

Mellin Moments of Proton Unpolarized GPDs at Nonzero Skewness From Lattice QCD With Neural Networks

- Manuel Colaço – PhD student
- Adam Mickiewicz University, Poznań

GPDs Mellin Moments

Lattice QCD

Artificial Neural Networks

Goal: internal structure of the nucleon

- Internal structure of hadrons \rightarrow quantified by partonic functions
- Partons \rightarrow quarks and gluons viewed in a high-energy scattering
- Independent constituents carrying fractions of the hadron momentum

Generalized Parton Distribution Functions (GPDs)

- GPDs \rightarrow different initial and final hadron states:

$$\text{GPD: } |P_i\rangle \neq |P_f\rangle$$

- Functions of:

$x \rightarrow$ fraction of the hadron's momentum carried by a parton

$t \equiv \Delta^2 \rightarrow$ square of the momentum transfer Δ^μ

$\xi \rightarrow$ skewness, quantifies the fraction of momentum transferred in the longitudinal direction

Unpolarized Proton GPDs definition

$$F_{s's}^{\gamma^+}(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle P', s' | \bar{q}(-\frac{z}{2}) \gamma^+ W(-\frac{z}{2}, \frac{z}{2}) q(\frac{z}{2}) | P, s \rangle \Big|_{z^+=0, \mathbf{z}_T=0}$$

- Wilson line $\rightarrow W\left(\frac{-z}{2}, \frac{z}{2}\right)$ $|P, s\rangle \rightarrow$ In this work, proton state
- γ^+ \rightarrow unpolarized quark sector

$$\text{Light-cone coordinates} \rightarrow z^\pm = \frac{z^0 \pm z^3}{\sqrt{2}}, \quad \vec{z}_T = (z^1, z^2)$$

$$z^+ = 0, \vec{z}_T = \vec{0}_T, z^- \neq 0 \implies z^2 = 0$$

Unpolarized Proton GPDs definition

$$F_{s's}^{\gamma^+}(x, \xi, t) = \int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle P', s' | \bar{q}\left(-\frac{z}{2}\right) \gamma^+ W\left(-\frac{z}{2}, \frac{z}{2}\right) q\left(\frac{z}{2}\right) | P, s \rangle \Big|_{z^+=0, \mathbf{z}_T=0}$$

- OPE + Parameterization on ME above \rightarrow twist-2 GPDs

$$F_{s's}^{\gamma^+}(x, \xi, t) = \frac{1}{2P^+} \bar{u}(P', s') \left[\gamma^+ H(x, \xi, t) + \frac{i\sigma^{+\alpha} \Delta_\alpha}{2m_N} E(x, \xi, t) \right] u(P, s)$$

Unpolarized proton twist-2 GPDs $\rightarrow H$ and E

Mellin moments and polynomiality

$$\text{Mellin moments of } H \rightarrow \int_{-1}^1 dx x^n H(x, \xi, t) = \sum_{\substack{i=0 \\ \text{even}}}^n (2\xi)^i A_{n+1,i}(t) + \text{mod}(n, 2) (2\xi)^{n+1} C_{n+1}(t)$$

$$\text{Mellin moments of } E \rightarrow \int_{-1}^1 dx x^n E(x, \xi, t) = \sum_{\substack{i=0 \\ \text{even}}}^n (2\xi)^i B_{n+1,i}(t) - \text{mod}(n, 2) (2\xi)^{n+1} C_{n+1}(t).$$

Generalized form factors (GFFs) $\rightarrow A_{n+1,i}(t), B_{n+1,i}(t), C_{n+1}(t)$

Physical meaning of GFFs

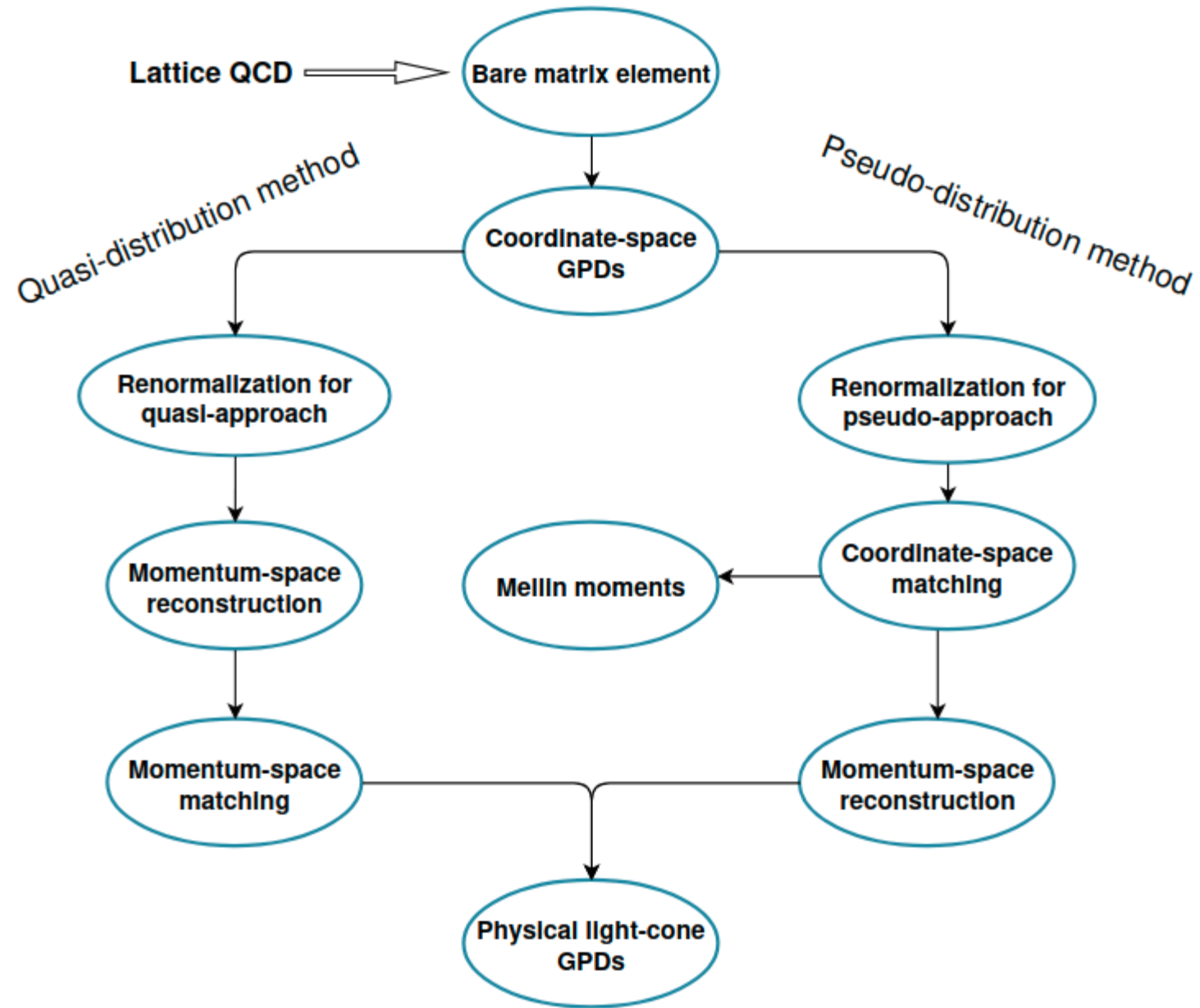
- A_{10} and B_{10} → information on the electromagnetic structure of the nucleon
- A_{20} and B_{20} , Ji's quark total angular momentum sum rule → $J = 1/2[A_{20}(0) + B_{20}(0)]$
- C_2 → only contributes at $\xi \neq 0$
 - related to the D -term structure of the energy-momentum tensor
 - information on "mechanical" structure inside hadrons

Internal structure of the nucleon

Methodology

How to use lattice QCD and ANNs compute these GFFs?

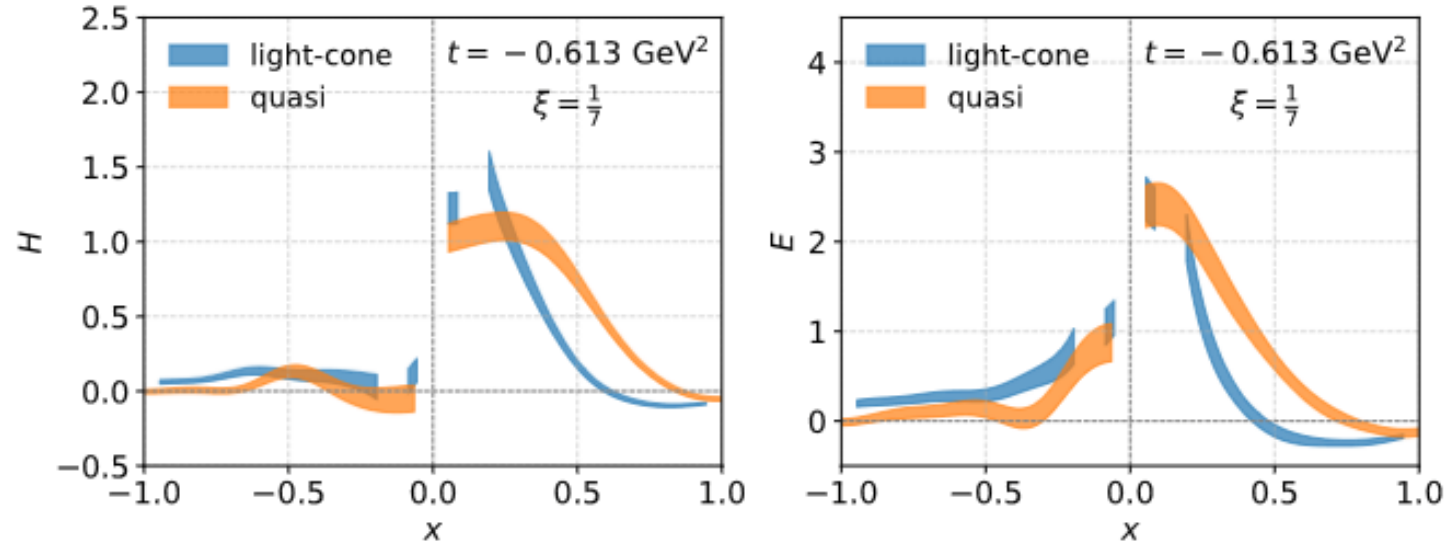
Methodology



Connection to Previous Quasi-GPD Nonzero-Skewness Work

$$\int_{-1}^1 dx x^n H(x, \xi, t) = \sum_{\substack{i=0 \\ \text{even}}}^n (2\xi)^i A_{n+1,i}(t) + \text{mod}(n, 2) (2\xi)^{n+1} C_{n+1}(t)$$

$$\int_{-1}^1 dx x^n E(x, \xi, t) = \sum_{\substack{i=0 \\ \text{even}}}^n (2\xi)^i B_{n+1,i}(t) - \text{mod}(n, 2) (2\xi)^{n+1} C_{n+1}(t).$$



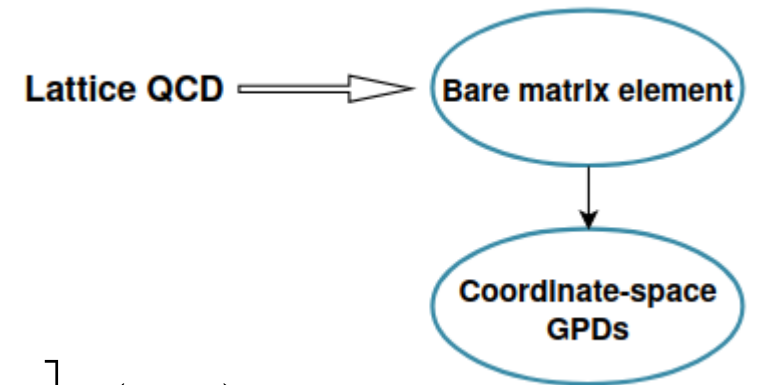
M.-H. Chu et al. “Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Unpolarized Quarks at Nonzero Skewness”. In: *Phys. Rev. D* 112 (2025), p. 094510. DOI: 10.1103/ts5s-hvb1. arXiv: 2508.17998 [hep-lat].

Mellin moments \rightarrow constrain GPDs \rightarrow help reconstruct these missing regions

Coordinate-space GPDs from Lattice Matrix Elements

- Coordinate-space GPDs:

$$\begin{aligned} F^\mu(z, P, \Delta) &= \langle P', s' | \bar{q}(-\frac{z}{2}) \gamma^\mu W(-\frac{z}{2}, \frac{z}{2}) q(\frac{z}{2}) | P, s \rangle \\ &= \bar{u}(P', s') \left[\frac{P^\mu}{m} A_1 + m z^\mu A_2 + \frac{\Delta^\mu}{m} A_3 + i m \sigma^{\mu z} A_4 \right. \\ &\quad \left. + \frac{i \sigma^{\mu \Delta}}{m} A_5 + \frac{P^\mu i \sigma^{z \Delta}}{m} A_6 + m z^\mu i \sigma^{z \Delta} A_7 + \frac{\Delta^\mu i \sigma^{z \Delta}}{m} A_8 \right] u(P, s) \end{aligned}$$



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Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

[Shohini Bhattacharya](#) ^{1,*}, [Krzysztof Cichy](#)², [Martha Constantinou](#) ^{3,†}, [Jack Dodson](#)³, [Xiang Gao](#)⁴, [Andreas Metz](#)³, [Swagato Mukherjee](#) ¹, [Aurora Scapellato](#)³, [Fernanda Steffens](#)⁵ *et al.*

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Coordinate-space GPDs from Lattice Matrix Elements

$$H = A_1 - 2\xi A_3 \quad \rightarrow \quad \text{Discrete and defined in coordinate-space}$$

$$E = -A_1 + 2\xi A_3 + 2A_5 + 2P_3 z A_6 - 4\xi P_3 z A_8$$

- Amplitudes are from found:

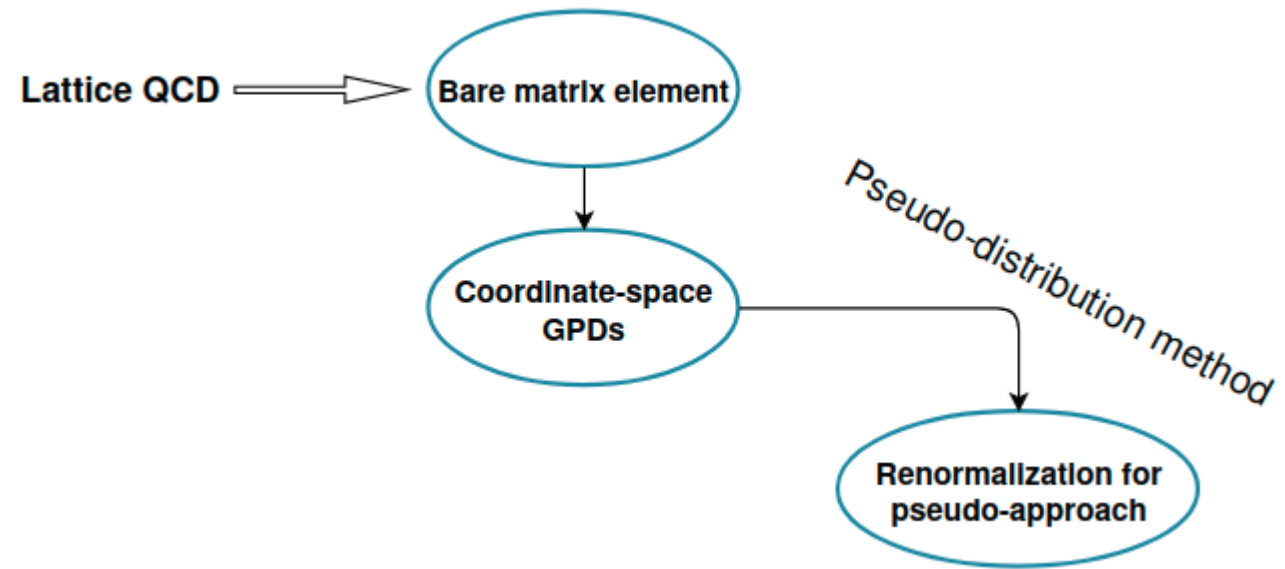
Matrix element $\rightarrow \Pi_\mu(\Gamma_\nu) = K \text{Tr} \left[\Gamma_\nu \left(\frac{-i\not{P}_f + m}{2m} \right) \tilde{F}^\mu \left(\frac{-i\not{P}_i + m}{2m} \right) \right] \rightarrow$ Linear combination A_i
System of linear equations

$$\Gamma_0 = \frac{1}{4} (1 + \gamma_0), \quad \Gamma_k = \frac{1}{4} (1 + \gamma_0) i\gamma_5 \gamma_k, \quad k = 1, 2, 3$$

$$K = \frac{2m^2}{\sqrt{E_f E_i (E_f + m)(E_i + m)}}$$

$$\tilde{F}^\mu(z, P, \Delta) = \left[\frac{P^\mu}{m} A_1 + m z^\mu A_2 + \frac{\Delta^\mu}{m} A_3 + i m \sigma^{\mu z} A_4 + \frac{i \sigma^{\mu \Delta}}{m} A_5 + \frac{P^\mu i \sigma^{z \Delta}}{m} A_6 + m z^\mu i \sigma^{z \Delta} A_7 + \frac{\Delta^\mu i \sigma^{z \Delta}}{m} A_8 \right]$$

Renormalization

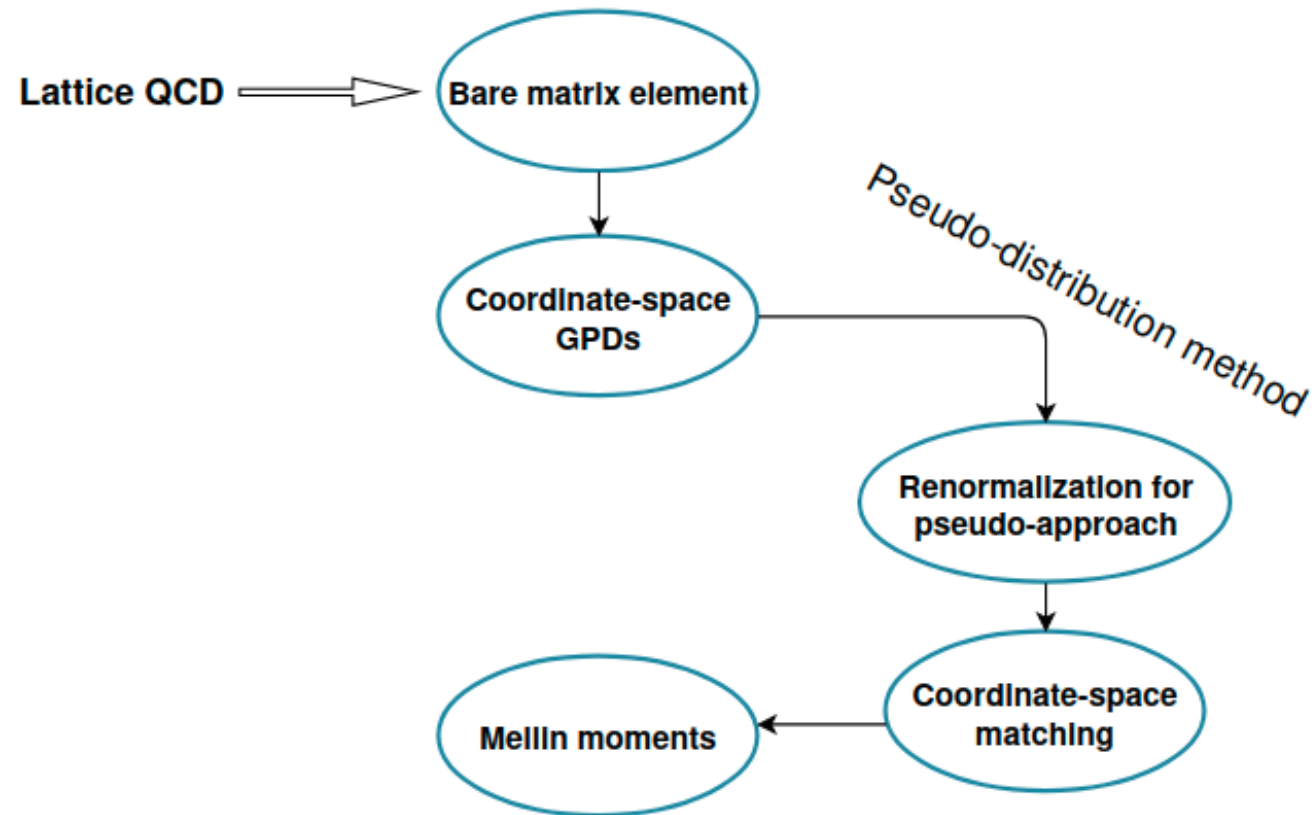


- Wilson-line operators contain UV divergences

$$\text{Wilson-line self-energy} \sim |z|/a$$

- Ratio scheme \rightarrow divide by PDF \rightarrow cancel Wilson-line self-energy factor

Reduced Ioffe-time Distributions (ITDs)



- Renormalized H and $E \rightarrow$ Reduced ITDs $\rightarrow F(\nu, t, \xi, z^2)$, $\nu := P \cdot z$, $P := (P_i + P_f)/2$

Pseudo-distribution – Short Distance Matching

$$\underbrace{F_n(\xi, t, z^2)}_{\substack{\text{finite-}z^2 \\ \text{lattice quantity}}} = \sum_m \underbrace{C_{nm}(z^2 \mu^2)}_{\substack{\text{Wilson matching} \\ \text{coefficient}}} \underbrace{\bar{F}_m(\xi, t)}_{\substack{\text{light-cone } z^2 = 0 \\ \text{GPD moment}}} + \underbrace{\mathcal{O}(z^2 \Lambda_{\text{QCD}}^2, z^2 m_N^2, z^2 t)}_{\substack{\text{finite-}z^2 \text{ power corrections} \\ \text{vanish as } z^2 \rightarrow 0}}$$

From Matching + Polynomiality and Truncation to GFFs

$$\operatorname{Re} F^{(H)}(\nu, t, \xi; z) = \left(c_{1,0} - \frac{\nu^2}{2} \xi^2 c_{3,2} \right) A_{1,0}(t) - \frac{\nu^2}{2} c_{3,0} A_{3,0}(t) - \frac{\nu^2}{2} \xi^2 c_{3,0} A_{3,2}(t) + O(\nu^4)$$

$$\begin{aligned} \operatorname{Im} F^{(H)}(\nu, t, \xi; z) &= \left(-\nu c_{2,0} + \frac{\nu^3}{6} \xi^2 c_{4,2} \right) A_{2,0}(t) + \left(-\nu \xi^2 c_{2,0} + \frac{\nu^3}{6} \xi^4 c_{4,2} \right) C_2(t) \\ &+ \frac{\nu^3}{6} c_{4,0} A_{4,0}(t) + \frac{\nu^3}{6} \xi^2 c_{4,0} A_{4,2}(t) + \frac{\nu^3}{6} \xi^4 c_{4,0} C_4(t) + O(\nu^5) \end{aligned}$$

$$\operatorname{Re} F^{(E)}(\nu, t, \xi; z) = \left(c_{1,0} - \frac{\nu^2}{2} \xi^2 c_{3,2} \right) B_{1,0}(t) - \frac{\nu^2}{2} c_{3,0} B_{3,0}(t) - \frac{\nu^2}{2} \xi^2 c_{3,0} B_{3,2}(t) + O(\nu^4)$$

$$\begin{aligned} \operatorname{Im} F^{(E)}(\nu, t, \xi; z) &= \left(-\nu c_{2,0} + \frac{\nu^3}{6} \xi^2 c_{4,2} \right) B_{2,0}(t) + \left(\nu \xi^2 c_{2,0} - \frac{\nu^3}{6} \xi^4 c_{4,2} \right) C_2(t) \\ &+ \frac{\nu^3}{6} c_{4,0} B_{4,0}(t) + \frac{\nu^3}{6} \xi^2 c_{4,0} B_{4,2}(t) - \frac{\nu^3}{6} \xi^4 c_{4,0} C_4(t) + O(\nu^5) \end{aligned}$$

Kinematics of the data points – correlation between (ν, t, ξ)

$$\text{Re } F^{(H)}(\nu, t, \xi; z) = \left(c_{1,0} - \frac{\nu^2}{2} \xi^2 c_{3,2} \right) A_{1,0}(t) - \frac{\nu^2}{2} c_{3,0} A_{3,0}(t) - \frac{\nu^2}{2} \xi^2 c_{3,0} A_{3,2}(t) \quad \text{Re } F^{(E)}(\nu, t, \xi; z) = \left(c_{1,0} - \frac{\nu^2}{2} \xi^2 c_{3,2} \right) B_{1,0}(t) - \frac{\nu^2}{2} c_{3,0} B_{3,0}(t) - \frac{\nu^2}{2} \xi^2 c_{3,0} B_{3,2}(t)$$

$$\begin{aligned} \text{Im } F^{(H)}(\nu, t, \xi; z) &= \left(-\nu c_{2,0} + \frac{\nu^3}{6} \xi^2 c_{4,2} \right) A_{2,0}(t) + \left(-\nu \xi^2 c_{2,0} + \frac{\nu^3}{6} \xi^4 c_{4,2} \right) C_2(t) \\ &\quad + \frac{\nu^3}{6} c_{4,0} A_{4,0}(t) + \frac{\nu^3}{6} \xi^2 c_{4,0} A_{4,2}(t) + \frac{\nu^3}{6} \xi^4 c_{4,0} C_4(t) \end{aligned} \quad \begin{aligned} \text{Im } F^{(E)}(\nu, t, \xi; z) &= \left(-\nu c_{2,0} + \frac{\nu^3}{6} \xi^2 c_{4,2} \right) B_{2,0}(t) + \left(\nu \xi^2 c_{2,0} - \frac{\nu^3}{6} \xi^4 c_{4,2} \right) C_2(t) \\ &\quad + \frac{\nu^3}{6} c_{4,0} B_{4,0}(t) + \frac{\nu^3}{6} \xi^2 c_{4,0} B_{4,2}(t) - \frac{\nu^3}{6} \xi^4 c_{4,0} C_4(t) \end{aligned}$$

- For $z = (0, 0, 0, z_3)$: $\nu = z_3 P_3$, $\xi = -\frac{\Delta_3}{2P_3}$, $t = (E_f - E_i)^2 - \mathbf{\Delta}^2$, $E_{f/i} = \sqrt{m_N^2 + \mathbf{P}_{f/i}^2}$

In lattice units ($z_3 = 1$) \rightarrow

P_3	Δ_3	ν	ξ	$t \text{ GeV}^2$
2	1	2	-1/4	-1.25
3	1	3	-1/6	-1.12
3	2	3	-1/3	-1.53

Changing P_3 changes ν and also changes t .

Changing Δ_3 changes ξ and also changes t .

Generally, data do not provide independent samples at fixed t

Artificial Neural Network and GFFs

$$\operatorname{Re} F^{(H)}(\nu, t, \xi; z) = \left(c_{1,0} - \frac{\nu^2}{2} \xi^2 c_{3,2} \right) A_{1,0}(t) - \frac{\nu^2}{2} c_{3,0} A_{3,0}(t) - \frac{\nu^2}{2} \xi^2 c_{3,0} A_{3,2}(t)$$

$$\operatorname{Re} F^{(E)}(\nu, t, \xi; z) = \left(c_{1,0} - \frac{\nu^2}{2} \xi^2 c_{3,2} \right) B_{1,0}(t) - \frac{\nu^2}{2} c_{3,0} B_{3,0}(t) - \frac{\nu^2}{2} \xi^2 c_{3,0} B_{3,2}(t)$$

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$$\begin{aligned} \operatorname{Im} F^{(E)}(\nu, t, \xi; z) = & \left(-\nu c_{2,0} + \frac{\nu^3}{6} \xi^2 c_{4,2} \right) B_{2,0}(t) + \left(\nu \xi^2 c_{2,0} - \frac{\nu^3}{6} \xi^4 c_{4,2} \right) C_2(t) \\ & + \frac{\nu^3}{6} c_{4,0} B_{4,0}(t) + \frac{\nu^3}{6} \xi^2 c_{4,0} B_{4,2}(t) - \frac{\nu^3}{6} \xi^4 c_{4,0} C_4(t) \end{aligned}$$

Parameterize GFFs $\rightarrow A_{1,0}, A_{2,0}, A_{3,0}, A_{3,2}, A_{4,0}, A_{4,2}, B_{1,0}, B_{2,0}, B_{3,0}, B_{3,2}, B_{4,0}, B_{4,2}, C_2, C_4,$

$$G^{\text{ANN}}(t) = \sum_{i=1}^M W_i^{(G)} \tanh(w_i^{(G)} t + b_i^{(G)}) + B^{(G)}$$

ANN is just a flexible global fit ansatz for the t-dependence of the GFFs

Artificial Neural Network and GFFs

$$\operatorname{Re} F^{(H)}(\nu, t, \xi; z) = \left(c_{1,0} - \frac{\nu^2}{2} \xi^2 c_{3,2} \right) A_{1,0}(t) - \frac{\nu^2}{2} c_{3,0} A_{3,0}(t) - \frac{\nu^2}{2} \xi^2 c_{3,0} A_{3,2}(t) \quad \operatorname{Re} F^{(E)}(\nu, t, \xi; z) = \left(c_{1,0} - \frac{\nu^2}{2} \xi^2 c_{3,2} \right) B_{1,0}(t) - \frac{\nu^2}{2} c_{3,0} B_{3,0}(t) - \frac{\nu^2}{2} \xi^2 c_{3,0} B_{3,2}(t)$$

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$$G^{\text{ANN}}(t) = \sum_{i=1}^M W_i^{(G)} \tanh(w_i^{(G)} t + b_i^{(G)}) + B^{(G)}$$

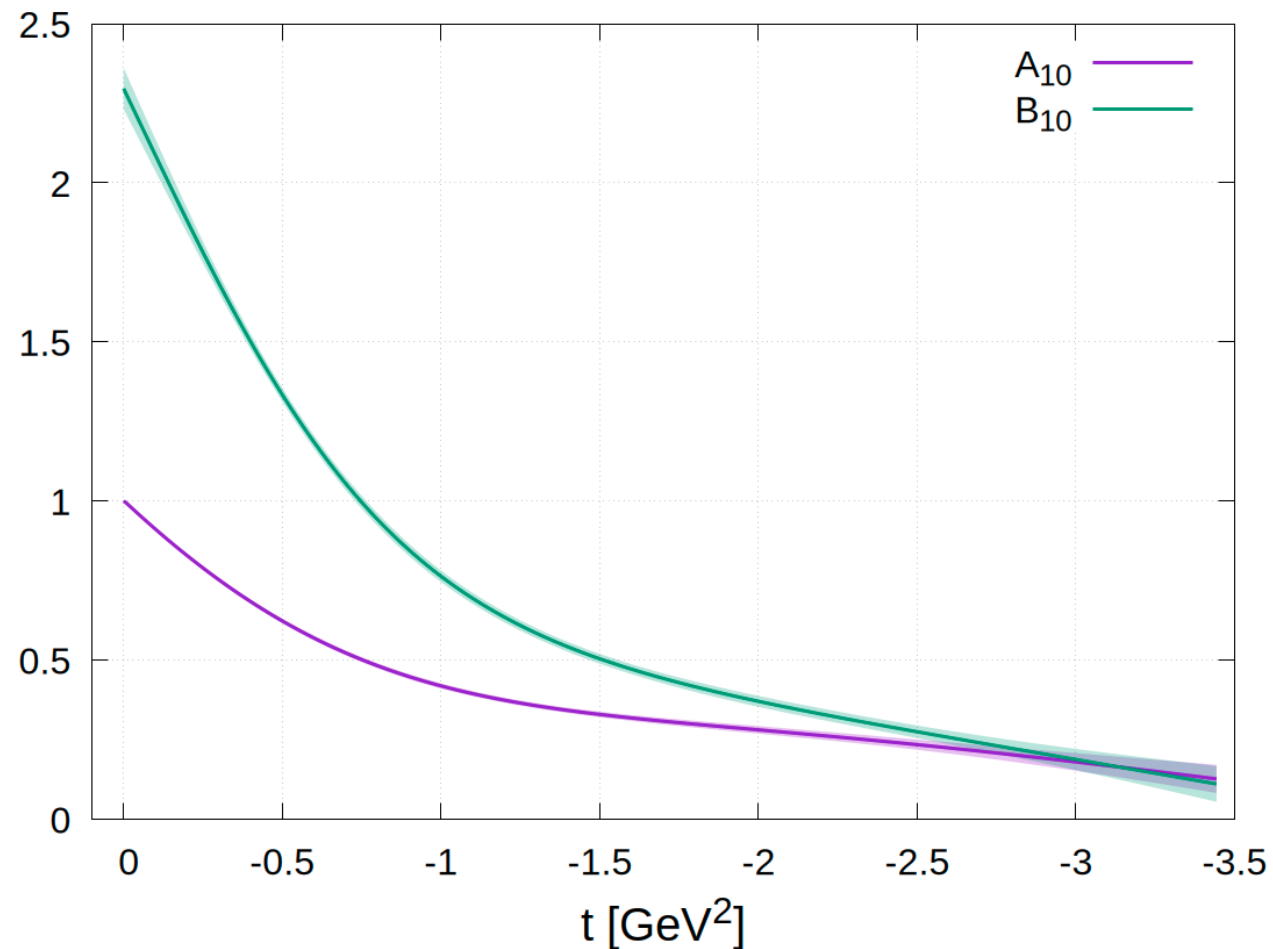
Train parameters $\rightarrow \left\{ W_i^{(G)}, w_i^{(G)}, b_i^{(G)}, B^{(G)} \right\} \iff \text{Minimize } \chi^2 \sim (\text{data} - \text{theory})^2 \leftarrow \text{Global fit}$

Results: isovector $u - d$, NLO, $\mu = 2 \text{ GeV}$, $\overline{\text{MS}}$

- Preliminary proof-of-concept results
 - Controlling lattice systematics (see next talk for details)
 - Checks of stability of GFFs under small ANN setup variations
 - Some of the data used for training may have large $\underbrace{\mathcal{O}(z^2)}_{\text{power corrections}}$
 - Large logs and α_s in matching \rightarrow breakdown of perturbation theory
vanish as $z^2 \rightarrow 0$
- Cross-checked for and checked self-consistency
- Soon publication-ready

Lowest GFFs: A_{10} and B_{10}

- A_{10}^{u-d} and B_{10}^{u-d} are related to nucleon electromagnetic structure

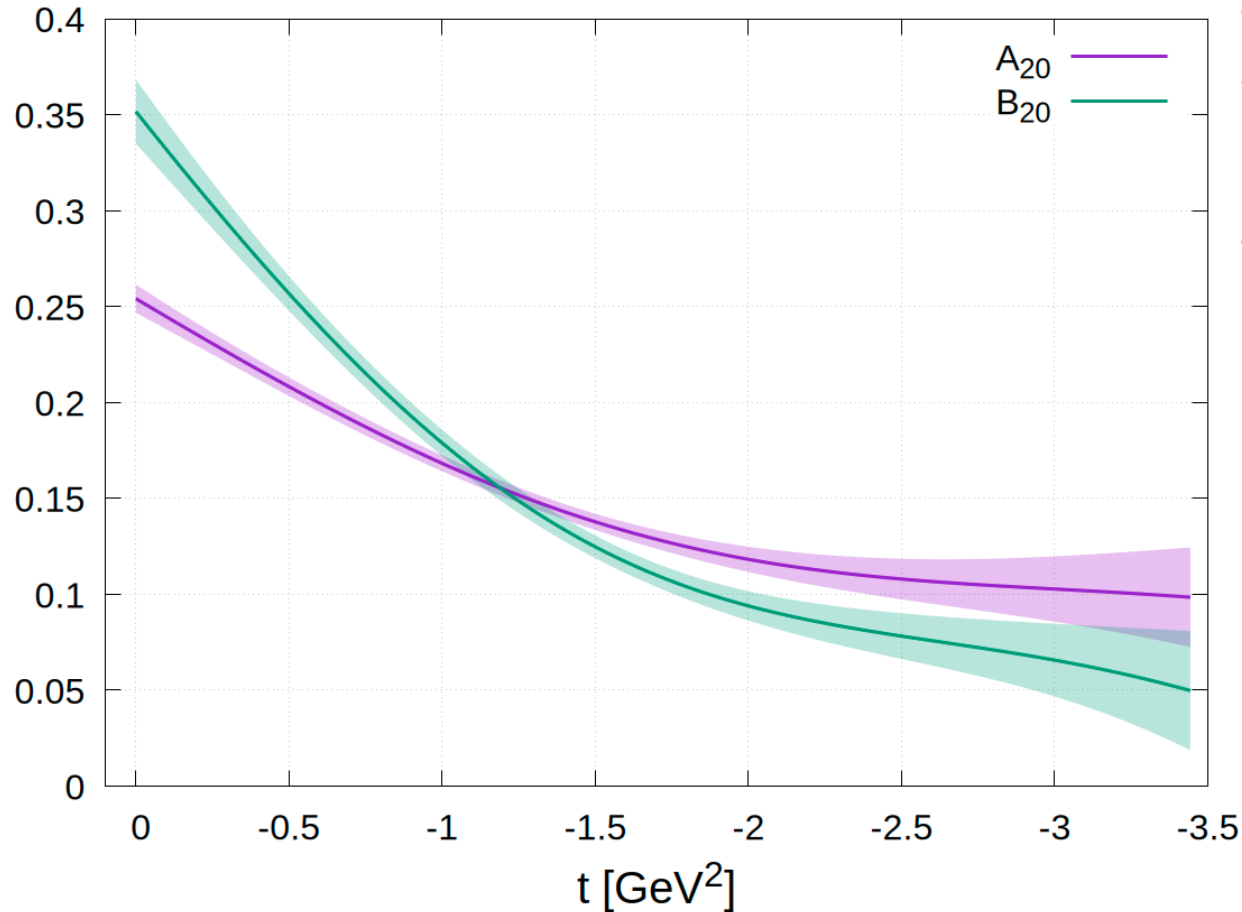


Source	$A_{10}^{u-d}(0)$	$B_{10}^{u-d}(0)$
This work	0.9937(16)	2.30(6)
Bhattacharya et al. 2023 [1]	0.98(5)(3)	2.54(22)(11)
Dutrieux et al. 2026 [2]	0.943(15)	3.17(6)

- [1] S. Bhattacharya et al. “Moments of proton GPDs from the OPE of nonlocal quark bilinears up to NNLO”. In: *Phys. Rev. D* 108.1 (2023), p. 014507. DOI: 10.1103/PhysRevD.108.014507. arXiv: 2305.11117 [hep-lat].
- [2] H. Dutrieux et al. *Reconstructing the full kinematic dependence of GPDs from pseudo-distributions*. 2026. arXiv: 2604.21476 [hep-lat].

Angular momentum sector: A_{20} and B_{20}

- $A_{20}^{u-d}(0)$ and $B_{20}^{u-d}(0) \rightarrow$ Ji's quark total angular momentum sum rule
 $\rightarrow J^{u-d} = 1/2[A_{20}^{u-d}(0) + B_{20}^{u-d}(0)] = J^u - J^d$

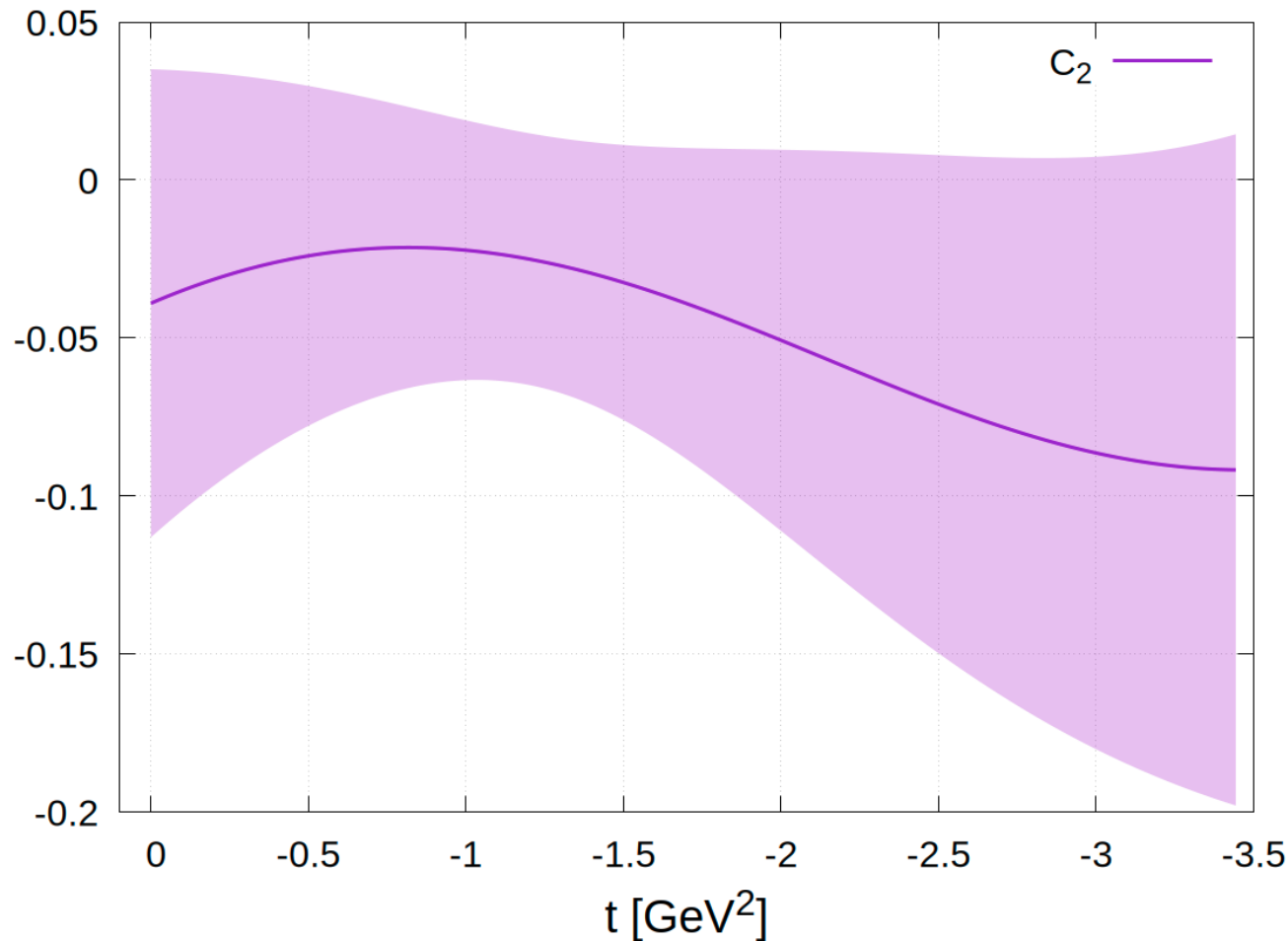


Source	$A_{20}^{u-d}(0)$	$B_{20}^{u-d}(0)$	J^{u-d}
This work	0.254(7)	0.352(17)	0.303(9)
Bhattacharya et al. 2023 [1]	0.267(13)(10)	0.30(4)(2)	0.281(21)(11)
Dutrieux et al. 2026 [2]	0.215(10)	0.38(2)	0.298(11)

- [1] S. Bhattacharya et al. “Moments of proton GPDs from the OPE of nonlocal quark bilinears up to NNLO”. In: *Phys. Rev. D* 108.1 (2023), p. 014507. DOI: 10.1103/PhysRevD.108.014507. arXiv: 2305.11117 [hep-lat].
- [2] H. Dutrieux et al. *Reconstructing the full kinematic dependence of GPDs from pseudo-distributions*. 2026. arXiv: 2604.21476 [hep-lat].

D-term sector: C_2

- C_2 is related to the D -term \rightarrow connected to “mechanical” information



Source	$C_2^{u-d}(0)$
This work	$-0.04(7)$
Dutrieux et al. 2024 [3]	$0.025(8)$

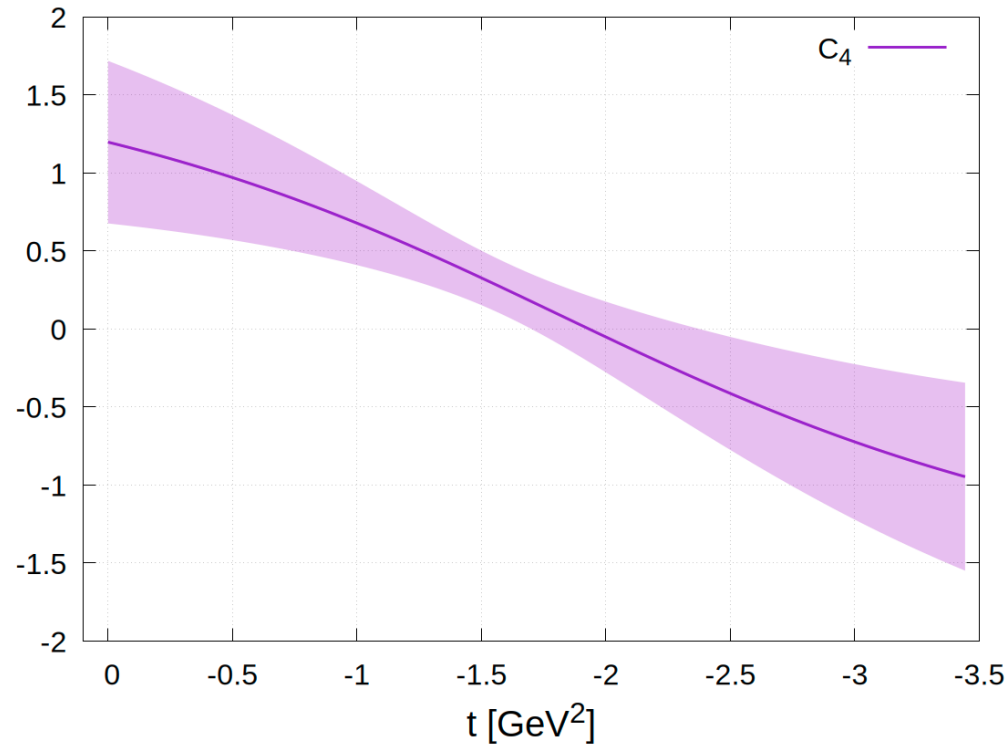
- [3] H. Dutrieux et al. “Towards unpolarized GPDs from pseudo-distributions”. In: *JHEP* 08 (2024), p. 162. DOI: 10.1007/JHEP08(2024)162. arXiv: 2405.10304 [hep-lat].

Higher-Type GFFs

- Higher GFFs \rightarrow less directly constrained by the data
- Enter through higher powers of ν and ξ
- More sensitive to fit-systematic effects
- Constraints on the Mellin expansion
- Included to reduce bias on the lower moments due to truncation

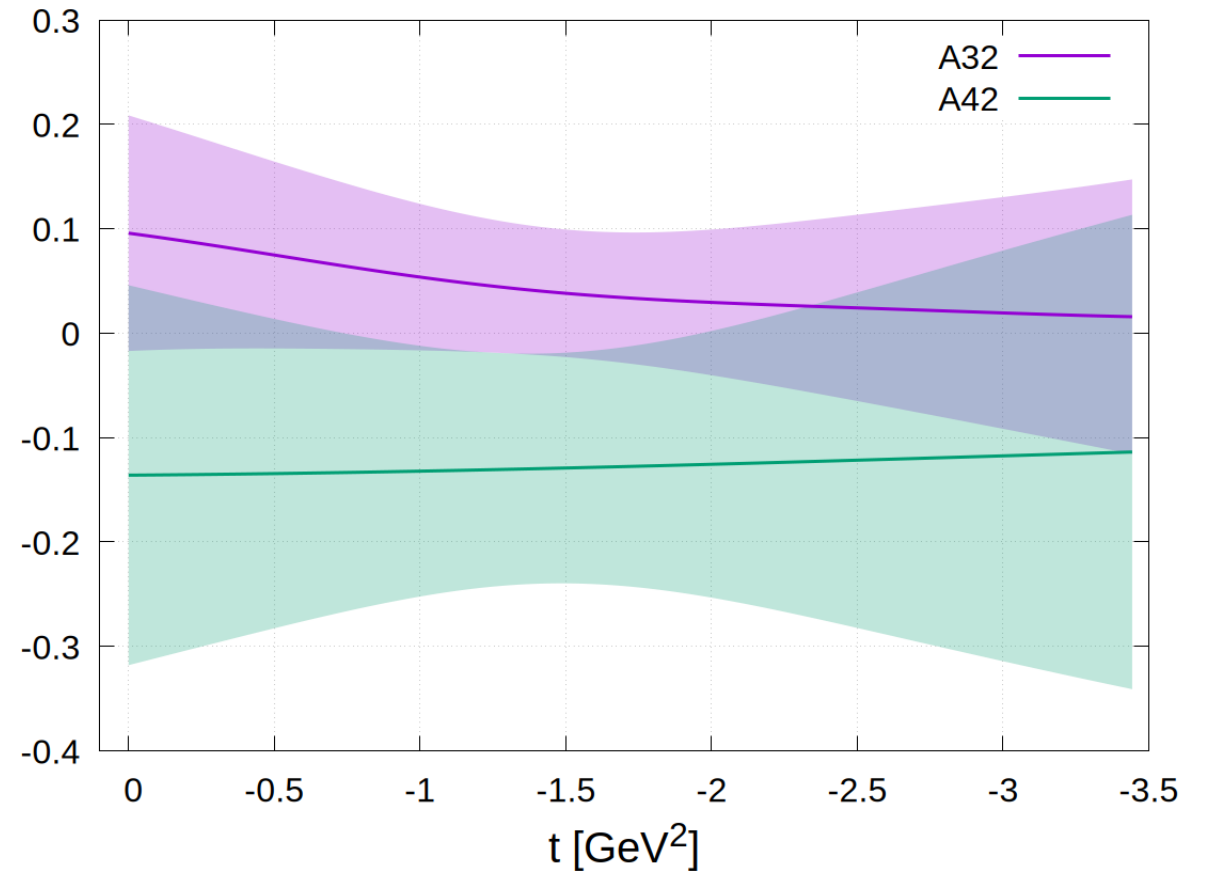
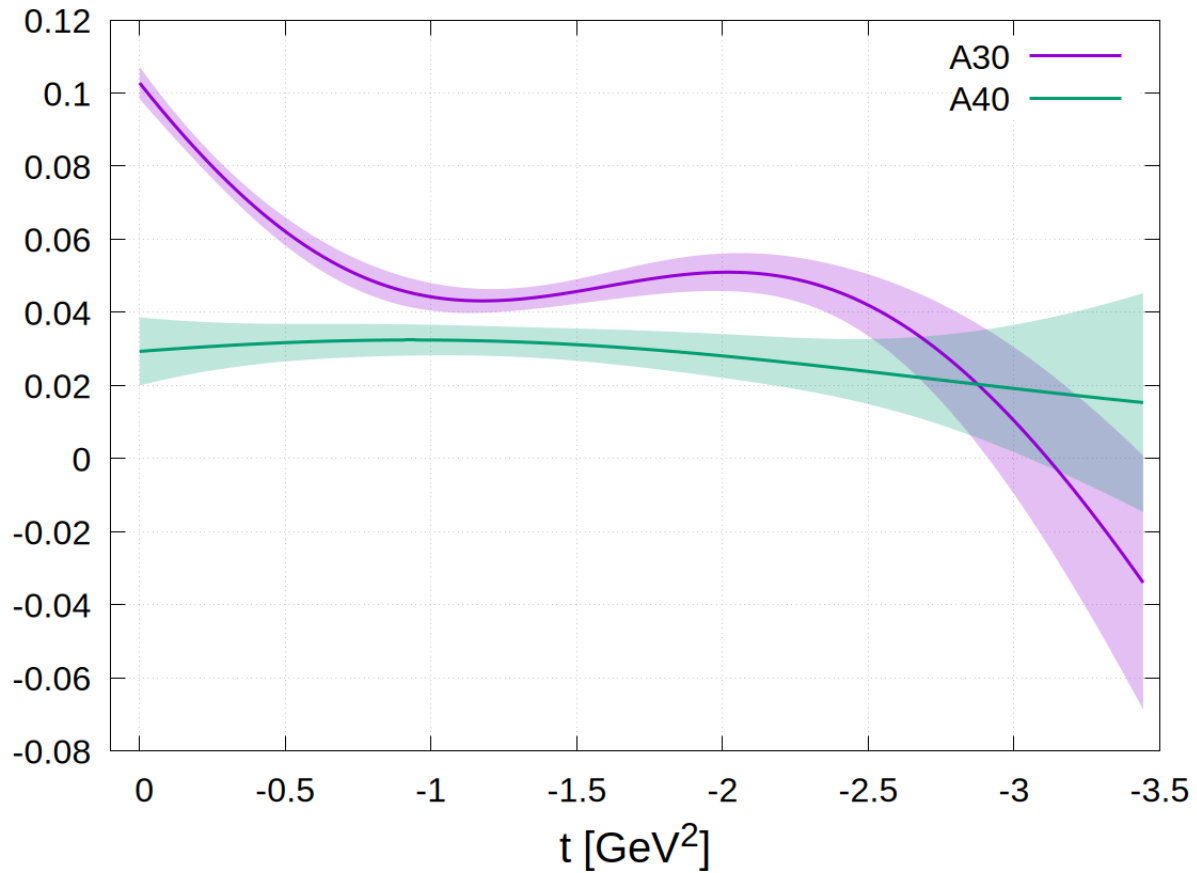
Higher-Type GFFs - C_4

- Disclaimer on lack of control on C_4
 - Highest-skewness coefficient (ξ^4)
 - Appears in both H and E sector
 - Included mainly as a truncation-control term



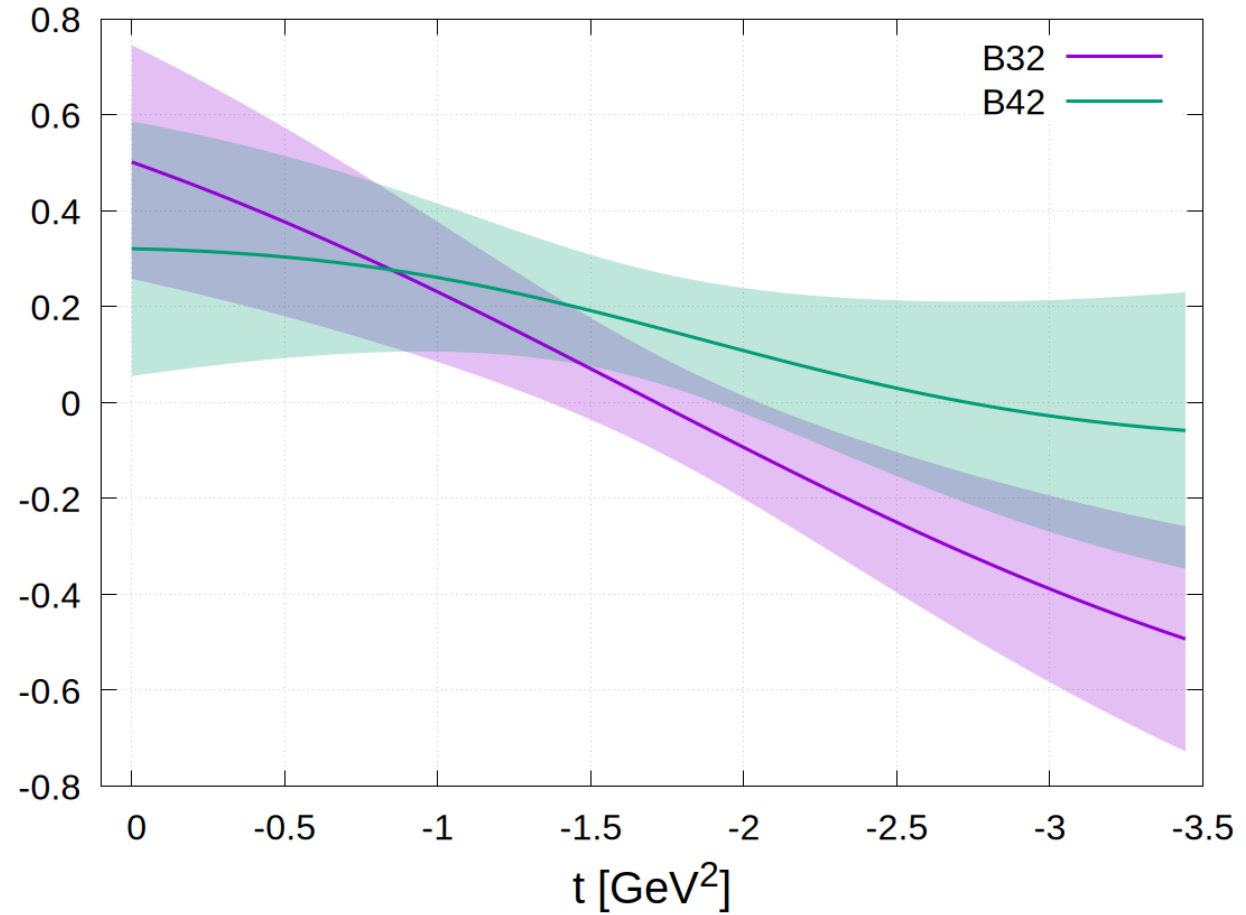
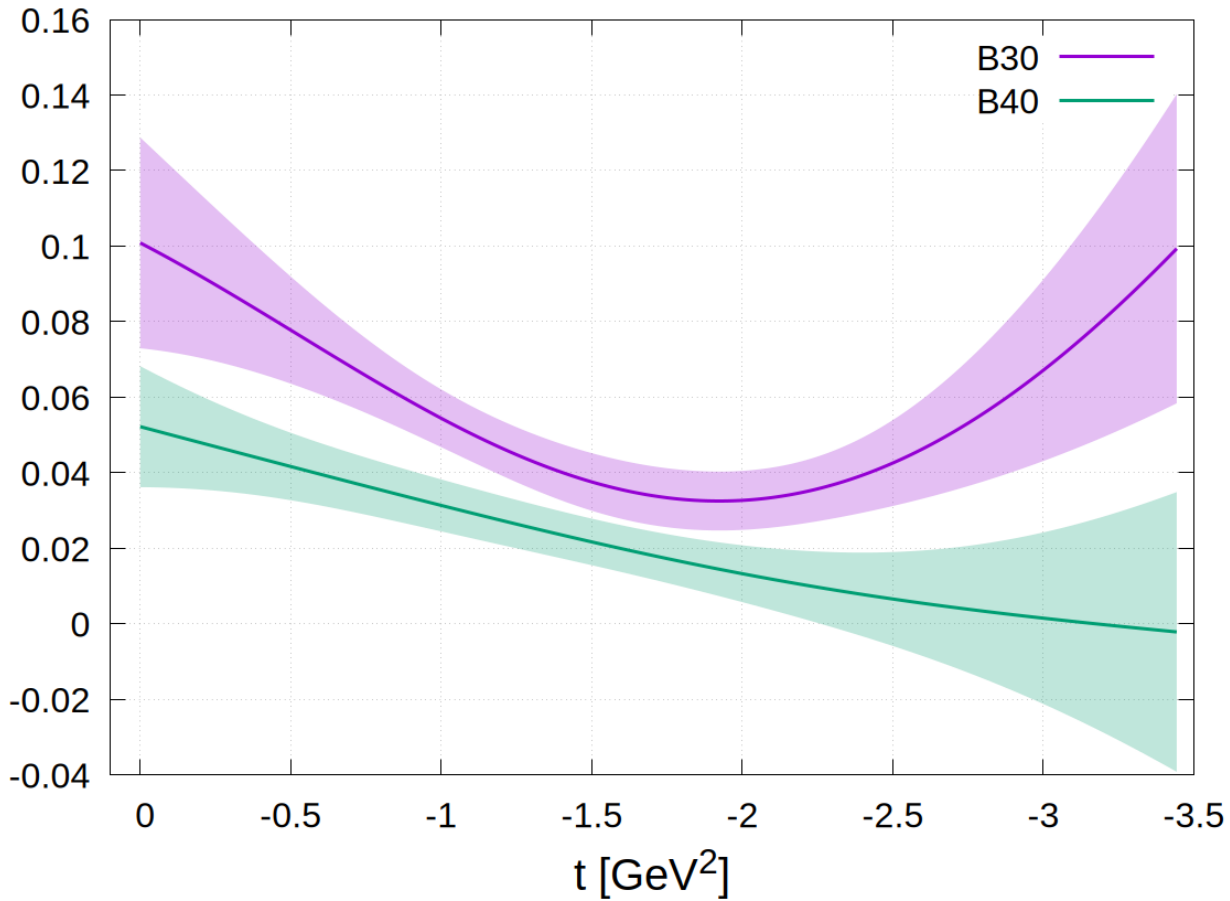
Higher A-type GFFs

- Skewness-dependent GFFs have larger relative uncertainties



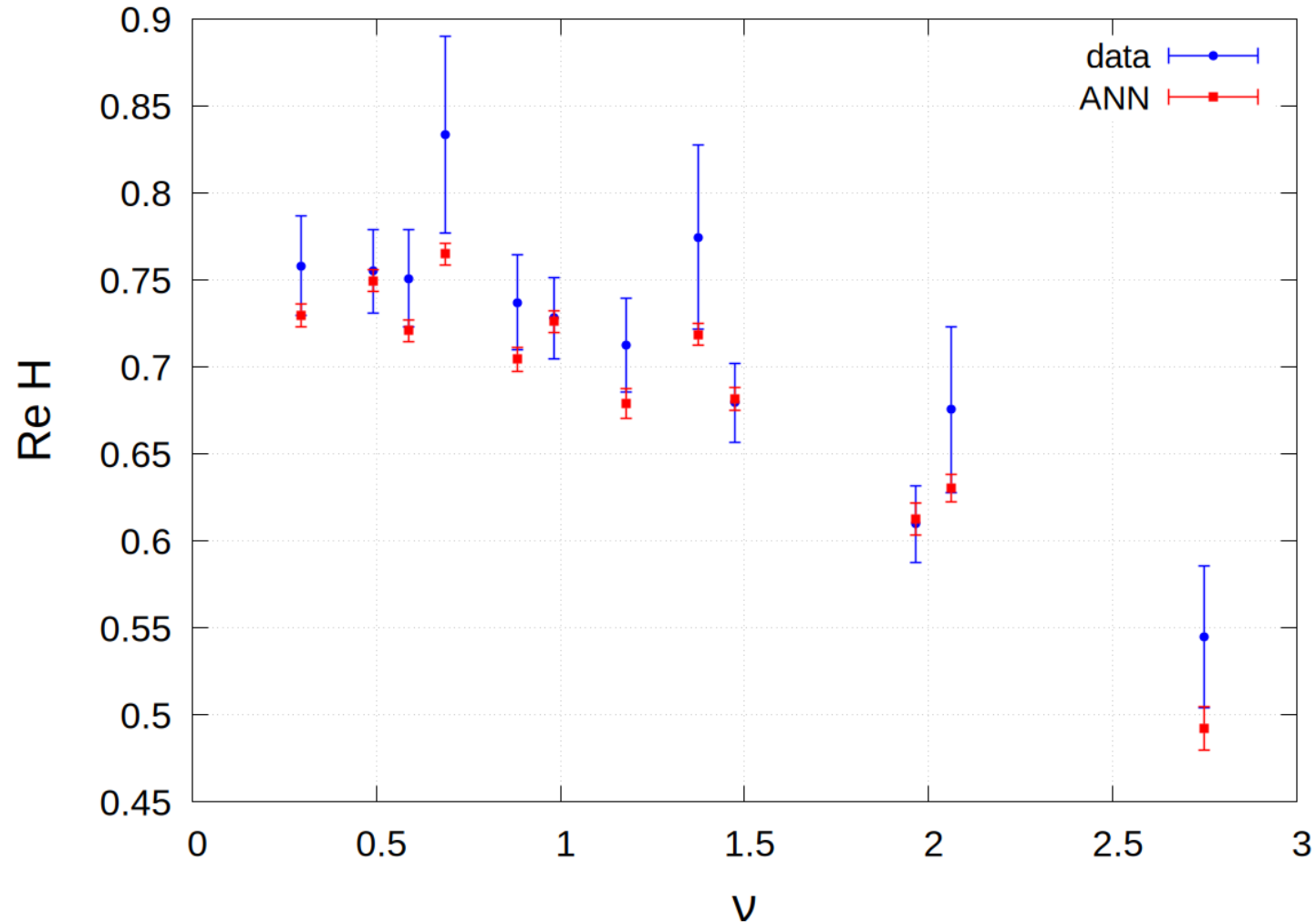
Higher B-type GFFs

- Skewness-dependent GFFs have larger relative uncertainties



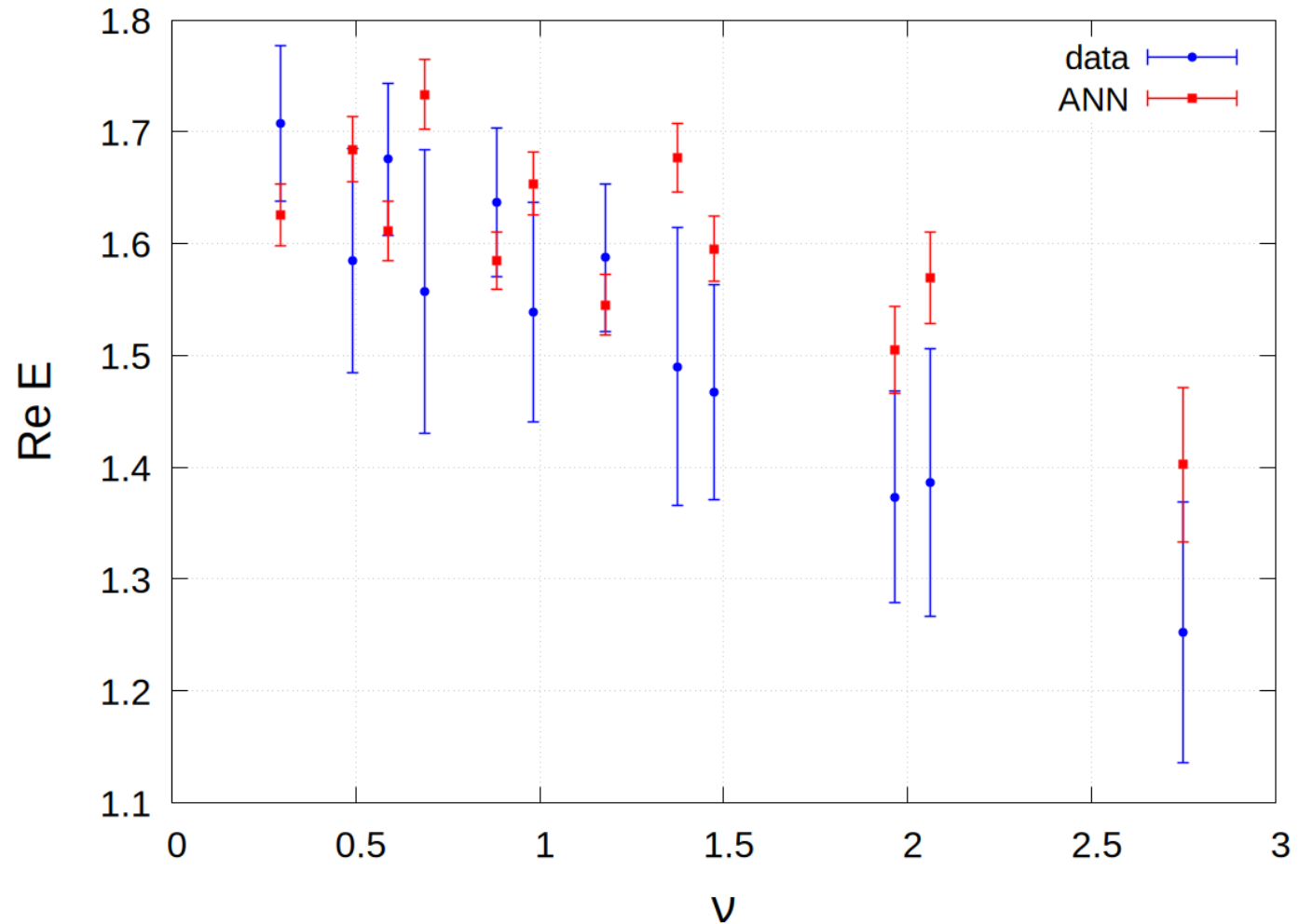
ANN Predictions $\Delta=(1,0,-1)$

$$\operatorname{Re} F^{(H)}(\nu, t, \xi; z) = \left(c_{1,0} - \frac{\nu^2}{2} \xi^2 c_{3,2} \right) A_{1,0}(t) - \frac{\nu^2}{2} c_{3,0} A_{3,0}(t) - \frac{\nu^2}{2} \xi^2 c_{3,0} A_{3,2}(t)$$



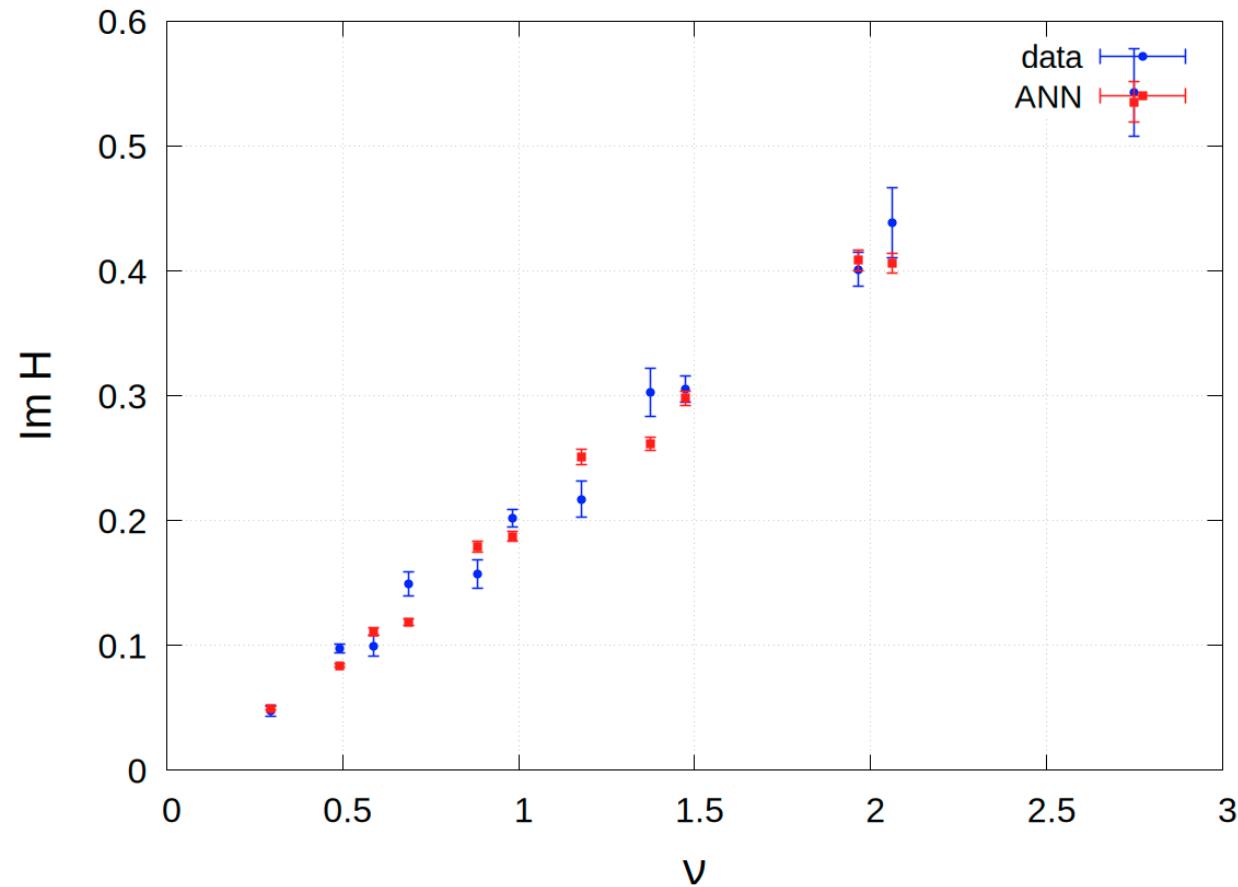
ANN Predictions $\Delta=(1,0,-1)$

$$\text{Re } F^{(E)}(\nu, t, \xi; z) = \left(c_{1,0} - \frac{\nu^2}{2} \xi^2 c_{3,2} \right) B_{1,0}(t) - \frac{\nu^2}{2} c_{3,0} B_{3,0}(t) - \frac{\nu^2}{2} \xi^2 c_{3,0} B_{3,2}(t)$$



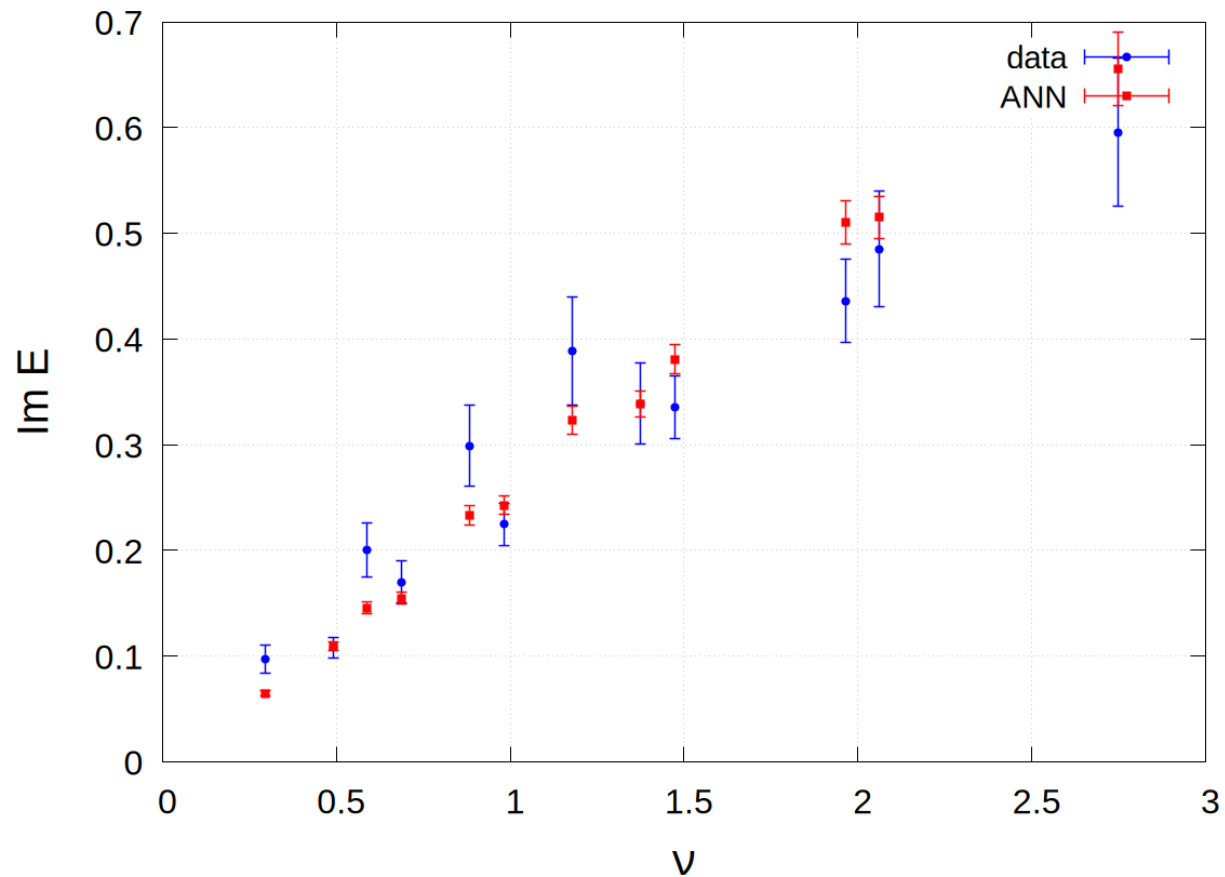
ANN Predictions $\Delta=(1,0,-1)$

$$\begin{aligned} \text{Im } F^{(H)}(\nu, t, \xi; z) = & \left(-\nu c_{2,0} + \frac{\nu^3}{6} \xi^2 c_{4,2} \right) A_{2,0}(t) + \left(-\nu \xi^2 c_{2,0} + \frac{\nu^3}{6} \xi^4 c_{4,2} \right) C_2(t) \\ & + \frac{\nu^3}{6} c_{4,0} A_{4,0}(t) + \frac{\nu^3}{6} \xi^2 c_{4,0} A_{4,2}(t) + \frac{\nu^3}{6} \xi^4 c_{4,0} C_4(t) \end{aligned}$$



ANN Predictions $\Delta=(1,0,-1)$

$$\begin{aligned} \text{Im } F^{(E)}(\nu, t, \xi; z) = & \left(-\nu c_{2,0} + \frac{\nu^3}{6} \xi^2 c_{4,2} \right) B_{2,0}(t) + \left(\nu \xi^2 c_{2,0} - \frac{\nu^3}{6} \xi^4 c_{4,2} \right) C_2(t) \\ & + \frac{\nu^3}{6} c_{4,0} B_{4,0}(t) + \frac{\nu^3}{6} \xi^2 c_{4,0} B_{4,2}(t) - \frac{\nu^3}{6} \xi^4 c_{4,0} C_4(t) \end{aligned}$$



Conclusions

- Physical Goal:
 - Extract Mellin moments of proton unpolarized GPDs at nonzero skewness
 - GFFs encode nucleon internal structure

Conclusions

- Methodology – pseudo-distributions:
 - Re-use lattice matrix elements from nonzero skewness quasi-GPD analysis
 - Complementary pseudo-distribution approach

Conclusions

- Methodology – ANNs:
 - Matching and polynomiality \rightarrow relation between GFFs and lattice data
 - ANNs parametrize these functions in a flexible way

Main message

- Consistent route to GFFs at nonzero skewness from lattice QCD matrix elements
- Preliminary results show the method works → can provide information on nucleon structure

Backup slides

Lattice setup

- $N_f = 2 + 1 + 1$ twisted-mass fermions with clover improvement
- Iwasaki-improved gauge action
- $a = 0.093 = \text{fm}$, $32^3 \times 64$, $m_\pi = 260\text{MeV}$
- Flavor non-singlet combination: $u - d$
- Source-sink separation:

$$t_s = 10a \simeq 0.93 \text{ fm}$$