Finite-t and target mass corrections to quasi(pseudo)GPDs

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based on: V.M. Braun, Hua-Yu Jiang, 2606.09732

Generalized parton distributions (GPDs)

- “Magnetic GPD” $M(x,\xi,t) = H(x,\xi,t) + E(x,\xi,t)$

$$O^{(\Gamma)}(z_1 n, z_2 n) = \bar q(z_1 n)\Gamma[z_1 n, z_2 n] q(z_2 n)$$

$$\langle p'|\mathcal{O}^{(\not n)}(z_1 n, z_2 n)|p\rangle = \\ = \int_{-1}^{1} dx\, e^{-i(Pn)[z_1(\xi - x) + z_2(x+\xi)]}\Big\{\bar u(p')\not n u(p) M(x,\xi,t) - \frac{2(Pn)}{2m_N}\bar u(p') u(p) E(x,\xi,t)\Big\}$$

- Kinematic variables

$$P_\mu = \frac{1}{2}(p+p')_\mu\,, \qquad \Delta_\mu = (p'-p)_\mu\,, \qquad t = \Delta^2, \qquad \xi = -\frac{(\Delta n)}{2(Pn)}$$

- Gegenbauer moments

$$O_N^{(\not n)}(y) = (-\partial_+)^N \bar q(y) C_N^{3/2}\left(\frac{\overrightarrow{D}_+ - \overleftarrow{D}_+}{\overrightarrow{D}_+ + \overleftarrow{D}_+}\right)\not n q(y)$$

$$(-i)^N \langle p'|O_N^{(\not n)}|p\rangle = \bar u(p')\not n u(p)\, M_N^{(n)}(t) - \frac{Pn}{m_N}\bar u(p') u(p)\, E_N^{(n)}(t)$$

- Polynomiality

$$M_N^{(n)}(t) = \sum_{m,\text{even}}^{N} (\Delta n)^m (Pn)^{N-m} M_{Nm}(t) \qquad \int_{-1}^{1} dx\, C_N^{3/2}\Big(\frac{x}{\xi}\Big) M(x,\xi,t) = \sum_{m,\text{even}}^{N} (-)^m (2\xi)^{m-N} M_{Nm}(t)$$

- Similar for $\widetilde H$ and $\widetilde E$; for E the sum extends to $m = N+1$ (D-term)

Quasi(pseudo) Distributions

$z_{12} = z_1 - z_2$
$v^\mu = (0,0,0,1)$

- QCD factorization of equal-time matrix elements

$$\langle p'|O^{(\Gamma)}(z_1 v, z_2 v)|p\rangle = C_\Gamma(z_1 v, z_2 v; \mu_F^2) \otimes \langle p'|Q^{(\not{n})}(z_1 n, z_2 n)|p\rangle^{\mu_F^2} + \mathcal{O}(z_{12}^2\Lambda_{\rm QCD}^2)$$

- Eight invariant functions for $\Gamma = \gamma_\mu$, similar for $\Gamma = \gamma_\mu\gamma_5$

$$\begin{aligned}\langle p'|Q^{(\gamma_\mu)}|p\rangle^V = {}& \bar u(p')\not{v} u(p)\left\{\frac{v^\mu}{v^2}\,\mathcal{M}^{(v)} + \left(\frac{P^\mu}{(Pv)} - \frac{v^\mu}{v^2}\right)\mathcal{M}^{(P)} + \left(\frac{\Delta_\perp^\mu}{(Pv)}\right)\mathcal{M}^{(\Delta)}\right\} \\ & + \bar u(p')\left(\gamma_\mu - \frac{P_\mu \not{v}}{(Pv)}\right)u(p)\mathcal{M}^{(\gamma)} \\ & - \frac{(Pv)}{m_N}\bar u(p')u(p)\left\{\frac{v^\mu}{v^2}\,\mathcal{E}^{(v)} + \left(\frac{P^\mu}{(Pv)} - \frac{v^\mu}{v^2}\right)\mathcal{E}^{(P)} + \left(\frac{\Delta_\perp^\mu}{(Pv)}\right)\mathcal{E}^{(\Delta)}\right\}\end{aligned}$$

[one more structure populated by contributions of axial-vector GPDs]

- Short-distance expansion, e.g.

$$\begin{aligned}\mathcal{M}^{(P)}(z,0) &= \sum_N \frac{\Gamma(3/2)[iz(Pv)]^N}{4^N\Gamma(N+3/2)}\left[\mathcal{M}_N^{(P,0)} + \frac{tv^2}{(Pv)^2}\mathcal{M}_N^{(P,t)} + \frac{m_N^2 v^2}{(Pv)^2}\mathcal{M}_N^{(P,m_N^2)} + \ldots\right] \\ &= \sum_N \frac{\Gamma(3/2)[i\tau]^N}{4^N\Gamma(N+3/2)}\left[\mathcal{M}_N^{(P,0)} + \frac{tz^2v^2}{\tau^2}\underbrace{\mathcal{M}_N^{(P,t)}}_{\text{this work}} + \frac{m_N^2 z^2 v^2}{\tau^2}\underbrace{\mathcal{M}_N^{(P,m_N^2)}}_{\text{this work}} + \ldots\right]\end{aligned}$$

$\tau = z(Pv)$ "Ioffe time

Example: Scalar target, $N=2$, the $\not{v}$-structure

$$\mathcal{O}_{\{\alpha\beta\gamma\}} = \text{Sym}_{\alpha\beta\gamma}\Big[\tfrac{15}{2}\bar{q}\gamma_\alpha \overset{\leftrightarrow}{D}_\beta \overset{\leftrightarrow}{D}_\gamma\, q - \tfrac{3}{2}\partial_\beta\partial_\gamma\bar{q}\gamma_\alpha q\Big] - \text{traces}$$

$$\langle \bar{q}(zv)\not{v}q(0)\rangle_{N=2} = v^\alpha v^\beta v^\gamma \left\{\frac{z^2}{60}\langle\mathcal{O}_{\{\alpha\beta\gamma\}}\rangle + \frac{z^2}{12}\langle\partial_{\{\alpha}O_{\beta\gamma\}}\rangle + \frac{3z^2}{20}\langle\partial_{\{\alpha}\partial_\beta O_{\gamma\}}\rangle\right\}$$
$$+ v^2 v_\alpha \frac{z^2}{36}\langle\partial^\mu\mathcal{O}_{\mu\alpha}\rangle + \text{genuine higher twist}$$

Matrix elements

$$\langle\mathcal{O}_{\{\alpha\beta\gamma\}}\rangle = -H_{20}\Big[P_\alpha P_\beta P_\gamma - \frac{P^2}{6}\Big(g_{\alpha\beta}P_\gamma + g_{\alpha\gamma}P_\beta + g_{\gamma\beta}P_\alpha\Big)\Big]$$
$$- \frac{1}{3}H_{22}\Big[\Big(\Delta_\alpha\Delta_\beta P_\gamma + \Delta_\alpha\Delta_\gamma P_\beta + \Delta_\gamma\Delta_\beta P_\alpha\Big) - \frac{\Delta^2}{6}\Big(g_{\alpha\beta}P_\gamma + g_{\alpha\gamma}P_\beta + g_{\gamma\beta}P_\alpha\Big)\Big],$$
$$\langle\partial_{\{\alpha}\mathcal{O}_{\beta\gamma\}}\rangle = -\frac{1}{3}H_{10}\Big[\Big(\Delta_\alpha P_\beta P_\gamma + \Delta_\gamma P_\alpha P_\beta + \Delta_\beta P_\alpha P_\gamma\Big) - \frac{P^2}{6}\Big(g_{\alpha\beta}\Delta_\gamma + g_{\alpha\gamma}\Delta_\beta + g_{\gamma\beta}\Delta_\alpha\Big)\Big],$$
$$\langle\partial_{\{\alpha}\partial_\beta\mathcal{O}_{\gamma\}}\rangle = -\frac{1}{3}H_{00}\Big[\Big(\Delta_\alpha\Delta_\beta P_\gamma + \Delta_\alpha\Delta_\gamma P_\beta + \Delta_\gamma\Delta_\beta P_\alpha\Big) - \frac{\Delta^2}{6}\Big(g_{\alpha\beta}P_\gamma + g_{\alpha\gamma}P_\beta + g_{\gamma\beta}P_\alpha\Big)\Big],$$
$$\langle\partial^\mu\mathcal{O}_{\{\mu\alpha\}}\rangle = i^2\Delta^\mu H_{10}\Big[P_\mu P_\alpha - \frac{1}{4}g_{\mu\alpha}P^2\Big] = \frac{1}{4}H_{10}\Delta_\alpha P^2.$$

$$P^2 = M^2 - \frac{\Delta^2}{4}$$

Collecting

$$\langle p'|\bar{q}(zv)\not{v}q(0)|p\rangle_{N=2} = \frac{(iz)^2}{60}(Pv)^3\Big\{H_{20} + 4\eta^2 H_{22} - 10\eta H_{10} + 36\eta^2 H_{00} \qquad \Leftarrow \text{Leading power}$$

$$+ \frac{1}{6}\frac{v^2}{(Pv)^2}\Big[-3P^2 H_{20} - \Delta^2 H_{22} + 10P^2\eta H_{10} - 9\Delta^2 H_{00}\Big] \qquad \Leftarrow \text{Nachtmann}$$

$$+ \frac{5}{6}\eta H_{10}\frac{P^2v^2}{(Pv)^2}\Big\} \qquad \Leftarrow \text{Twist-four operator}$$

"lattice skewedness parameter"

$$\eta = -\frac{\Delta v}{2(Pv)} \qquad \text{cf.} \quad \xi = -\frac{\Delta n}{2(Pn)}$$

- H_{22} contributes also in the $\eta = 0$ limit
- The contribution from twist-four operator vanishes for $\eta = 0$

The twist-four contribution is nontrivial, vanishes for free quarks

$$\begin{aligned}
\partial^\mu \mathcal{O}_{\{\mu\alpha\}} &= 2i\bar{q} gF_{\alpha\mu}\gamma^\mu q\,,\\
\frac{4}{5}\partial^\mu \mathcal{O}_{\{\mu\alpha\beta\}} &= -12 i\bar{q}\gamma^\rho \left\{ gF_{\rho\beta}\overrightarrow{D}_\alpha - \overleftarrow{D}_\alpha gF_{\rho\beta} + (\alpha\leftrightarrow\beta)\right\} q - 4\partial^\rho \bar{q}(\gamma_\beta g\widetilde{F}_{\alpha\rho} + \gamma_\alpha g\widetilde{F}_{\beta\rho})\gamma_5 q\\
&\quad - \frac{8}{3}\,\partial_\beta \bar{q}\gamma^\sigma g\widetilde{F}_{\sigma\alpha}\gamma_5 q - \frac{8}{3}\,\partial_\alpha \bar{q}\gamma^\sigma g\widetilde{F}_{\sigma\beta}\gamma_5 q + \frac{28}{3}\, g_{\alpha\beta}\partial_\rho \bar{q}\gamma^\sigma g\widetilde{F}_{\sigma\rho} q\,,\\
&\ldots
\end{aligned}$$

The “kinematic” contribution can be separated from “dynamical power corrections by requiring autonomous scale dependence

V.B., A.N. Manashov, PRL 107 (2011) 202001

V.B., A.N. Manashov, JHEP 01 (2012) 085

$\mathcal{M}^{(P)}(z,0)$

- Twist-two (Nachtmann)

$$(\mathcal{M}^{(P)})_{t2} = \sum_{N=0}^{\infty} 2(2N+3) \sum_{k=0}^{\infty} \frac{(izP_v)^{N+k}}{k!} \frac{\Gamma(N+k+1)}{\Gamma(2N+k+4)} (\mathcal{M}^{(P)}_{Nk})_{t2}$$

$$(\mathcal{M}^{(P)}_{Nk})_{t2} = (N+k+1) \sum_{\substack{m=0,\\ \text{even}}}^{N} (-2\eta)^{m+k} M_{Nm} - \frac{(N+k-1)}{4(N+k+1)} \frac{v^2}{P_v^2} \sum_{\substack{m=-2,\\ \text{even}}}^{N-2} (-2\eta)^{m+k} \mathbb{M}^{(4)}_{Nkm},$$

- Twist-three

$$(\mathcal{M}^{(P)})_{t3} = \sum_{N=0}^{\infty} \frac{(2N+3)}{(N+1)(N+2)} \sum_{k=0}^{\infty} \frac{(izP_v)^{N+k+1}}{k!} \frac{\Gamma(N+k+2)}{\Gamma(2N+k+4)} (\mathcal{M}^{(P)}_{Nk})_{t3}$$

$$(\mathcal{M}^{(P)}_{Nk})_{t3} = \frac{1}{(N+k+2)} \frac{v^2}{P_v^2} \sum_{\substack{m=-2,\\ \text{even}}}^{N-2} (-2\eta)^{m+k+1} \Big\{ \mathbb{M}^{(4)}_{Nkm} - (N+k)(m+k+2) t M_{Nm+2} \Big\},$$

where

$$\mathbb{M}^{(4)}_{Nkm} = (m+k+1)(m+k+2)t\, M_{Nm+2} + (N-m)(N-m-1)\Big(m_N^2 - \frac{t}{4}\Big) M_{Nm}$$

- Twist-four

$$(\mathcal{M}^{(P)})_{t4} = \frac{1}{2}\sum_{N=0}^{\infty}\frac{(2N+3)}{(N+1)^2(N+2)^2}\sum_{k=0}^{\infty}\frac{(izP_v)^{N+k+1}}{k!}\frac{\Gamma(N+k+2)}{\Gamma(2N+k+4)}\mathfrak{T}_{Nk}(\mathcal{M}^{(P)}_{Nk})_{t4}$$

$$(\mathcal{M}^{(P)}_{Nk})_{t4} = \frac{v^2}{P_v^2}\sum_{\substack{m=0,\\ \text{even}}}^{N-2}(N+k)(-2\eta)^{m+k+1}(\partial\mathbb{M})_{Nm}\,,$$

where

$$(\partial\mathbb{M})_{Nm} = (m+2)(2N-m+1)tM_{N\,m+2} - (N-m)(N-m-1)\left(m_N^2 - \frac{t}{4}\right)M_{Nm}$$

$$\mathfrak{T}_{Nk} = \text{complicated}$$

- most likely not possible to resum in terms of GPDs

[some extra terms involving $\widetilde{H}$ and $\widetilde{E}$ not shown (twist-three)]

GPD model (GK12)

- A simple model based on the DD ansatz

$$M^q(x,\xi,t) = \iint\limits_{|\beta|+|\alpha|\le 1} d\alpha d\beta\, \delta(x-\beta-\xi\alpha) q(\beta,t) h(\beta,\alpha)$$

$$q(\beta,t) = \theta(\beta)\beta^{-0.5-\alpha' t}(1-\beta)^3 e^{Bt}\,, \qquad \alpha' = 0.9 \text{ GeV}^{-2}$$

$$h(\beta,\alpha) = \frac{3}{4}\frac{(1-|\beta|)^2-\alpha^2}{(1-|\beta|)^3}$$

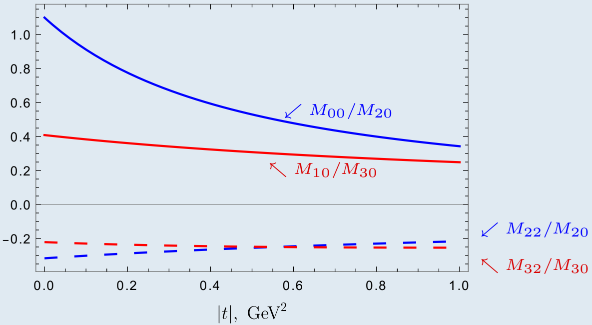
Important: Higher moments have a weaker t-dependence

Test case: $\eta = 0$, $(Pv) = 2$ GeV, $|t| = 1$ GeV^2

- Test case: $\eta = 0$, $(Pv) = 2$ GeV, $|t| = 1$ GeV^2 $\qquad P^2 = m_N^2 - t/4$

$$\mathcal{M}_2^{(P)} = M_{20}(t)\left[1 - \frac{1}{18}\frac{v^2P^2}{P_v^2} - 3\frac{v^2t}{P_v^2}\frac{M_{00}(t)}{M_{20}(t)} - \frac{1}{18}\frac{v^2t}{P_v^2}\frac{M_{22}(t)}{M_{20}(t)}\right],$$

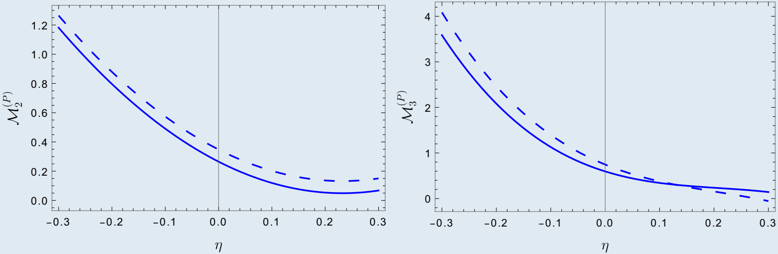
$$\mathcal{M}_3^{(P)} = M_{30}(t)\left[1 - \frac{3}{16}\frac{v^2P^2}{P_v^2} - \frac{25}{6}\frac{v^2t}{P_v^2}\frac{M_{10}(t)}{M_{30}(t)} - \frac{1}{16}\frac{v^2t}{P_v^2}\frac{M_{32}(t)}{M_{30}(t)}\right]$$



One obtains a power correction $\simeq 24\%$ for $\mathcal{M}_2^{(P)}$ and $\simeq 20\%$ for $\mathcal{M}_3^{(P)}$, respectively

Test case: $\eta \neq 0$, $(Pv) = 2$ GeV, $|t| = 1$ GeV^2

- The same for nonzero skewedness



Solid: with power corrections; Dashed: without power corrections

Translation invariance

$$O^{(\Gamma)}(z_1, z_2) = \bar{q}(z_1 v)\Gamma[z_1 v, z_2 v]q(z_2 v)$$

- Consequence of Lorentz invariance

$$\langle p'|O^{(\Gamma)}(z_1+\delta, z_2+\delta)|p\rangle = e^{i\delta(\Delta v)}\langle p'|O^{(\Gamma)}(z_1, z_2)|p\rangle$$

- Violated by the twist expansion, restored by adding kinematic power corrections

$$\langle p'|[O^{(\Gamma)}(z_1+\delta, z_2+\delta)]_{t2}|p\rangle = e^{i\delta(\Delta v)}\langle p'|[O^{(\Gamma)}(z_1, z_2)]_{t2}|p\rangle + \mathcal{O}\left(\frac{1}{(Pv)}\right)$$

$$\langle p'|[O^{(\Gamma)}(z_1+\delta, z_2+\delta)]_{t2+t3}|p\rangle = e^{i\delta(\Delta v)}\langle p'|[O^{(\Gamma)}(z_1, z_2)]_{t2+t3}|p\rangle + \mathcal{O}\left(\frac{1}{(Pv)^2}\right)$$

V.B., 2308.04270

$$\langle p'|[O^{(\Gamma)}(z_1+\delta, z_2+\delta)]_{t2+t3+t4}|p\rangle = e^{i\delta(\Delta v)}\langle p'|[O^{(\Gamma)}(z_1, z_2)]_{t2+t3+t4}|p\rangle + \mathcal{O}\left(\frac{1}{(Pv)^3}\right)$$

This work

[The problem is unrelated to the separation of Lorentz-invariant amplitudes]

- Summary
  - Finite t and nucleon mass corrections calculated for all invariant structures for $\Gamma = \gamma_\mu$ and $\Gamma = \gamma_\mu\gamma_5$
  - Results presented for $z_1 = z$, $z_2 = 0$, can be extended to arbitrary positions if necessary
  - Results presented in the form of the short-distance expansion (in Ioffe time)
    — general expressions for all moments and separately for $N = 1, 2, 3$
  - Estimated size of power corrections: 20-25% for $(Pv) = 2$ GeV, $|t| = 1$ GeV^2
    The largest corrections come from twist-three contributions; nucleon mass corrections are small
  - Overall structure of "genuine" power corrections from quark-gluon correlations — renormalons

V.B., M. Koller, J. Schoenleber, 2401.08012

- Comments
  - Most likely, the power series in Ioffe time is convergent and can be resummed (numerically)
  - Separation of Lorentz-invariant amplitudes not necessary. Can reduce stochastic noise?

Supplementary Slides

Light-ray Operator Product Expansion

$$[\bar{q}(z_1 v)\gamma^\mu q(z_2 v)]_{t4} = 2v^\mu [A(v; z_1, z_2)]_{lt} + 2v^2 \partial^\mu [B(v; z_1, z_2)]_{lt}\,,$$

$$\begin{aligned}
A(n; z_1, z_2) &= \frac{1}{4}\int_0^1 du\, \Big\{ u^2 \ln u\, z_1 z_2\, \nabla^2 \Big[\bar{q}(uz_1 n)\not{n} q(uz_2 un)\Big] \\
&\quad + \Big(z_2\partial_{z_2} - \frac{z_1}{z_{12}} - \ln u\, z_2 \partial^2_{z_2} z_{12}\Big) R_2(uz_1 n, uz_2 n) \\
&\quad - \Big(z_1\partial_{z_1} - \frac{z_2}{z_{21}} - \ln u\, z_1 \partial^2_{z_1} z_{21}\Big) \bar{R}_2(uz_1 n, uz_2 n) \Big\} \\
B(n; z_1, z_2) &= \frac{1}{8}\int_0^1 \frac{du}{u^2}\, \Big\{ u^2(1-u^2+u^2\ln u)\, z_1 z_2\, \nabla^2 \Big[\bar{q}(uz_1 n)\not{n} q(uz_2 un)\Big] \\
&\quad - \Big[(1-u^2)\Big(z_2\partial_{z_2} - \frac{z_1}{z_{12}}\Big) + (1-u^2+u^2\ln u)\, z_2 \partial^2_{z_2} z_{12}\Big] R_2(uz_1 n, uz_2 n) \\
&\quad + \Big[(1-u^2)\Big(z_1\partial_{z_1} - \frac{z_2}{z_{21}}\Big) + (1-u^2+u^2\ln u)\, z_1 \partial^2_{z_1} z_{21}\Big] \bar{R}_2(uz_1 n, uz_2 n) \Big\} \\
&\quad + \frac{1}{8}\int_0^1 \frac{du}{u^2}\Big[R_1(uz_1 n, uz_2 n) - \bar{R}_1(uz_1 n, uz_2 n)\Big]
\end{aligned}$$

The R-operators

$$R_i(z_1 n, z_2 n) = \int_{z_2}^{z_1} dw\, (w - z_2)\, Q_i(z_1 n, wn, z_2 n)$$

where Q_1 and Q_2 are the components of twist-four quark-gluon operators with different properties under conformal $SL(2,R)$ transformations

Twist-four coefficients

$$\mathfrak{T}_{Nk} = \frac{1}{N+k}\left\{\frac{1}{2}N(N+3) + \frac{\Gamma(N+2)}{\Gamma(N+k+2)}\frac{S(N,k)-\bar{S}(N,k)}{\gamma_N}\right\}$$

$$S(N,k) = \frac{1}{2}(N+2)\sum_{m=0}^{N-1}\left\{\frac{(-1)^{N-m+1}\Gamma(N+2)}{\Gamma(m+2)\Gamma(N-m+2)} + 1 - \frac{(N+3)(N-m)}{(N+1)(N+2)}\right\}$$
$$\times \frac{\Gamma(m+k+2)}{\Gamma(m+1)}\frac{1}{N-m+1}\,{}_3F_2\left(\begin{matrix}-k, N-m+1, N-m+1\\ -m-k-1, N-m+2\end{matrix};1\right),$$

$$\overline{S}(N,k) = (N+2)\sum_{m=0}^{N-1}\left\{\frac{(-1)^{N-m+1}\Gamma(N+1)}{\Gamma(m+1)\Gamma(N-m+3)} + \frac{1}{2}\right\}\left\{\frac{(m+1)}{(N-m)}\frac{\Gamma(N+k+2)}{\Gamma(N+2)}\right.$$
$$\left. - \frac{\Gamma(m+k+2)}{\Gamma(m+1)}\frac{1}{N-m+1}\,{}_3F_2\left(\begin{matrix}-k, N-m+1, N-m+1\\ -m-k-1, N-m+2\end{matrix};1\right)\right\}.$$