Contribution ID: **52** Type: **Invited talk**

Lukasiewicz logic and Tsallis entropy connected with free projections in the free and conditionally free probability

Wednesday, 17 September 2025 11:50 (20 minutes)

In my talk we consider the following topics:

1.Free and C-free probaility and completely positive maps.

2.Free independent projections as a model of Jozef Lukasiewicz
n-valued logic , $n > 2$ and also model of continuous logic of Lukasiewicz-Tarski.

3.Main Theorem: : If q is real number and x, y are from interval $(0, 1)$, then the Tsallis entropy is defined as

$\mathrm{T}_q(x, y) = [x^{1-q} + y^{1-q} - 1]_+^{1/(1-q)}$.

Then we have: If $\mathbf{P}$ and $\mathbf{Q}$ are free independent in some probability space $(\mathbf{A}, \mathrm{tr})$ with trace tr state on $\mathbf{A}$, and $\mathrm{tr}(\mathbf{P}) = x$, $\mathrm{tr}(yQ) = y$, then $\mathrm{tr}(\mathbf{P^Q}) = T_0(x, y)$, if $\mathbf{P}$ and $\mathbf{Q}$ are Boolean independent, then $\mathrm{tr}(\mathbf{P^Q}) = T_2(x, y)$ and relations with Dagum distributions, which are called log-logistic distributions in many statistics models.

If $\mathbf{P}$ and $\mathbf{Q}$ are classical independent then $\mathrm{tr}(\mathbf{P^Q}) = T_1(x, y) = \lim$
$T_s(x, y)$, as s tends to 1.

Here the projection $\mathbf{P^Q}$ is the smallest projections on the closed linear span of Im($\mathbf{P}$) and Im($\mathbf{Q}$).
The generalizations of cases of Tsallis entropy T_q, for q in (0,1)
we will use conditionally free independent projections.

4.Remarks on the free product of qunatum channels.

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Primary author: BOŻEJKO, Marek (Instytut Matematyczny Uniwersytet Wrocławski)

Presenter: BOŻEJKO, Marek (Instytut Matematyczny Uniwersytet Wrocławski)

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