Maximally entangled proton and charged hadron multiplicity in Deep Inelastic Scattering

The Henryk Niewodniczański
Institute of Nuclear Physics
Polish Academy of Sciences

Krzysztof Kutak

Based on

Eur.Phys.J.C 82 (2022) 2, 111
M. Hentschinski, K. Kutak

Eur.Phys.J.C 82 (2022) 12, 1147
M. Hentschinski, K.Kutak, R. Straka

Arxiv:2305.03069
H. Hentschinski, D. Kharzeev. K. Kutak, Z. Tu

Maximally entangled proton and charged hadron multiplicity in Deep Inelastic Scattering and Diffractive Deep Inelastic Scattering

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Motivation for studies of EE

- bounds and properties of EE may provide some new insight on behavior of pdfs
- links to other areas (thermodynamics, gravity, quantum information, conformal field theory)
- Interesting in context of parton saturation and thermalization problem of Quark Gluon Plasma

Various approaches to entropy in the low x limit: entropy of gluon density, thermodynamic entropy, momentum space entanglement, coordinate space entanglement, Wehrl entropy,...

Boltzman and von Neuman entropy formulas – reminder

The entropy S of macrostate is given by the log of number W of distinct microstates that compose it

$$S = -\sum_{i=1}^{W} p(i) \ln p(i)$$ Gibbs entropy

For uniform distribution $p(i) = \frac{1}{W}$ the entropy is maximal

Boltzmann entropy

$$S = \ln W$$

Since partons are introduced as the microscopic constituents that compose the macroscopic state of the proton, it is natural to evaluate the corresponding entropy or entropy corresponding to parton density.

K. Kutak '11, Peschanski'12
A. Kovner, M. Lublinsky '15
D. Kharzeev, E. Levin '17,...

But proton as a whole is a pure state and the von Neuman entropy is 0. Can one get any nontrivial result?

For pure state (one state) density matrix is:

$$\rho = |\psi\rangle\langle\psi|$$

$$S_{VN} = -Tr[\rho \ln \rho] = -1 \ln 1 = 0$$

For mixed state i.e. classical statistical mixture

$$\rho = \sum p(i) |\psi_i\rangle\langle\psi_i|$$

$$S_{VN} \neq 0$$

Kharzeev, Levin '17

Entanglement entropy in DIS – the idea

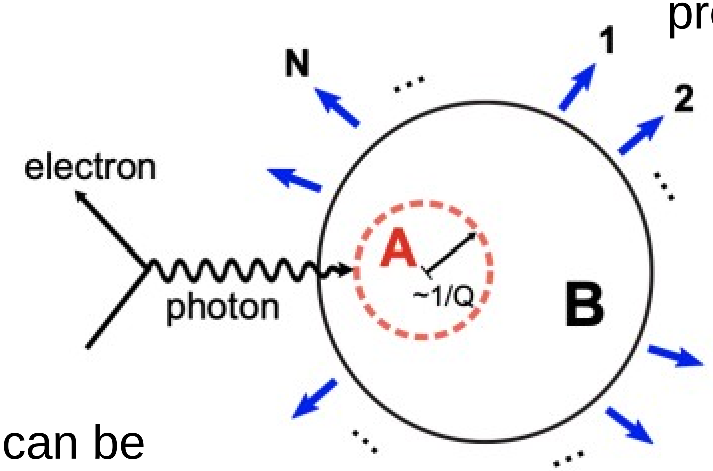
The composite system is described by $\mathcal{H}_A \otimes \mathcal{H}_B$

$|\Psi_{AB}\rangle$ in $A \cap B$

physical state in A
physical state in B

$$|\Psi_{AB}\rangle = \sum_{i,j} c_{ij} |\varphi_i^A\rangle \otimes |\varphi_j^B\rangle$$

proton's rest frame



entangled
if the product can not be expressed as separable product state

$$|\Psi_{AB}\rangle = \sum_{i,j} c_{ij} |\varphi_i^A\rangle \otimes |\varphi_j^B\rangle$$

separable
if the product can be expressed as separable product state

$$|\Psi_{AB}\rangle = |\varphi^A\rangle \otimes |\varphi^B\rangle$$

$\mathcal{H}_B$ of dimension n_B.

$\mathcal{H}_A$ of dimension n_A

Kharzeev, Levin '17

See also
Dumitru,Kovner,Skokov'23

Schmidt decomposition allows to

$$|\Psi_{AB}\rangle = \sum \alpha_n |\Psi_n^A\rangle |\Psi_n^B\rangle$$

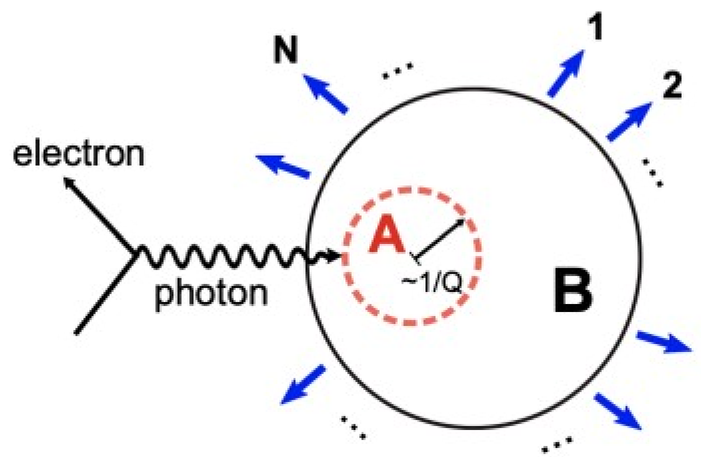
orthonormal states belonging to A

orthonormal states bolonging to B

related to matrix C

Entanglement entropy in DIS – the idea

$$|\Psi_{AB}\rangle = \sum_n \alpha_n |\Psi_n^A\rangle |\Psi_n^B\rangle$$



proton's rest frame

$$\rho_{AB} = |\Psi_{AB}\rangle\langle\Psi_{AB}|$$

$$\rho_A = \mathrm{tr}_B \rho_{AB} = \sum_n \alpha_n^2 |\Psi_n^A\rangle\langle\Psi_n^A|$$

The density matrix of the mixed state probed in region A

$\alpha_n^2 \equiv p_n$ probability of state with *n* partons or dipoles

Kharzeev, Levin '17

$$S = -\sum_n p_n \ln p_n$$

"entropy results from the entanglement between the regions A and B, and can thus be interpreted as the entanglement entropy. Entropy of region A is the same as entropy in region B."

See talk by D. Kharzeev

Partonic, dipole cascade

$$p_n = P_n$$

$$\frac{dP_n(Y)}{dY} = -\lambda n P_n(Y) + (n-1)\lambda P_{n-1}(Y)$$

set of partons is described by set of dipoles with fixed sizes ,Y is rapidity and is related to energy Mueller 95, Lublinsky, Levin '03

depletion of the probability to find n dipoles due to the splitting into (n + 1) dipoles.

the growth due to the splitting of (n − 1) dipoles into n dipoles.

$$P_n(Y) = e^{-\lambda Y}\left(1 - e^{-\lambda Y}\right)^{n-1}$$

$$S = -\sum_n p_n \ln p_n$$

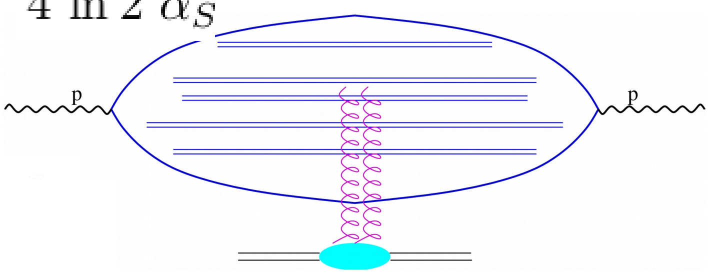
$$S(Y) = \ln\left(e^{\lambda Y} - 1\right) + e^{\lambda Y} \ln\left(\frac{1}{1 - e^{-\lambda Y}}\right)$$

$S(Y) \approx \lambda Y$ where $Y = \ln 1/x$

$$\langle n \rangle = \sum_n n P_n(Y) = \left(\frac{1}{x}\right)^{\lambda}$$

$4 \ln 2\, \bar{\alpha}_S$

See also Kovner, Levin, Lublinsky, JHEP 05 (2022) 019;



Kharzeev, Levin '17

Assumption

$$xg(x) = \langle n \rangle$$

Natural object for equations with rapidity evolution variable

See for in 1+1 density matrix and 2+1 dimensional case in Liu, Nowak, Zahed '22 See talk by Y. Liu

$$S(x) = \ln(xg(x))$$

$$S(x, Q^2) \approx \ln(xg(x, Q^2))$$

KL entropy formula - interpretation

$$S(x) = \ln(xg(x))$$

$$P_n(Y) = e^{-\lambda Y}(1 - e^{-\lambda Y})^{n-1}$$

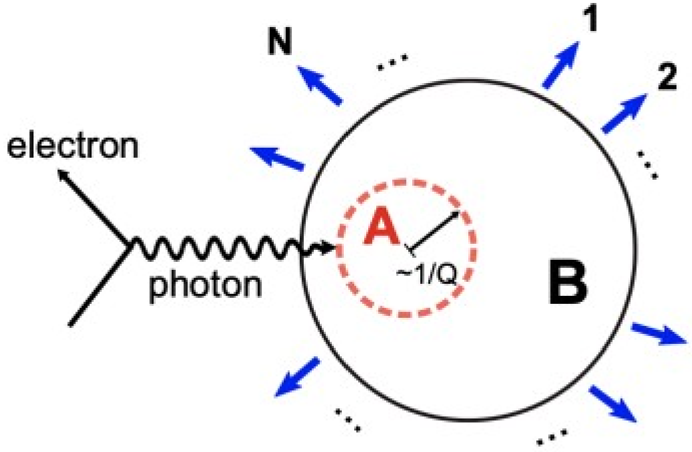
At low x partonic microstates have equal probabilities.

In this equipartitioned state the entropy is maximal – the partonic state at small x is maximally entangled.

Entanglement entropy – calculation and measurement

For DIS at high energies, this entanglement entropy can be calculated using

$$S(x, Q^2) = \ln \left\langle n \left(\ln \frac{1}{x}, Q \right) \right\rangle$$

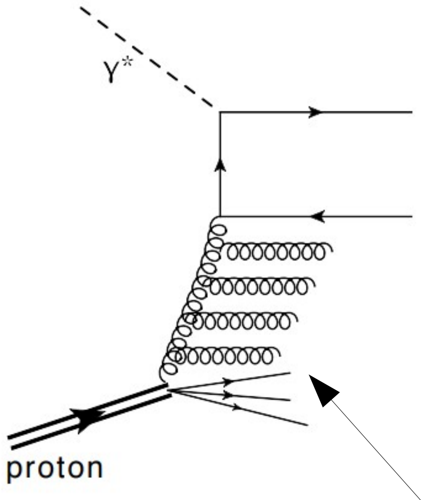


$$S_{hadron} = \sum P(N) \ln P(N)$$

The charged particle multiplicity distribution is measured in either the current fragmentation region or the target fragmentation region.

Fraction of events with charged hadron. Measurement performed in rapidity bins.

Gluon and quark distribution



Hentschinski, Kutak'21

$$\left\langle n\left(\ln\frac{1}{x},Q\right)\right\rangle = xg(x,Q)+x\Sigma(x,Q)$$

In the linear regime obeys BFKL equation. In our calculations we use NLO BFKL with kinematical improvements and running coupling. The gluon density has been fitted to F_2 data (exact kinematics was used)

Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
Phys.Rev.Lett. 110 (2013) 4, 041601

We calculate the sea quarks distribution using

See also Kharzeev and Levin
Phys. Rev. D 104, 031503 (2021)

$$x\Sigma(x,Q)=P_{qg}(Q,\mathbf{k})\otimes\mathcal{F}(x,\mathbf{k}^2)$$

$$xg(x,Q)=\int_0^{Q^2}d\mathbf{k}^2\mathcal{F}(x,\mathbf{k}^2)$$

Transverse momentum dependent splitting function
Catani, Hautmann
Nucl.Phys. B427 (1994) 475-524

Other methods for resummation:
KMS (Kwiecinski, Martin, Stasto);
CCSS (Colferai, Ciafaloni, Staśto, Salam)

Gluon distribution

NLO BFKL with collinear resummation

$$\mathcal{F}\left(x,\boldsymbol{k}^2,Q\right) = \frac{1}{\boldsymbol{k}^2}\int_{\frac{1}{2}-i\infty}^{\frac{1}{2}+i\infty}\frac{d\gamma}{2\pi i}\;\hat{g}\left(x,\frac{Q^2}{Q_0^2},\gamma\right)\;\left(\frac{\boldsymbol{k}^2}{Q_0^2}\right)^{\gamma}$$

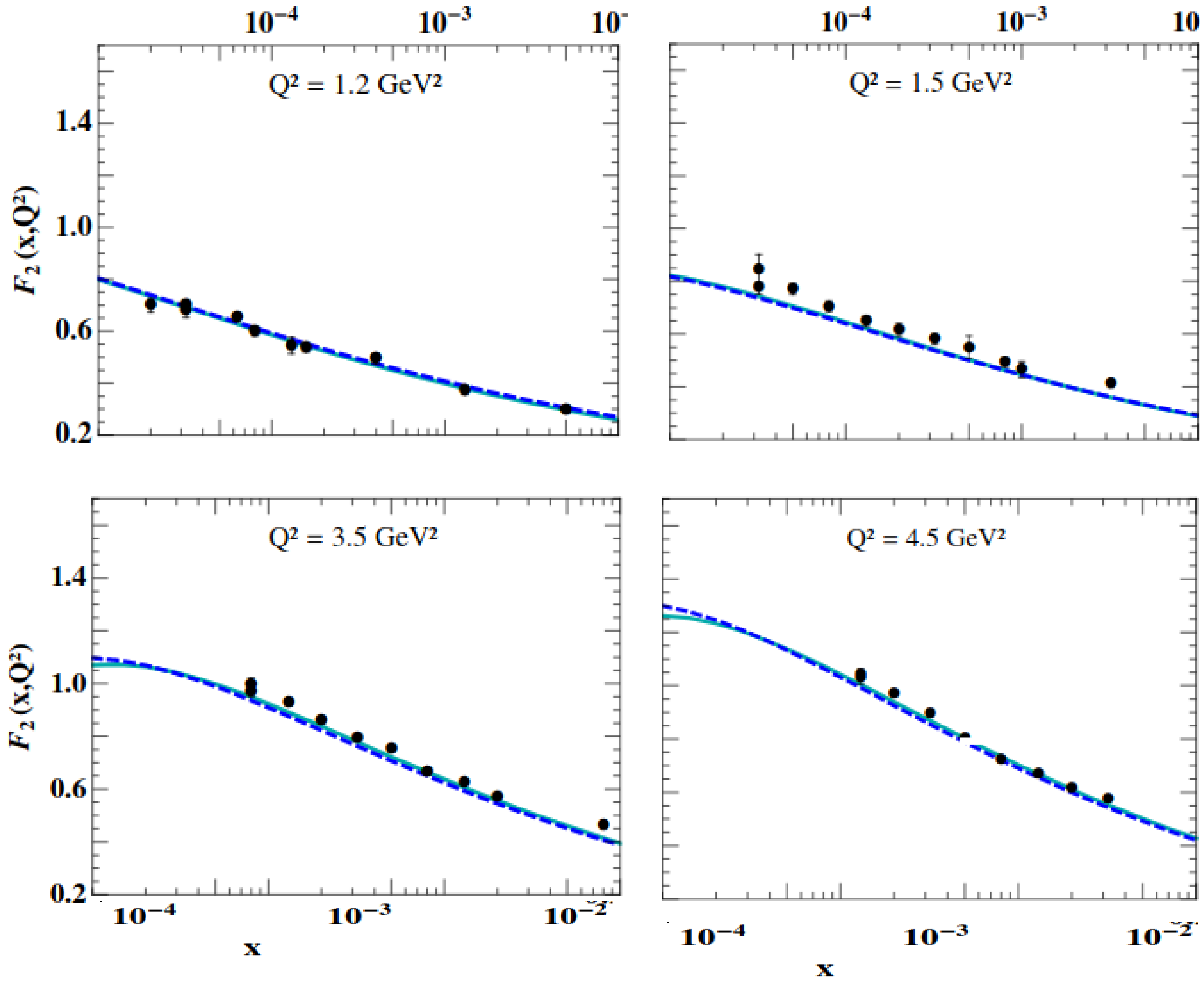
$$\hat{g}\left(x,\frac{Q^2}{Q_0^2}\gamma\right) = \frac{\mathcal{C}\cdot\Gamma(\delta-\gamma)}{\pi\Gamma(\delta)}\cdot\left(\frac{1}{x}\right)^{\chi(\gamma,Q,Q)}\;\left\{1+\frac{\bar{\alpha}_s^2\beta_0\chi_0\left(\gamma\right)}{8N_c}\log\left(\frac{1}{x}\right)\left[-\psi\left(\delta-\gamma\right)+\log\frac{Q^2}{Q_0^2}-\partial_\gamma\right]\right\}$$

the low x growth

Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
Phys.Rev.Lett. 110 (2013) 4, 041601

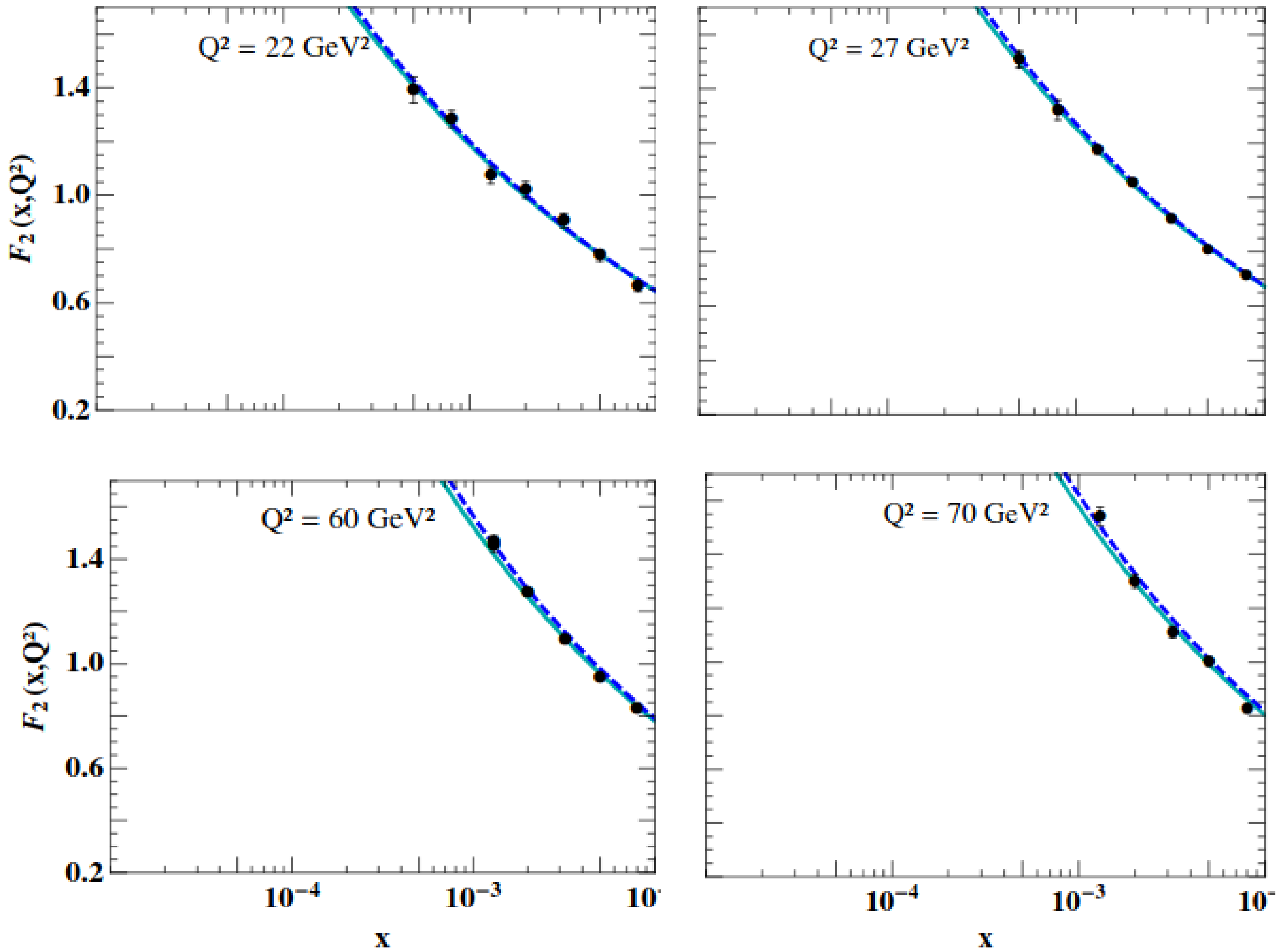
Proton structure function from HSS fit

F_2 data description

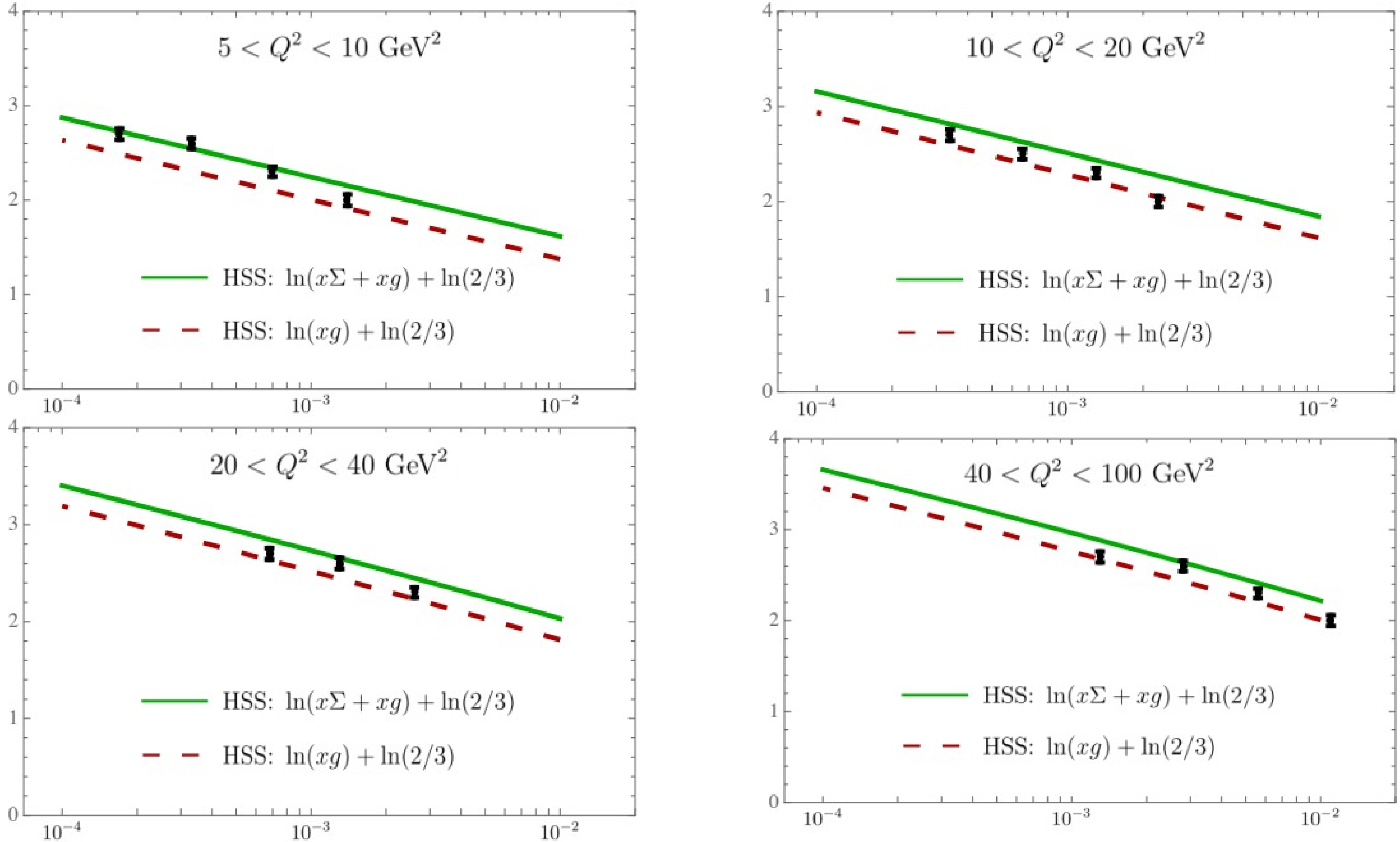


Proton structure function from HSS fit

F_2 data description

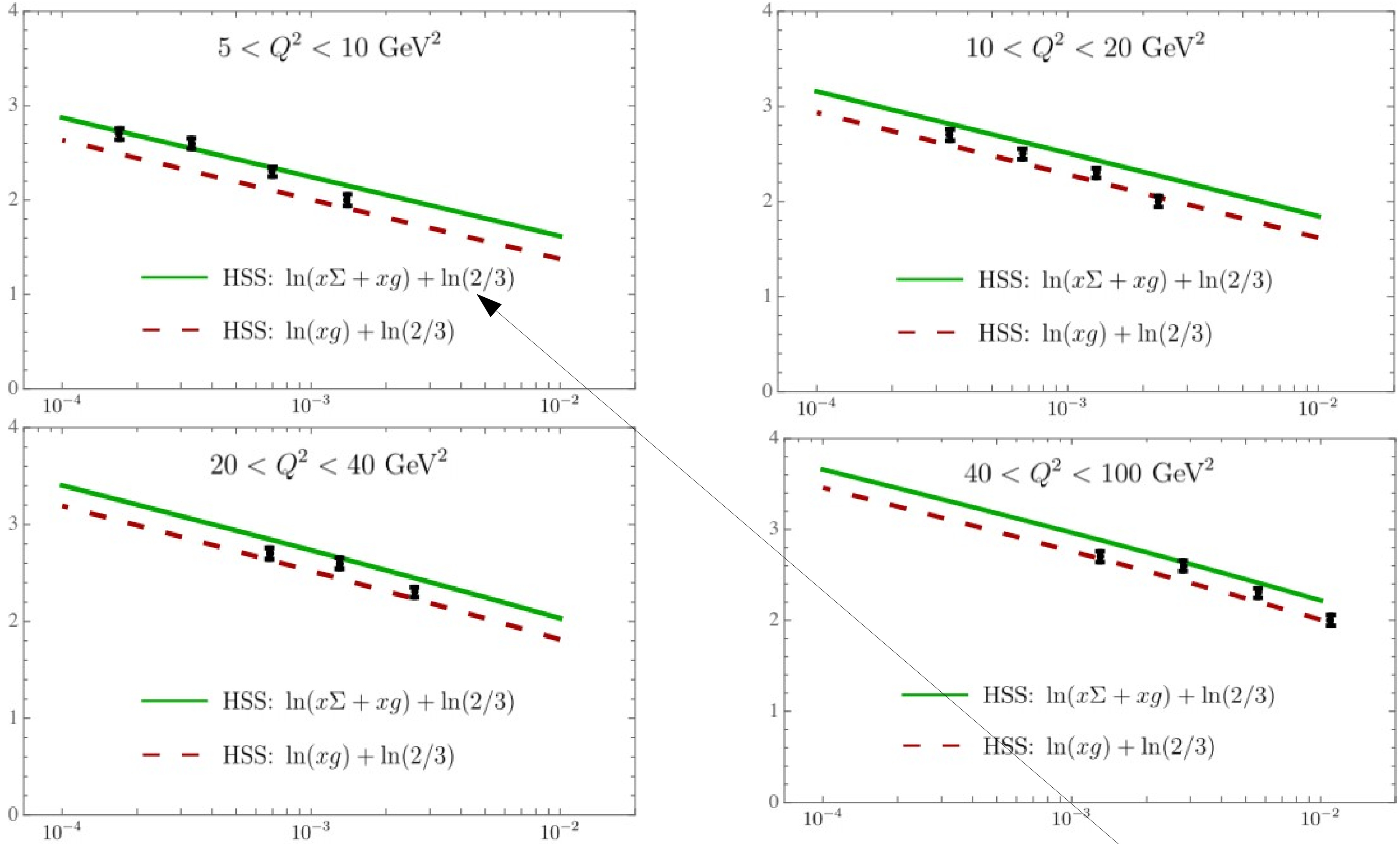


Results



Hint that the general idea works. Gluon dominates over quarks.
One has to also take into account that only charged hadrons were measured.

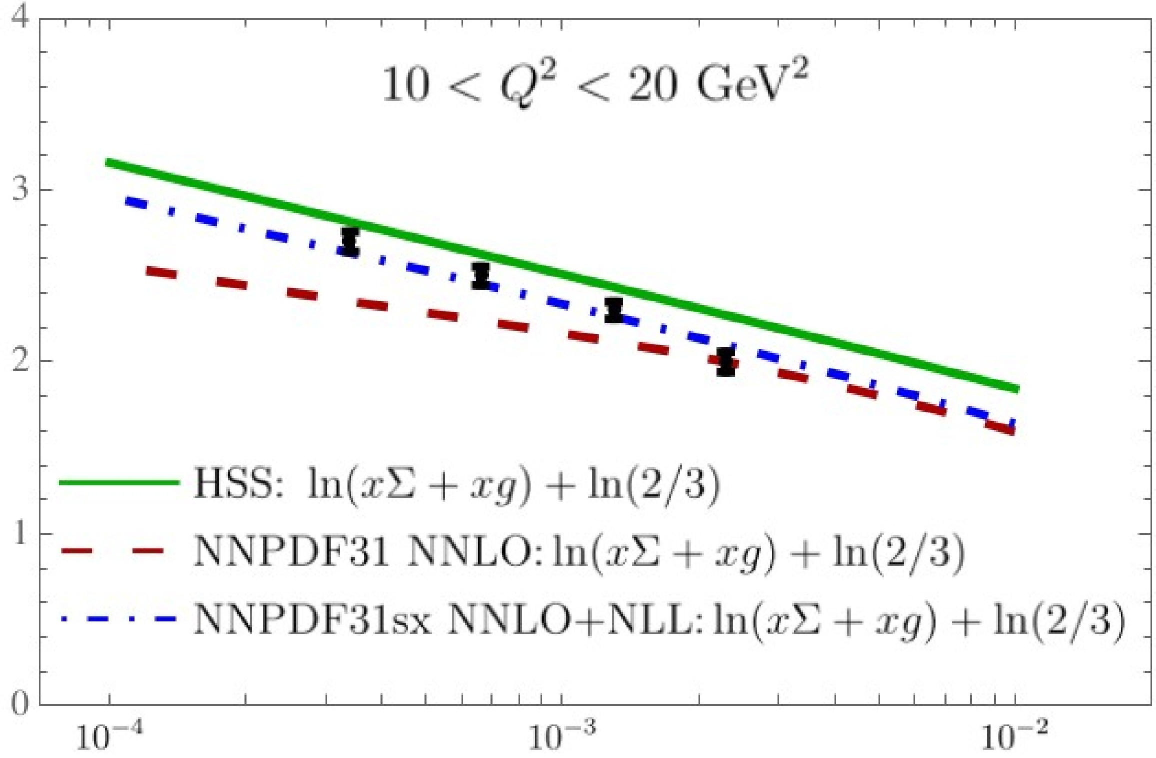
Results



Hint that the general idea works. Gluon dominates over quarks.
One has to also take into account that only charged hadrons were measured i.e 2/3 of partons contribute

Results

Eur.Phys.J.C 82 (2022) 2, 111 Hentschinski, Kutak



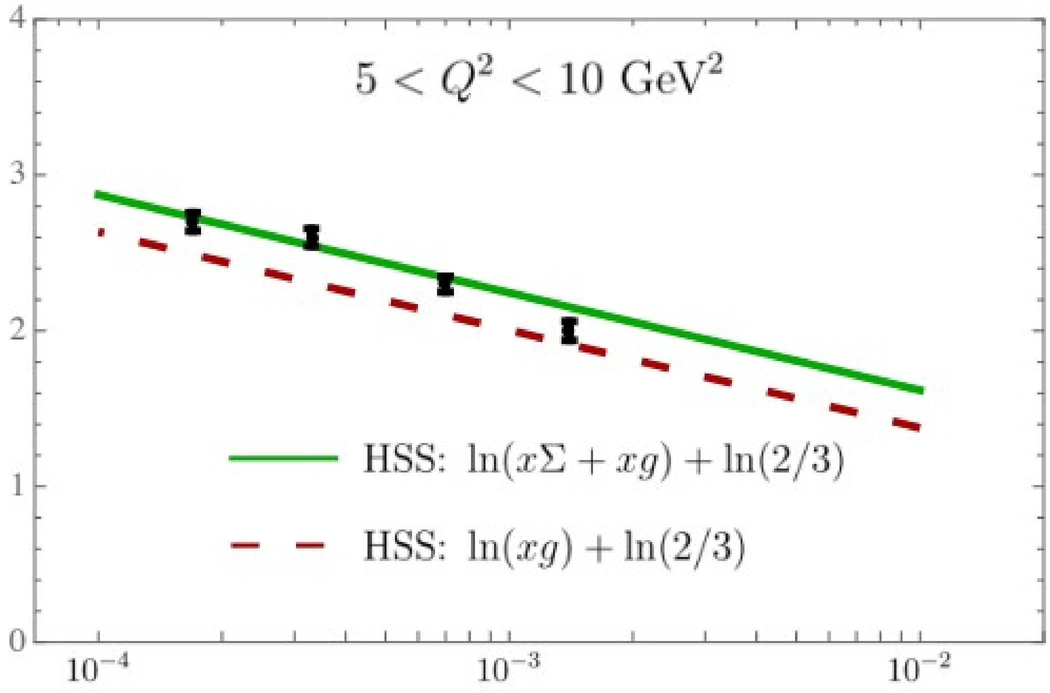
Low x resummation is essential

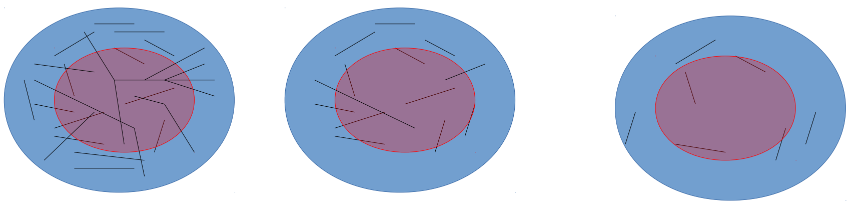
HSS gluon density used
i.e. NLO BFKL + kinematical Improvements
Hentschinski, Sabio-Vera, Salas.
Phys.Rev.D 87 (2013) 7, 076005
Phys.Rev.Lett. 110 (2013) 4, 041601

Large uncertainities of pdfs. In this study we did not take them into account.

NNPDF 31 → DGLAP
NNPDF 31sx → DGLAP + low x resummation

Dipoles and mechanism of entanglement





segments – dipoles, color singlets
maximally entangled states

red circle – resolved area defined by photon

entanglement arises because of dipoles that are partially in the red circle and partially in blue.

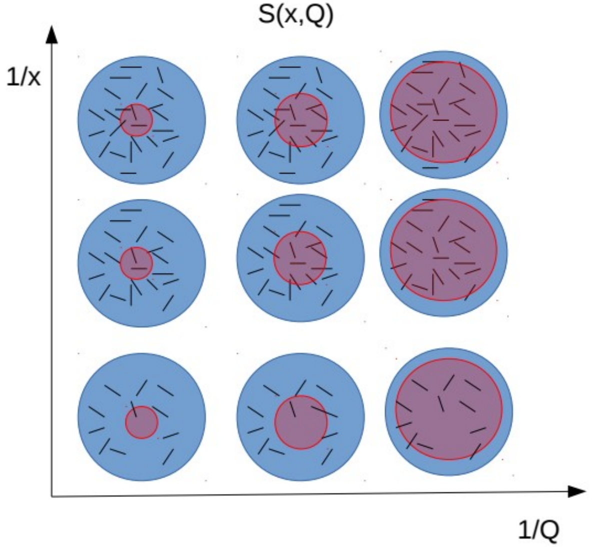
The broken dipoles contribute to final state hadron multiplicity and entropy of proton

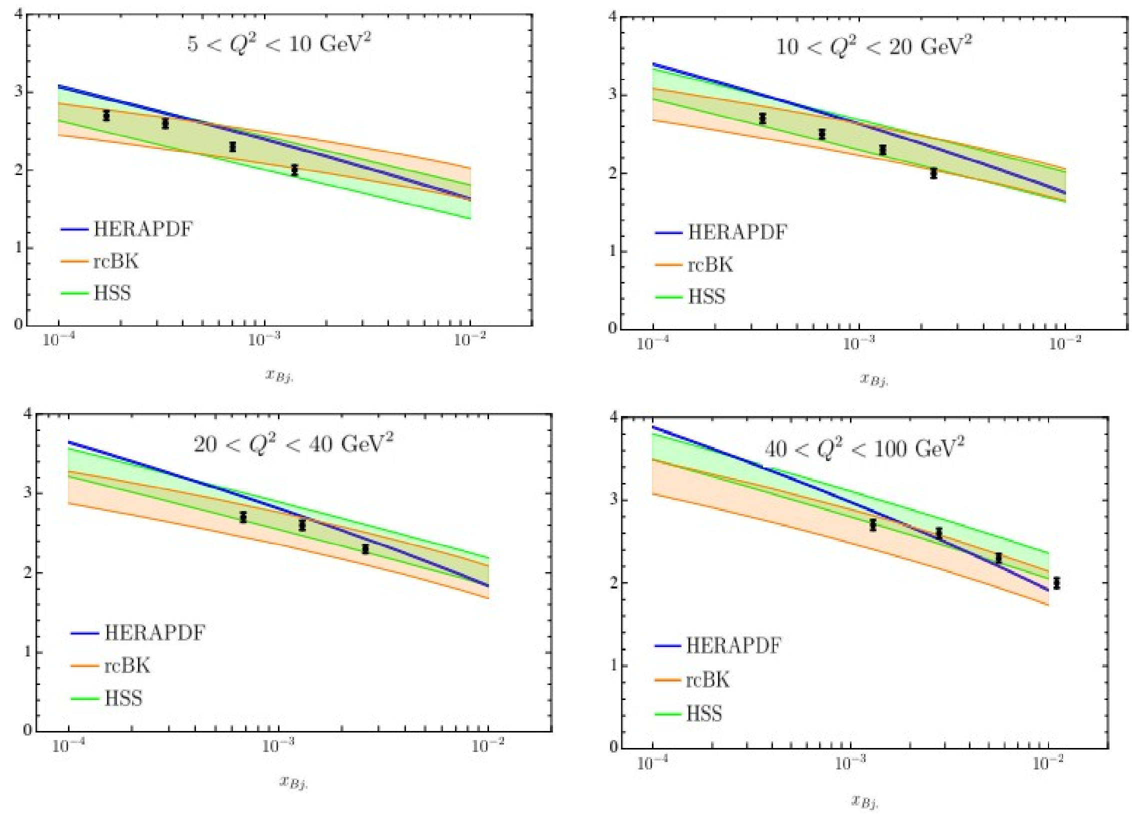
If we go to lower x we have more and more dipoles that cross the red line and entanglement grows

"Entanglement of predictions arises from the fact that the two bodies at some earlier time from in the true sense one system that is were interacting and have left behind choices on each other."

E. Schrodinger

Large scales - description

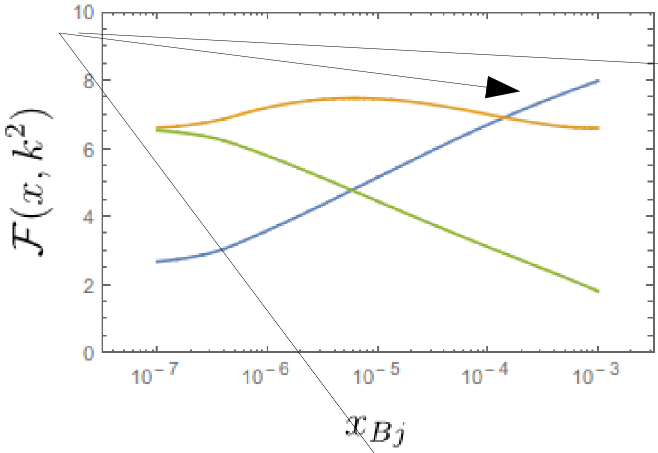


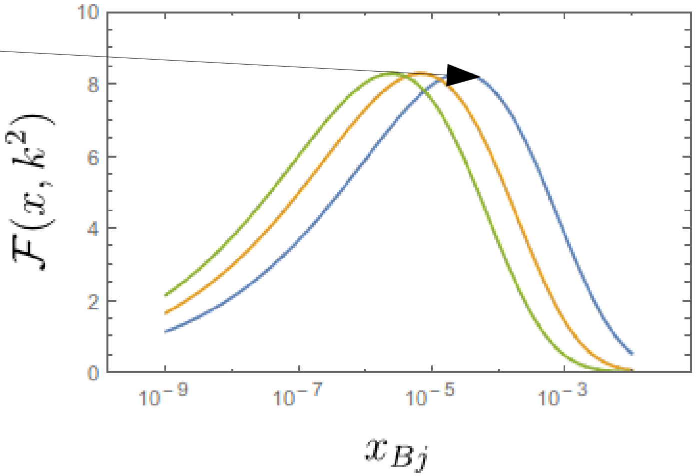


Dipole gluon density with nonlinearity– x depend.

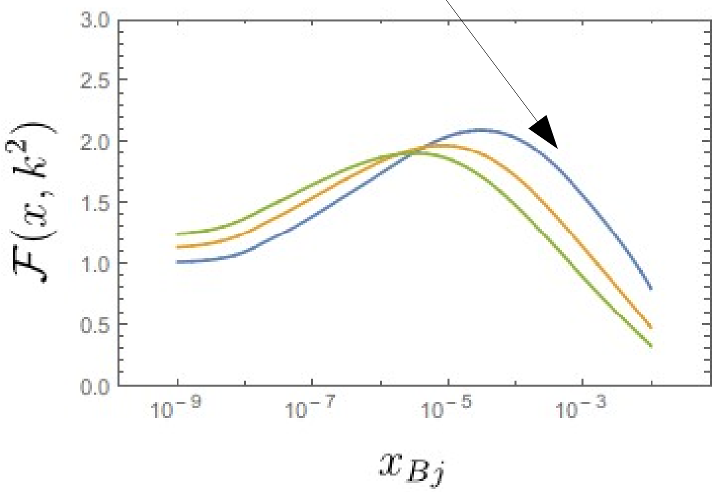
small kT

rcBK





GBW



KS gluon i.e. BK + kinematical corrections

$$\mathcal{F}(x,k^2) = \frac{N_c k^2 S_\perp}{8\pi^2 \alpha_s} \int dr^2 (1 - N(r,x)) J_0(r^2 k^2)$$

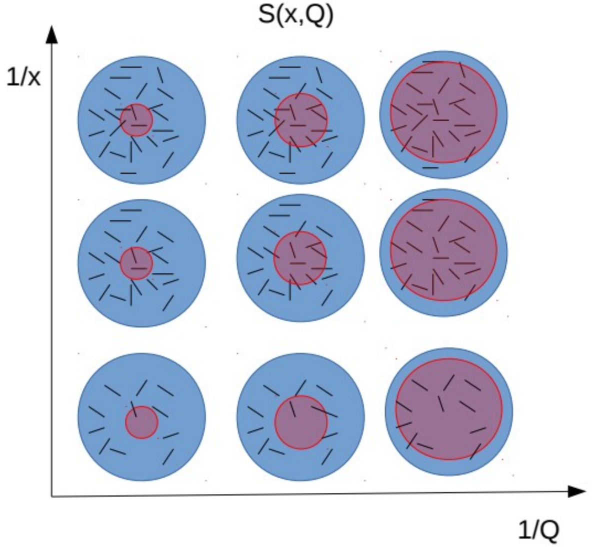
$$\mathcal{F}(x,k^2) = \frac{N_c S_\perp}{\alpha_s 8\pi^2} \frac{k^2}{Q_s^2} e^{-k^2/Q_s^2}$$

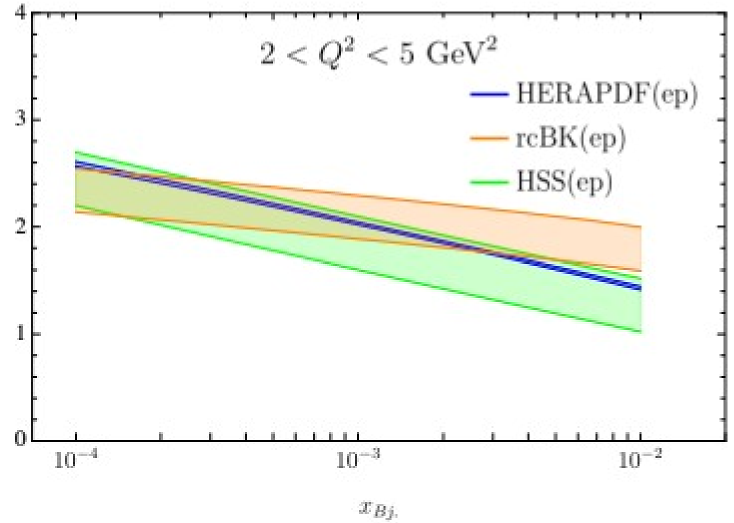
Similar plots in

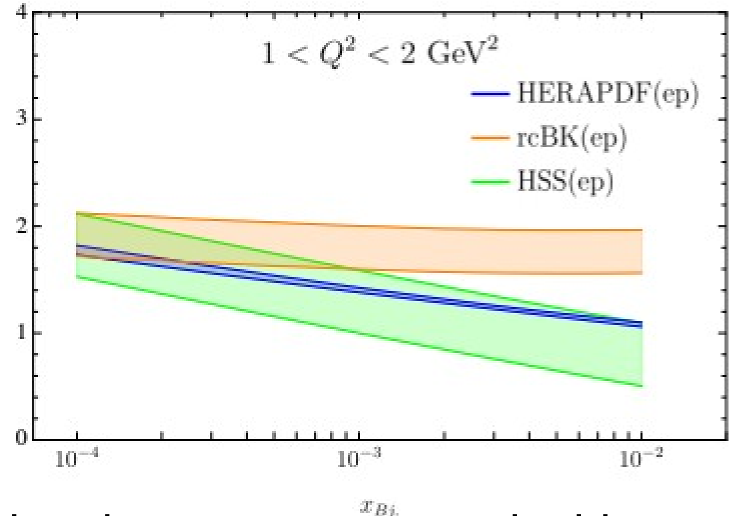
hep-ph/0703068

KK PhD thesis

Small scales - prediction

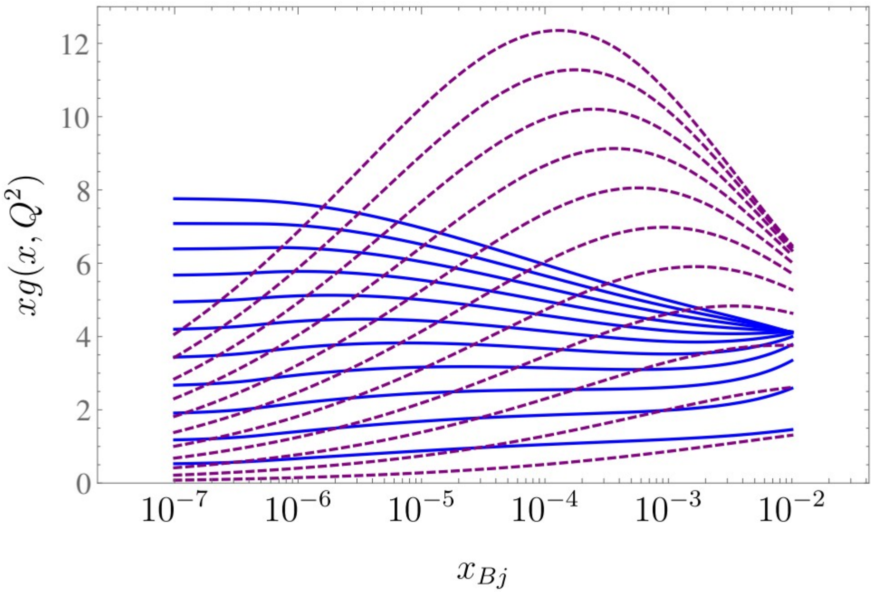


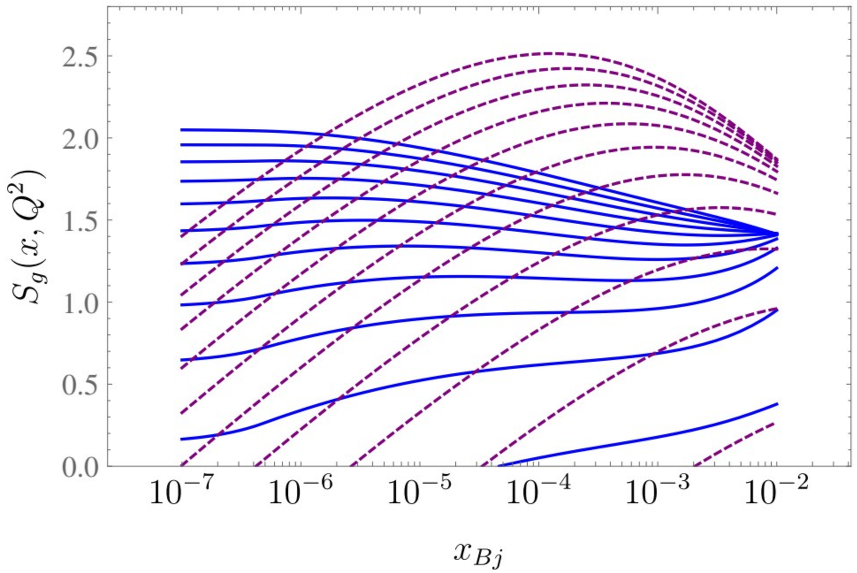




See also Hagivara, Hatta, Xiao ‘ 18
The genaralized KL model is used and entropy saturates in this approach
and Nowak, Liu, Zahed ‘22

Integrated gluon and entropy





$$\lim_{Q^2 \gg Q_s^2} S(x,Q^2) = \ln\left(S_\perp Q_s^2(x)\right) + \ln\frac{N_c}{8\alpha_s\pi^2} = \lambda \ln\frac{1}{x} + \text{const}$$

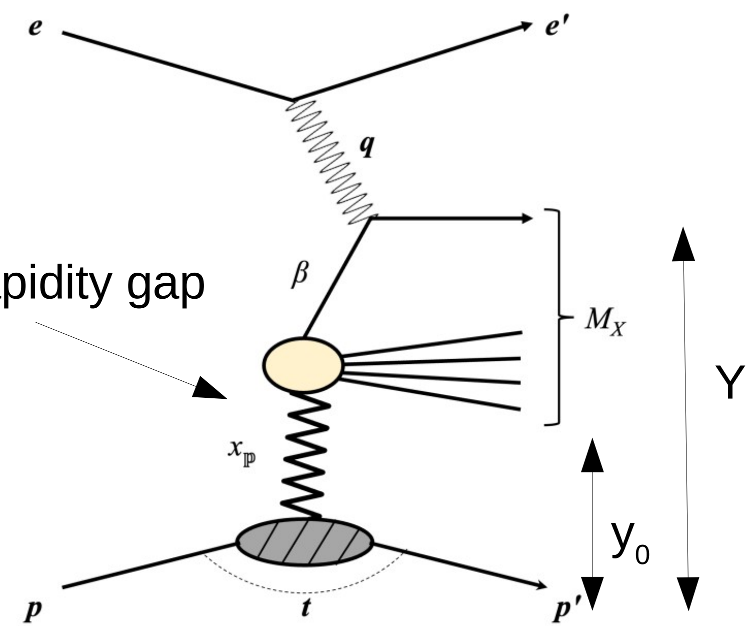
$$\lim_{Q^2 \ll Q_s^2} S(x,Q^2) = \ln\left(\frac{S_\perp Q^4}{Q_s^2(x)}\right) + \ln\frac{N_c}{16\alpha_s\pi^2}$$

Photon can not resolve proton anymore therefore the EE vanishes.
But it might be that the formalism breaks down for low scales.
There might be another source of entropy that keep the total entropy not vanishing → generalized second law Bekenstein

H. Hentschinski, D. Kharzeev. K. Kutak, Z. Tu ‘23

EE in Diffractive Deep Inelastic Scattering

rapidity gap



$x_{\mathbb{P}}$ proton’s momentum fraction carried by the Pomeron

β denotes the Pomeron’s momentum fraction carried by the quark interacting with the virtual photon

$x = \beta \cdot x_{\mathbb{P}}$ Bjorken x

$y_0 \simeq \ln 1/x_{\mathbb{P}}$ size of rpidity gap

$Y = \ln 1/x$

$y_X = Y - y_0 \simeq \ln 1/\beta$

Analogous evolution equation as for non-diffractive case but Initial conditions are different and there is delay because of rapidity gap.
Munier, Mueller Phys. Rev. D 98, 034021 (2018)

See also Peschanski, Seki’19 for entanglement in diffraction in p-p

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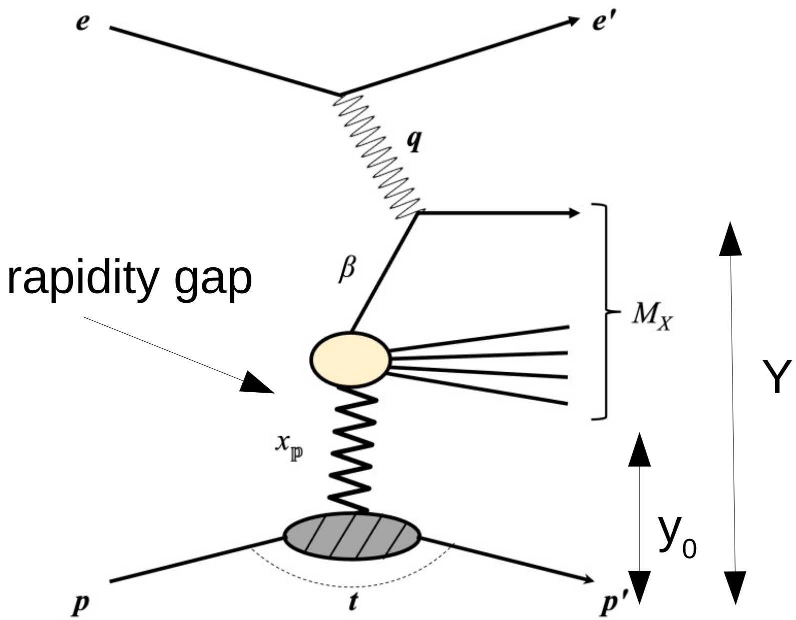
EE in Diffractive Deep Inelastic Scattering



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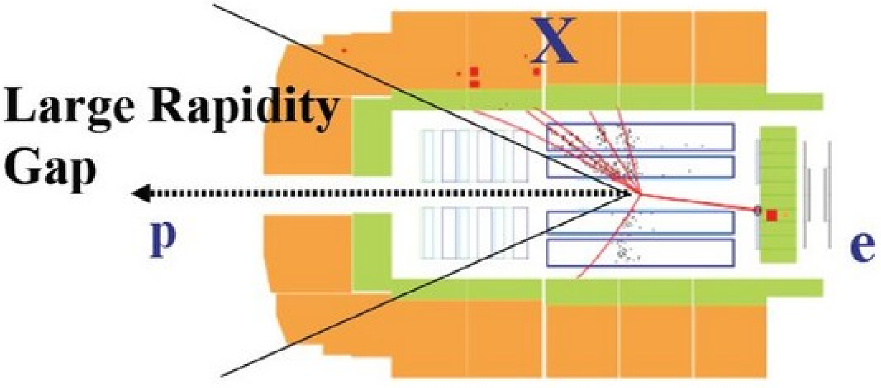
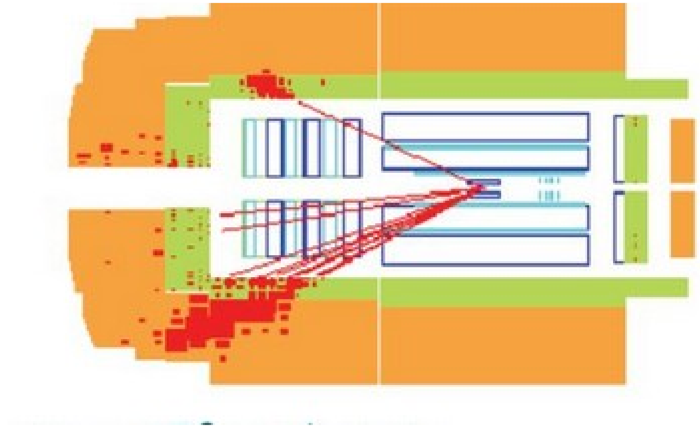
Diffraction vs. nondiffraction





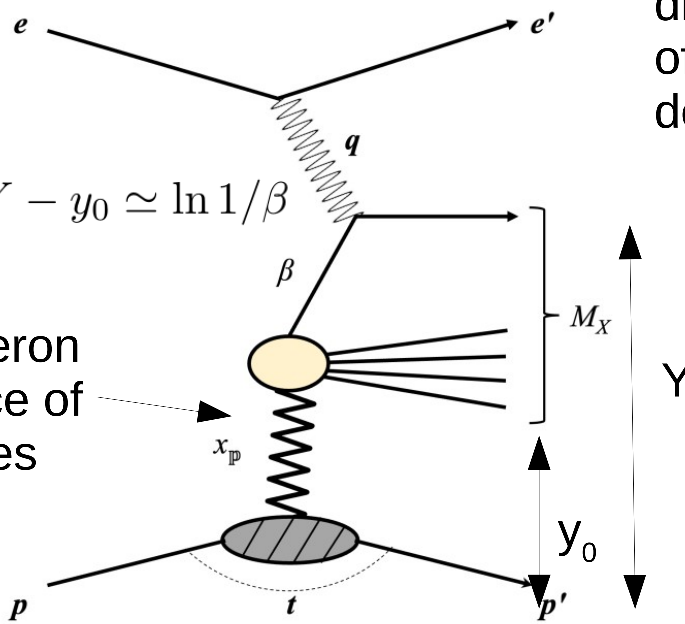
H1 detector

H. Hentschinski, D. Kharzeev. K. Kutak, Z. Tu ‘23

EE in DDIS

For large invariant mass M_X or small values of β of the diffractive system X, using factorization and the limit of large number of colors, the diffractive system can be described as a set of color dipoles.

$$y_X = Y - y_0 \simeq \ln 1/\beta$$



Pomeron source of dipoles

$$\frac{\partial p_n^D(y_X)}{\partial y_X} = -n\Delta p_n^D(y_X) + (n-1)\Delta p_{n-1}^D(y_X)$$

$$p_n^D(y_X) = \frac{1}{C} e^{-\Delta y_X} \left(1 - \frac{1}{C} e^{-\Delta y_X}\right)^{n-1}$$

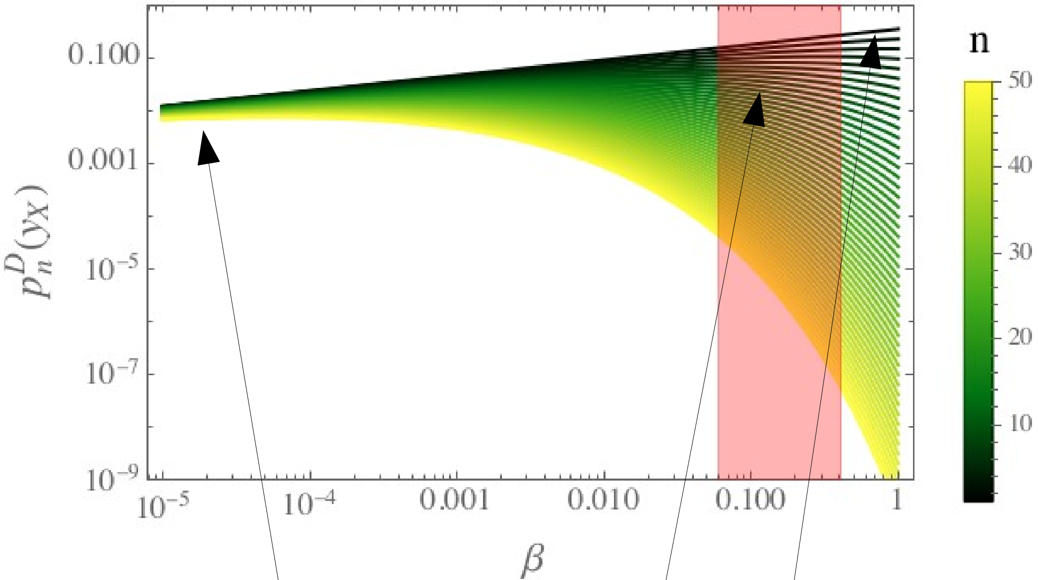
$$\left\langle \frac{dn(\beta)}{d\ln 1/\beta} \right\rangle = \sum_n n p_n^D(y_X) = C \left(\frac{1}{\beta}\right)^{\Delta}$$

We constrain the parameters at low values of β by fit to diffractive pdfs

M. Goharipour, H. Khanpour, and V. Guzey, Eur. Phys. J. C 78, 309 (2018)

$$\left\langle \frac{dn(\beta)}{d\ln 1/\beta} \right\rangle = \frac{1}{Q^2_{\max} - Q^2_{\min}} \int_{Q^2_{\min}}^{Q^2_{max}} dQ^2 \int_{x_{\mathbb{P},\min}}^{x_{\mathbb{P},\max}} dx_{\mathbb{P}} \;\; \beta \left[f^D_{\Sigma/p}\left(\beta, x_{\mathbb{P}}, Q^2\right) + f^D_{g/p}\left(\beta, x_{\mathbb{P}}, Q^2\right) \right]$$

EE in DDIS



$$\left\langle \frac{dn(\beta)}{d\beta} \right\rangle_{\text{charged}} \simeq \frac{2}{3} \left\langle \frac{dn(\beta)}{d\beta} \right\rangle$$

$$p_n^D(y_X) = \frac{1}{C} e^{-\Delta y_X} \left(1 - \frac{1}{C} e^{-\Delta y_X}\right)^{n-1}$$

$$p_n \equiv 1/Z$$

H1 data region

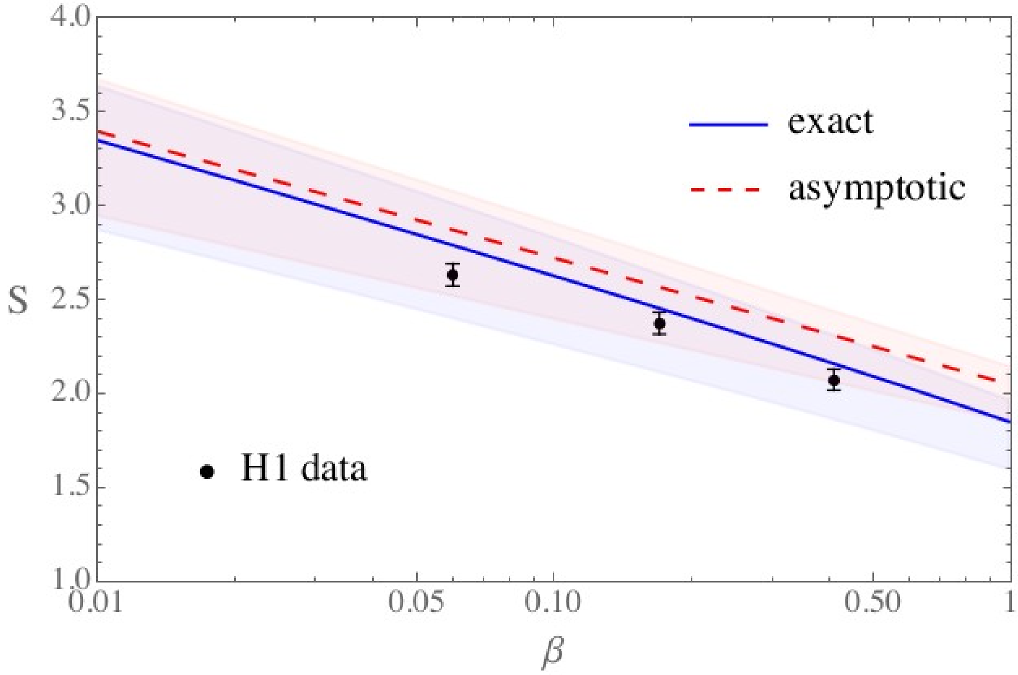
$$S(Z) = -\sum_n p_n \ln p_n = (1-Z) \ln \frac{Z-1}{Z} + \ln Z$$

Asymptotic or maximal entangled region. All configurations have the same probability

$$S_{\text{asym.}}(Z) = \ln Z + 1 + \mathcal{O}(1/Z)$$

High probability of configurations with few partons

EE in DDIS



$$\left\langle \frac{dn(\beta)}{d\beta} \right\rangle_{\text{charged}} \simeq \frac{2}{3} \left\langle \frac{dn(\beta)}{d\beta} \right\rangle$$

$$p_n^D(y_X) = \frac{1}{C} e^{-\Delta y_X} \left(1 - \frac{1}{C} e^{-\Delta y_X}\right)^{n-1}$$

$$p_n \equiv 1/Z$$

Entropy of hadrons calculated using Gibbs entropy formula

$$S(Z) = -\sum_n p_n \ln p_n = (1-Z)\ln\frac{Z-1}{Z} + \ln Z$$

$$S_{\text{hadron}} = -\sum P_N \log P_N$$

number of charged hadrons

$$S_{\text{asym.}}(Z) = \ln Z + 1 + \mathcal{O}(1/Z)$$

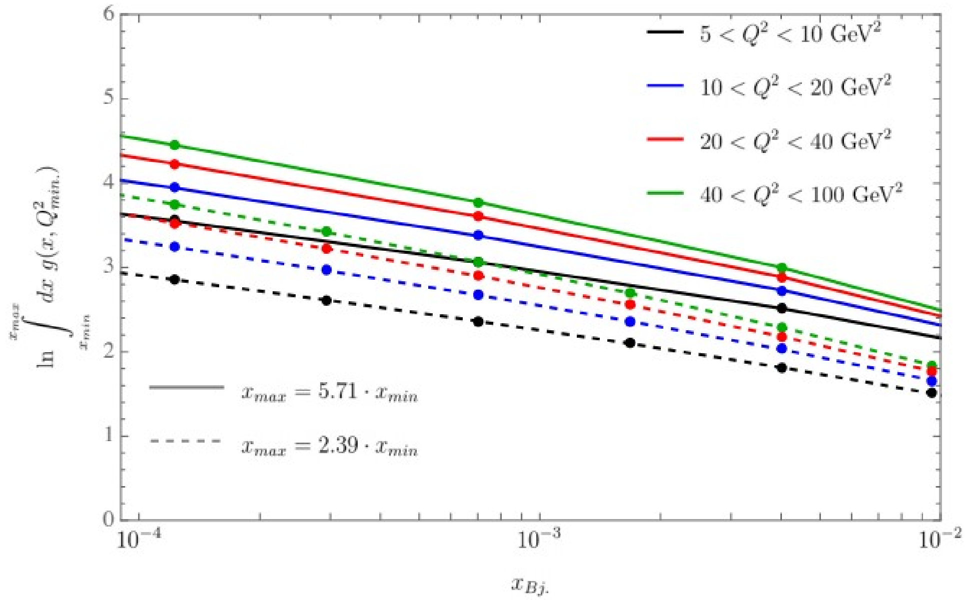
Eur. Phys. J. C 5, 439 (1998)

Conclusions and outlook

- We show evidences for the Kharzeev and Levin proposal for low x maximal entanglement entropy of proton constituents .
- It can be systematically improved (quark contributions, NLO BFKL, rc BK) and can describe successfully H1 data.
- We obtain saturation of entropy at small resolution scales.
- We demonstrate that the proposal works for DDIS and that it can be used to study onset of maximal entanglement.

Backup

Bining and KL formula



plot showing dependence of the result on the sie of bins if binig is naive

$$n_g(Q^2) = \int_0^1 dx g(x, Q^2)$$

Formal definition of number of gluons

$$n_g(\bar{x}) = \int_{x_{\min}}^{x_{\max}} dx g(x, Q^2) \qquad \bar{x} \in [x_{\min}, x_{\max}]$$

$$\bar{x} = \frac{\int_{x_{\min}}^{x_{\max}} dx\, x g(x, Q^2)}{\int_{x_{\min}}^{x_{\max}} dx g(x, Q^2)}$$

average *x*

Data binning takes place in rapidity

$$\bar{n}_g(\bar{x}) = \frac{1}{y_{max} - y_{min}} \int_{y_{min}}^{y_{max}} dy \frac{dn_g}{dy} = \frac{n_g(y_{max}) - n_g(y_{min})}{y_{max} - y_{min}}$$

$$y_{max,min} = \ln 1/x_{min,max}$$

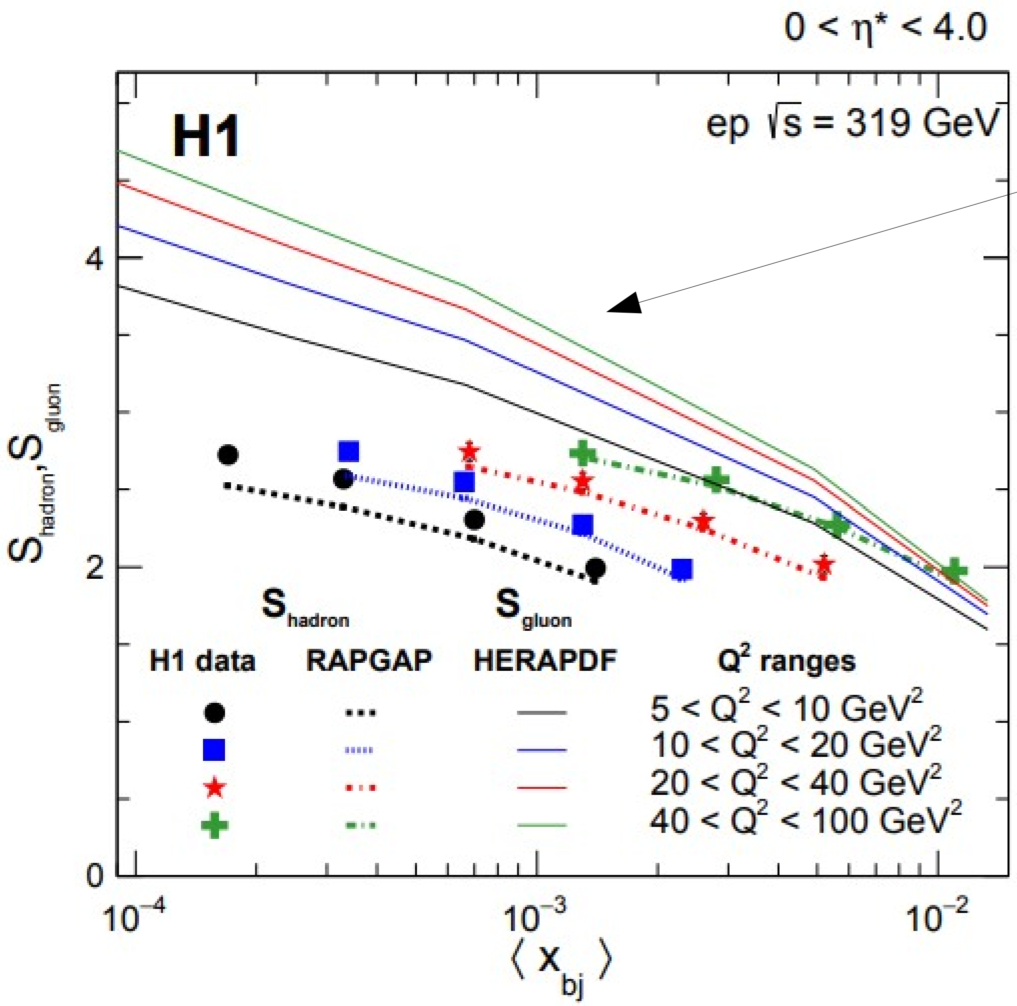
for small bins

$$\bar{n}_g(x, Q^2) = \frac{dn_g}{d\ln(1/x)} = xg(x, Q^2)$$

$$\langle \bar{n}(x, Q^2) \rangle_{Q^2} = \frac{1}{Q^2_{\max} - Q^2_{\min}} \int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2 \left[xg(x, Q^2) + x\Sigma(x, Q^2) \right]$$

$$\langle S(x, Q^2) \rangle_{Q^2} = \ln \langle \bar{n}(x, Q^2) \rangle_{Q^2}$$

Monte Carlo, KL formula, and data



HERA pdf used

See also Kharzeev and Levin
Phys. Rev. D 104, 031503 (2021)

H1

Eur.Phys.J.C 81 (2021) 3, 212

See also Z. Tu, D. Kharzeev, T. Ulrich ’20
for calculations of EE in p-p.