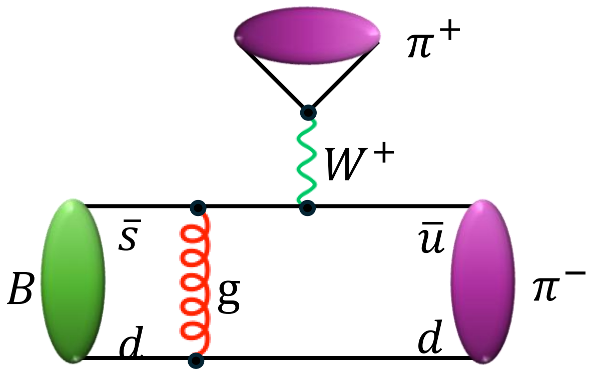


- **LCDAs**: richer QCD dynamical information in **Exclusive processes**

$\gamma\gamma^* \to \pi$



$B \to \pi^+\pi^-$



Why baryon LCDAs ?

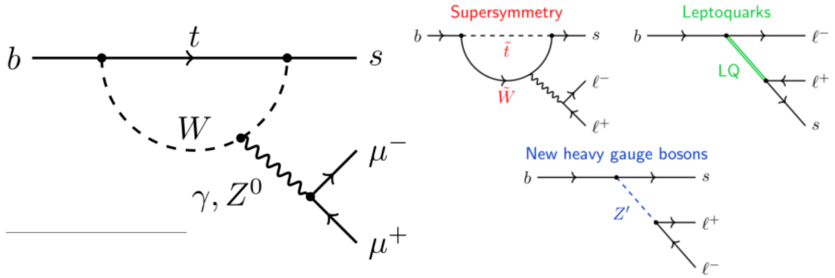
Heavy Baryon Decays

Establish CPV in baryonic decays

- First bservation of CPV in **Baryon**:

$\Lambda_b^0 \to p$

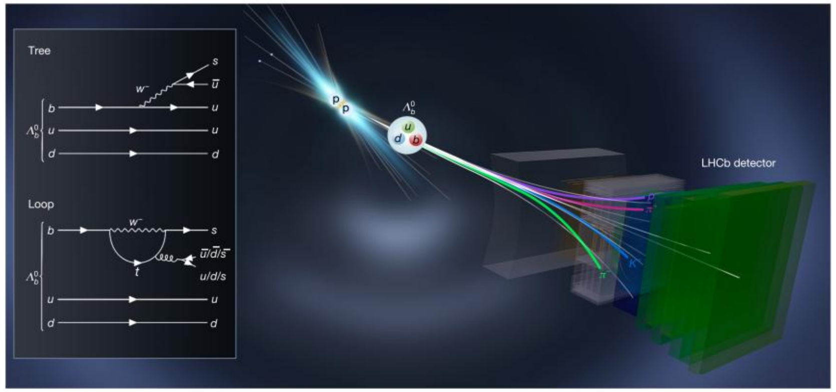
Searching New Physics beyond SM



Observation of charge–parity symmetry breaking in baryon decays

LHCb Collaboration

Nature **643**, 1223–1228 (2025) | Cite this article



Multi-dimensional baryon structure

Nucleon form factors, …

LHCb, Nature (2025); J.J.Han, et.al. PRL 134, 221801 (2025); Y.Jia, et al, PRL 135, 131903 (2025); Y.M.Wang et.al, PRL 135, 061901 (2025)

How to access baryon LCDAs ?

Research methods: (1983 - now)

- **Asymptotic form**

  Chernyak, Zhitnitsky, 1983; ……

- **QCD Sum Rules**

  King, Sachrajda, 1987;

  Stefanis, Bergmann, 1993;

  Braun, et al, 2000, 2006; ……

- **Models parametrization**

  Bell, et al, 2013; ……

- **Moments from Lattice QCD**

  QCDSF, 2008, 2009;

  RQCD, 2016, 2019, 2025; ……

- **x-dependent LCDAs from Lattice QCD**

  LPC, PRD 111, 034510 (2025);

  LPC, PRD 112, 114515 (2025);

  LPC, arXiv:2606·30387:

  Baryon Light-Cone Distribution Amplitudes from Lattice QCD: Formalism, Renormalization, Extrapolation, and Matching

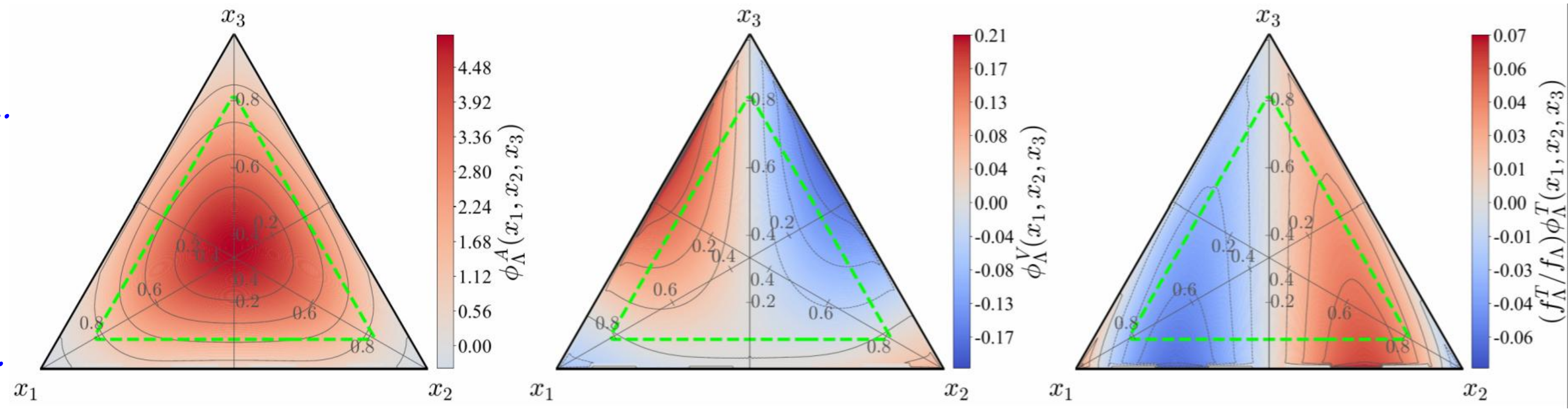
  LPC, arXiv:2606·29597:

  Complete Access to Leading-Twist Λ-Baryon Light-Cone Distribution Amplitudes from Lattice QCD

How to access baryon LCDAs ?

Research methods: (1983 - now)

LPC, arXiv:2606.29597;



- **Asymptotic form**
  - *Chernyak, Zhitnitsky, 1983; ……*
- **QCD Sum Rules**
  - *King, Sachrajda, 1987;*
  - *Stefanis, Bergmann, 1993;*
  - *Braun, et al, 2000, 2006; ……*
- **Models parametrization**
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  - *QCDSF, 2008, 2009;*
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- **x-dependent LCDAs from Lattice QCD**
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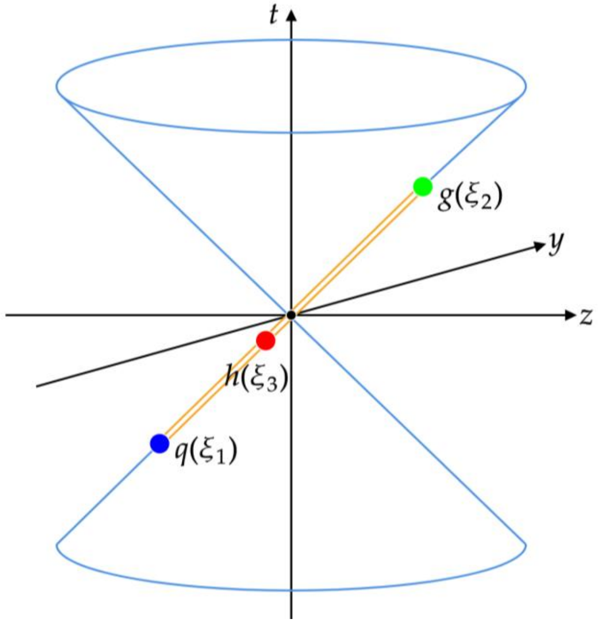
How to access baryon LCDAs ?

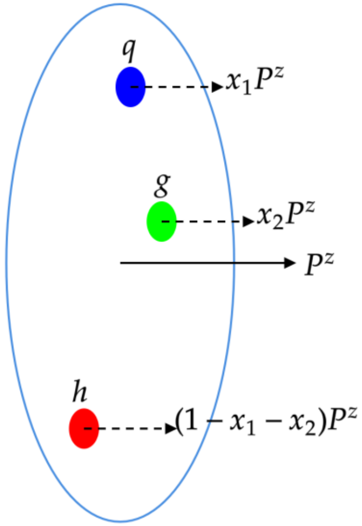


- **Operator definition of baryon LCDAs:**

$$\phi_B^X(x_1,x_2;\mu) = (n\cdot P)^2 \int \frac{d\xi_1}{2\pi}\frac{d\xi_2}{2\pi}\, e^{i(x_1\xi_1+x_2\xi_2)n\cdot P}$$
$$\times \epsilon^{ijk}\langle 0| q_\alpha^{i'}(\xi_1 n)U_{i'i}(\xi_1 n,\xi_0 n)$$
$$\times g_\beta^{j'}(\xi_2 n)U_{j'j}(\xi_2 n,\xi_0 n)$$
$$\times h_\gamma^{k'}(\xi_3 n)U_{k'k}(\xi_3 n,\xi_0 n)|B(P)\rangle^R$$

- **Leading twist for octet baryon:**

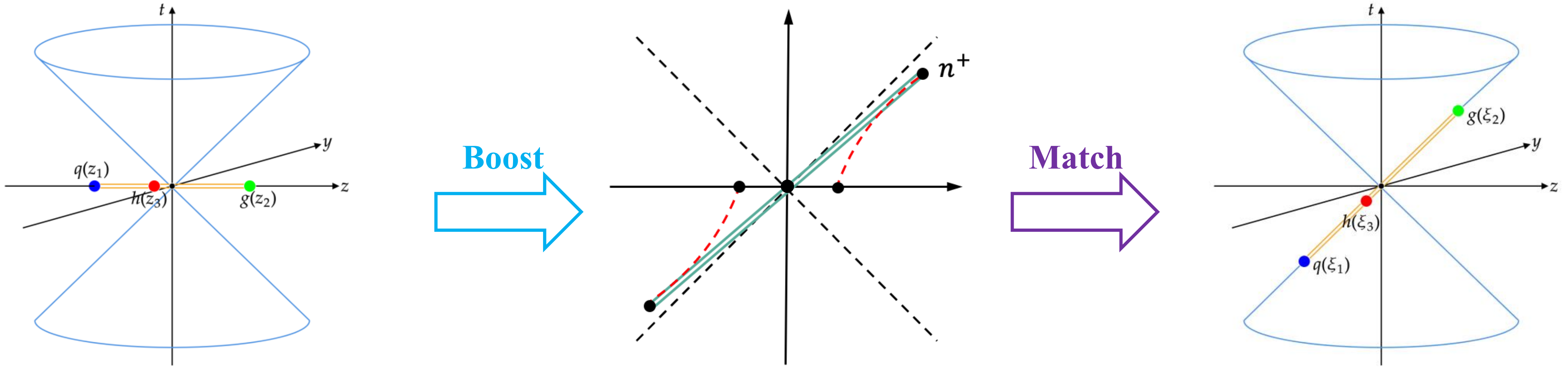
$$\langle 0|f_\alpha(\xi_1 n)g_\beta(\xi_2 n)h_\gamma(\xi_3 n)|B(P)\rangle^R$$
$$=\frac{1}{4}f_B\Big[(\not{P}C)_{\alpha\beta}(\gamma^5 u_B)_\gamma \Phi_B^V(\xi_\ell n\cdot P,\mu)$$
$$+(\not{P}\gamma^5 C)_{\alpha\beta}(u_B)_\gamma \Phi_B^A(\xi_\ell n\cdot P,\mu)\Big]$$
$$+\frac{1}{4}f_B^T(i\sigma_{\mu\nu}P^\nu C)_{\alpha\beta}(\gamma^\mu\gamma^5 u_B)_\gamma \Phi_B^T(\xi_\ell n\cdot P,\mu)\ .$$



How to access baryon LCDAs ?

Large-momentum Effective Theory

X.Ji, PRL 110, 262002 (2013); X.Ji, Sci.China Phys.Mech.Astron. 57 (2014)



LaMET expansion:

LCDA — Matching kernel — quasi-DA

$$\phi(x_1,x_2;\mu) = \int \mathrm{d}y_1\mathrm{d}y_2\, \mathcal{C}(x_1,x_2;y_1,y_2;P^z,\mu)\,\boxed{\tilde{\phi}(y_1,y_2;P^z,\mu)}$$

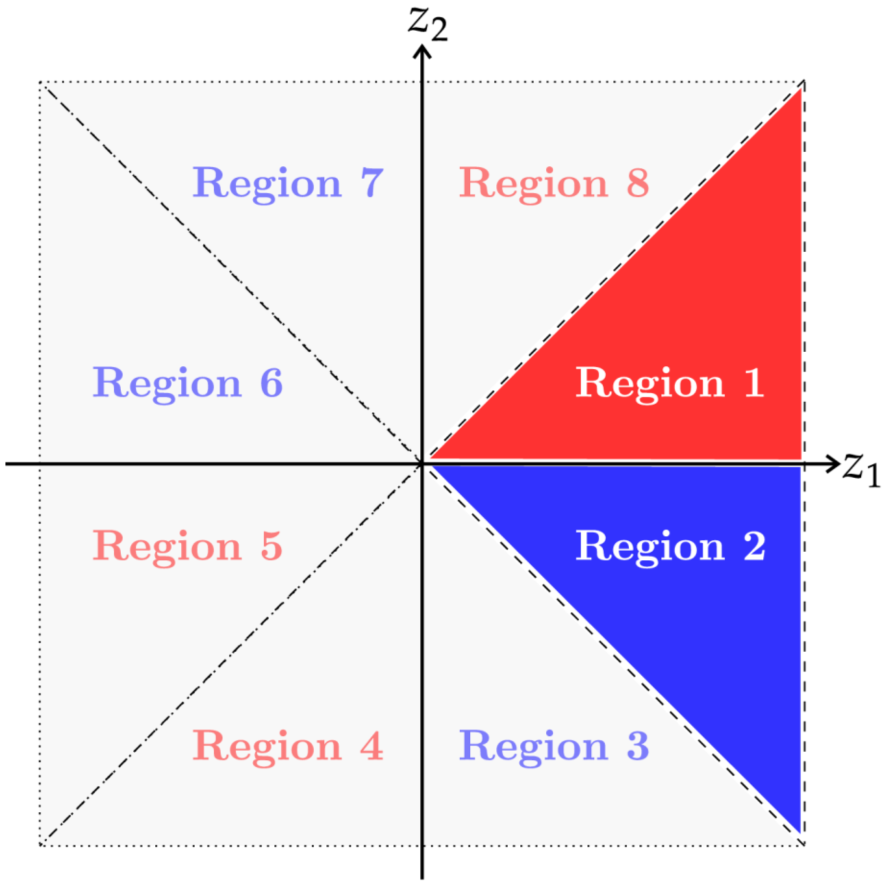
$$+\,\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{(x_1P^z)^2},\frac{\Lambda_{\mathrm{QCD}}^2}{(x_2P^z)^2},\frac{\Lambda_{\mathrm{QCD}}^2}{[(1-x_1-x_2)P^z]^2}\right)$$

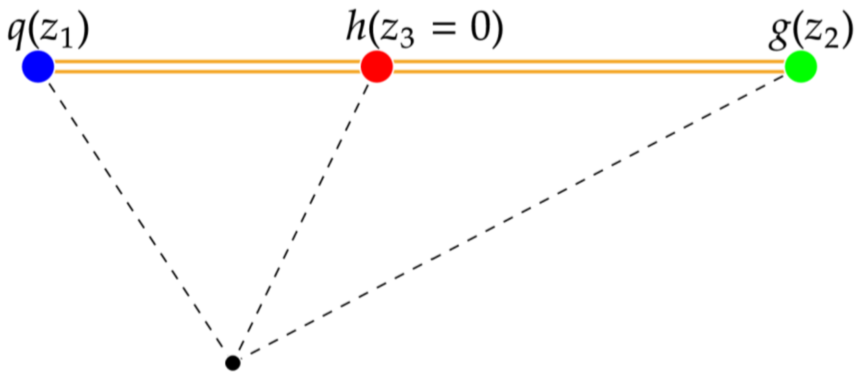
power corrections

X.Ji, PRL 110, 262002 (2013); X.Ji, Sci.China Phys.Mech.Astron. 57 (2014)

Symmetries on (z_1, z_2) plane

Genuinely 2-dimensional distribution

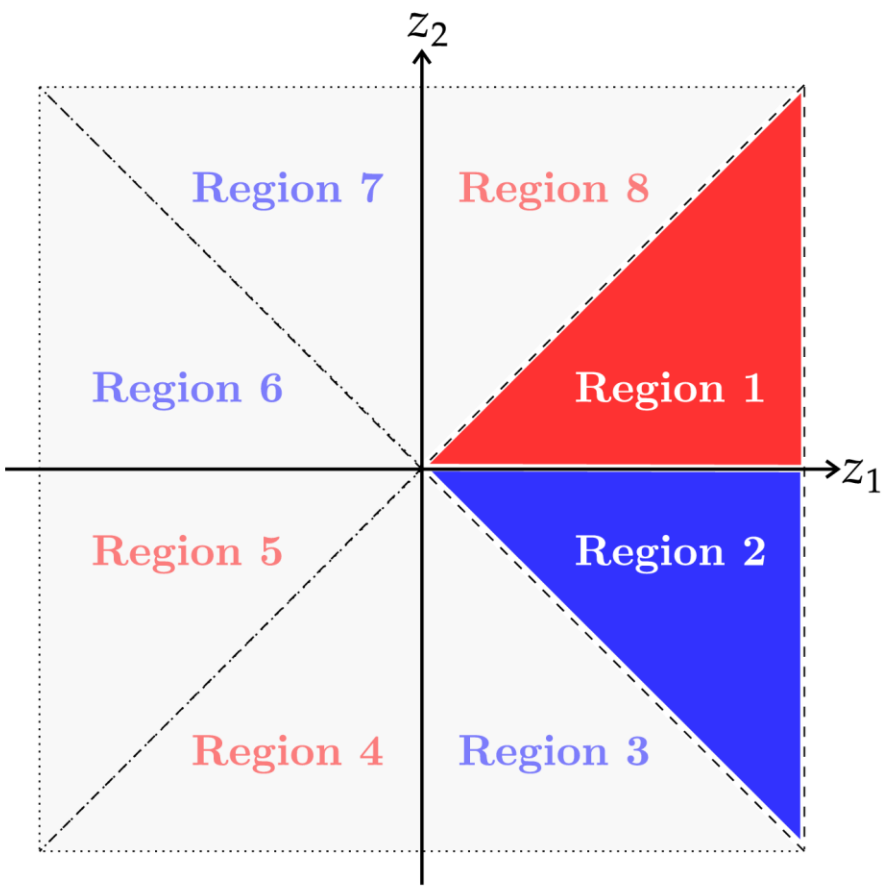


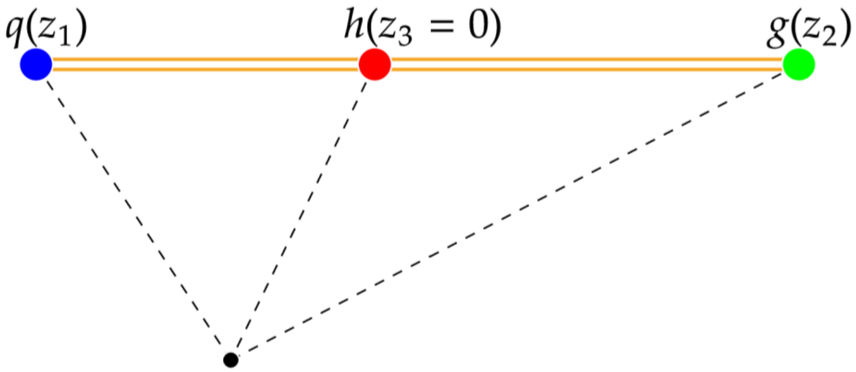


- 3-quark operator with 2 geometrically intertwined Wilson-line lengths $\boldsymbol{z_1}$ and $\boldsymbol{z_2}$;
- Genuinely 2-dimensional distribution among whole $(\boldsymbol{z_1}, \boldsymbol{z_2})$ plane.

Symmetries on (z_1, z_2) plane

Genuinely 2-dimensional distribution



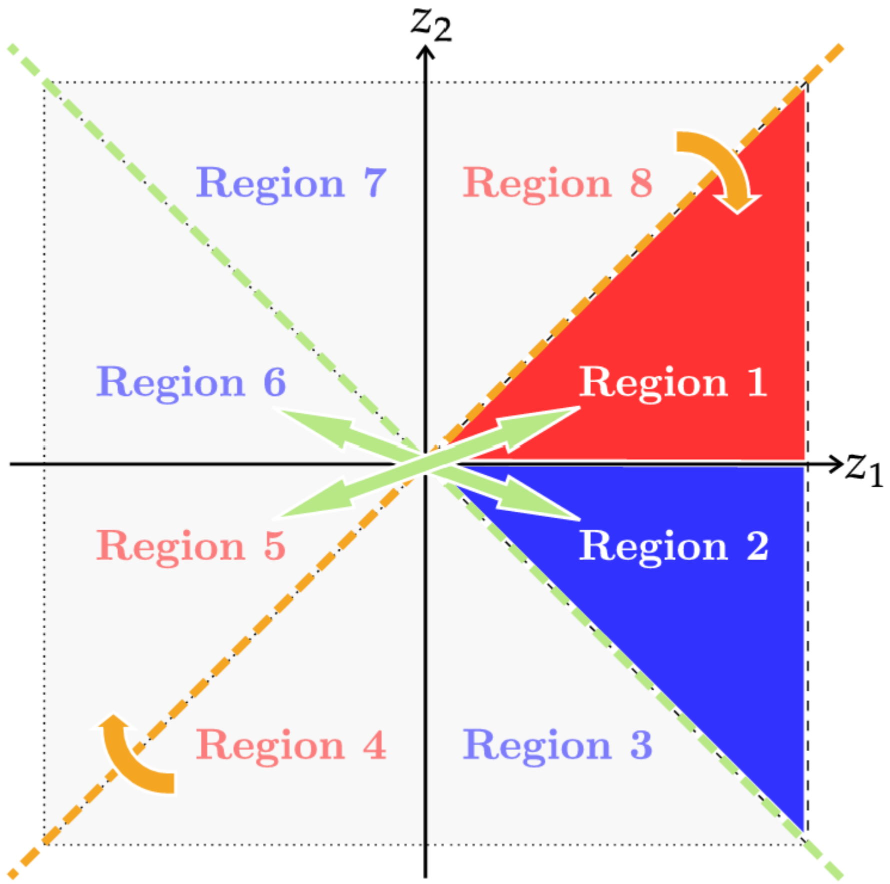


- 3-quark operator with 2 geometrically intertwined Wilson-line lengths $\boldsymbol{z_1}$ and $\boldsymbol{z_2}$;
- Genuinely 2-dimensional distribution among whole $(\boldsymbol{z_1}, \boldsymbol{z_2})$ plane.

Central challenge !

Symmetries on (z_1, z_2) plane

Genuinely 2-dimensional distribution



- **Exchange symmetry of $x_1 \leftrightarrow x_2$**

for Lambda:

$$\widetilde{\phi}_{\Lambda}^{V,T}(x_1, x_2) = -\widetilde{\phi}_{\Lambda}^{V,T}(x_2, x_1) ,$$
$$\widetilde{\phi}_{\Lambda}^{A}(x_1, x_2) = +\widetilde{\phi}_{\Lambda}^{A}(x_2, x_1) ,$$

other octets:

$$\widetilde{\phi}_{B\neq\Lambda}^{V,T}(x_1, x_2) = +\widetilde{\phi}_{B\neq\Lambda}^{V,T}(x_2, x_1) ,$$
$$\widetilde{\phi}_{B\neq\Lambda}^{A}(x_1, x_2) = -\widetilde{\phi}_{B\neq\Lambda}^{A}(x_2, x_1) .$$

- **Hermiticity condition**

$$\mathrm{Re}\ \widetilde{\phi}_{B}^{X}(x_1, x_2) = \widetilde{\phi}_{B}^{X}(x_1, x_2) ,$$
$$\mathrm{Im}\ \widetilde{\phi}_{B}^{X}(x_1, x_2) = 0 ,$$

$\Rightarrow$ $$\widetilde{\Phi}_{B}^{X}(-z_1, -z_2) = \left[\widetilde{\Phi}_{B}^{X}(z_1, z_2)\right]^* ,$$

Lattice simulation

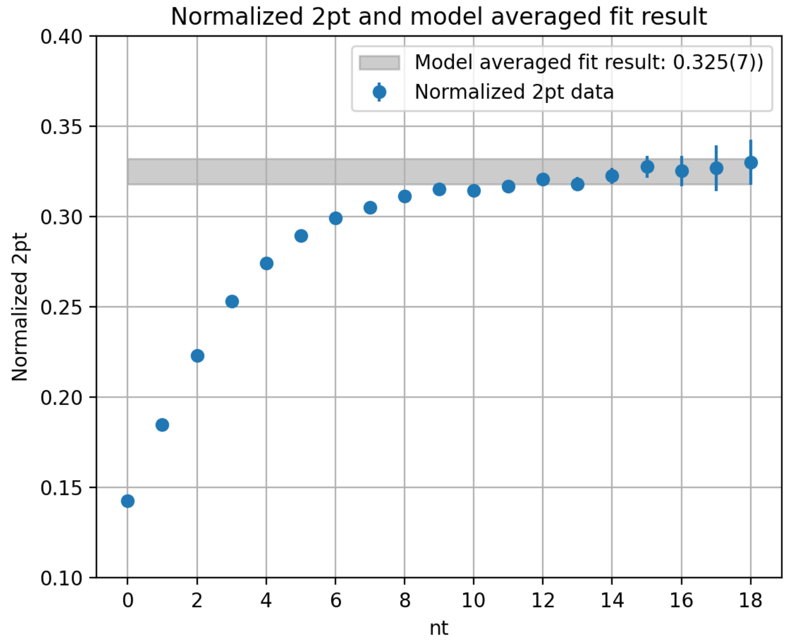
Nonlocal 2-point function related to baryon quasi-DAs:

$$C_2(z_1, z_2; t, P^z) = \int \mathrm{d}^3x\ \mathrm{e}^{-\mathrm{i}\vec{x}\cdot\vec{P}} \langle 0| \mathcal{O}^{\gamma'}_{\mathrm{Sink}}(\vec{x}, t; z_1, z_2) \overline{\mathcal{O}}^{\gamma}_{\mathrm{Src}}(0, 0; 0, 0) T_{\gamma'\gamma} |0\rangle$$

- **Extract ground-state quasi-DA from 2-state fit**

LPC, PRD 112, 114515 (2025);

$$\frac{C_{\mathrm{2pt}}(\tau, P^z; z_1, z_2)}{C_{\mathrm{2pt}}(\tau, P^z; 0, 0)} = \underbrace{\widehat{M}(z_1, z_2; P^z)}_{\text{ground-state matrix element}} \left[1 + \underbrace{\Delta C_1(z_1, z_2; P^z)\ \mathrm{e}^{-\Delta E_1(z_1, z_2; P^z)\tau}}_{\text{excited-states contribution}}\right]$$



Lattice simulation: Improvements

Nonlocal 2-point function related to baryon quasi-DAs:

$$C_2(z_1, z_2; t, P^z) = \int d^3x \; e^{-i\vec{x}\cdot\vec{P}} \langle 0 | \underbrace{\mathcal{O}^{\gamma'}_{\rm Sink}(\vec{x}, t; z_1, z_2)}_{\text{Determined by DA}} \underbrace{\overline{\mathcal{O}}^{\gamma}_{\rm Src}(0,0;0,0) T_{\gamma'\gamma}}_{\text{Up to our choice !}} | 0 \rangle$$

- **Interpolators and projection operators**

□ Sink-side Interpolators

$$\mathcal{O}^{\mathrm{snk},A}(z_1, z_2) = q^{\mathrm{T}}(z_1)\left(C\gamma^5\gamma^t\right) g(z_2) h(x) \ ,$$

$$\mathcal{O}^{\mathrm{snk},V}(z_1, z_2) = q^{\mathrm{T}}(z_1)\left(C\gamma^t\right) g(z_2)\gamma^5 h(x) \ ,$$

$$\mathcal{O}^{\mathrm{snk},T}(z_1, z_2) = q^{\mathrm{T}}(z_1)(\tfrac{1}{2}C[\gamma^t, \gamma^{x,y}]) g(z_2)\gamma^5\gamma_{x,y} h(x) \ .$$

□ Projector

$$T = I, \gamma^t, \gamma^z \quad ?$$

□ Source-side Interpolators

$$\mathcal{O}^{\rm src}_{\Lambda} = \frac{2u^{\mathrm{T}}(C\gamma^5)ds + u^{\mathrm{T}}(C\gamma^5)sd + s^{\mathrm{T}}(C\gamma^5)du}{\sqrt{6}} \ ,$$

$$\mathcal{O}^{\rm src}_{\rm p} = u^{\mathrm{T}}(C\gamma^5)du \ . \quad ?$$

Lattice simulation: Improvements

Improvement from interpolator: Kinematically-enhancement

- Traditional interpolator: $N_{\gamma_5} = \epsilon^{ijk}(f_i^T \; C\gamma_5 \; g_j)h_k$

  $\langle 0|N_{\gamma_5}|N(P)\rangle = \lambda u(P)$

  for static baryon states

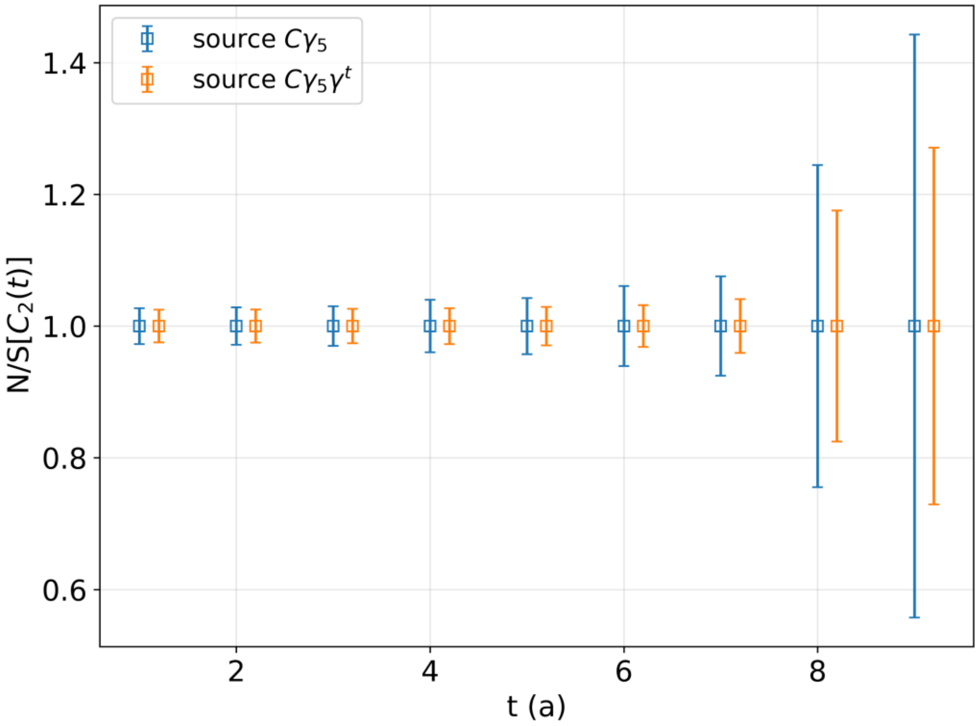
- Enhanced interpolator : $N_{\gamma_5\gamma_\mu} = \epsilon^{ijk}(f_i^T \; C\gamma_5\gamma_\mu \; g_j)h_k$

  $\langle 0|N_{\gamma_5\gamma_\mu}|N(P)\rangle = \alpha P_\mu u(P) + \beta\gamma_\mu u(P)$

  better overlap with boosted states

$$\Rightarrow \quad \mathrm{SNR}(C_{2\mathrm{pt}}(t\to\infty)) \propto \frac{P_\mu}{M_0} \mathrm{e}^{-E' t}$$

LPC, PRD 112, 114515 (2025);



R.Zhang et.al, PRD 112, L051502 (2025); D.Reitinger et.al, arXiv:2606.02447

Lattice simulation: Improvements

Improvement from projector

- Traditional parity projector

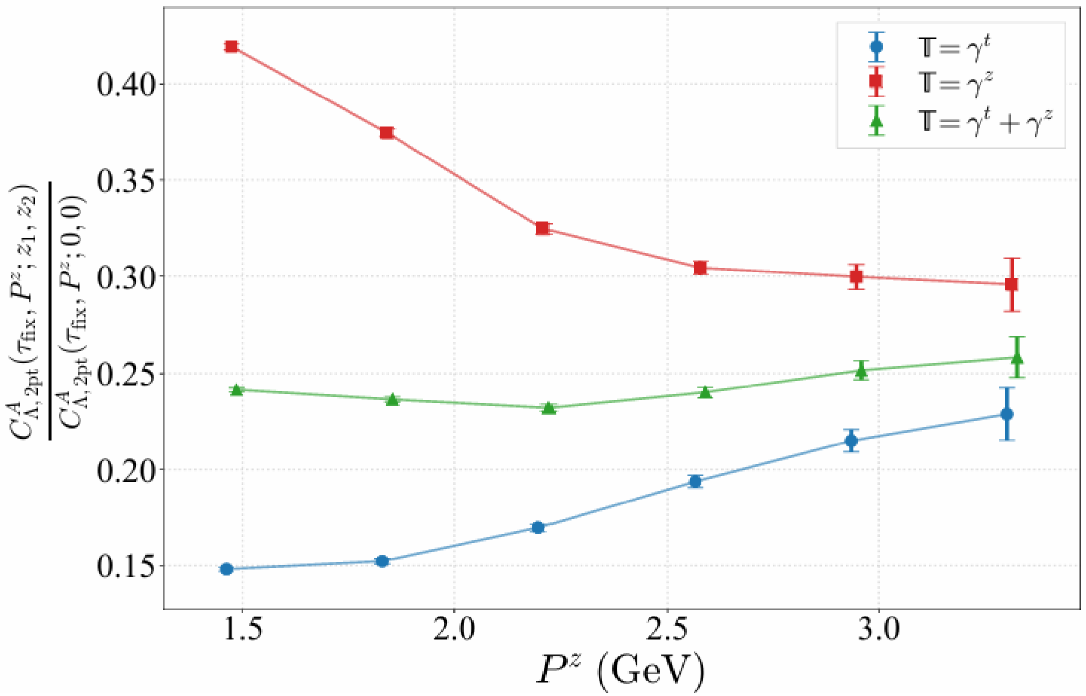
$$\mathbb{T}_{\text{parity}} = \frac{1+\gamma^t}{2}$$

Parity is not a good quantum number when boosted !

- consider spinor structure $\text{tr}\left[u_B(n\cdot P)\not{n}u_B(n\cdot P)\right]$

$$\mathbb{T} = \not{n} = \gamma^t + \gamma^z$$

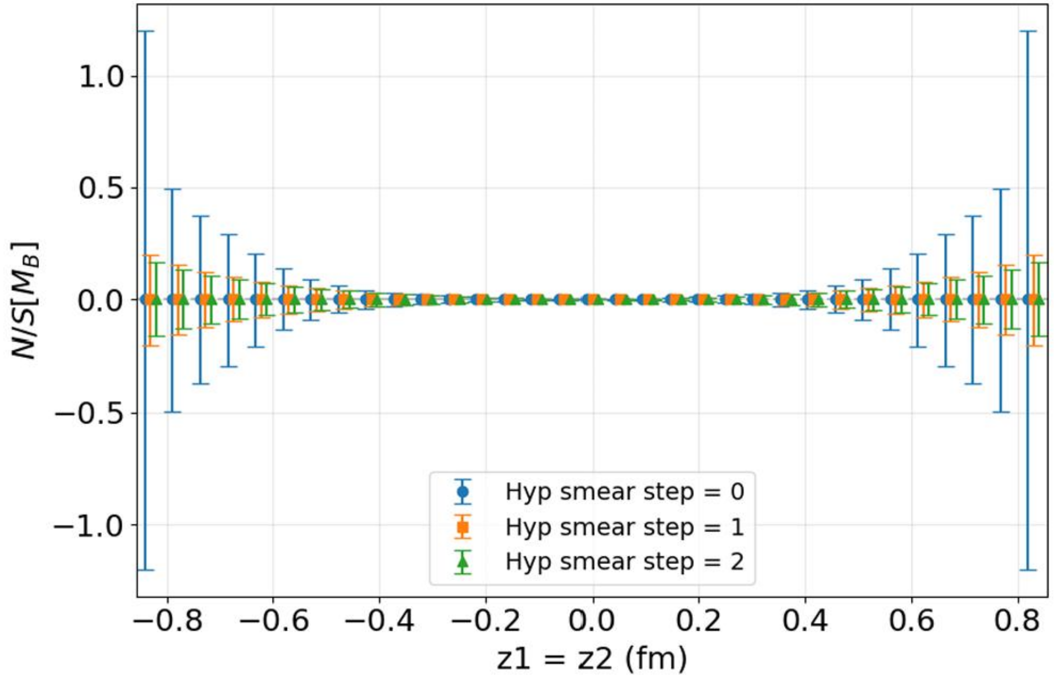
- *for leading-twist contribution;*
- *suppress sub-leading powers*



Lattice simulation: Improvements

Other improvement strategies

- **for gauge link: HYP smearing**



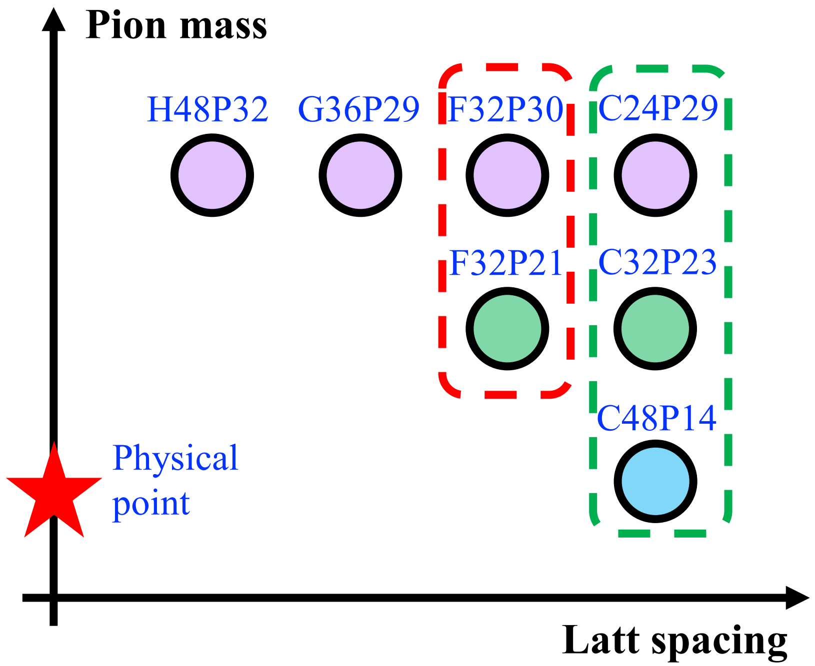
- **for propagator: momentum smearing**

LPC, PRD 112, 114515 (2025)

Lattice simulation

CLQCD Ensembles

- ☐ 4 lattice spacings for **continuum** limit
- ☐ 3 pion masses for **physical mass** limit
- ☐ multi momenta for $P_z \to \infty$ limit



Ensemble	Lattice spacing	π mass	measurements	Pz / GeV
C24P29	0.105 fm	292 MeV	864 *4 src *9 nt	0, 1.96, 2.45, 2.94
C32P23	0.105 fm	228 MeV	954 *4 src *8 nt	0, 1.84, 2.21, 2.57, 2.94
C48P14	0.105 fm	136 MeV	302 *4 src *16 nt	0, 1.96, 2.45, 2.94
F32P30	0.078 fm	300 MeV	777 *4 src *8 nt	0, 2.00, 2.49, 2.99
F32P21	0.078 fm	210 MeV	459 *4 src *16 nt	0, 2.00, 2.49, 2.99
G36P29	0.069 fm	297 MeV	656 *6 src *8 nt	0, 2.00, 2.50, 3.00
H48P32	0.052 fm	317 MeV	550 *6 src *9 nt	0, 1.98, 2.48, 2.98

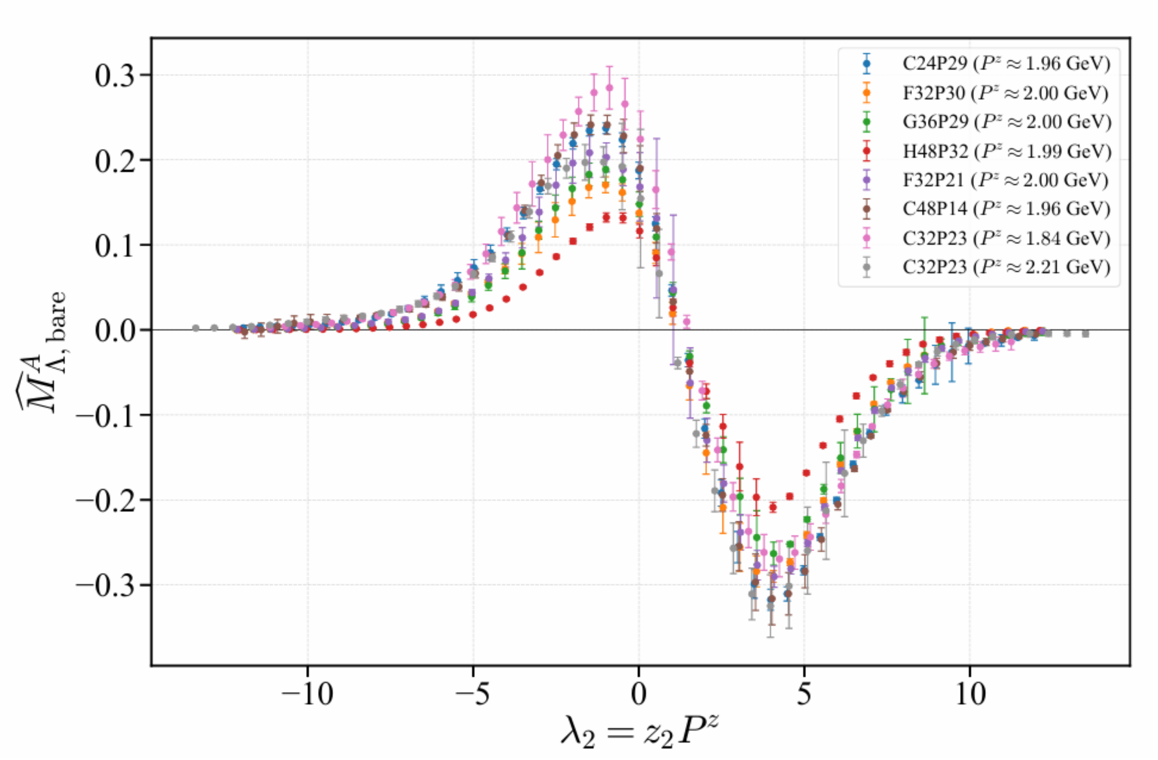
02 Renormalization for nonlocal operators

Hybrid Renormalization

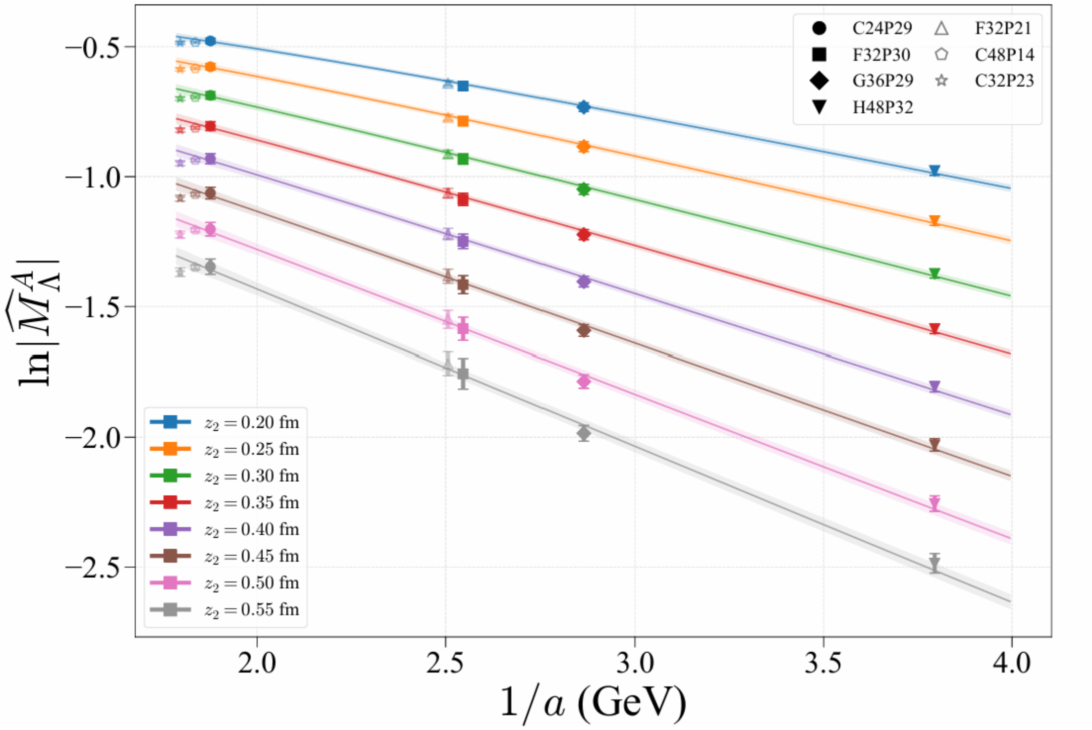
Linear divergences in Bare quasi-DAs

$$M(z_1, z_2; P^z, a) \propto \exp\left[\left(\frac{k}{a\ln(a\Lambda_{\rm QCD})} + m_0\right)\tilde{z}\right] m(z_1, z_2, P^z)$$

bare matrix elements



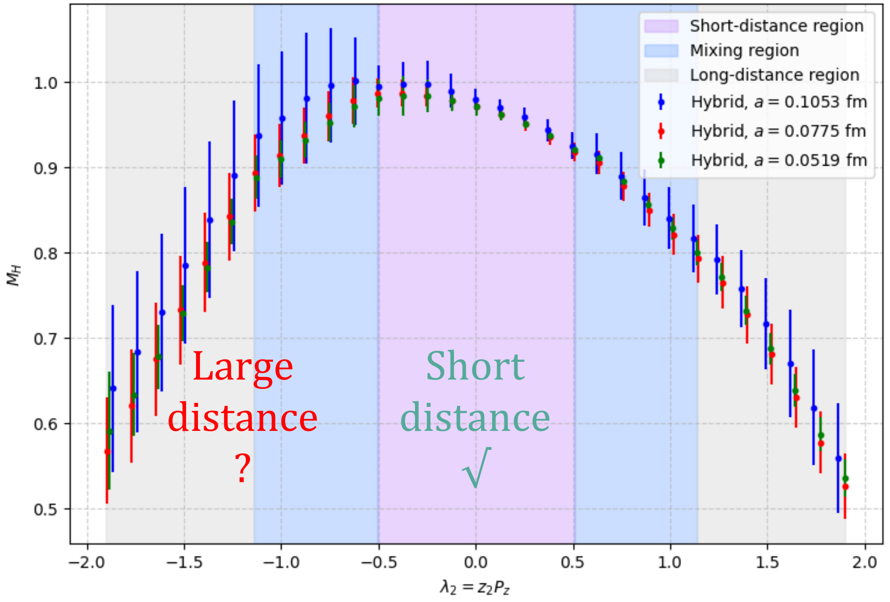
log M v.s. $1/a$

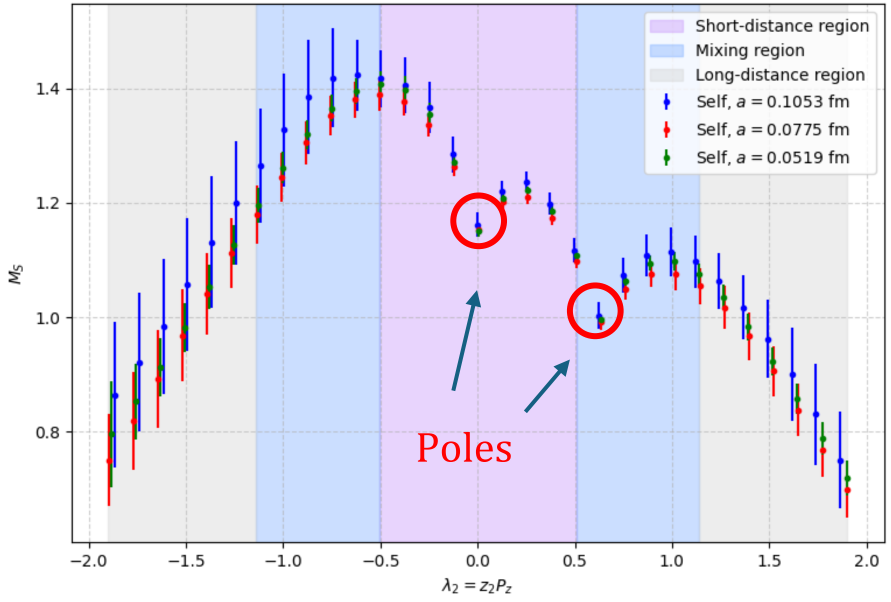


Hybrid Renormalization

Hybrid Renormalization:

adopt different schemes in different regions





- **Ratio prescription:**

  cancel divergences at short distance

- **Self-Renormalization:**

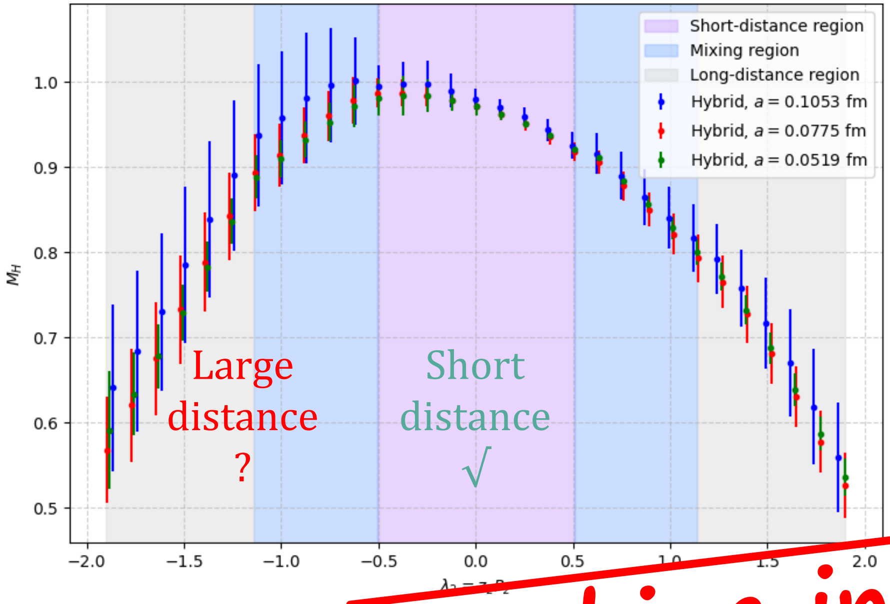
  control long-distance behavior

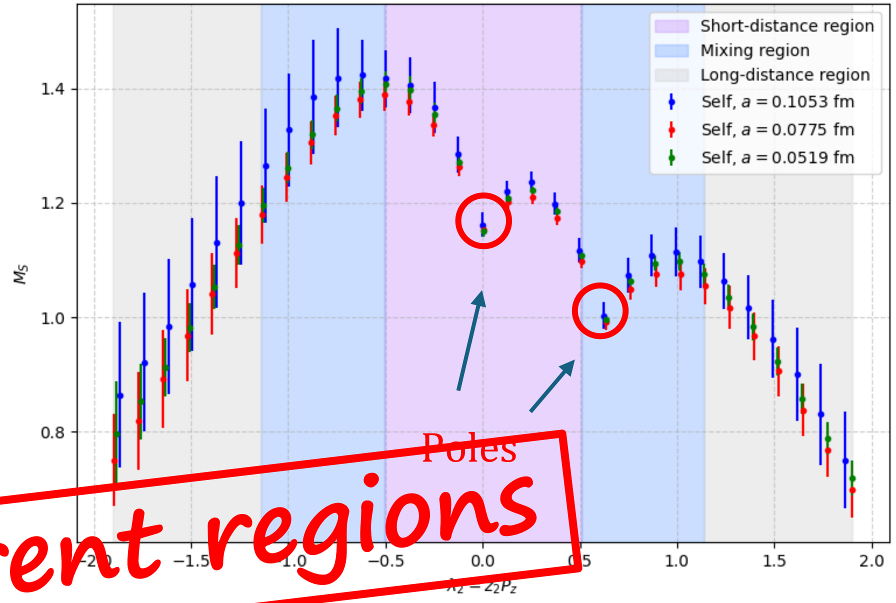
X.Ji et.al, NPB 2021; Y.K.Huo et.al, NPB 2021; Radyushkin et.al, PRD 2017;

Hybrid Renormalization

Hybrid Renormalization:

adopt different schemes in different regions





combine in different regions

- **Ratio prescription:**

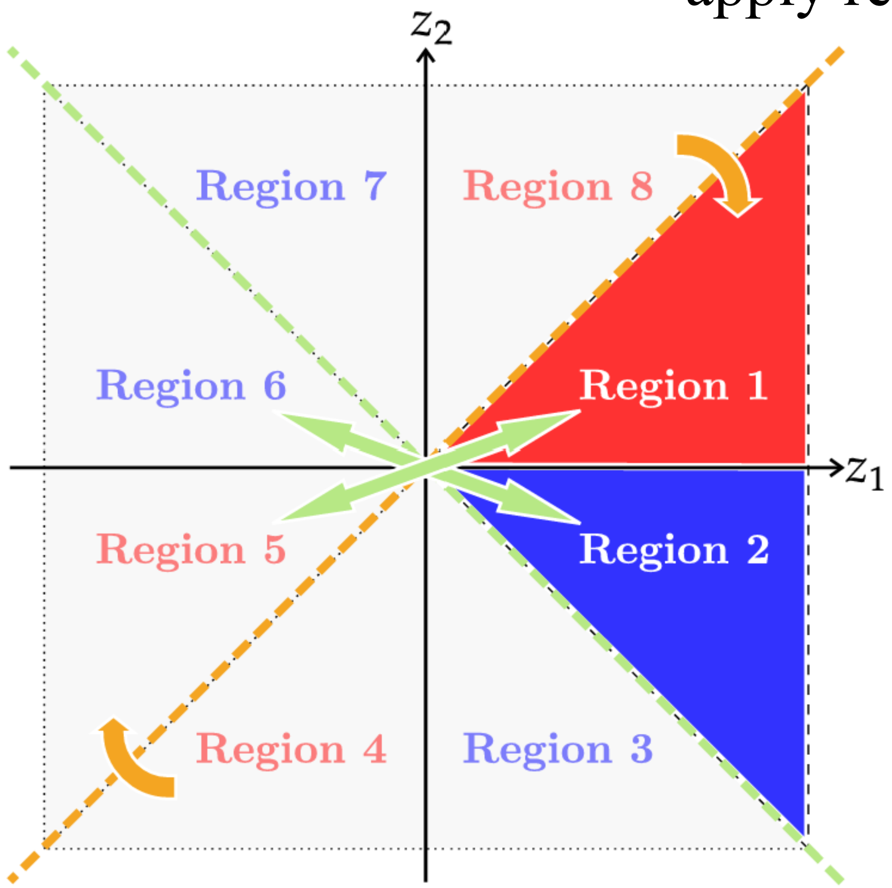
  cancel divergences at short distance

- **Self-Renormalization:**

  control long-distance behavior

2-Dimensional Hybrid Scheme

—— apply renormalization uniformly over full (z1,z2) plane

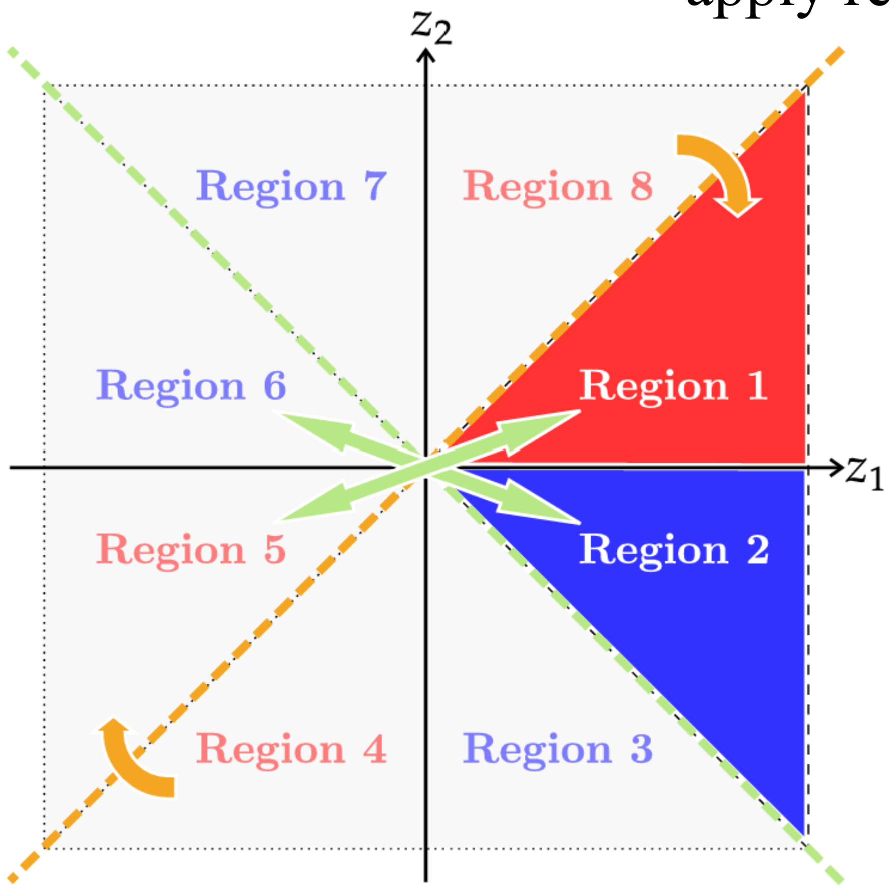


region partition by short-long distances:

| z1 |, | z2 |, | z1-z2 |

2-Dimensional Hybrid Scheme

—— apply renormalization uniformly over full (z1,z2) plane



region partition by

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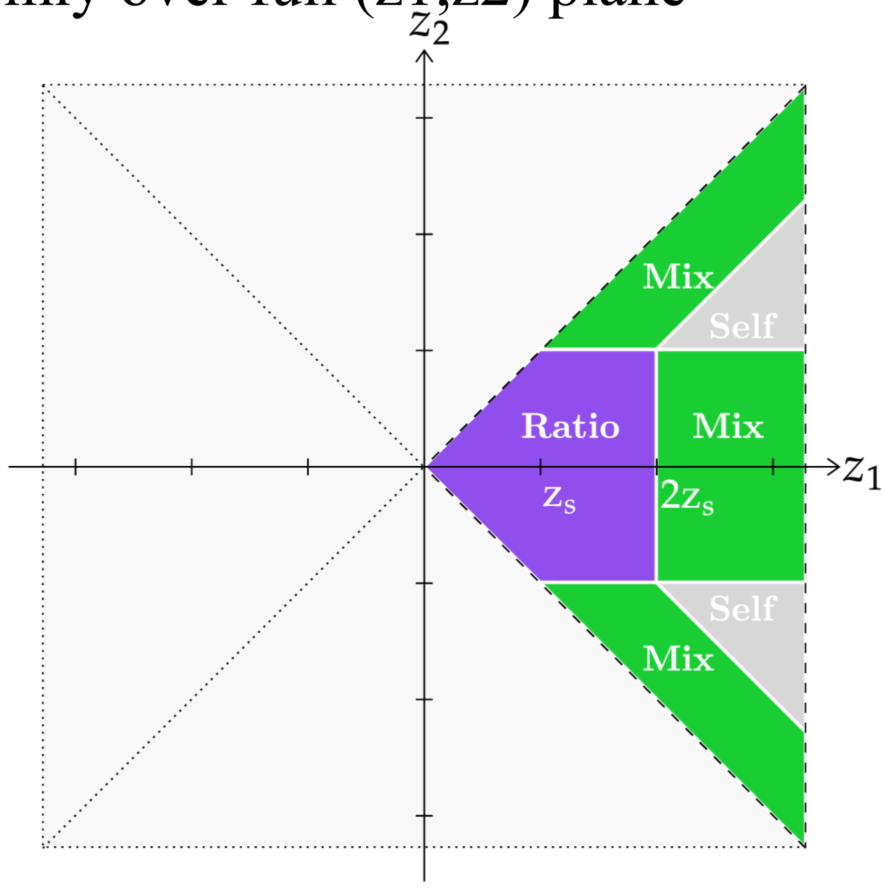
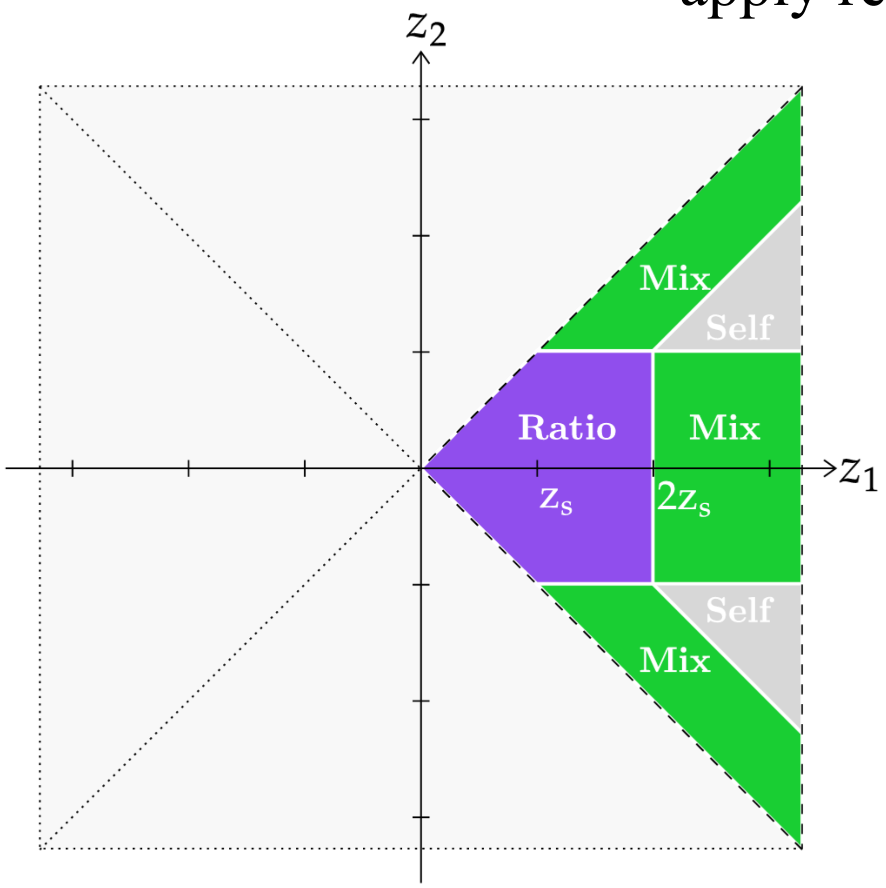
| z1 |, | z2 |, | z1-z2 |



2-Dimensional Hybrid Scheme

—— apply renormalization uniformly over full (z1,z2) plane



- **1) Short-distance regions: Ratio**

$$\frac{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, 0, \mu)}$$

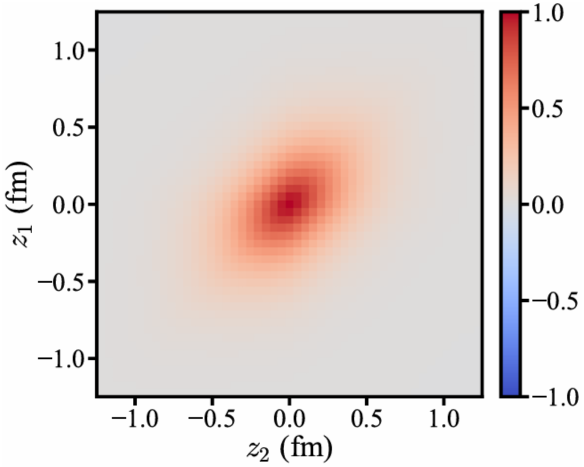
- **2) Long-distance regions: Self-renormalization**

$$\frac{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\mathrm{MS}}}(\mathrm{sign}(z_1)z_s, \mathrm{sign}(z_2)2z_s, 0, 0, \mu)}$$

- **3) Mixing regions**

$$\frac{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\mathrm{MS}}}(z_1, \mathrm{sign}(z_2)2z_s, 0, 0, \mu)} \cdot \theta(z_s - |z_1|)\theta(|z_2| - 2z_s)$$

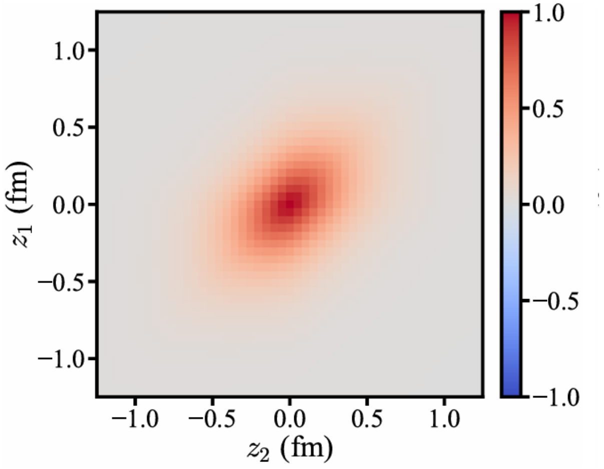
Hybrid Renormalization



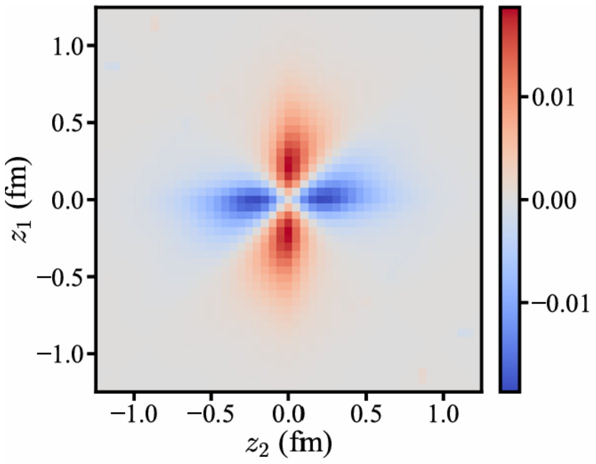
(a) Λ-baryon A

LPC, PRD 112, 114515 (2025)
for symmetric amplitude

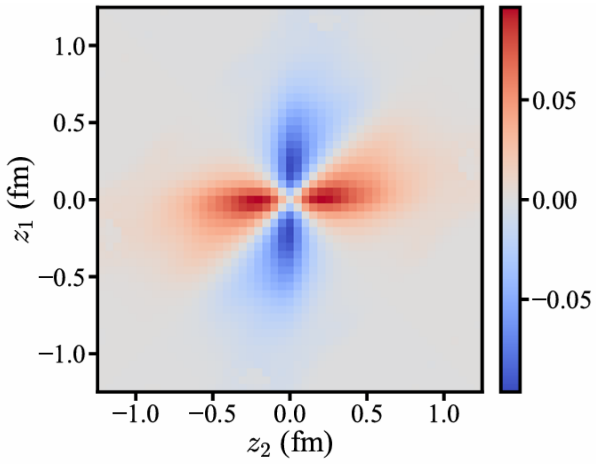
Hybrid Renormalization



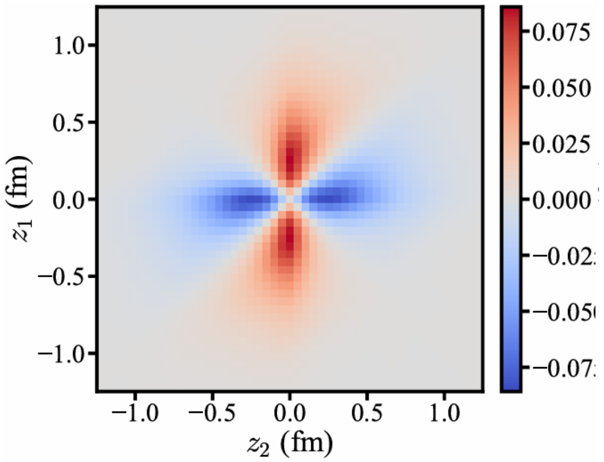
(a) Λ-baryon A



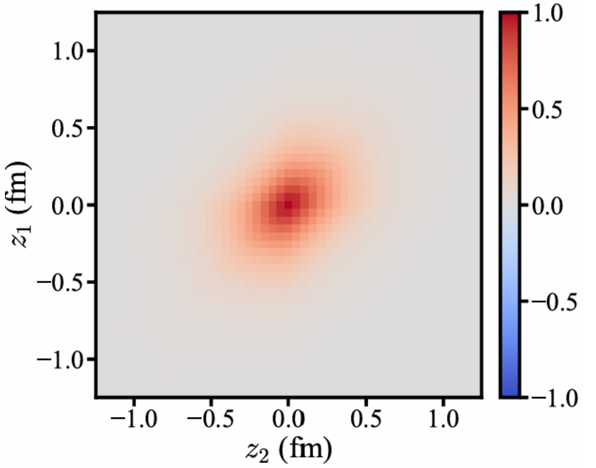
(b) Λ-baryon V



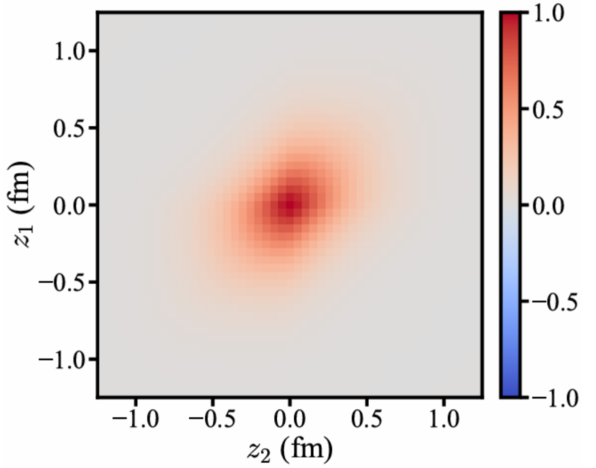
(c) Λ-baryon T



(d) proton A

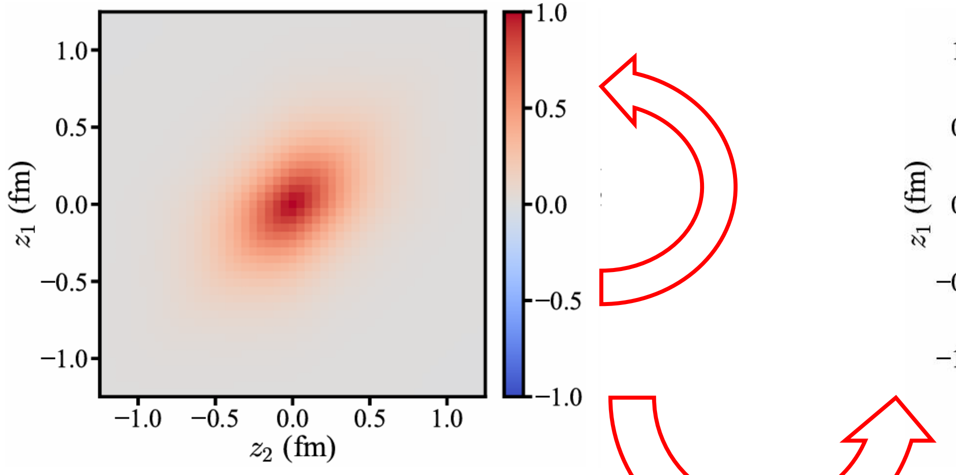


(e) proton V

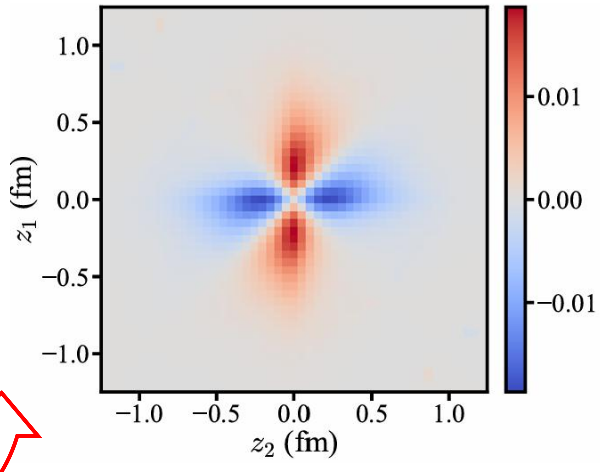


(f) proton T

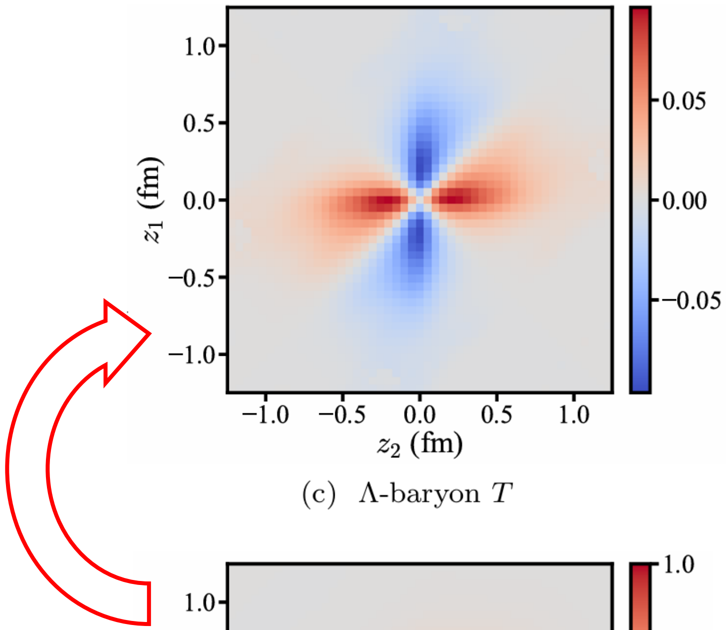
Hybrid Renormalization



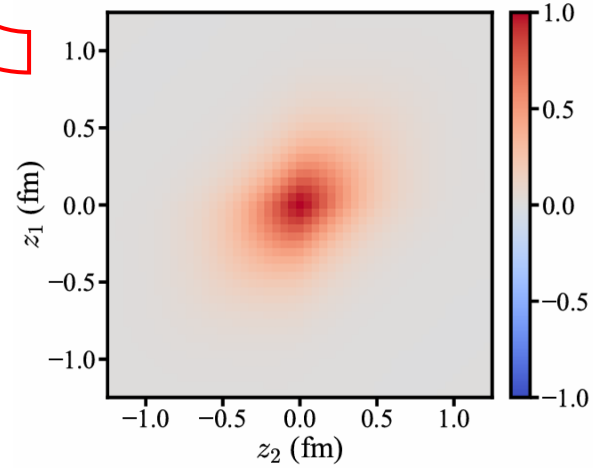
(a) Λ-baryon A



(b) Λ-baryon V



(c) Λ-baryon T



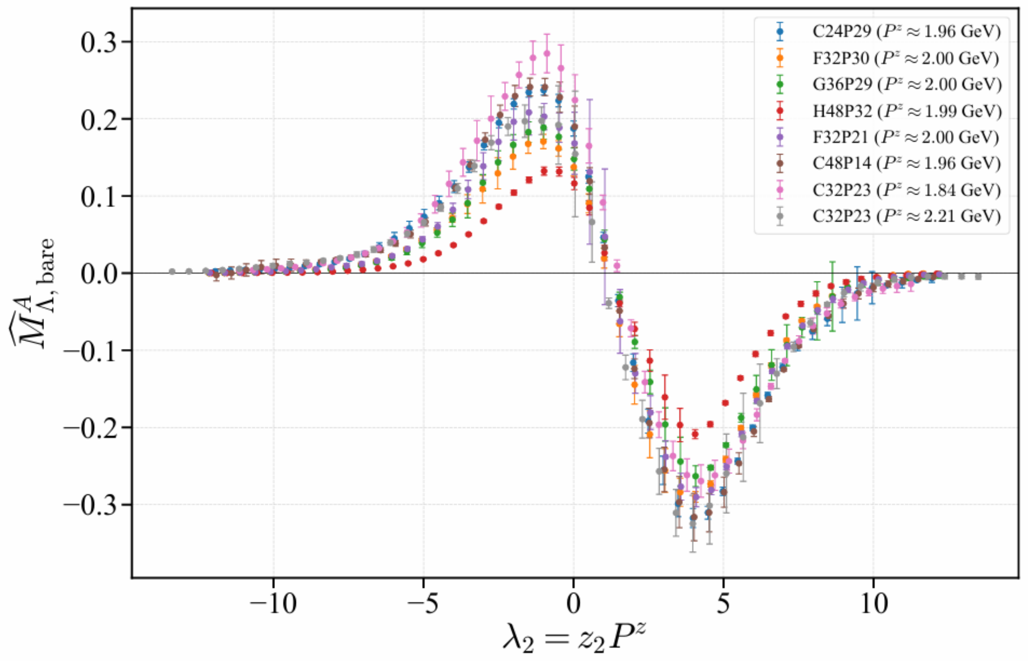
(f) proton T

$$
\begin{aligned}
\widehat{M}^{V,A}_{\overline{\rm MS}}(z_1,z_2;P^z=0,\mu) &= 1+\frac{\alpha_s C_F}{2\pi}\left(\frac{7}{8}\ln\frac{z_1^2\mu^2 e^{2\gamma_E}}{4}+\frac{7}{8}\ln\frac{z_2^2\mu^2 e^{2\gamma_E}}{4}+\frac{3}{4}\ln\frac{(z_1-z_2)^2\mu^2 e^{2\gamma_E}}{4}+4\right)\\
\widehat{M}^{T}_{\overline{\rm MS}}(z_1,z_2;P^z=0,\mu) &= 1+\frac{\alpha_s C_F}{2\pi}\left(\frac{7}{8}\ln\frac{z_1^2\mu^2 e^{2\gamma_E}}{4}+\frac{7}{8}\ln\frac{z_2^2\mu^2 e^{2\gamma_E}}{4}+\frac{1}{2}\ln\frac{(z_1-z_2)^2\mu^2 e^{2\gamma_E}}{4}+\frac{13}{4}\right)
\end{aligned}
\tag{44}
$$

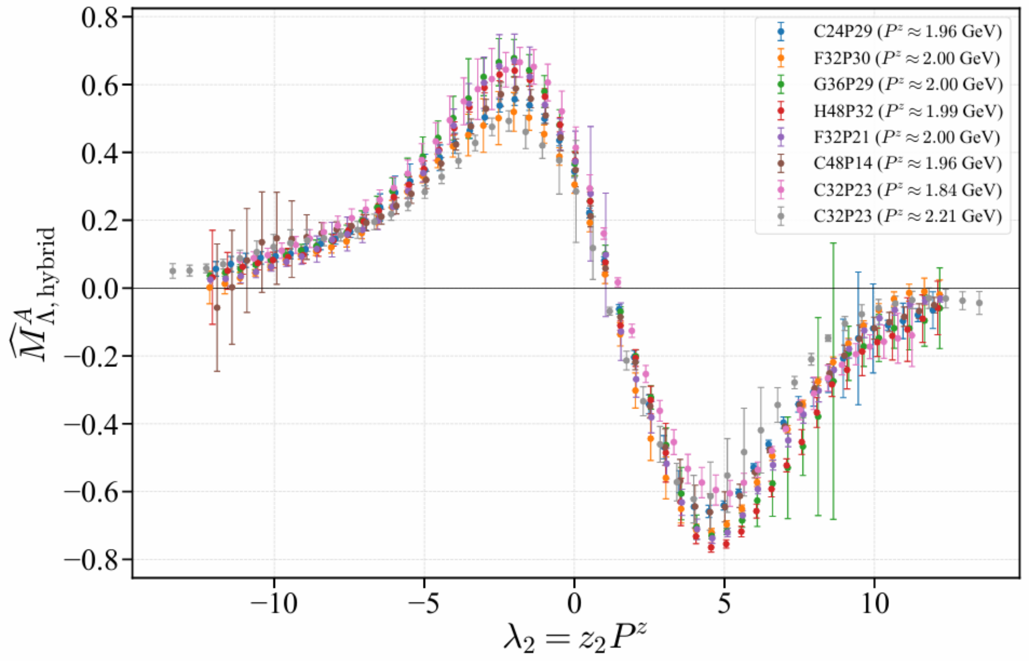
Due to symmetry properties and vanishing local limits, we introduce **different treatment for different amplitudes;** details see article arXiv:2606.30387

Hybrid Renormalization

Bare Matrix Element



Hybrid Renormalized result

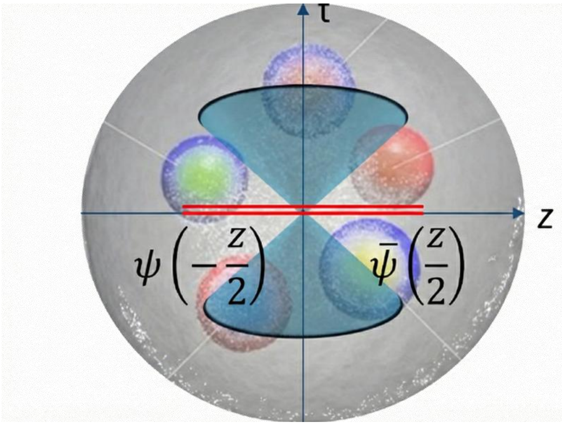


- Satisfyingly eliminate linear divergence
- Well defined at short-, long- & mixing-regions
- Smooth between different regions

03 Large-distance Extrapolation

Large-distance Extrapolation

Asymptotic expansion



$$z \to +\infty \quad \sim \quad \frac{e^{-\Lambda z} e^{i\phi(z)}}{z^{\alpha}}$$

Asymptotic Long-Distance Expansion of Euclidean Correlators in Lattice Parton Applications

X. Ji, Y. Liu, Y. Su, arXiv:2601.12189

$$Q(x)\bar{Q}(0) \longrightarrow \underline{U(x,0)}\, D_Q(x^2, m_Q^2)\, \frac{i\not{x} + \sqrt{-x^2}}{2\sqrt{-x^2}} \left[1 + \mathcal{O}\left(\frac{1}{m_Q\sqrt{-x^2}}, \frac{\Lambda_{\rm QCD}}{m_Q}\right)\right] ,$$

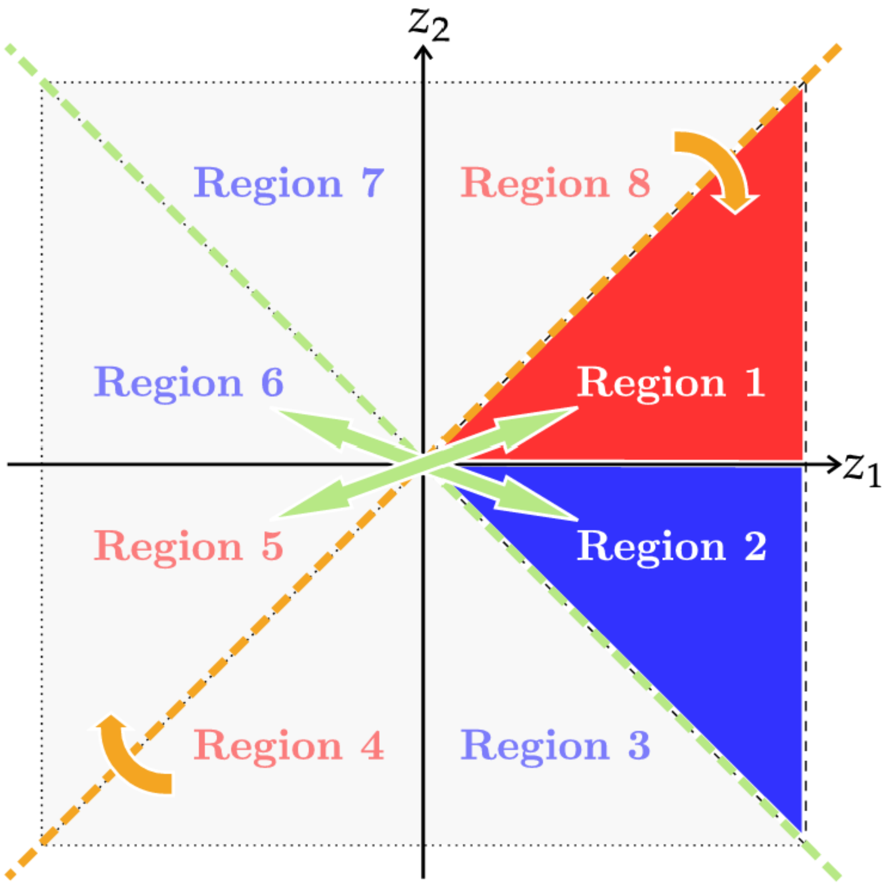
Expand in large Euclidean separation |*z*|:

$$\tilde{h}(z, P^z) = \sum_{\Lambda^{J^P}} \mathrm{e}^{-\Lambda^{J^P}|z|} \times \left[\left(\mathcal{H}_1 \mathrm{e}^{-\mathrm{i}zP^z} + \mathcal{H}_2\right) + \left(\mathcal{H}'_1 \mathrm{e}^{-\mathrm{i}zP^z} + \mathcal{H}'_2\right)\frac{1}{|z|} + \cdots\right] ,$$

LA NLA

Large-distance Extrapolation

Generalize to baryon-DA case:

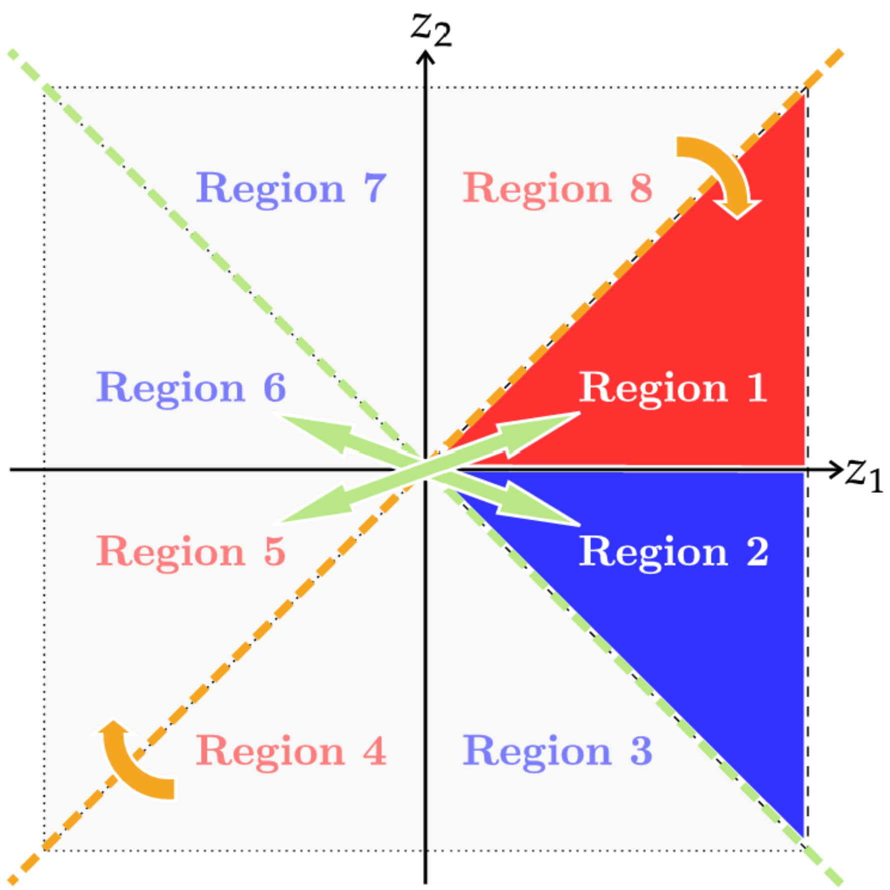


3 relevant distances:

| z1 |, | z2 |, | z1-z2 |

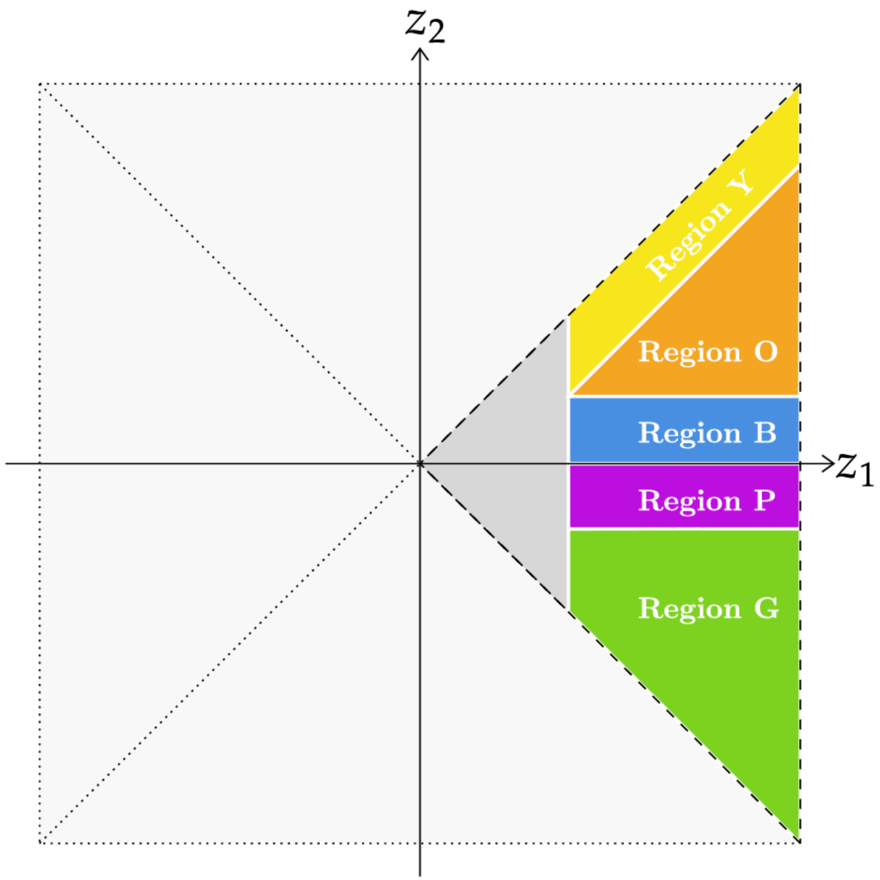
Large-distance Extrapolation

Generalize to baryon-DA case:



3 relevant distances:

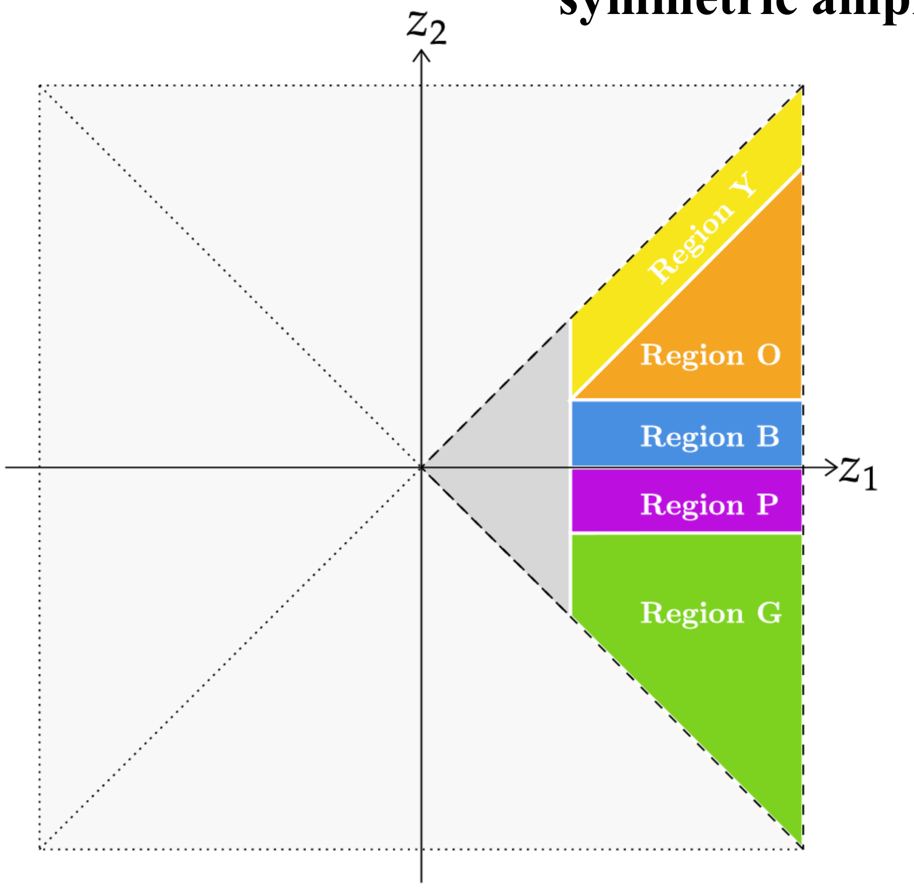
| z1 |, | z2 |, | z1-z2 |



Large-distance Extrapolation

for Lambda A:

symmetric amplitude with good signal quality



- **Region G**: $z_1 z_2 < 0$, $|z_1| \to \infty$, $|z_2| \to \infty$ and $|z_1 - z_2| \to \infty$

$$\widetilde{A}^{\rm NLA}(z_1, z_2, P^z) = e^{-\Lambda_0 \cdot |z_1|} e^{-\Lambda_0 \cdot |z_2|} \left[G_A(i\hat{z}_1, i\hat{z}_2, P^z) + G'_A(i\hat{z}_1, i\hat{z}_2, P^z) \frac{1}{|z|} \right] ;$$

- **Region P**: $z_1 z_2 < 0$, $|z_1| \to \infty$, $|z_1 - z_2| \to \infty$ while $|z_2|$ kept finite

$$\widetilde{A}^{\rm NLA}(z_1, z_2, P^z) = e^{-\Lambda_0 \cdot |z_2|} e^{i z_1 P^z} \left[P_{A,1}(i z_1, i\hat{z}_2, P^z) + P'_{A,1}(i z_1, i\hat{z}_2, P^z) \frac{1}{|z|} \right] + e^{-\Lambda_0 \cdot |z_2|} \left[P_{A,2}(i z_1, i\hat{z}_2, P^z) + P'_{A,2}(i z_1, i\hat{z}_2, P^z) \frac{1}{|z|} \right] ;$$

- **Region B**: $z_1 z_2 > 0$, $|z_1| \to \infty$, $|z_1 - z_2| \to \infty$ while $|z_2|$ kept finite

$$\widetilde{A}^{\rm NLA}(z_1, z_2, P^z) = e^{-\Lambda_0 \cdot |z_2 - z_1|} e^{i z_1 P^z} \left[B_{A,1}(-i z_1, i\hat{z}_2, P^z) + B'_{A,1}(-i z_1, i\hat{z}_2, P^z) \frac{1}{|z|} \right] + e^{-\Lambda_0 \cdot |z_2 - z_1|} \left[B_{A,2}(-i z_1, i\hat{z}_2, P^z) + B'_{A,2}(-i z_1, i\hat{z}_2, P^z) \frac{1}{|z|} \right] ;$$

- **Region O**: $z_1 z_2 > 0$, $|z_1| \to \infty$, $|z_2| \to \infty$ and $|z_1 - z_2| \to \infty$

$$\widetilde{A}^{\rm NLA}(z_1, z_2, P^z) = e^{-\Lambda_0 \cdot |z_1|} e^{-\Lambda_0 \cdot |z_2 - z_1|} e^{i z_1 P^z} \left[O_A(i\hat{z}_1, i\hat{z}_2, P^z) + O'_A(i\hat{z}_1, i\hat{z}_2, P^z) \frac{1}{|z|} \right] ;$$

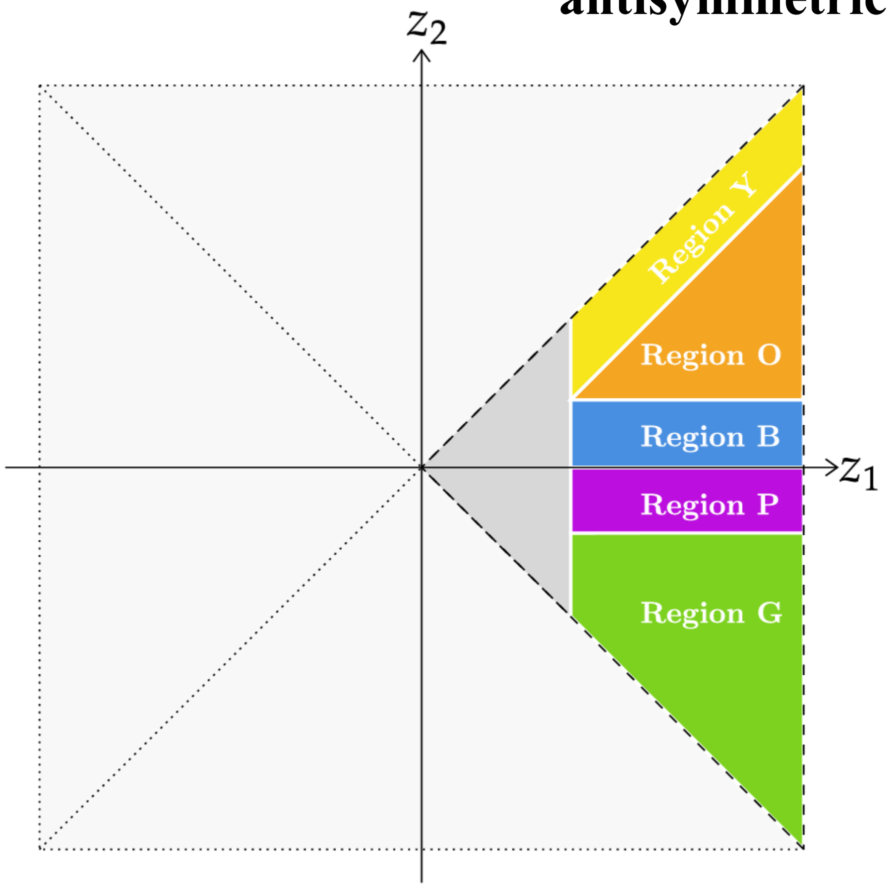
- **Region Y**: $z_1 z_2 > 0$, $|z_1| \to \infty$, $|z_2| \to \infty$ while $|z_1 - z_2|$ kept finite

$$\widetilde{A}^{\rm NLA}(z_1, z_2, P^z) = e^{-\Lambda_0 \cdot |z_1|} e^{i z_2 P^z} \left[Y_{A,1}(i\hat{z}_1, i(z_2 - z_1), P^z) + Y'_{A,1}(i\hat{z}_1, i(z_2 - z_1), P^z) \frac{1}{|z|} \right] + e^{-\Lambda_0 \cdot |z_1|} e^{i z_1 P^z} \left[Y_{A,2}(i\hat{z}_1, i(z_2 - z_1), P^z) + Y'_{A,2}(i\hat{z}_1, i(z_2 - z_1), P^z) \frac{1}{|z|} \right]$$

Large-distance Extrapolation

for Lambda V/T:

antisymmetric amplitudes with poor signal qualities



- **Region G**: $z_1 z_2 < 0$, $|z_1| \to \infty$, $|z_2| \to \infty$ and $|z_1 - z_2| \to \infty$

$$\widetilde{V}^{\mathrm{LA}}/\widetilde{T}^{\mathrm{LA}}(z_1, z_2, P^z) = e^{-\Lambda_{0-}|z_1|} e^{-\Lambda_{0+}|z_2|} G_{V/T}(i\hat{z}_1, i\hat{z}_2, P^z) - e^{-\Lambda_{0+}|z_1|} e^{-\Lambda_{0-}|z_2|} G^*_{V/T}(i\hat{z}_1, i\hat{z}_2, P^z) \; ;$$

- **Region P**: $z_1 z_2 < 0$, $|z_1| \to \infty$, $|z_1 - z_2| \to \infty$ while $|z_2|$ kept finite

$$\widetilde{V}^{\mathrm{LA}}/\widetilde{T}^{\mathrm{LA}}(z_1, z_2, P^z) = e^{-\Lambda_{0-}|z_2|} e^{iz_1 P^z} P_{V/T,1}(iz_1, i\hat{z}_2, P^z) + e^{-\Lambda_{0-}|z_2|} P_{V/T,2}(iz_1, i\hat{z}_2, P^z) \; ;$$

- **Region B**: $z_1 z_2 > 0$, $|z_1| \to \infty$, $|z_1 - z_2| \to \infty$ while $|z_2|$ kept finite

$$\widetilde{V}^{\mathrm{LA}}/\widetilde{T}^{\mathrm{LA}}(z_1, z_2, P^z) = e^{-\Lambda_{0-}|z_2 - z_1|} e^{iz_1 P^z} B_{V/T,1}(-iz_1, i\hat{z}_2, P^z) + e^{-\Lambda_{0-}|z_2 - z_1|} B_{V/T,2}(-iz_1, i\hat{z}_2, P^z) \; ;$$

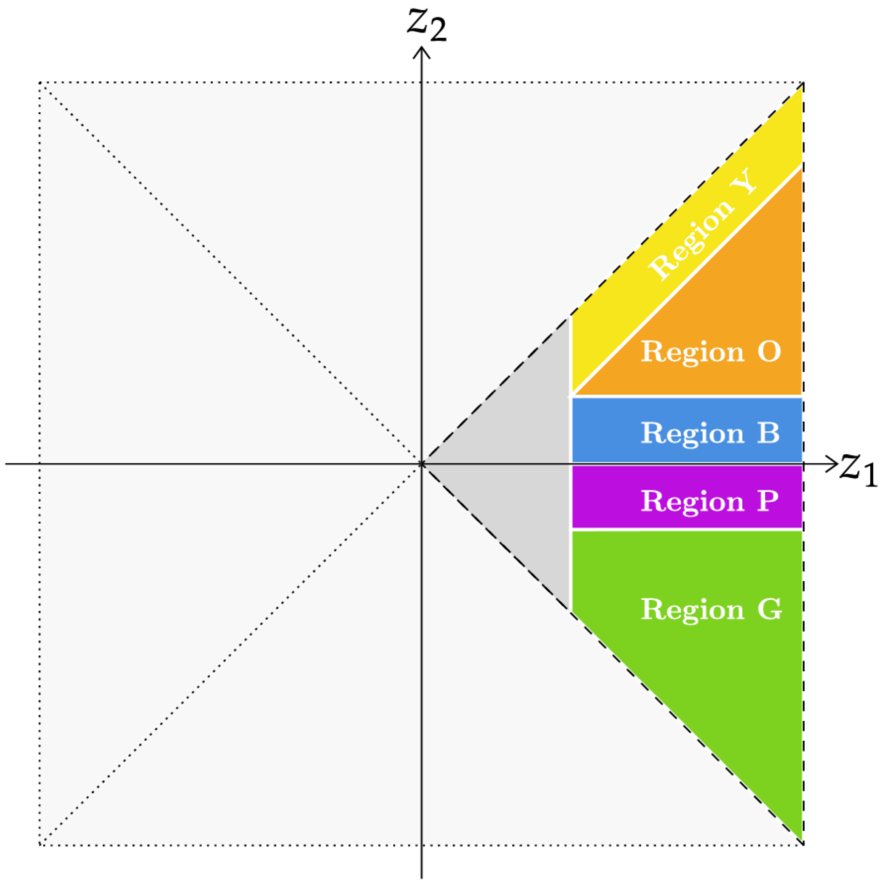
- **Region O**: $z_1 z_2 > 0$, $|z_1| \to \infty$, $|z_2| \to \infty$ and $|z_1 - z_2| \to \infty$

$$\widetilde{V}^{\mathrm{LA}}/\widetilde{T}^{\mathrm{LA}}(z_1, z_2, P^z) = e^{-\Lambda_{0-}|z_1|} e^{-\Lambda_{0-}|z_2 - z_1|} e^{iz_1 P^z} O_{V/T}(i\hat{z}_1, i\hat{z}_2, P^z) \; ;$$

- **Region Y**: $z_1 z_2 > 0$, $|z_1| \to \infty$, $|z_2| \to \infty$ while $|z_1 - z_2|$ kept finite

$$\widetilde{V}^{\mathrm{LA}}/\widetilde{T}^{\mathrm{LA}}(z_1, z_2, P^z) = e^{-\Lambda_{0-}|z_1|} e^{iz_2 P^z} Y_{V/T,1}(i\hat{z}_1, i(z_2 - z_1), P^z) + e^{-\Lambda_{0-}|z_1|} e^{iz_1 P^z} Y_{V/T,2}(i\hat{z}_1, i(z_2 - z_1), P^z) \; ;$$

Large-distance Extrapolation



- **for Lambda A:**

  symmetric amplitude with good signal quality

$$\widetilde{A}^{\mathrm{NLA}}(z_1, z_2, P^z) = e^{-\Lambda_{0-}|z_1|}e^{-\Lambda_{0-}|z_2|}\left[G_A(i\hat{z}_1, i\hat{z}_2, P^z) + G'_A(i\hat{z}_1, i\hat{z}_2, P^z)\frac{1}{|z|}\right] \;;$$

NLA ansaetze

- **for Lambda V / T:**

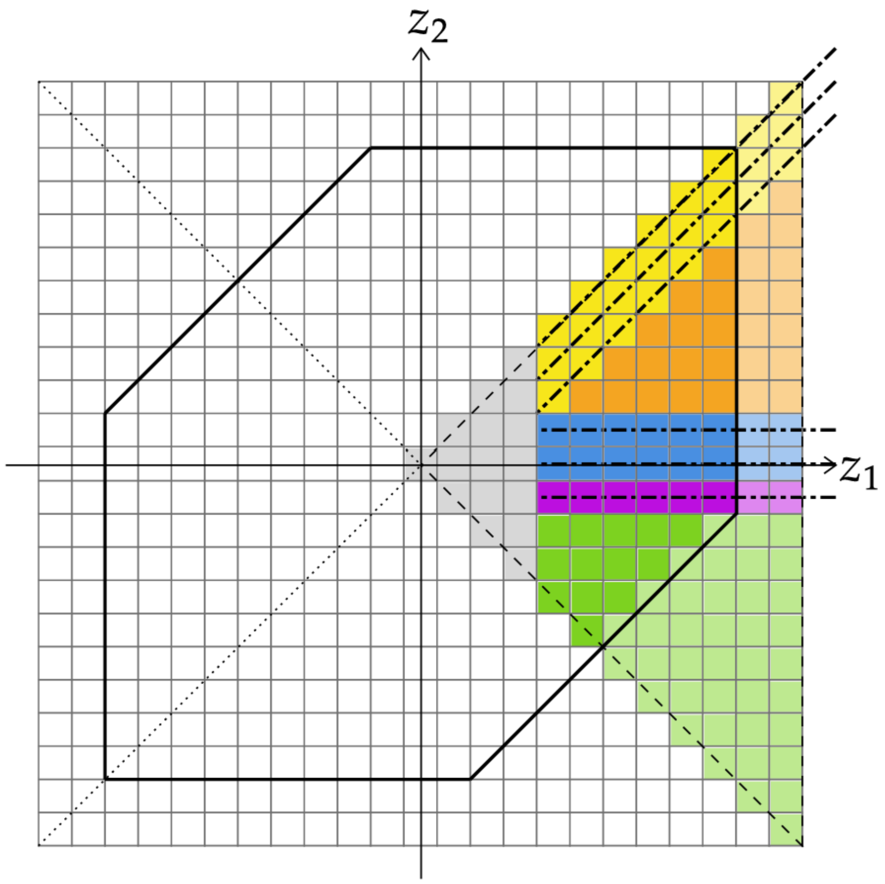
  antisymmetric amplitudes with poor signal qualities

$$\widetilde{V}^{\mathrm{LA}}/\widetilde{T}^{\mathrm{LA}}(z_1, z_2, P^z) = e^{-\Lambda_{0-}|z_1|}e^{-\Lambda_{0+}|z_2|}G_{V/T}(i\hat{z}_1, i\hat{z}_2, P^z)$$

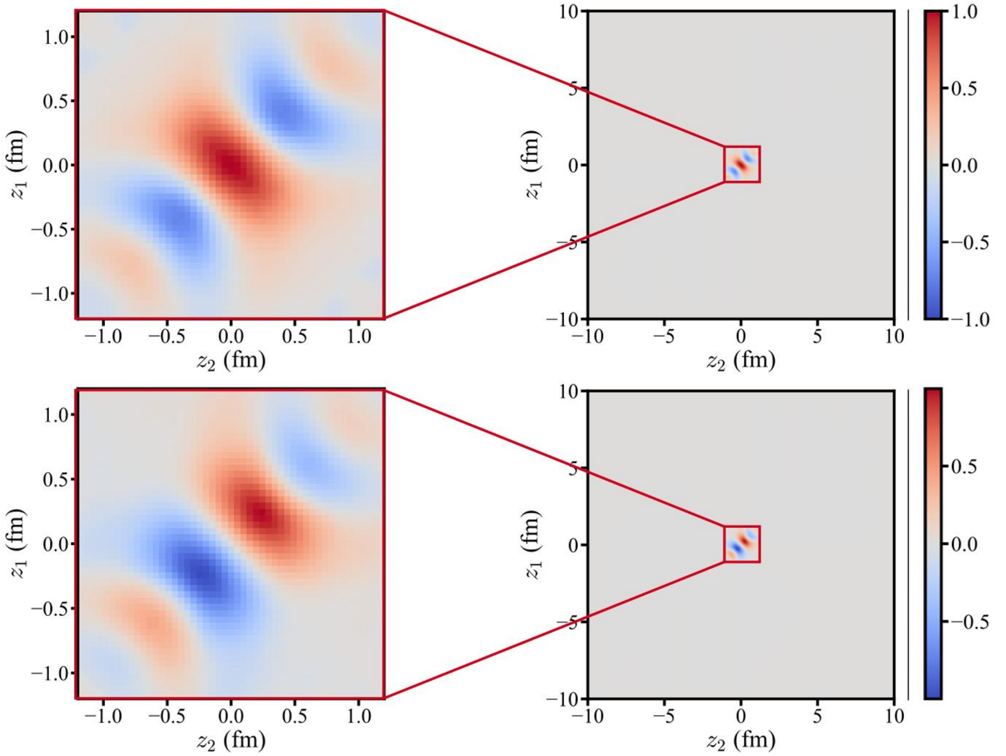
LA ansaetze

Large-distance Extrapolation

Enlarge the data region sufficient for 2-dimensional FT:



1.2 fm ×1.2 fm



10 fm ×10 fm

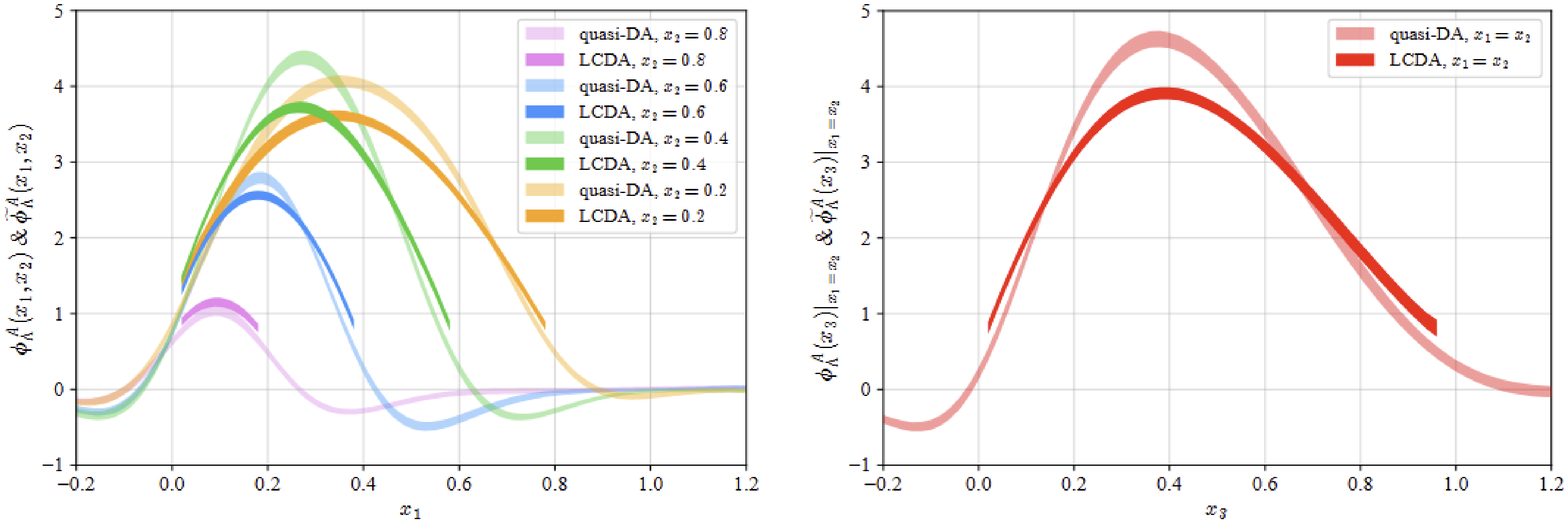
04 Matching & Physical-limit extrapolation

Matching in hybrid scheme

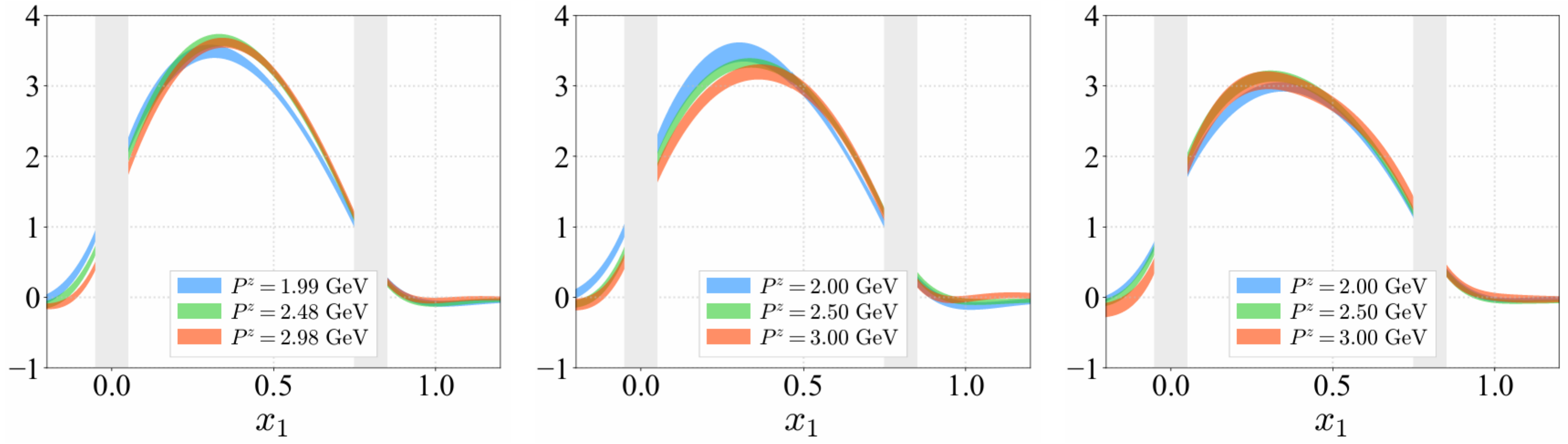
$$\phi(x_1, x_2; \mu) = \int \mathrm{d}y_1 \mathrm{d}y_2\, \mathcal{C}(x_1, x_2; y_1, y_2; P^z, \mu)\, \tilde{\phi}(y_1, y_2; P^z, \mu)$$
$$+ \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{(x_1 P^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(x_2 P^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{[(1 - x_1 - x_2) P^z]^2} \right)$$

$\overline{\mathrm{MS}}$ kernel — hybrid counter-term

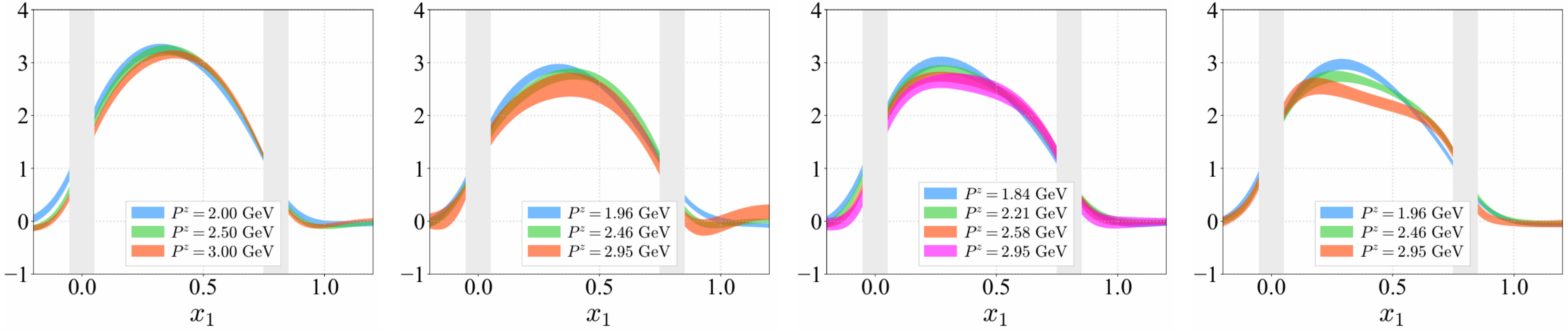
$$\mathcal{C}^{(1)}(x_1, x_2; y_1, y_2; P_B^z, \mu) = \left[\mathcal{C}^{(1)}_{\overline{\mathrm{MS}}}(x_1, x_2; y_1, y_2; P_B^z, \mu) + \delta\mathcal{C}^{(1)}_H(x_1, x_2; y_1, y_2; P_B^z, \mu) \right]_{\oplus} .$$



Matching in hybrid scheme

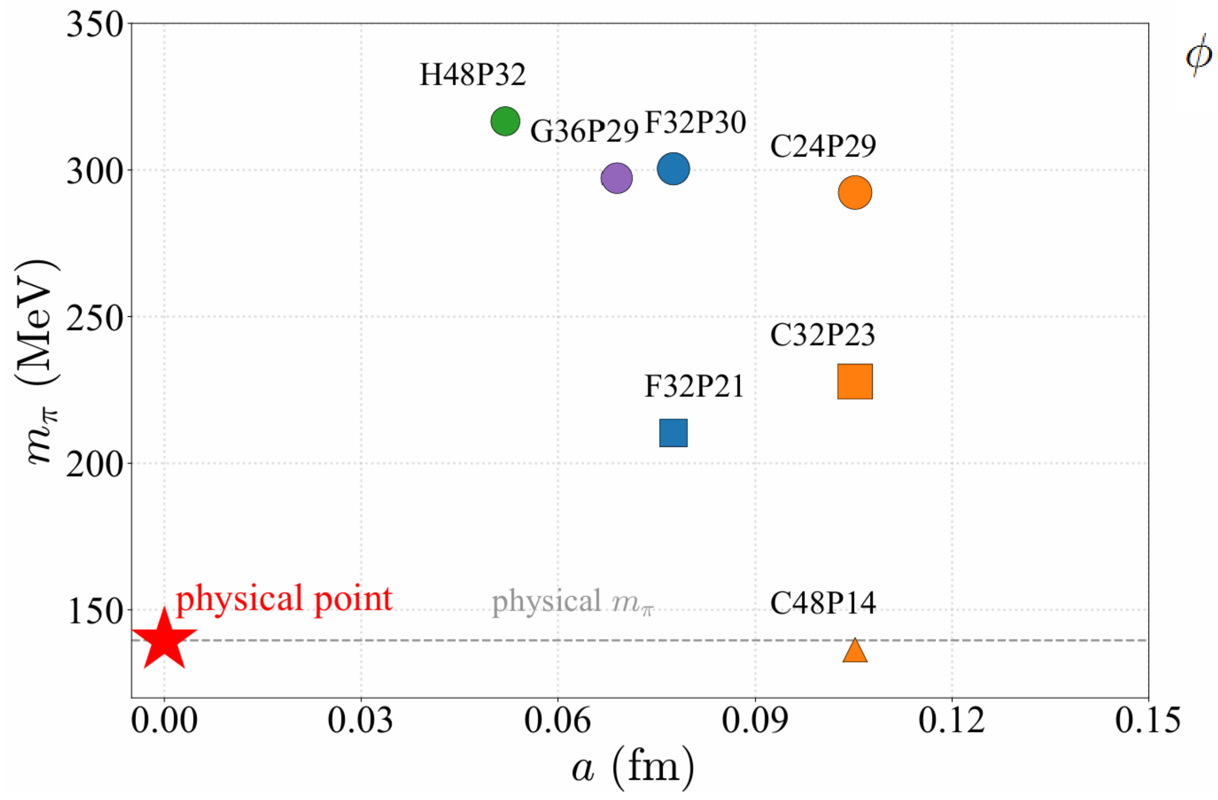


(a) H48P32 (b) G36P29 (c) F32P30



(d) F32P21 (e) C48P14 (f) C32P23 (g) C24P29

Physical Extrapolation



$$\phi(x_1, x_2)|_{a, m_\pi, P^z} = \phi_{\text{phys}}(x_1, x_2)$$

$$+ \frac{A(x_1, x_2)}{(P^z)^2}$$ LaMET power corrections

$$+ \left(m_\pi^2 - m_{\pi,\text{phys}}^2\right) B(x_1, x_2)$$ m_π-dependence

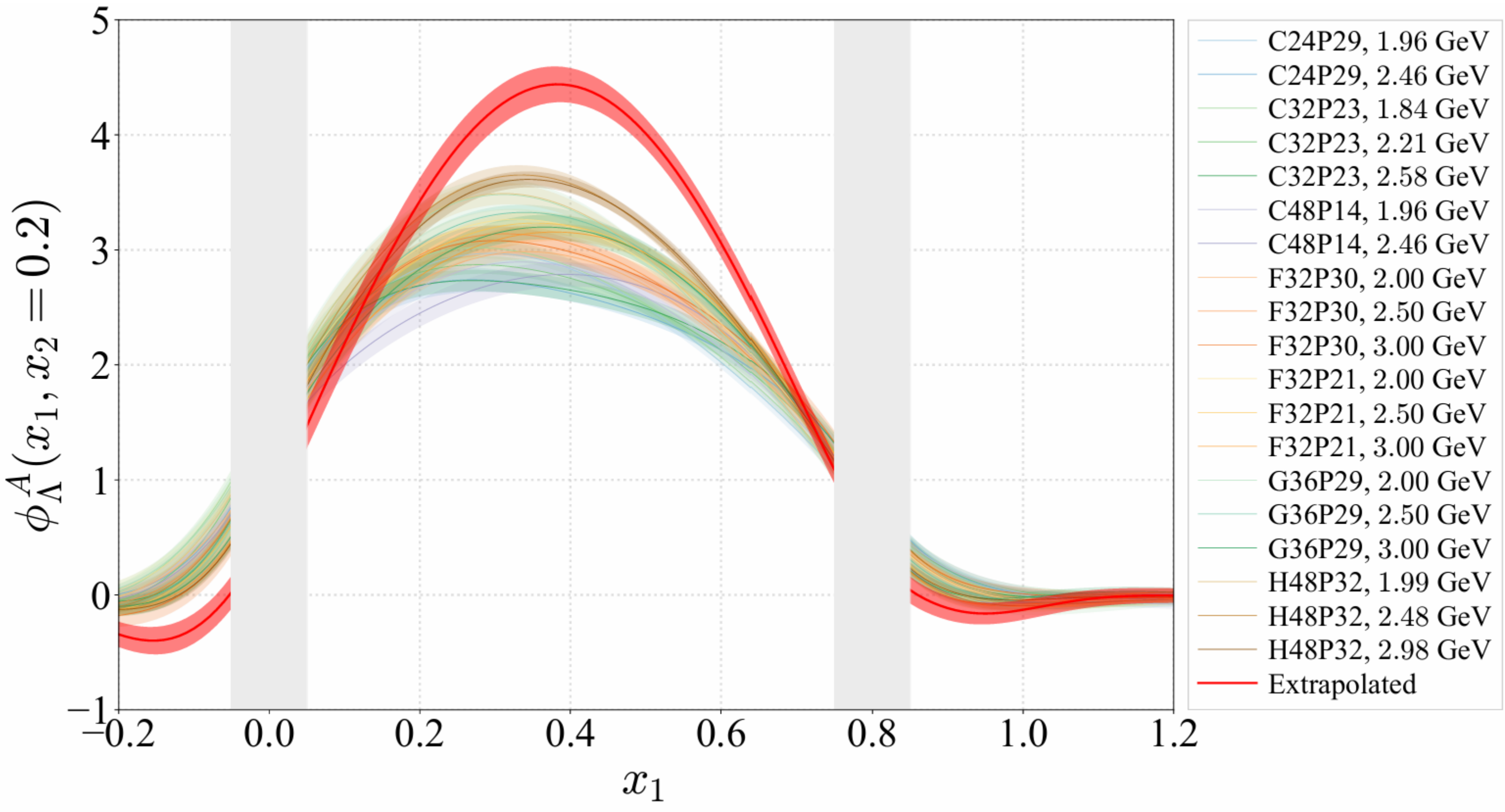
$$+ a^2 \left[C_1(x_1, x_2) + C_2(x_1, x_2)(P^z)^2\right] \; ,$$

discretization effects, including:

- P^z-independent artifacts, and
- P^z-dependent artifacts $a^2(P^z)^2$

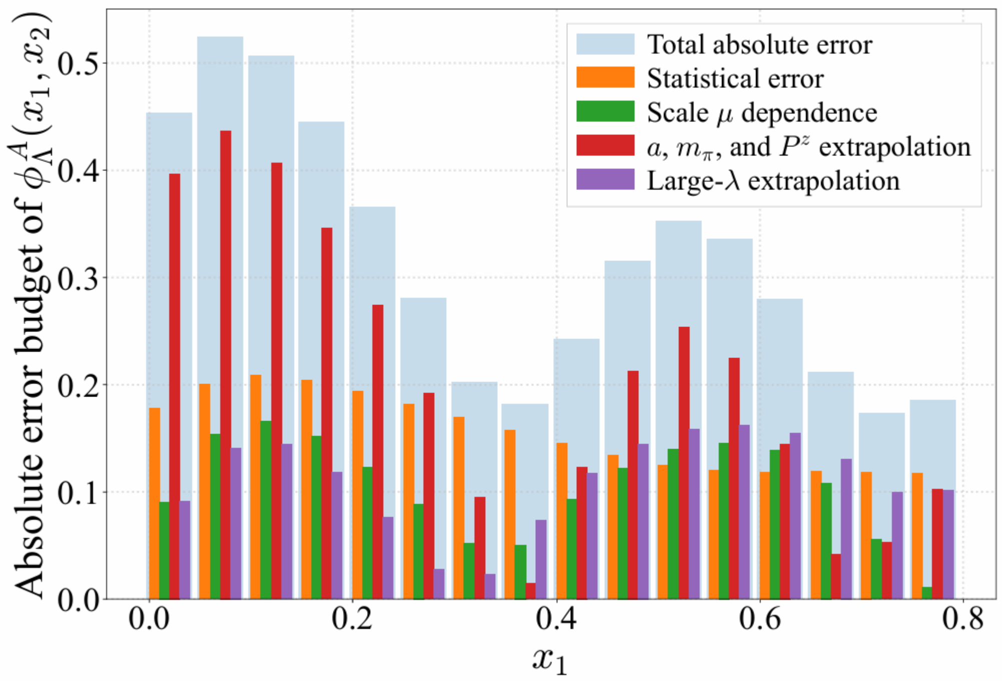
Physical Extrapolation

$$\phi(x_1, x_2)|_{a,m_\pi,P^z} = \phi_{\rm phys}(x_1, x_2) + \frac{A(x_1, x_2)}{(P^z)^2} + \left(m_\pi^2 - m_{\pi,\rm phys}^2\right) B(x_1, x_2)$$
$$+ a^2 \left[C_1(x_1, x_2) + C_2(x_1, x_2)(P^z)^2\right] \ ,$$



Systematic uncertainty

$$\delta_{\text{total}}(x_1,x_2) = \sqrt{[\delta_{\text{stat}}(x_1,x_2)]^2 + [\delta_{\text{sys}}(x_1,x_2)]^2}$$
$$= \sqrt{[\delta_{\text{stat}}(x_1,x_2)]^2 + [\delta^{\mu}_{\text{sys}}(x_1,x_2)]^2 + [\delta^{\lambda}_{\text{sys}}(x_1,x_2)]^2 + [\delta^{a,m_\pi,P^z}_{\text{sys}}(x_1,x_2)]^2}\ .$$



- **Scale μ dependence in perturbation treatments**

  $\mu = \mu_R = \mu_F$

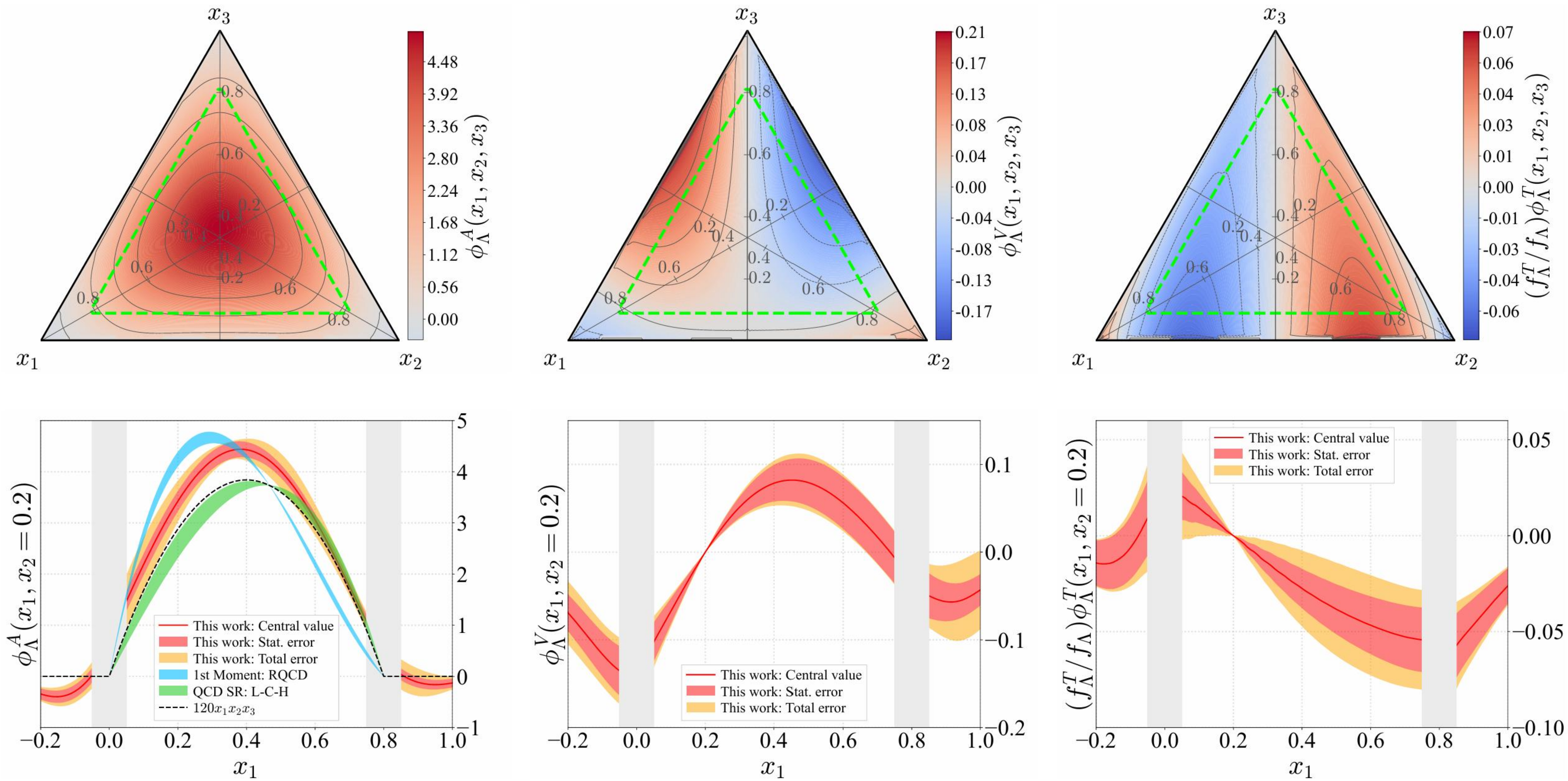
- **Large-distance extrapolation uncertainty**

  NLA / LA extrapolation ansaetze

- **Physical-limit extrapolation uncertainty**

  Combine / Sequential extrapolation

Final results



Summary

- ***LPC, PRD 111, 034510***: prove the lattice calculation of baryon LCDAs from LaMET
- ***LPC, PRD 112, 114515***: details for Hybrid renormalization of baryon matrix elements
- ***arXiv:2606.30387***: Article to establish the **framework** for multi-dimensional distributions
  - *2-dimensional hybrid renormalization;*
  - *Extrapolation using HQET-based asymptotic expansion;*
  - *Convolution matching with hybrid counter-terms;*
- ***arXiv:2606.29597***: Letter to report the first complete numerical results for **Λ LCDAs**
  - *All 3 **leading-twist** x-dependent baryon LCDAs;*
  - ***7 ensembles** to physical point: $a \to 0$ + physical m_π;*
  - *$P^z \to \infty$ limit with $P^z_{\max} \sim 3$ GeV;*

Thanks for your attention!