Hainan Normal University

Heavy-Hadron Light-Cone Distribution Amplitudes: Heavy-Quark Symmetry, Factorization, and Lattice Access

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based on: Phys. Rev. D 110, 114006 (2024) and JHEP 05 (2026) 212

2026 July 7th, Jagiellonian University, Kraków

The XIIIth Meeting on Lattice Parton Physics from Large Momentum Effective Theory (LaMET 2026)

Outline

Why is LCDA important?

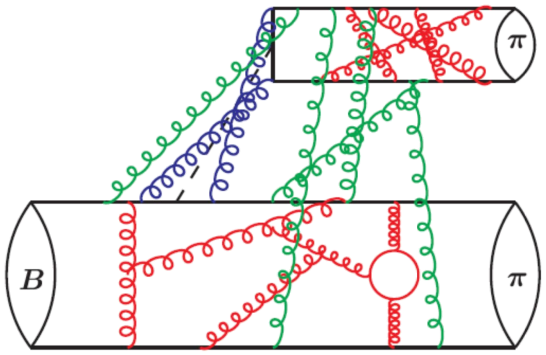
Understand the strong interactions of heavy quark decay

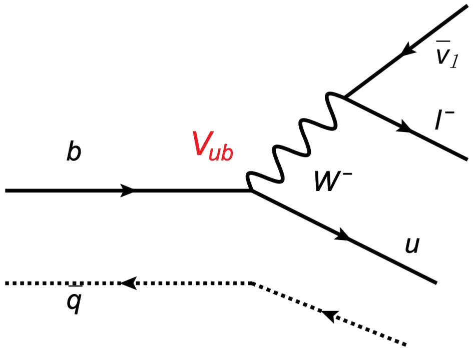
$B \to \pi\ \pi$ Phys. Rev. Lett. 83, 1914 (1999)
$B \to \pi\ K$ Nucl. Phys. B 606, 245 (2001)
$B \to \pi\ D$ Phys. Rev. D 69, 112002 (2004)

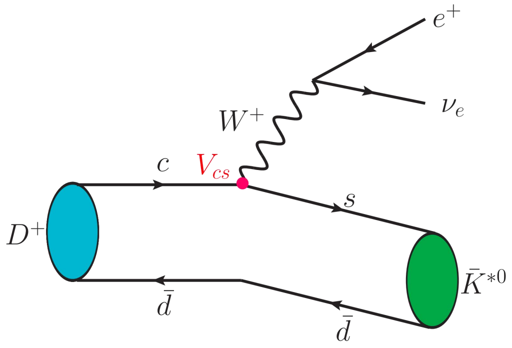
Accurate measurement of SM parameters : $Vub\ V$cs
$B \to \pi\ \ell\ \nu$ PLB, 633(2006)61, 240+ citations
$D \to$ K $\ell\ \nu$ ZPC, 29 (1985) 637, 1900+ citations

Precise measurement of CP violation parameters ： A_{CP}

$$A_{CP}(B^+ \to \pi^+\pi^0) \qquad A_{CP}(B^+ \to D^0\ell^+\nu_\ell)$$
$$A_{CP}(B^+ \to K^+\pi^0) \qquad A_{CP}(B^+ \to \overline{D}^0\pi^+)$$







Why is LCDA important?

Test of lepton universality with $B^0 \to K^{*0}\ell^+\ell^-$ decays

LHCb Collaboration • R. Aaij (CERN) et al. (May 16, 2017)

Published in: *JHEP* 08 (2017) 055 • e-Print: 1705.05802 [hep-ex]

pdf links DOI cite datasets claim

reference search 1,344 citations

Angular analysis of the $B^0 \to K^{*0}\mu^+\mu^-$ decay using 3 fb^{-1} of integrated luminosity

LHCb Collaboration • Roel Aaij (CERN) et al. (Dec 14, 2015)

Published in: *JHEP* 02 (2016) 104 • e-Print: 1512.04442 [hep-ex]

pdf links DOI cite datasets claim

reference search 985 citations

Measurement of the Differential Branching Fraction and Forward-Backward Asymmetry for $B \to K^{(*)}\ell^+\ell^-$

Belle Collaboration • J.-T. Wei (Taiwan, Natl. Taiwan U.) et al. (Apr, 2009)

Published in: *Phys.Rev.Lett.* 103 (2009) 171801 • e-Print: 0904.0770 [hep-ex]

pdf DOI cite claim

reference search 630 citations

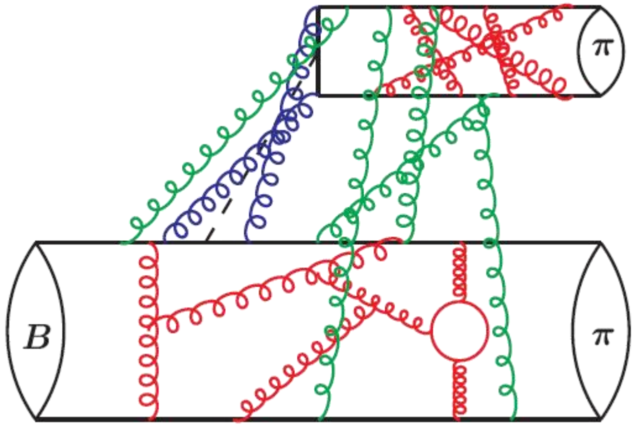
Lepton Flavor Universality

Angular Analysis and P_5'

Forward-backward Asymmetry

Without reliable (precise) knowledge on LCDAs, it is hard to probe NP

Why is LCDA important?



$$\langle \pi(p')\,\pi(q)\,|Q_i|\,\bar{B}(p)\rangle = f^{B\to\pi}(q^2)\int_0^1 dx T_i^{\mathrm{I}}(x)\phi_\pi(x) + \int_0^1 d\xi dx dy T_i^{\mathrm{II}}(\xi,x,y)\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)$$

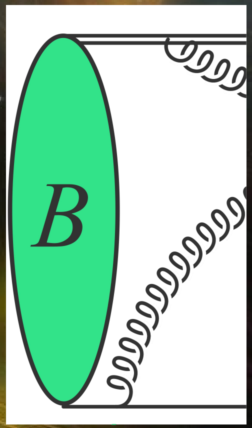
B → π form factor

Hard kernel

B-meson LCDA

QCD Factorization: BBNS, PRL 83, 1914 (1999)
For PQCD, See: Keum, Li, Sanda PRD 63,054008 (2001)

Heavy quark spin symmetry



The heavy meson LCDAs: QCD LCDA

The heavy meson QCD LCDA

$$\mathcal{O}_C^P(u) = \int \frac{dt}{2\pi} e^{-iutn_+\cdot p} \bar{\xi}_C^{(Q)}(0) \not{n}_+ \gamma_5 \left[0, tn_+\right] \xi_C(tn_+) = \int \frac{dt}{2\pi} e^{-iutn_+\cdot p} \bar{Q}(0) \not{n}_+ \gamma_5 \left[0, tn_+\right] q(tn_+),$$

$$\mathcal{O}_C^{\parallel}(u) = \int \frac{dt}{2\pi} e^{-iutn_+\cdot p} \bar{\xi}_C^{(Q)}(0) \not{n}_+ \left[0, tn_+\right] \xi_C(tn_+) = \int \frac{dt}{2\pi} e^{-iutn_+\cdot p} \bar{Q}(0) \not{n}_+ \left[0, tn_+\right] q(tn_+),$$

$$\mathcal{O}_C^{\perp\mu}(u) = \int \frac{dt}{2\pi} e^{-iutn_+\cdot p} \bar{\xi}_C^{(Q)}(0) \not{n}_+ \gamma_\perp^\mu \left[0, tn_+\right] \xi_C(tn_+) = \int \frac{dt}{2\pi} e^{-iutn_+\cdot p} \bar{Q}(0) \not{n}_+ \gamma_\perp^\mu \left[0, tn_+\right] q(tn_+),$$

$$\langle H(p_H)|\mathcal{O}_C^P(u)|0\rangle = -if_P\phi_P(u),$$

$$\langle H^*(p_H,\eta)|\mathcal{O}_C^{\parallel}(u)|0\rangle = f_\parallel \frac{m_H}{n_+\cdot p_H} n_+\cdot\eta^*\phi_\parallel(u),$$

$$\langle H^*(p_H,\eta)|\mathcal{O}_C^{\perp\mu}(u)|0\rangle = f_\perp(\mu)\eta_\perp^{*\mu}\phi_\perp(u).$$

$$q(x) = \xi_C(x) + \eta_C(x),$$

$$Q(x) = \xi_C^{(Q)}(x) + \eta_C^{(Q)}(x),$$

$$\xi_C(x) = \frac{\not{n}_-\not{n}_+}{4} q(x),$$

$$\xi_C^{(Q)}(x) = \frac{\not{n}_-\not{n}_+}{4} Q(x),$$

$$\eta_C(x) = \frac{\not{n}_+\not{n}_-}{4} q(x),$$

$$\eta_C^{(Q)}(x) = \frac{\not{n}_+\not{n}_-}{4} Q(x).$$

The heavy meson LCDAs: HQET LCDA

In heavy-quark limit: $$\mathcal{L}_{\rm HQET} = \bar{h}_v(x) i v\cdot D h_v(x)$$

The leading-twist heavy-meson LCDA in HQET [Grozin and Neubert, 96]

$$\langle H(p_H)|O_v^P(tn_+)|0\rangle = -i\tilde{f}_H m_H n_+\cdot v\int_0^\infty d\omega e^{i\omega t n_+\cdot v}\varphi_+(\omega;\mu),$$

$$\langle H^*(p_H,\eta)|O_v^{||}(tn_+)|0\rangle = \tilde{f}_H m_H n_+\cdot \eta^*\int_0^\infty d\omega e^{i\omega t n_+\cdot v}\varphi_+(\omega;\mu),$$

$$\langle H^*(p_H,\eta)|O_v^{\perp\mu}(tn_+)|0\rangle = \tilde{f}_H m_H n_+\cdot v\eta_\perp^{*\mu}\int_0^\infty d\omega e^{i\omega t n_+\cdot v}\varphi_+(\omega;\mu),$$

$$\begin{aligned} O_v^P(tn_+) &= \bar{h}_v(0)\not{n}_+\gamma_5[0,tn_+]q_s(tn_+), \\ O_v^{||}(tn_+) &= \bar{h}_v(0)\not{n}_+[0,tn_+]q_s(tn_+), \\ O_v^{\perp\mu}(tn_+) &= \bar{h}_v(0)\not{n}_+\gamma_\perp^\mu[0,tn_+]q_s(tn_+), \end{aligned}$$

$$h_v(x) = e^{im_Q v\cdot x}\frac{1+\not{v}}{2}Q(x)$$

The heavy meson LCDAs: bHQET LCDA

HQET LCDAs is boost invariant and can also be defined in boosted HQET (bHQET)

$$\langle H(p_H)|\mathcal{O}_b^P(\omega)|0\rangle = -i\tilde{f}_H\varphi_+(\omega;\mu),$$
$$\langle H^*(p_H,\eta)|\mathcal{O}_b^{||}(\omega)|0\rangle = \tilde{f}_H\frac{n_+\cdot\eta^*}{n_+\cdot v}\varphi_+(\omega;\mu)$$
$$\langle H^*(p_H,\eta)|\mathcal{O}_b^{\perp\mu}(\omega)|0\rangle = \tilde{f}_H\eta_\perp^{*\mu}\varphi_+(\omega;\mu).$$

$$\mathcal{O}_b^P(\omega) = \frac{1}{m_H}\int\frac{dt}{2\pi}e^{-it\omega n_+\cdot v}\sqrt{\frac{n_+\cdot v}{2}}\bar{h}_n(0)\not{n}_+\gamma_5[0,tn_+]\xi_{sc}(tn_+),$$
$$\mathcal{O}_b^{||}(\omega) = \frac{1}{m_H}\int\frac{dt}{2\pi}e^{-it\omega n_+\cdot v}\sqrt{\frac{n_+\cdot v}{2}}\bar{h}_n(0)\not{n}_+[0,tn_+]\xi_{sc}(tn_+),$$
$$\mathcal{O}_b^{\perp\mu}(\omega) = \frac{1}{m_H}\int\frac{dt}{2\pi}e^{-it\omega n_+\cdot v}\sqrt{\frac{n_+\cdot v}{2}}\bar{h}_n(0)\not{n}_+\gamma_\perp^\mu[0,tn_+]\xi_{sc}(tn_+),$$

$$h_n(x) \equiv \sqrt{\frac{2}{n_+\cdot v}}e^{im_Q v\cdot x}\frac{\not{n}_-\not{n}_+}{4}Q(x)$$

The soft-collinear field describes the light anti-quark in the heavy meson in the boosted frame.

$$\xi_{sc} = \frac{\not{n}_-\not{n}_+}{4}q_{sc}(x)$$

The heavy meson LCDAs

The heavy meson QCD LCDA

u: The light quark momentum fraction

$$\mathcal{O}^i_{\mathrm{QCD}}(u) = \int \frac{dt}{2\pi} e^{-iutn_+\cdot p_H} \bar{Q}(0) \not{n}_+ \Gamma_i [0, tn_+] q(tn_+)$$

The heavy meson HQET LCDA

$$\Gamma_i = \gamma^5, 1, \gamma^\mu_\perp$$

$$O^i_v(\omega) = \int \frac{dt}{2\pi} e^{-it\omega n_+\cdot v} \bar{h}_v(0) \not{n}_+ \Gamma_i [0, tn_+] q_s(tn_+)$$

$$\langle Q(p_Q)\bar{q}(p_q)|\mathcal{O}^i_{\mathrm{QCD}}(u)|0\rangle = \frac{1}{n_+ p_H} \int_0^\infty d\omega \mathcal{J}^i(u,\omega) \langle Q(p_Q)\bar{q}(p_q)|\mathcal{O}^i_v(\omega)|0\rangle$$

The heavy meson LCDAs

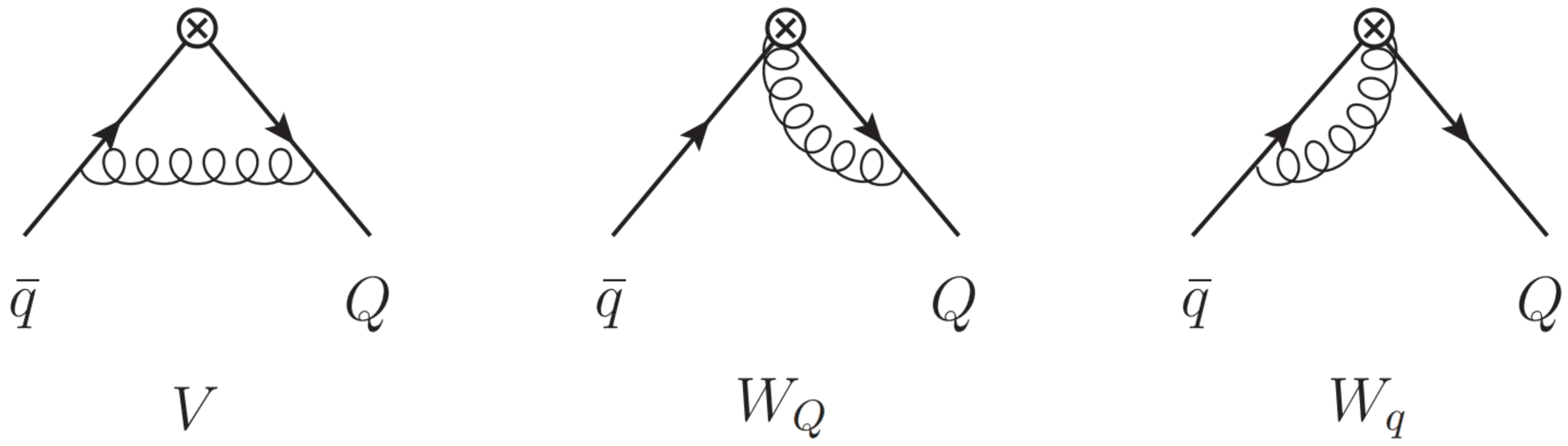


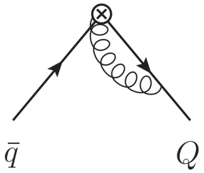
FIG. 1. The one-loop diagrams relevant for calculating the jet function.

$$\phi_i(u) = \frac{\tilde{f}_H}{f_i} \mathcal{J}^i(u,\omega) \otimes \varphi_+(\omega), \qquad u \sim \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_Q}\right)$$ peak region

Heavy quark spin symmetry

$$W^{\rm QCD}_{Q,i}(u,s) = \int \frac{D^dq}{(2\pi)^d}\,\frac{-ig_{\mu\nu}\delta^{ab}}{q^2+i\epsilon}\,\frac{\delta_{ik}\bar u(p_Q)(igT^a_{ij}\gamma^\mu)\frac{i(\not p_Q-\not q+m_Q)}{(p_Q-q)^2-m_Q^2+i\epsilon}(ign_+^\nu T^b_{jk})\frac{i}{n_+\cdot q+i\epsilon}\Gamma_i v(p_q)}{\bar u(p_Q)\Gamma_i v(p_q)}\left(\delta\Big(u-s-\frac{n_+\cdot q}{n_+\cdot p_H}\Big)-\delta(u-s)\right)$$

$$= -ig_s^2C_F\int\frac{D^dq}{(2\pi)^d}\,\frac{2n_+\cdot(p_Q-q)\big(\delta(u-s-\frac{n_+\cdot q}{n_+\cdot p_H})-\delta(u-s)\big)}{[(p_Q-q)^2-m_Q^2+i\epsilon]\,[n_+\cdot q+i\epsilon]\,[q^2+i\epsilon]},$$



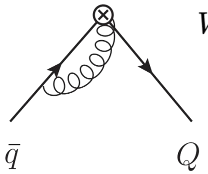
W_Q

$$W^{{\rm QCD},h}_{Q,i}(u,s) = ig_s^2C_F\int\frac{D^dq}{(2\pi)^d}\,\frac{2n_+\cdot(p_Q-q)\delta(s-u)}{[(p_Q-q)^2-m_Q^2+i\epsilon]\,[n_+\cdot q+i\epsilon]\,[q^2+i\epsilon]},$$

$$W^{{\rm QCD},s}_{Q,i}(u,s) = -ig_s^2C_F\int\frac{D^dq}{(2\pi)^d}\,\frac{2n_+\cdot p_Q\big(\delta(u-s-\frac{n_+\cdot q}{n_+\cdot p_H})-\delta(u-s)\big)}{[-2p_Q\cdot q+i\epsilon]\,[n_+\cdot q+i\epsilon]\,[q^2+i\epsilon]}+\mathcal{O}(\lambda),$$

$$W^{\rm QCD}_{q,i}(u,s) = \int \frac{D^dq}{(2\pi)^d}\,\frac{-ig_{\mu\nu}\delta^{ab}}{q^2+i\epsilon}\,\frac{\delta_{ik}\bar u(p_Q)\frac{i}{n_+\cdot q+i\epsilon}(-igT^a_{ij}n_+^\mu)\Gamma_i\frac{i(\not p_q-\not q)}{(p_q-q)^2+i\epsilon}(ig\gamma^\nu T^b_{jk})v(p_q)}{\bar u(p_Q)\Gamma_i v(p_q)}\left(\delta\Big(s-u-\frac{n_+\cdot q}{n_+\cdot p_H}\Big)-\delta(s-u)\right)$$

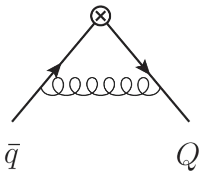
$$= ig_s^2C_F\int\frac{D^dq}{(2\pi)^d}\,\frac{2n_+\cdot(p_q-q)\big(\delta(s-u-\frac{n_+\cdot q}{n_+\cdot p_H})-\delta(s-u)\big)}{[(p_q-q)^2+i\epsilon]\,[n_+\cdot q+i\epsilon]\,[q^2+i\epsilon]}.$$



W_q

$$W^{{\rm QCD},h}_{q,i}(u,s) = -\delta(u-s)ig_s^2C_F\int\frac{D^dq}{(2\pi)^d}\,\frac{2n_+\cdot(-q)}{[q^2+i\epsilon]\,[n_+\cdot q+i\epsilon]\,[q^2+i\epsilon]}=0$$

$$W^{{\rm QCD},s}_{q,i}(u,s) = ig_s^2C_F\int\frac{D^dq}{(2\pi)^d}\,\frac{2n_+\cdot(p_q-q)\big(\delta(s-u-\frac{n_+\cdot q}{n_+\cdot p_H})-\delta(s-u)\big)}{[(p_q-q)^2+i\epsilon]\,[n_+\cdot q+i\epsilon]\,[q^2+i\epsilon]}+\mathcal{O}(\lambda).$$



V

$$V_i^{\rm QCD}(u,s)=\begin{cases} ig_s^2C_F\displaystyle\int\frac{D^dq}{(2\pi)^d}\,\frac{\delta(u-s-\frac{n_+\cdot q}{n_+\cdot p_H})\Big[(d-2)q_\perp^2+\boxed{2q^+p_Q^-+2p_q^+p_Q^-}\Big]}{[(p_Q-q)^2-m_Q^2+i\epsilon]\,[(p_q+q)^2+i\epsilon]\,[q^2+i\epsilon]}, & \text{for}\quad \Gamma_i=\not n_+\gamma_5,\not n_+ \\ ig_s^2C_F\displaystyle\int\frac{D^dq}{(2\pi)^d}\,\frac{\delta(u-s-\frac{n_+\cdot q}{n_+\cdot p_H})\Big[(d-4)q_\perp^2+\boxed{2q^+p_Q^-+2p_q^+p_Q^-}\Big]}{[(p_Q-q)^2-m_Q^2+i\epsilon]\,[(p_q+q)^2+i\epsilon]\,[q^2+i\epsilon]}, & \text{for}\quad \Gamma_i=\not n_+\gamma_\perp. \end{cases}$$

$$V_i^{{\rm QCD},h}(u,s) = 0$$

$$V_i^{{\rm QCD},s}(u,s) = ig_s^2C_F\int\frac{D^dq}{(2\pi)^d}\,\frac{\delta(u-s-\frac{n_+\cdot q}{n_+\cdot p_H})\Big[2q^+p_Q^-+2p_q^+p_Q^-\Big]}{[-2p_Q\cdot q+i\epsilon]\,[(p_q+q)^2+i\epsilon]\,[q^2+i\epsilon]}.$$

Heavy quark spin symmetry

$$W^{\rm HQET}_{Q,i}(\omega,\omega_0) = \int \frac{D^dq}{(2\pi)^d}\frac{-ig_{\mu\nu}\delta^{ab}}{q^2+i\epsilon}\frac{\delta_{ik}\bar u(p_Q)(igT^a_{ij}v^\mu)\frac{1+\slashed{v}}{2}\frac{i\Gamma_i}{v\cdot k+i\epsilon}(ign^\nu_+T^b_{jk})\frac{i}{n_+\cdot q+i\epsilon}v(p_q)}{\bar u(p_Q)\Gamma_i v(p_q)}\left(\delta(\omega-\frac{n_+\cdot(p_q+q)}{n_+\cdot v})-\delta(\omega-\frac{n_+\cdot p_q}{n_+\cdot v})\right)$$
$$= -ig_s^2C_Fn_+\cdot v\int\frac{D^dq}{(2\pi)^d}\frac{\left(\delta(\omega-\frac{n_+\cdot(p_q+q)}{n_+\cdot v})-\delta(\omega-\frac{n_+\cdot p_q}{n_+\cdot v})\right)}{\left[-v\cdot q+i\epsilon\right]\left[n_+\cdot q+i\epsilon\right]\left[q^2+i\epsilon\right]}.$$

$$W^{\rm HQET}_{q,i}(\omega,\omega_0) = \int \frac{D^dq}{(2\pi)^d}\frac{-ig_{\mu\nu}\delta^{ab}}{q^2+i\epsilon}\frac{\delta_{ik}\bar u(p_Q)\frac{i}{n_+\cdot q+i\epsilon}(-igT^a_{ij}n^\mu_+)\Gamma_i\frac{i(\slashed{p}_q-\slashed{q})}{(p_q-q)^2+i\epsilon}(ig\gamma^\nu T^b_{jk})v(p_q)}{\bar u(p_Q)\Gamma_i v(p_q)}\left(\delta(\omega-\frac{n_+\cdot(p_q-q)}{n_+\cdot v})-\delta(\omega-\frac{n_+\cdot p_q}{n_+\cdot v})\right)$$
$$= ig_s^2C_F\int\frac{D^dq}{(2\pi)^d}\frac{2n_+\cdot(p_q-q)\left(\delta(\omega-\frac{n_+\cdot(p_q-q)}{n_+\cdot v})-\delta(\omega-\frac{n_+\cdot p_q}{n_+\cdot v})\right)}{\left[(p_q-q)^2+i\epsilon\right]\left[n_+\cdot q+i\epsilon\right]\left[q^2+i\epsilon\right]}.$$

$$V^{\rm HQET}_{i}(\omega,\omega_0) = \int \frac{D^dq}{(2\pi)^d}\frac{-ig_{\mu\nu}\delta^{ab}}{q^2+i\epsilon}\frac{\delta_{ik}\bar u(p_Q)(igT^a_{ij}v^\mu)\frac{1+\slashed{v}}{2}\frac{i\Gamma_i}{v\cdot k+i\epsilon}\frac{i(\slashed{p}_q+\slashed{q})}{(p_q+q)^2+i\epsilon}(ig\gamma^\nu T^b_{jk})v(p_q)\delta(\omega-\frac{n_+\cdot(p_q+q)}{n_+\cdot v})}{\bar u(p_Q)\Gamma_i v(p_q)}$$
$$= -ig_s^2C_F\int\frac{D^dq}{(2\pi)^d}\frac{1}{q^2+i\epsilon}\frac{1}{-v\cdot q+i\epsilon}\frac{1}{(p_q+q)^2+i\epsilon}\frac{\bar u(p_Q)\Gamma_i(\slashed{p}_q+\slashed{q})\slashed{v}v(p_q)\delta(\omega-\frac{n_+\cdot(p_q+q)}{n_+\cdot v})}{\bar u(p_Q)\Gamma_i v(p_q)}$$
$$= ig_s^2C_F\int\frac{D^dq}{(2\pi)^d}\frac{\delta(\omega-\frac{n_+\cdot(p_q+q)}{n_+\cdot v})\left[2q^+p_Q^-+2p_q^+p_Q^-\right]}{\left[-2p_Q\cdot q+i\epsilon\right]\left[(p_q+q)^2+i\epsilon\right]\left[q^2+i\epsilon\right]}.$$

QCD HQET

$$i\frac{\slashed{p}+m_Q}{p^2-m_Q^2+i\varepsilon} = i\frac{m_Q\slashed{v}+m_Q+\slashed{k}}{2m_Qv\cdot k+k^2+i\varepsilon}\to i\frac{1+\slashed{v}}{2v\cdot k+i\varepsilon}$$

$$\gamma^\mu\to\frac{1+\slashed{v}}{2}\gamma^\mu\frac{1+\slashed{v}}{2}=v^\mu\frac{1+\slashed{v}}{2}\to v^\mu$$

The soft region contribution of QCD is diagram by diagram the same as the HQET results.

$$W^{{\rm QCD},s}_{Q,i}(u,\frac{\omega}{m_H}) = m_HW^{\rm HQET}_{Q,i}(um_H,\omega)$$

$$W^{{\rm QCD},s}_{q,i}(u,\frac{\omega}{m_H}) = m_HW^{\rm HQET}_{q,i}(um_H,\omega)$$

$$V^{{\rm QCD},s}_{i}(u,\frac{\omega}{m_H}) = m_HV^{\rm HQET}_{i}(um_H,\omega)$$

Heavy quark spin symmetry

$$\mathcal{J}^i(u,\omega) = \theta(m_H - \omega)\delta(u - \frac{\omega}{m_H})\left(1 + \frac{\alpha_s C_F}{4\pi}\mathcal{J}^{(1)}_{\text{peak}}(m_H) + \mathcal{O}(\alpha_s^2)\right), \qquad i = P,\ \parallel,\ \perp$$

$$\mathcal{J}^{(1)}_{\text{peak}}(m_H) = \frac{1}{2}\ln^2\frac{\mu^2}{m_H^2} + \frac{1}{2}\ln\frac{\mu^2}{m_H^2} + \frac{\pi^2}{12} + 2.$$

Independent of hadron spin!

$$W^{\text{SCET}}_{Q,hc} = W^{\text{QCD}}_{Q,h} = 2ig_s^2 C_F \int \frac{d^D l}{(2\pi)^D}\frac{n_+\cdot(p_Q - l)\delta(u-s)}{(l^2 - 2l\cdot p_Q + i\epsilon)(l^2 + i\epsilon)(n_+\cdot l + i\epsilon)}$$

$$W^{\text{SCET}}_{q,hc} = W^{\text{QCD}}_{q,h} = 0$$

$$V^{\text{SCET}}_{hc} = V^{\text{QCD}}_{h} = 0.$$

$$W^{\text{HQET}}_{Q,i} = W^{\text{bHQET}}_{Q,i},\ W^{\text{HQET}}_{q,i} = W^{\text{bHQET}}_{q,i},\ \text{and}\ V^{\text{HQET}}_{i} = V^{\text{bHQET}}_{i}$$

Quasi DAs to QCD LCDAs

$$\hat{\mathcal{O}}_i(x) = \int \frac{dz n_z \cdot P}{2\pi} e^{-ixzn_z \cdot P} \bar{Q}(0)\Gamma_i[0, zn_z]q(zn_z)$$

$\Gamma_i = \gamma^{z/t}\gamma_5, \gamma^{z/t}, \gamma^{z/t}\gamma_\perp^\mu$ for $i = P, \|, \perp$

$$\begin{aligned}
\langle H(p_H)|\hat{\mathcal{O}}_P(x)|0\rangle &= -i\hat{f}_P P^{z/t}\hat{\phi}_P(x), \\
\langle H^*(p_H,\eta)|\hat{\mathcal{O}}_{||}(x)|0\rangle &= \hat{f}_{||} m_{H^*}\eta^{*,z/t}\hat{\phi}_{||}(x), \\
\langle H^*(p_H,\eta)|\hat{\mathcal{O}}_\perp^\mu(x)|0\rangle &= \hat{f}_\perp P^{z/t}\eta_\perp^{*\mu}\hat{\phi}_\perp(x).
\end{aligned}$$

Quasi DAs

Lattice QCD calculable!

$$\hat{\phi}_i(x) = \int dy \left[\left(\delta(x-y) + C_i^{(1)}(x,y,\mu) - C_{CT}^{(1)} \right) \phi_i(y,\mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{(yP^z)^2}, \frac{m_H^2}{(P^z)^2} \right) \right]$$

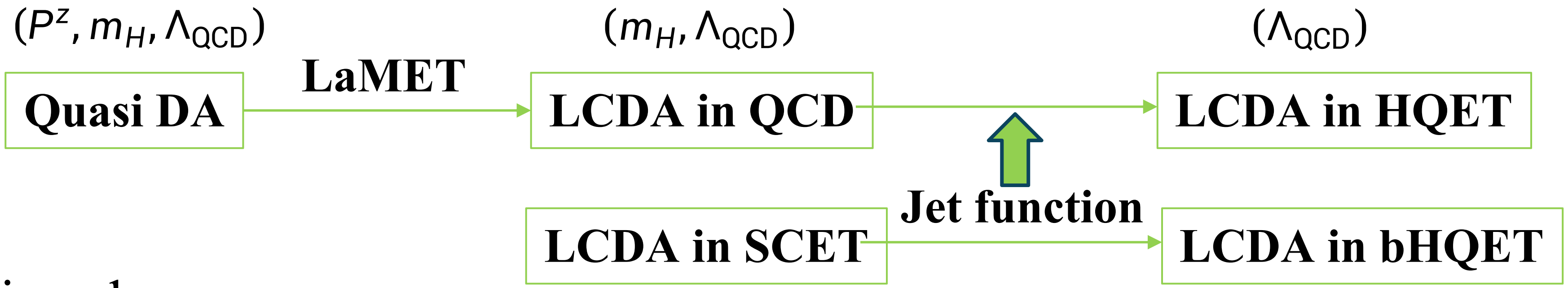
LaMET matching

J. Xu, Q. A. Zhang and S. Zhao, PRD 97 (2018) 114026
Y. S. Liu, W. Wang, J. Xu, Q. A. Zhang, S. Zhao and Y. Zhao, PRD, 99(2019) 094036

The LPC recent work: Phys.Rev.D 111 (2025) 11, L111503, Phys.Rev.D 111 (2025) 3, 034503

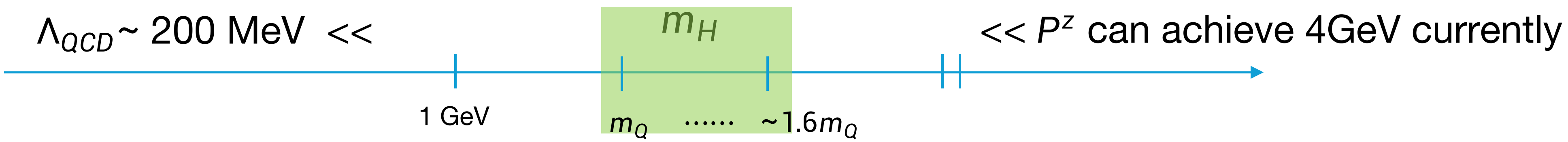
A two-step matching method

- Start from Quasi DA, calculable from LQCD



- A multi-scale processes:
  1. LaMET requires Λ_{QCD}, $m_H \ll P^z$ and finally integrate out P^z;
  2. HQET requires $\Lambda_{QCD} \ll m_H$ and integrate out m_H;

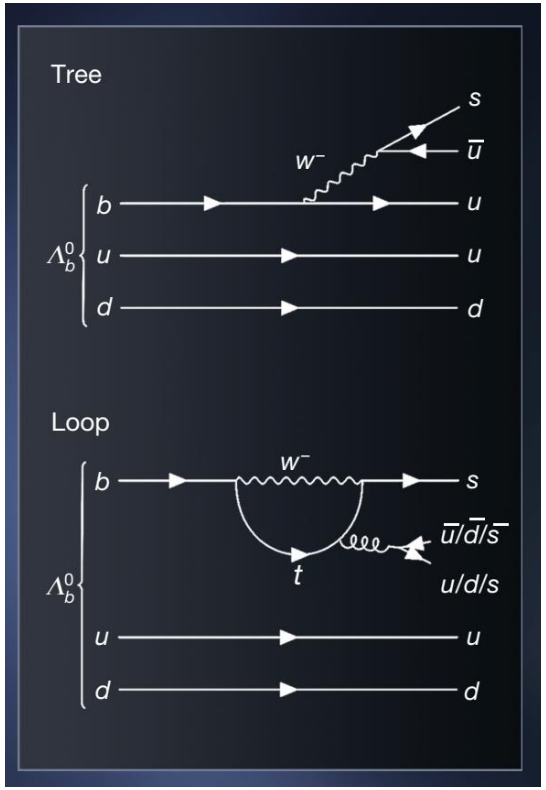
$\Rightarrow$ **Hierarchy $\boldsymbol{\Lambda_{QCD} \ll m_H \ll P^z}$:** A big challenge for lattice simulation but still calculable on the lattice



Brief summary

- The LCDAs at for a heavy pseudoscalar and vector meson within the framework of HQET become indistinguishable.
- The leading-twist HQET LCDA can be determined through lattice simulations of quasi-DAs with a large momentum.
- One can make use of three different equal time matrix elements and determine the same HQET LCDA.

Observation of CP symmetry breaking in heavy baryon decays



Nature 643 (2025) 8074, 1223-1228

Decay topology	Mass region (GeV/c²)	$\mathcal{A}_{CP}$
$\Lambda_b^0 \to R(pK^-)R(\pi^+\pi^-)$	$m_{pK^-} < 2.2$	(5.3±1.3±0.2)%
	$m_{\pi^+\pi^-} < 1.1$	
$\Lambda_b^0 \to R(p\pi^-)R(K^-\pi^+)$	$m_{p\pi^-} < 1.7$	(2.7±0.8±0.1)%
	$0.8 < m_{\pi^+K^-} < 1.0$	
	or $1.1 < m_{\pi^+K^-} < 1.6$	
$\Lambda_b^0 \to R(p\pi^+\pi^-)K^-$	$m_{p\pi^+\pi^-} < 2.7$	(5.4±0.9±0.1)%
$\Lambda_b^0 \to R(K^-\pi^+\pi^-)p$	$m_{K^-\pi^+\pi^-} < 2.0$	(2.0±1.2±0.3)%

- Heavy baryon decays have attracted increasing interest owing to the search for CP violation
- Precision calculations are therefore necessary
- LCDA is a crucial input parameter for inclusive/exclusive processes in high-energy

Heavy baryon LCDAs

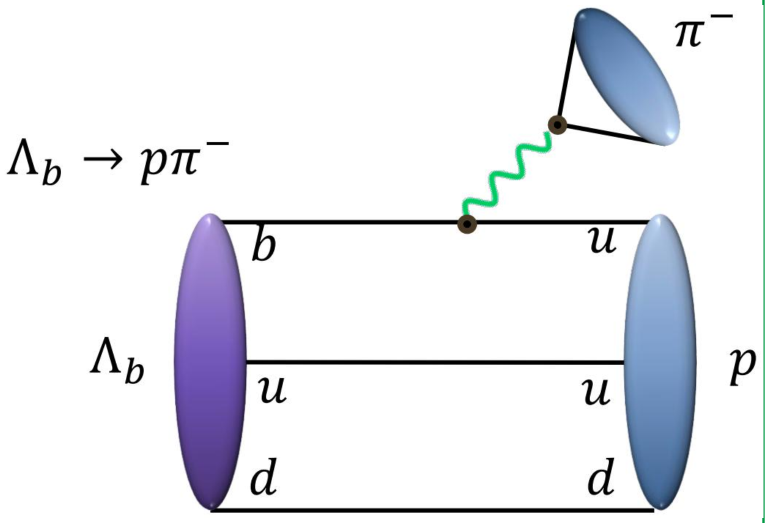
- QCD factorization (QCDF)
- soft-collinear effective theory (SCET)
- perturbative QCD (PQCD)
- light-cone sum rules (LCSR)

Use Λ_b LCDA as inputs

For example, PQCD calculation for: $\Lambda_b \to p$

$$\mathcal{A} = \underline{\Psi_{\Lambda_b}(x_i, b_i, \mu)} \otimes \underline{H(x_i, b_i, x_i', b_i', \mu)} \otimes \underline{\Psi_P(x_i', b_i', \mu)}$$

Λ_b LCDA — **Hard kernel** — Proton LCDA



see Jun Hua's talk

Precise determination of the Λ_b LCDAs is crucial for reliable theoretical predictions!

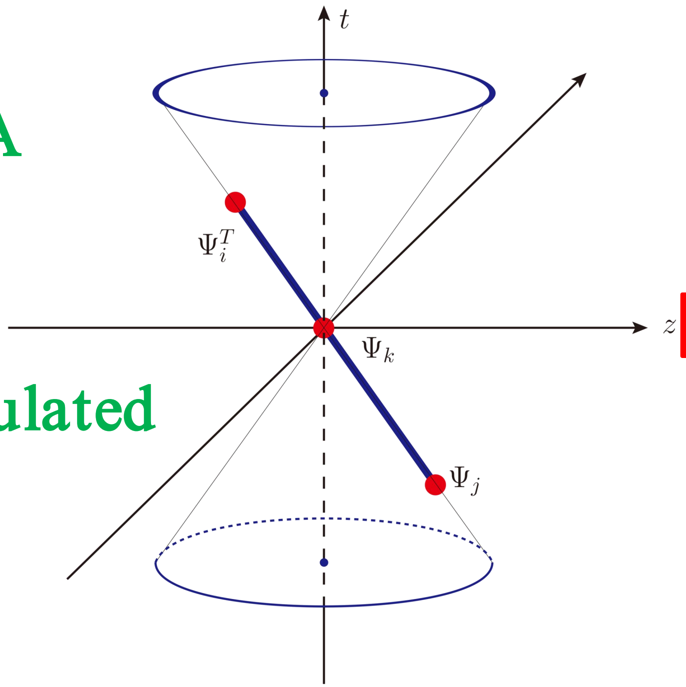
Light-cone definition in the heavy quark limit:

$$\Phi_{\mathrm{HQET}}(\omega_1,\omega_2)\tilde{f}_{\Lambda_Q}u_{\Lambda_Q}(p) = \langle 0|\mathcal{O}_h(\omega_1,\omega_2)|\Lambda_Q(p)\rangle$$

$$\mathcal{O}_h(\omega_1,\omega_2) =(v^+)^2\int\frac{dt_1}{2\pi}\int\frac{dt_2}{2\pi}e^{i\omega_1 v^+ t_1+i\omega_2 v^+ t_2}$$
$$\times\, \epsilon_{ijk}W_{ii'}(t_1 n)u^T_{i'}(t_1 n)\Gamma W_{jj'}(t_2 n)d_{j'}(t_2 n)W_{kk'}(0)h_{v,k'}(0)$$

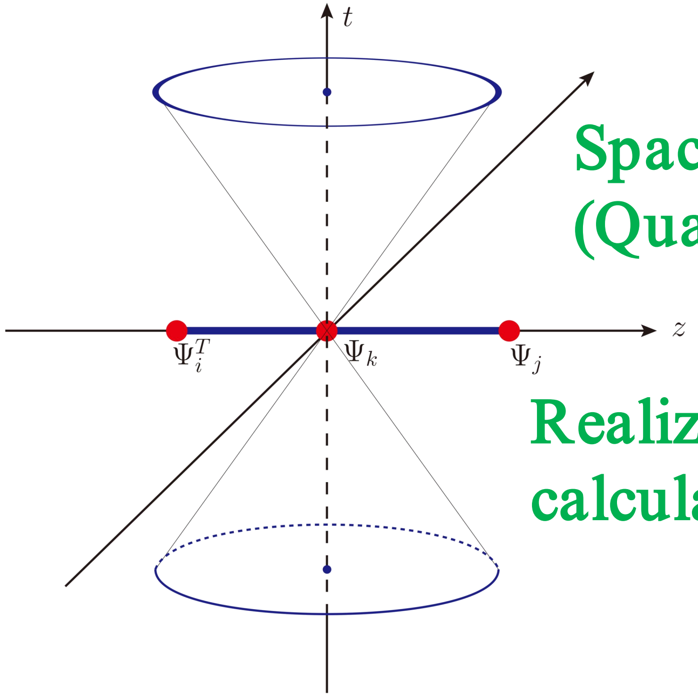
Light-like DA (LCDA)

cannot be calculated in lattice QCD



Matching

LaMET



Space-like DA (Quasi-DA)

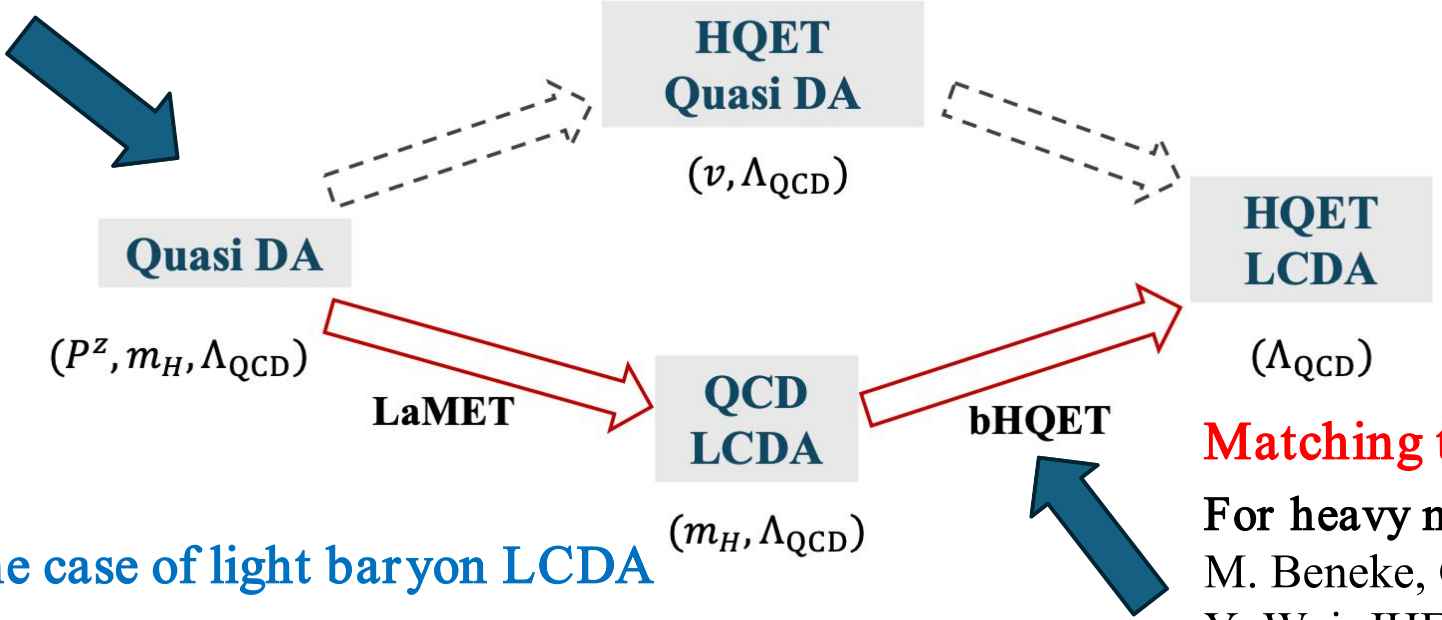
Realize the lattice calculation

Heavy baryon LCDAs: Two-step matching scheme

- However, lattice QCD cannot handle the effective heavy quark field h_v
- To obtain the Λ_b LCDA in HQET, a two-step matching scheme is proposed

- X. Y. Han, J. Hua, X. Ji, C. D. Lu, W. Wang, J. Xu, Q. A. Zhang and S. Zhao, Phys. Rev. D 111, no.11, L111503 (2025)
- X. Y. Han et al. [Lattice Parton], Phys. Rev. D 111, no.3, 034503 (2025)

One uses the heavy quark field in QCD for Lattice calculation



The same as the case of light baryon LCDA

Z. F. Deng, C. Han, W. Wang* , **J. Zeng*** and J. L. Zhang, JHEP 07, 191 (2023)

Matching to the boosted HQET

For heavy meson:

M. Beneke, G. Finauri, K. K. Vos and Y. Wei. JHEP 09, 066 (2023)

QCD LCDA in the boosted frame

Heavy baryon LCDAs: Leading twist $\mathbf{\Lambda_b}$ LCDA in QCD

In the boosted frame, the Leading twist QCD LCDA is described by massive SCET

$$\mathcal{L}_{\rm SCET} = \bar{\xi}(x)\left[2i\bar{n}\cdot D + (i\not{D}_\perp - m_Q)\frac{1}{in\cdot D}(i\not{D}_\perp + m_Q)\right]\frac{\not{n}}{2}\xi(x)$$

Definition of QCD LCDA :

$$\begin{aligned}&\Phi_{\rm QCD}(x_1,x_2)f_{\Lambda_Q}u_{\Lambda_Q}(p)\\ =&(p^+)^2\int\frac{dt_1}{2\pi}\int\frac{dt_2}{2\pi}e^{ix_1p^+t_1+ix_2p^+t_2}\\ &\times\epsilon_{ijk}\langle 0|W_{ii'}(t_1n)u^T_{i'}(t_1n)\Gamma W_{jj'}(t_2n)d_{j'}(t_2n)W_{kk'}(0)Q_{k'}(0)|\Lambda_Q(p)\rangle\end{aligned}$$

Decay constant: $f_{\Lambda_Q}u_{\Lambda_Q}(p) = \epsilon_{ijk}\langle 0|u^T_i(0)\Gamma d_j(0)Q_k(0)|\Lambda_Q(p)\rangle$

Baryon momentum: $p^\mu = m_H v = p^+\bar{n}^\mu + (m_H^2/2p^+)n^\mu \qquad p^+\sim Q\gg m_H$

Heavy baryon LCDAs: Leading twist Λ_b LCDA in QCD

In the boosted frame, the Leading twist HQET LCDA is described by bHQET

The Lagrangian of bHQET can be derived from that of massive SCET by the redefinition of the collinear field

$$\xi(x) = \sqrt{\frac{n\cdot v}{\sqrt{2}}} e^{-im_Q v\cdot x} h_n(x).$$

$$\mathcal{L}_{\rm SCET} \Longrightarrow \mathcal{L}_{\rm bHQET} = \bar{h}_n (iv\cdot D)\frac{\not{\bar{n}}}{\sqrt{2}} h_n + \mathcal{O}(\lambda_h) \qquad \lambda_h = \Lambda_{\rm QCD}/m_Q$$

Heavy quark momentum: $p_Q = m_Q v + k$

$v^\mu \sim (1/b, 1, b) \qquad k^\mu \sim (1/b, 1, b)\Lambda_{\rm QCD}$ boost parameter : $b = m_H/Q \ll 1$

Definition of bHQET LCDA :

$$\begin{aligned}
&\Phi_{\rm bHQET}(\omega_1,\omega_2)\bar{f}_{\Lambda_Q} u_{\Lambda_Q}(p)\\
=&(v^+)^2 \int \frac{dt_1}{2\pi}\int \frac{dt_2}{2\pi} e^{i\omega_1 v^+ t_1 + i\omega_2 v^+ t_2}\\
&\times \epsilon_{ijk}\langle 0| W_{ii'}(t_1 n) u^T_{i'}(t_1 n)\Gamma W_{jj'}(t_2 n) d_{j'}(t_2 n) W_{kk'}(0) h_{n,k'}(0)|\Lambda_Q(p)\rangle.
\end{aligned}$$

Heavy baryon LCDAs: Factorization

- Peak region $x_{1,2} \sim \lambda_h$: Small momentum fraction
- Tail regio $x_1 + x_2 \sim 1$ Large momentum fraction

Factorization formula:

$$\Phi_{\rm QCD}(x_1,x_2) f_{\Lambda_Q} = \begin{cases} \mathcal{J}_{\rm peak}(x_1,x_2,\omega_1,\omega_2) \otimes \Phi_{\rm bHQET}(\omega_1,\omega_2)\bar{f}_{\Lambda_Q}, & x_{1,2} \sim \lambda_h, \\ \mathcal{J}_{\rm tail}(x_1,x_2), & x_1 + x_2 \sim 1 \end{cases}$$

- ➢ QCD LCDA does not match the form of the bHQET LCDA in the tail region
- ➢ Main task is to matching for the jet function in the peak regio $\mathcal{J}_{\rm peak}$

Heavy baryon LCDAs: One-loop calculation

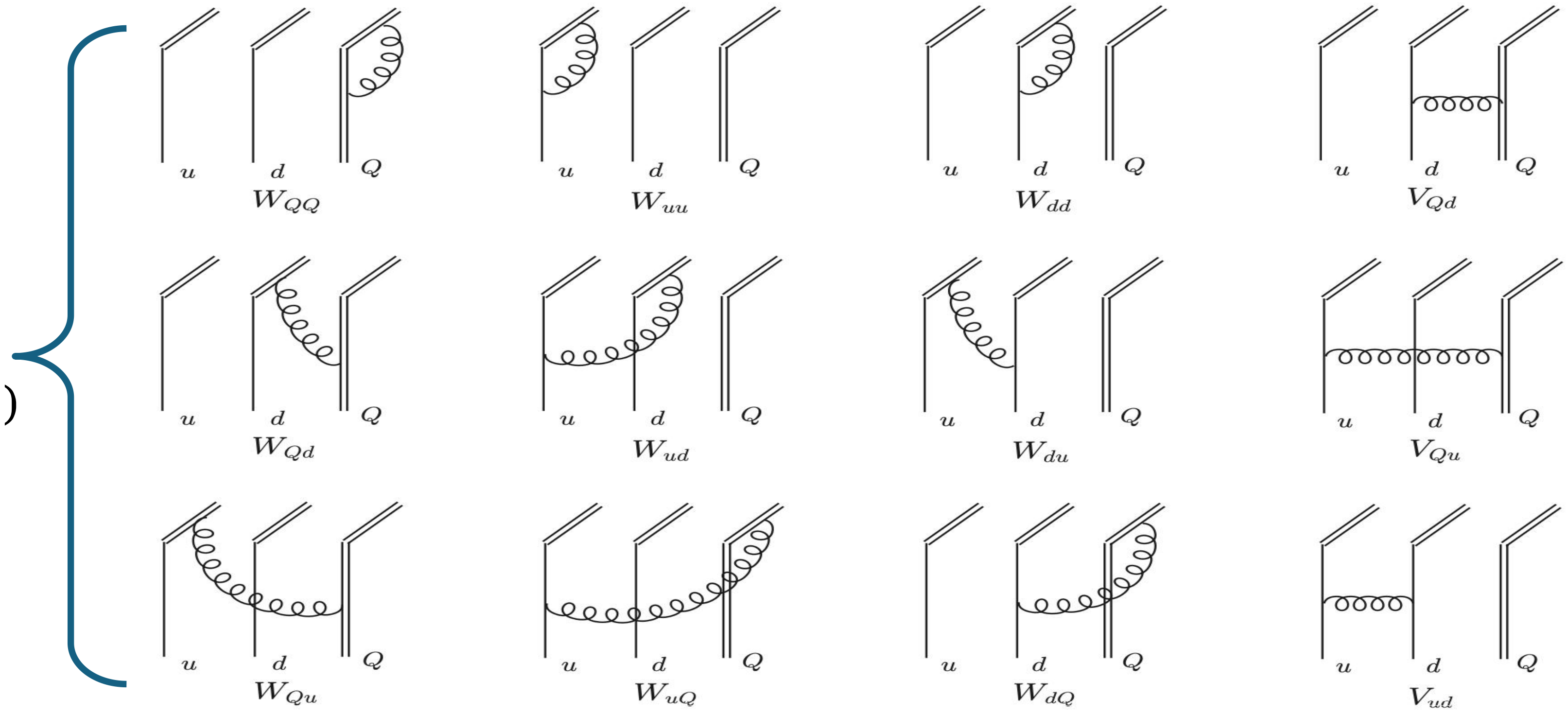
One-loop correction to the jet function

$$\mathcal{J}_{\text{peak}}(x_1, x_2; \omega_1, \omega_2)$$
$$= \theta\left(m_H - \omega_1 - \omega_2\right)\left[\delta\left(x_1 - \frac{\omega_1}{m_H}\right)\delta\left(x_2 - \frac{\omega_2}{m_H}\right) + \frac{\alpha_s C_F}{4\pi}\mathcal{J}^{(1)}(x_1, x_2; \omega_1, \omega_2)\right]$$

$$\mathcal{J}^{(1)}(x_1, x_2; \omega_1, \omega_2) = M^{(1)}(x_1, x_2; \frac{\omega_1}{m_H}, \frac{\omega_2}{m_H}) - m_H^2 N^{(1)}(x_1 m_H, x_2 m_H; \omega_1, \omega_2)$$

QCD amplitudes: $\boldsymbol{M}^{(1)}$

bHQET amplitudes: $\boldsymbol{N}^{(1)}$



Heavy baryon LCDAs: One-loop calculation

$$M^{(1)}(x_1,x_2;x_{1,0},x_{2,0}) = \left[W_Q^c + W_q^c + V_Q^c\right](x_1,x_2;x_{1,0},x_{2,0}) + \frac{1}{2}Z_Q^{(1)}\delta(x_1-x_{1,0})\delta(x_2-x_{2,0})$$
$$+ Z_{\rm SCET}^{(1)}(x_1,x_2;x_{1,0},x_{2,0}),$$

$$N^{(1)}(\omega_1,\omega_2;\nu_1,\nu_2) = \left[W_Q^h + W_q^h + V_Q^h\right](\omega_1,\omega_2;\nu_1,\nu_2) + \frac{1}{2}Z_h^{(1)}\delta(\omega_1-\nu_1)\delta(\omega_2-\nu_2)$$
$$+ Z_{\rm bHQET}^{(1)}(\omega_1,\omega_2;\nu_1,\nu_2),$$

Field renormalization $\quad Z_Q^{(1)} = -\frac{3}{\epsilon} - 3\ln\frac{\mu^2}{m_Q^2} - 4, \qquad Z_h^{(1)} = 0.$

$$W_Q^{c,h} = W_{QQ}^{c,h} + W_{Qd}^{c,h} + W_{Qu}^{c,h},$$
$$W_q^{c,h} = W_{uQ}^{c,h} + W_{dQ}^{c,h},$$
$$V_Q^{c,h} = V_{Qu}^{c,h} + V_{Qd}^{c,h}.$$

Actually, not all the diagrams contribute to the matching kernel

Heavy baryon LCDAs: Expansion by regions

The integration of loop momentum can be separated into integrations in the hard-collinear region and the soft-collinear region

$$\int \frac{D^d q}{(2\pi)^4} \begin{cases} q_{\rm hc} \sim (1, b, b^2)Q & \text{hard-collinear} \\ q_{\rm sc} \sim (1, b, b^2)Q\lambda_h & \text{soft-collinear} \end{cases}$$

Separate the QCD amplitudes:

$$W_Q^c = W_{Q,\rm hc}^c + W_{Q,\rm sc}^c,$$
$$V_Q^c = V_{Q,\rm hc}^c + V_{Q,\rm sc}^c.$$

All diagrams containing a gluon linking the light-quark and heavy-quark sectors yield vanishing hard-collinear amplitudes

$$x_2 - x_{2,0} \sim \lambda_h$$
$$q_{\rm hc}^+/p^+ \sim 1$$

$$\frac{\alpha_s C_F}{4\pi} W_{Qd}^c = -\frac{C_F}{2} i g_s^2 \int \frac{D^d q}{(2\pi)^4} \delta(x_1 - x_{1,0}) \delta\left(x_2 - x_{2,0} + \frac{q^+}{p^+}\right) \frac{1}{(q \cdot n - i\varepsilon)}$$
$$\times \frac{1}{q^2 + i\varepsilon} \frac{1}{(p_Q - q)^2 - m_Q^2 + i\varepsilon} u^T(k_1)\Gamma d(k_2)(\not{p}_Q - \not{q} + m_Q)\not{n}\, Q(p_Q)$$

$$\Rightarrow \quad W_{Qu,\rm hc}^c = W_{Qd,\rm hc}^c = V_{Qu,\rm hc}^c = V_{Qd,\rm hc}^c = 0$$

Heavy baryon LCDAs: Expansion by regions

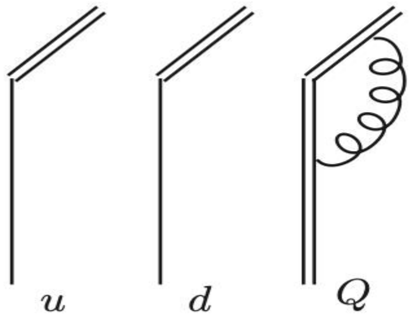
The soft-collinear part is

$$\frac{\alpha_s C_F}{4\pi} W^c_{Qd,\text{sc}} = \frac{C_F}{2}(n\cdot v) i g_s^2 \delta(x_1 - x_{1,0}) \times \int \frac{D^d q}{(2\pi)^4} \delta\left(x_2 - x_{2,0} + \frac{q^+}{p^+}\right) \frac{1}{q\cdot n + i\varepsilon} \frac{1}{q^2 + i\varepsilon} \frac{1}{v\cdot q + i\varepsilon}.$$

which is exactly equivalent to the corresponding bHQET amplitude:

$$W^c_{Q,\text{sc}}(x_1, x_2; \omega_1, \omega_2) = m_H^2 W^h_{Q,\text{sc}}(x_1 m_H, x_2 m_H; \omega_1, \omega_2),$$
$$V^c_{Q,\text{sc}}(x_1, x_2; \omega_1, \omega_2) = m_H^2 V^h_{Q,\text{sc}}(x_1 m_H, x_2 m_H; \omega_1, \omega_2).$$

$$\mathcal{J}^{(1)}(x_1, x_2; \omega_1, \omega_2) = W^c_{QQ,\text{hc}}\left(x_1, x_2; \frac{\omega_1}{m_H}, \frac{\omega_2}{m_H}\right) + \frac{1}{2} Z^{(1)}_Q \delta\left(x_1 - \frac{\omega_1}{m_H}\right)\delta\left(x_2 - \frac{\omega_2}{m_H}\right) + Z^{(1),\text{peak}}_{\text{SCET}}\left(x_1, x_2; \frac{\omega_1}{m_H}, \frac{\omega_2}{m_H}\right) - m_H^2 Z^{(1)}_{\text{bHQET}}(x_1 m_H, x_2 m_H; \omega_1, \omega_2)$$



W_{QQ}

Only one heavy quark virtual diagram contributes to the matching

The UV part of other diagrams are also necessary

Heavy baryon LCDAs: Expansion by regions

$$W^{c}_{QQ,\mathrm{hc}} = \delta(x_1 - x_{1,0})\delta(x_2 - x_{2,0})\left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\ln\frac{\mu^2}{m_Q^2} + \frac{2}{\epsilon} + \frac{1}{2}\ln^2\frac{\mu^2}{m_Q^2} + 2\ln\frac{\mu^2}{m_Q^2} + \frac{\pi^2}{12} + 4\right]$$

Renormalization factor (UV part) in the peak regi $x_i \sim x_{i,0} \sim \lambda_h \ll 1$

$$\begin{aligned}
&Z^{(1),\mathrm{peak}}_{\mathrm{SCET}}(x_1, x_2; x_{1,0}, x_{2,0})\\
&= -\left[W^{c,\mathrm{peak}}_{Q,UV} + W^{c,\mathrm{peak}}_{q,UV} + V^{c,\mathrm{peak}}_{Q,UV}\right](x_1, x_2; x_{1,0}, x_{2,0}) - \delta(x_1 - x_{1,0})\,\delta(x_2 - x_{2,0})\,\frac{1}{2}Z^{UV}_{Q}\frac{1}{\epsilon}\\
&= -\frac{1}{2}\delta(x_1 - x_{1,0})\frac{2}{\epsilon}\left[\frac{\theta(x_2 - x_{2,0})}{x_2 - x_{2,0}} + \frac{x_2}{x_{2,0}}\frac{\theta(x_{2,0} - x_2)}{x_{2,0} - x_2}\right]_{x_{2,0}+}\\
&\quad -\frac{1}{2}\delta(x_2 - x_{2,0})\frac{2}{\epsilon}\left[\frac{\theta(x_1 - x_{1,0})}{x_1 - x_{1,0}} + \frac{x_1}{x_{1,0}}\frac{\theta(x_{1,0} - x_1)}{x_{1,0} - x_1}\right]_{x_{1,0}+}\\
&\quad -\frac{1}{2}\delta(x_1 - x_{1,0})\delta(x_2 - x_{2,0})\frac{2}{\epsilon}\left(4 + \frac{1}{2}Z^{UV}_{Q} + \ln x_{1,0} + \ln x_{2,0}\right)\\
&\quad -\delta(x_1 - x_{1,0})\delta(x_2 - x_{2,0})\frac{2}{\epsilon}\int_0^1 d\lambda\frac{1-\lambda}{\lambda},
\end{aligned}$$

Heavy baryon LCDAs: Expansion by regions

$$\mathcal{J}^{(1)}(x_1,x_2;\omega_1,\omega_2) = W^c_{QQ,\mathrm{hc}}\left(x_1,x_2;\frac{\omega_1}{m_H},\frac{\omega_2}{m_H}\right) + \frac{1}{2}Z_Q^{(1)}\delta\left(x_1-\frac{\omega_1}{m_H}\right)\delta\left(x_2-\frac{\omega_2}{m_H}\right) + Z^{(1),\mathrm{peak}}_{\mathrm{SCET}}\left(x_1,x_2;\frac{\omega_1}{m_H},\frac{\omega_2}{m_H}\right) - \underline{m_H^2 Z^{(1)}_{\mathrm{bHQET}}\left(x_1 m_H, x_2 m_H;\omega_1,\omega_2\right)}$$

The Renormalization factor of bHQET amplitude:

$$\begin{aligned}
&Z^{(1)}_{\mathrm{bHQET}}(\omega_1,\omega_2;\nu_1,\nu_2)\\
&= -\left[W^{h,\mathrm{peak}}_{Q,UV} + W^{h,\mathrm{peak}}_{q,UV} + V^{h,\mathrm{peak}}_{Q,UV}\right](\omega_1,\omega_2;\nu_1,\nu_2) - \delta\left(\omega_1-\nu_1\right)\delta\left(\omega_2-\nu_2\right)\frac{1}{2}Z_h^{UV}\frac{1}{\epsilon}\\
&= -\frac{1}{2}\delta(\omega_1-\nu_1)\frac{2}{\epsilon}\left[\frac{\theta(\omega_2-\nu_2)}{\omega_2-\nu_2} + \frac{\omega_2\theta(\nu_2-\omega_2)}{\nu_2(\nu_2-\omega_2)}\right]_{\nu_2+}\\
&\quad -\frac{1}{2}\delta(\omega_2-\nu_2)\frac{2}{\epsilon}\left[\frac{\theta(\omega_1-\nu_1)}{\omega_1-\nu_1} + \frac{\omega_1\theta(\nu_1-\omega_1)}{\nu_1(\nu_1-\omega_1)}\right]_{\nu_1+}\\
&\quad +\frac{1}{2}\delta\left(\omega_1-\nu_1\right)\delta\left(\omega_2-\nu_2\right)\left[\frac{2}{\epsilon^2} - \frac{4}{\epsilon} - Z_h^{UV}\frac{1}{\epsilon} + \frac{1}{\epsilon}\ln\frac{\mu^2}{\nu_1^2} + \frac{1}{\epsilon}\ln\frac{\mu^2}{\nu_2^2}\right]\\
&\quad -\delta(\omega_1-\nu_1)\delta(\omega_2-\nu_2)S^{(0)}\frac{2}{\epsilon}\int_0^1 d\lambda\frac{1-\lambda}{\lambda}.
\end{aligned}$$

Heavy baryon LCDAs: The jet function in the peak region

$$\begin{aligned}&\mathcal{J}_{\text{peak}}(x_1,x_2;\omega_1,\omega_2)\\&=\theta\left(m_H-\omega_1-\omega_2\right)\delta\left(x_1-\frac{\omega_1}{m_H}\right)\delta\left(x_2-\frac{\omega_2}{m_H}\right)J_{\text{peak}}(m_Q^2),\end{aligned}$$

$$J_{\text{peak}}(m_Q^2)=1+\frac{\alpha_s C_F}{4\pi}\left[\frac{1}{2}\ln^2\frac{\mu^2}{m_Q^2}+\frac{1}{2}\ln\frac{\mu^2}{m_Q^2}+\frac{\pi^2}{12}+2\right].$$

This result is the same as that of B meson

The locality of J_{peak} with respect to the momentum fractions to all orders in perturbation theory

$$\mathcal{O}_c(x_1,x_2)=\underline{\mathcal{J}_{\text{peak}}(x_1,x_2,\omega_1,\omega_2)\otimes\mathcal{O}_h(\omega_1,\omega_2),}\qquad x_{1,2}\sim\lambda_h.$$

$$\begin{aligned}&J_{\text{peak}}(m_Q^2)\cdot(p^+)^2\int\frac{dt_1}{2\pi}\int\frac{dt_2}{2\pi}e^{ix_1p^+t_1+ix_2p^+t_2}\\&\times\epsilon_{ijk}W_{ii'}(t_1n)u_{i'}^T(t_1n)\Gamma W_{jj'}(t_2n)d_{j'}(t_2n)W_{kk'}(0)h_{n,k'}(0).\end{aligned}$$

$$W_{kk'}Q_{k'}\to J_{\text{peak}}(m_Q^2)W_{kk'}h_{n,k'}.$$

J_{peak} serves as a matching coefficient that connects the SCET collinear quark field to the bHQET field.

Heavy baryon LCDAs: matching of the decay constant

$$\Phi_{\rm QCD}(x_1,x_2) = \int d\omega_1 d\omega_2 \frac{\bar{f}_{\Lambda_Q}}{f_{\Lambda_Q}} \mathcal{J}_{\rm peak}(x_1,x_2;\omega_1,\omega_2)\Phi_{\rm bHQET}(\omega_1,\omega_2).$$

Matching the matrix elements of local operators while replacing the external baryon state with free quark states:

$$\begin{cases} \epsilon_{ijk}\langle 0|u_i^T(0)\Gamma d_j(0)Q_k(0)|\overline{u(0)d(0)}Q(p_Q)\rangle, \\ \epsilon_{ijk}\langle 0|u_i^T(0)\Gamma d_j(0)h_{n,k}(0)|\overline{u(0)d(0)}Q(p_Q)\rangle. \end{cases}$$

Diagrams for matching the local operators:

$$\frac{\bar{f}_{\Lambda_Q}}{f_{\Lambda_Q}} = 1 + \frac{\alpha_s C_F}{4\pi}\left(2\ln\frac{\mu^2}{m_Q^2} + 3\right)$$

Heavy baryon LCDAs: Tail region

The tail region of the QCD LCDA and be can be calculated perturbatively

Tail region: $x_1+x_2 \sim 1 \qquad x_{1,0}, x_{2,0} \sim \lambda_h$

$$\langle 0|\mathcal{O}_c(x_1,x_2)|\overline{u(k_1)d(k_2)}Q(p_Q)\rangle$$

$$= S^{(0)}\left[\underline{\delta(x_1-x_{1,0})\delta(x_2-x_{2,0})} + \frac{\alpha_s C_F}{4\pi} M^{(1)}(x_1,x_2;x_{1,0},x_{2,0})\right]$$

=0 since $x_i - x_{i,0} \sim 1$

$$\Rightarrow \quad \Phi^{\text{tail}}_{\text{QCD}}(x_1,x_2) = \lim_{x_{1,0},x_{2,0}\to 0} \frac{\alpha_s C_F}{4\pi} M^{(1)}(x_1,x_2;x_{1,0},x_{2,0}).$$

At one-loop level: $M^{(1)} \propto \begin{cases} \delta(x_1-x_{1,0})\theta(\bar{x}_{1,0}-x_2)\theta(x_2-x_{2,0}) \\ \text{or} \\ \delta(x_2-x_{2,0})\theta(\bar{x}_{2,0}-x_1)\theta(x_1-x_{1,0}) \end{cases} \to 0$

The complete asymptotic behavior in tail region can only be determined by including two-loop contributions.

Heavy baryon LCDAs: Tail region

Boundary regions: $x_1 \sim 1,\ x_2 \sim \lambda_h$ or $x_2 \sim 1,\ x_1 \sim \lambda_h$

Amplitudes with a single delta function survive in the boundary region

$$\Phi^{\rm tail}_{\rm QCD}(x_1 \sim \lambda_h, x_2) \propto \frac{\alpha_s C_F}{4\pi}\frac{\bar{x}_2}{x_2}\left[2(x_2+2)\ln\frac{\mu}{x_2 m_Q} - x_2 + 2\right]$$

The radiative tail of the $\mathbf{\Lambda_b}$ HQET LCDA

$$\phi^{\rm asy}(\omega) = \frac{\alpha_s C_F}{2\pi}\frac{1}{\omega}\left(2\ln\frac{\mu}{\omega}+1\right) + \mathcal{O}\left(\frac{1}{\omega^2}\right)$$

T. Feldmann and D. Vladimirov, JHEP 07, 108 (2025)

Consistent: $m_H\phi^{\rm asy}(x_2 m_H) \propto \Phi^{\rm tail}_{\rm QCD}(x_1 \sim \lambda_h, x_2), \quad x_2 \ll 1$

Summary

1. We established a factorization framework connecting the heavy meson (and heavy baryon) QCD LCDA and bHQET LCDA, enabling non-perturbative extraction from lattice QCD.
2. We computed one-loop corrections and derived a universal jet function with logarithmic structure in heavy baryon which identical to that of heavy meson.
3. Full asymptotic behavior of the Λ_b QCD LCDA in the tail region requires two-loop calculations, while boundary behavior agrees with existing results.
4. This work provides a foundation for lattice QCD determination of hevay-hadron LCDA and precision phenomenology of hevay-hadron weak decays.

Thank you!

Welcome to Hainan Island

Hainan Normal University