Entanglement Entropy in High Energy Collisions

Michael Lublinsky

Ben-Gurion University of the Negev
Israel

Q: How do the systems produced in high energy collisions thermalise/"hydronise"?

Exploratory approach based on first principles QM w/o assuming any form of thermalisation.
Similar question (formation of black holes/entropy from collisions of pure states)

The wave-function of an incoming projectile hadron (eigenstate of QCD LC Hamiltonian) has many entangled gluon Fock components

After collision with a target, some of these gluons will be decohered and observed as final state particles

Initially, these gluons had an entanglement entropy (with respect to other Fock components in the incoming wave-function).

This entropy gets "released" by the scattering in the form of final state entropy

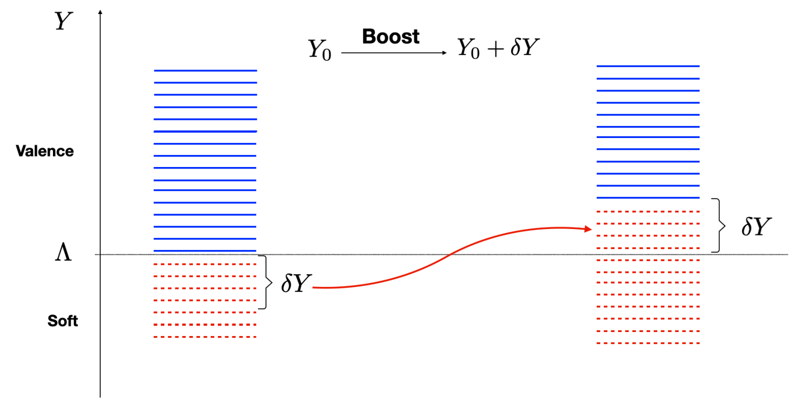
We hope to understand the dynamical mechanism of entropy production

We also hope to be able to shad light on the question to what extent the final state of the collision resembles a thermal state.

Light Cone Wave Function in Born-Oppenheimer approximation

$$\mathbf{H}^{\rm LC}_{\rm QCD}\,|\boldsymbol{\Psi}\rangle \;=\; \mathbf{E}\,|\boldsymbol{\Psi}\rangle$$

BO: split the modes into hard and soft.
The hard (valence) modes with $k^+ \;>\; \Lambda$
They act as an external background current
$j^+_a \;=\; \delta(x^-)\,\rho^a$ for the soft modes.



$$\mathbf{H}^{\rm LC}_{\rm QCD} \;=\; \mathbf{H}[\rho,\,\mathbf{a},\,\mathbf{a}^\dagger] \;=\; \mathbf{H}_{\rm V}[\rho] + \mathbf{H}_{\rm free}[\mathbf{a},\,\mathbf{a}^\dagger] + \mathbf{H}_{\rm int}[\rho,\,\mathbf{a},\,\mathbf{a}^\dagger]\,; \qquad \mathbf{H}_{\rm int} \sim \;\mathbf{g}\,''\rho\,\mathbf{a}'' + \mathbf{g}\,''\mathbf{aaa}''$$

LCWF with no soft modes

$$\mathbf{H}_{\rm V}\,|\mathbf{v},\,\mathbf{0_a}\rangle \;=\; \mathbf{E}_0\;|\mathbf{v},\,\mathbf{0_a}\rangle\,; \qquad \mathbf{a}\,|\mathbf{v},\,\mathbf{0_a}\rangle \;=\; \mathbf{0}\,; \qquad \mathbf{E}_0 \;=\; \mathbf{0}$$

LCWF with soft gluon/quark dressing

$$|\boldsymbol{\Psi}\rangle \;=\; \boldsymbol{\Omega}(\rho,\,\mathbf{a},\,\mathbf{a}^\dagger)\,|\mathbf{v},\,\mathbf{0_a}\rangle\,; \qquad \boldsymbol{\Omega}^\dagger\;(\mathbf{H}_{\rm free} + \mathbf{H}_{\rm int})\;\boldsymbol{\Omega} \;=\; \mathbf{H}_{\rm diagonal}$$

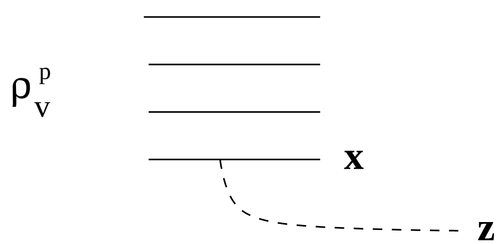
Find Ω in perturbation theory

In the dilute limit $\rho \;\sim\; 1$; gluon emission $\sim \alpha_s\,\rho$, LO = one gluon, NLO = 2 gluons

LCWF in Dilute Limit

Gluon coherent field operator in the dilute limit

$$\Omega_Y(\rho \to 0) \equiv \mathcal{C}_Y = \mathrm{Exp}\left\{ i \int d^2z\, b_i^a(z) \int_{e^{Y_0}\Lambda}^{e^Y \Lambda} \frac{dk^+}{\pi^{1/2}|k^+|^{1/2}} \left[a_i^a(k^+, z) + a_i^{\dagger a}(k^+, z)) \right] \right\}$$



Emission amplitude is given by the Weizsaker-Williams field

$$b_i^a(z) = \frac{g}{2\pi} \int d^2x \frac{(x - z)_i}{(x - z)^2} \rho^a(x)$$

The operator $\mathcal{C}$ dresses the valence charges by a cloud of the WW gluons

Density Matrix of soft modes

The wave function coming into the collision region at time $t = 0$

$$|\boldsymbol{\Psi}_{\mathbf{in}}\rangle \;=\; \boldsymbol{\Omega}_{\mathbf{Y}}\,|\rho, \mathbf{0_a}\rangle\,.$$

Define the reduced density matrix of soft modes

$$\hat{\rho} \;=\; \int \mathbf{D}\rho\,\mathbf{W}[\rho]\,|\boldsymbol{\Psi}_{\mathbf{in}}\rangle\,\langle\boldsymbol{\Psi}_{\mathbf{in}}|$$

McLerran-Venugopalan model for dense systems:

$$\mathbf{W}^{\mathbf{MV}}[\rho] \;=\; \mathcal{N}\,\exp\left[-\int_{\mathbf{k}}\frac{\mathbf{1}}{\mathbf{2}\mu^{\mathbf{2}}(\mathbf{k})}\rho(\mathbf{k})\rho(-\mathbf{k})\right] \qquad \text{where } \mathbf{Q_s^2} = \frac{\mathbf{g}^{\mathbf{4}}}{\pi}\mu^{\mathbf{2}}$$

Q_s denotes Saturation Scale – a typical semi-hard transverse momentum in a dense nucleus. At the same time Q_s measures average gluon density.

"Dilute/Dense mix approximation": $\Omega = \mathcal{C}$ and $W = W^{MV}$ (Gaussian), $\hat{\rho}$ is computable analytically

T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and ML, arXiv:1503.07126

$$\hat{\rho} = \sum_n \frac{1}{n!} e^{-\frac{1}{2}\phi_i M_{ij}\phi_j} \left[\prod_{m=1}^{n} M_{i_m j_m} \phi_{i_m} |0\rangle \, \langle 0| \phi_{j_m} \right] e^{-\frac{1}{2}\phi_i M_{ij}\phi_j}$$

Here we have introduced compact notations:

$$\phi_{\mathbf{i}} \equiv \left[\mathbf{a}_{\mathbf{i}}^{\dagger \mathbf{a}}(\mathbf{x}) + \mathbf{a}_{\mathbf{i}}^{\mathbf{a}}(\mathbf{x}) \right] \; ; \qquad \mathbf{M}_{\mathbf{ij}} \equiv \frac{\mathbf{g}^2}{4\pi^2} \int_{\mathbf{u},\mathbf{v}} \mu^2(\mathbf{u},\mathbf{v}) \frac{(\mathbf{x}-\mathbf{u})_{\mathbf{i}}}{(\mathbf{x}-\mathbf{u})^2} \frac{(\mathbf{y}-\mathbf{v})_{\mathbf{j}}}{(\mathbf{y}-\mathbf{v})^2} \delta^{\mathbf{ab}}$$

M bears two polarisation, colour, and coordinate indices, collectively denoted as $\{ij\}$.

Entanglement Entropy

Alex Kovner and ML, arXiv:1506.05394

Entanglement Entropy of soft modes

$$\sigma^{\mathbf{E}} \;=\; -\,\mathbf{tr}[\hat{\rho}\ln\hat{\rho}]$$

How to calculate $\ln$? The "replica trick":

$$\ln\hat{\rho} = \lim_{\epsilon\to\mathbf{0}}\frac{\mathbf{1}}{\epsilon}\left(\hat{\rho}^{\epsilon} - \mathbf{1}\right)$$

Calculate ρ^N and take $N \to 0$. N copies of the field - replicas.

The result

$$\sigma^{\mathbf{E}} = \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tr}\left\{\ln\frac{\mathbf{M}}{\pi} + \sqrt{\mathbf{1}+\frac{\mathbf{4M}}{\pi}}\ln\left[\mathbf{1}+\frac{\pi}{\mathbf{2M}}\left(\mathbf{1}+\sqrt{\mathbf{1}+\frac{\mathbf{4M}}{\pi}}\right)\right]\right\}$$

Translationally invariant limit ($\mu = const$):

$$\mathbf{M^{ab}_{ij}(p) = g^2 \mu^2 \frac{p_i p_j}{p^4} \delta^{ab}}$$

For small M, or the UV contribution (formally UV divergent)

$$\sigma^{E}_{UV} = \mathbf{tr}\left[\frac{\mathbf{M}}{\pi}\ln\frac{\pi\mathbf{e}}{\mathbf{M}}\right] = \frac{\mathbf{Q_s^2}}{\mathbf{4\pi g^2}}(\mathbf{N_c^2 - 1})\mathbf{S}\left[\ln^2\frac{\mathbf{g^2\Lambda^2_{UV}}}{\mathbf{Q_s^2}} + \ln\frac{\mathbf{g^2\Lambda^2_{UV}}}{\mathbf{Q_s^2}}\right]$$

$\mathbf{\Lambda_{UV} \sim M e^{Y_0} \gg M}$, where eikonal approximation breaks down

The large M, IR contribution is

$$\sigma^{E}_{IR} \simeq \frac{\mathbf{1}}{\mathbf{2}}\mathbf{tr}[\ln\frac{\mathbf{e^2 M}}{\pi}] == \frac{\mathbf{3(N_c^2 - 1)}}{\mathbf{8\pi g^2}}\mathbf{S Q_s^2}$$

But not quite what we would like to know.

We need to address scattering process

Density Matrix of produced soft gluons

The wave function coming into the collision region at time $t = 0$

$$|\mathbf{\Psi_{in}}\rangle \; = \; \mathbf{\Omega_Y} \, |\rho, \mathbf{0_a}\rangle \; .$$

The system emerges from the collision region with the wave function

$$|\mathbf{\Psi_{out}}\rangle \; = \; \hat{\mathbf{S}} \, \mathbf{\Omega_Y} \; |\rho, \mathbf{0_a}\rangle \; .$$

Eikonal scattering approximation

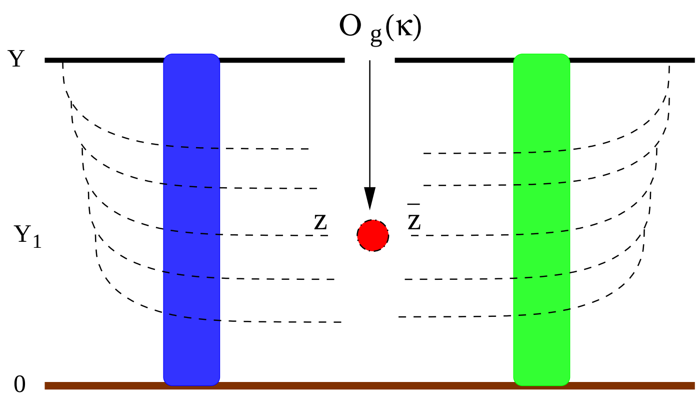
The system keeps evolving after the collision to the asymptotic time $t \rightarrow +\infty$, at which point measurements are made

The final state density matrix

$$\hat{\rho}_{\mathbf{P}} \; = \; \int \mathbf{D}\rho \, \mathbf{W}[\rho] \, \mathbf{\Omega_Y^\dagger} \, |\mathbf{\Psi_{out}}\rangle \, \langle \mathbf{\Psi_{out}}| \, \mathbf{\Omega_Y} \; = \; \hat{\rho}[\mathbf{M^P} \; \rightarrow \; \mathbf{M}]$$

Here extra Ω corresponds to final state radiation. It could be also viewed as change of basis projecting into eigenstates of free Hamiltonian.

Single inclusive gluon production



The observable

$$\hat{\mathcal{O}}_g \;\sim\; a_i^{\dagger\,a}(k)\, a_i^a(k)$$

$$\frac{dN}{d^2k\,d\eta} \;=\; \langle\, tr_a[\hat{\rho}_P\,\hat{\mathcal{O}}_g]\rangle_T \;=\; \int_{x,y} e^{ik(x-y)}\,\langle M^P_{ii}\rangle_T$$

$$M^P_{ij} \equiv g^2\int_{u,v}\mu^2(u,v)\frac{(x-u)_i}{(x-u)^2}\frac{(y-v)_j}{(y-v)^2}\left[(S(u)-S(x))(S^\dagger(v)-S^\dagger(y))\right]^{ab}$$

$\langle\ldots\rangle_T = \int\,[\mathcal{D}S]\;W[S]$ **corresponds to averaging over target fields** S **and is equivalent to event-by-event statistical ensemble average**

Produced Entropy

$$\sigma^{\mathbf{P}} \;=\; -\,\langle\,\mathbf{tr}[\hat{\rho}_{\mathbf{P}}\,\ln\hat{\rho}_{\mathbf{P}}]\,\rangle_{\mathbf{T}}$$

$$\sigma^{\mathbf{P}} = \frac{\mathbf{1}}{\mathbf{2}}\langle\mathbf{tr}\left\{\ln\frac{\mathbf{M^P}}{\pi} + \sqrt{\mathbf{1}+\frac{\mathbf{4M^P}}{\pi}}\,\ln\left[\mathbf{1}+\frac{\pi}{\mathbf{2M^P}}\left(\mathbf{1}+\sqrt{\mathbf{1}+\frac{\mathbf{4M^P}}{\pi}}\right)\right]\right\}\rangle_{\mathbf{T}}$$

T-averaging is complicated **Let expand σ^P around $\bar{\mathrm{M}} \equiv \langle \mathrm{M^P}\rangle_{\mathbf{T}}$ (dilute projectile limit)**

$$\bar{\mathbf{M}}_{\mathbf{ij}} \;=\; \delta^{\mathrm{ab}}\frac{\mathbf{Q_s^2}\pi}{\mathbf{g^2}}\int_{\mathbf{z}}\frac{(\mathbf{x}-\mathbf{z})_{\mathbf{i}}}{(\mathbf{x}-\mathbf{z})^{\mathbf{2}}}\frac{(\mathbf{y}-\mathbf{z})_{\mathbf{j}}}{(\mathbf{y}-\mathbf{z})^{\mathbf{2}}}[\mathbf{P_A}(\mathbf{x},\mathbf{y})+\mathbf{1}-\mathbf{P_A}(\mathbf{x},\mathbf{z})-\mathbf{P_A}(\mathbf{z},\mathbf{y})]$$

$P_A(x,y) \equiv \langle tr[S(x)S^\dagger(y)]\rangle_T$ - **S-matrix of an adjoint dipole**

$\bar{M}$ is *almost* single inclusive gluon, but it is not summed over ij

$$\sigma^{\mathbf{P}} = \mathbf{tr}\left[\frac{\bar{\mathbf{M}}}{\pi}\ln\frac{\pi\mathbf{e}}{\bar{\mathbf{M}}}\right] - \frac{\mathbf{1}}{\mathbf{2}\pi}\mathbf{tr}\left[\left\{\langle(\mathbf{M}^{\mathbf{P}} - \bar{\mathbf{M}})\ (\mathbf{M}^{\mathbf{P}} - \bar{\mathbf{M}})\rangle_{\mathbf{T}}\right\}\bar{\mathbf{M}}^{-1}\right]\ldots.$$

First term is $almost\ -n\ln n$**, where** n **is a multiplicity per unit rapidity (**$dN/d\eta$**)**

it depends on the production probabilities of longitudinally and transversely (with respect to the direction of their transverse momentum) polarized gluons separately

Second term - $almost$ **correlated part of double inclusive gluon production.**

Correlations between gluons decrease entropy of the produced state.

For a parametrically large number of produced particles ($\alpha_s dN/d\eta \sim 1$**), the entropy is parametrically of order** $1/\alpha_s$

"Temperature" of produced system

We can naturally define temperature through:

$$\mathbf{T^{-1} \;=\; \frac{d\sigma}{dE_T}}$$

$$\mathbf{E_\perp \;\propto\; \int d^2k\,|k|\,M^P(k) \;\propto\; (N_c^2-1)\,S\,\frac{Q_P^2}{g^2}Q_T}$$

Keeping only mean field term in the entropy:

$$\mathbf{T \;=\; \frac{\pi}{2}\,\langle k_T\rangle}$$

.

$$\mathbf{\langle k_T\rangle \;=\; E_\perp/N_{total}} \qquad\qquad \mathbf{N_{total} \;=\; \int d^2k\,M^P(k)}$$

Entropy in a Single Event?

A. Kovner, ML, and M. Serino, Phys. Lett. B 792, 4 (2019)

A given event corresponds to a fixed configuration of the projectile color charges and target fields. Averaging over these degrees of freedom corresponds to averaging over the event ensemble.

Intuitively it is clear that one should be able to ascribe entropy even to a single event.

The state of soft gluons emerging from a collision ($t = 0$) in a given event is a pure state.

$$|\psi(\mathbf{t=0})\rangle \;=\; \hat{\mathbf{S}}\,\mathbf{\Omega}|\mathbf{0}\rangle$$

After time t

$$|\psi(\mathbf{t})\rangle = \mathbf{\Omega}^\dagger\,\mathbf{U}(\mathbf{t})\,\hat{\mathbf{S}}\,\mathbf{\Omega}|\mathbf{0}\rangle\,, \qquad \mathbf{U}(\mathbf{t}) = \mathbf{e}^{\mathbf{i\,H\,t}}\,, \qquad \mathbf{\Omega} = \mathbf{U}(\infty)$$

At fixed ρ and S, $|\psi(\mathrm{t})\rangle$ is a pure state and it has vanishing von Neuman entropy.

Measurement uncertainty as a source for entropy

$|\psi(\mathbf{t})\rangle$ **is a superposition of many eigenstates with very different energies.**

$$|\psi(\mathbf{t})\rangle = \sum_{\mathbf{n}} \mathbf{e}^{-\mathbf{i}\mathbf{E}_\mathbf{n}\mathbf{t}} \mathbf{c}_\mathbf{n} |\psi_\mathbf{n}\rangle \; ,$$

The phases of these eigenstates are not infinitely resolvable by a real experimental apparatus, because of its finite resolution power. As a result the information about relative phases of the different eigenstates is scrambled by the measurement. This implies that the pure state produced by a scattering event will always be known - at best - in terms of a density matrix.

The density matrix of this state in the energy eigen-basis

$$\hat{\rho}(\mathbf{t}) = |\psi(\mathbf{t})\rangle\langle\psi(\mathbf{t})| = \begin{pmatrix} |c_1|^2 & c_1 c_2^* \, e^{i(E_1 - E_2)t} & \dots \\ c_2 c_1^* \, e^{i(E_2 - E_1)t} & |c_2|^2 & \dots \\ \dots & \dots & \dots \end{pmatrix} .$$

The entropy arises due to decoherence of eigenstates with different energies during the time evolution after the collisions with the target.

We study the decoherence in time of the soft gluon state emerging from the scattering event by interpreting it as a consequence of an entanglement of the final state particles with an imaginary experimental apparatus.

If one performs a measurements on the system which takes time T with $T \gg |E_1 - E_2|^{-1}$, such measurement effectively is sensitive only to the time average of the density matrix over T. Thus all the off diagonal matrix elements between states with large enough energy differences effectively vanish for the purpose of such measurement, the density matrix is equivalent to

$$\hat{\rho} \sim \begin{pmatrix} |c_1|^2 & 0 & \dots \\ 0 & |c_2|^2 & \dots \\ \dots & \dots & \dots \end{pmatrix},$$

This is a mixed density matrix and it has a non vanishing entropy.

Energy resolution and the calorimeter "white noise"

Introduce an apparatus as a separate degree of freedom coupled to the soft gluons. Suppose the apparatus is a "calorimeter", i.e. measures particle energies, then the additional degree of freedom should directly couple to the energy of the state. The state of the "calorimeter degree of freedom" contains information about the time resolution via some typical time scale T. Define a proper time averaged density matrix of soft gluons by reducing it over the calorimeter degree of freedom.

The extended density matrix now defined on a larger Hilbert space contains an additional degree of freedom ξ

$$\hat{\rho}_\xi(\mathbf{T}) = \mathbf{\Omega}^\dagger \mathbf{e}^{-\mathbf{i}\xi\,\mathbf{H}}\,\hat{\mathbf{S}}\,\mathbf{\Omega}\,|\mathbf{G}\rangle \otimes\ |\rho, \mathbf{0_a}\rangle\,\langle\rho, \mathbf{0_a}|\ \otimes \langle\mathbf{G}|\,\mathbf{\Omega}^\dagger\,\hat{\mathbf{S}}^\dagger\,\mathbf{e}^{\mathbf{i}\xi\,\mathbf{H}}\,\mathbf{\Omega}$$

with

$$\langle\xi|\mathbf{G}\rangle = \mathbf{e}^{-\frac{\xi^2}{2\mathbf{T}^2}}\,; \qquad\qquad \mathbf{\Omega}^\dagger\,\mathbf{H}\,\mathbf{\Omega}\ =\ \mathbf{H}_{\mathbf{diagonal}}\ =\ \mathbf{H}_0$$

The reduced density matrix of the soft gluons $\hat{\rho}'\ =\ tr_\xi[\hat{\rho}_\xi]$:

$$\hat{\rho}'(\mathbf{T}) = \int_\xi \frac{\mathbf{1}}{\sqrt{\pi}\,\mathbf{T}}\,\mathbf{e}^{-\frac{\xi^2}{\mathbf{T}^2}}\mathbf{e}^{-\mathbf{i}\mathbf{H}_0\,\xi}\,\mathbf{\Omega}^\dagger\hat{\mathbf{S}}\,\mathbf{\Omega}\,|\rho, \mathbf{0_a}\rangle\,\langle\rho, \mathbf{0_a}|\,\mathbf{\Omega}^\dagger\hat{\mathbf{S}}\,\mathbf{\Omega}\,\mathbf{e}^{\mathbf{i}\mathbf{H}_0\,\xi}$$

Event-by-event entropy in the weak field approximation

The single event entropy is expressed in terms of the leading eigenvalue $(1-\delta_1)$

$$\sigma^{\mathbf{E}}_{\text{weak field}} \simeq -\delta_1 \log \delta_1 + \text{corrections}$$

$$\delta_1 = \int_{\mathbf{q}} \mathbf{M}^{\mathbf{P}}_{\rho}(\mathbf{q}) \left(\mathbf{1} - \mathbf{e}^{-\frac{\mathbf{E}^2_{\mathbf{q}}\mathbf{T}^2}{\mathbf{2}}}\right)$$

$$\mathbf{M}^{\mathbf{P}}_{\rho}(\mathbf{q}) \equiv \frac{\mathbf{g}^2}{\mathbf{q}^+}\int_{\mathbf{u},\mathbf{v},\mathbf{x},\mathbf{y}} \mathbf{e}^{\mathbf{iq}(\mathbf{x}-\mathbf{y})}\rho^{\mathbf{a}}(\mathbf{u})\rho^{\mathbf{a}}(\mathbf{v})\frac{(\mathbf{x}-\mathbf{u})_{\mathbf{i}}}{(\mathbf{x}-\mathbf{u})^2}\frac{(\mathbf{y}-\mathbf{v})_{\mathbf{i}}}{(\mathbf{y}-\mathbf{v})^2}\left[(\mathbf{S}(\mathbf{u})-\mathbf{S}(\mathbf{x}))(\mathbf{S}^\dagger(\mathbf{v})-\mathbf{S}^\dagger(\mathbf{y}))\right]^{\mathbf{ab}}$$

The total multiplicity of soft gluons produced in a given event (at fixed ρ^a and S)

$$\mathbf{n} = \int_{\mathbf{q}} \mathbf{M}^{\mathbf{P}}_{\rho}(\mathbf{q})\,.$$

The simplest naïve expression for Boltzman entropy of a system of n particles

$$\sigma = -\mathbf{n}\log\mathbf{n}\,.$$

We interpret T as the number of particles produced up to time T

$$\mathbf{n(T)} = \int_{\mathbf{q}} \mathbf{M}^{\mathbf{P}}_{\rho}(\mathbf{q})\left(\mathbf{1} - \mathbf{e}^{-\frac{\mathbf{E_q^2 T^2}}{\mathbf{2}}}\right) \qquad \rightarrow \qquad \sigma^{\mathbf{E}}(\mathbf{T}) = -\mathbf{n(T)}\log\mathbf{n(T)}\,.$$

At $T=0$ our state has vanishing entropy, since it is just the pure state which emerges from the scattering region. No individual gluon state can be resolved. Up to time T only those particles are actually produced, which have energies $E_q > 1/T$. If the time T is short, only very energetic gluons are produced, while if one waits infinite amount of time, all gluons that are present in the wave function immediately after scattering, decohere from each other and are therefore produced in the final state.

Time dependent entropy for the ensemble of events

$$\hat{\rho}_{\mathbf{P},\xi,\rho} = \int [\mathcal{D}\rho]\,[\mathcal{D}\mathbf{S}]\;\mathbf{W}[\mathbf{S}]\;\hat{\rho}'(\mathbf{T})\;\mathbf{e}^{-\int_{\boldsymbol{q}} \frac{\rho^{\mathbf{a}}(\boldsymbol{q})\rho^{\mathbf{a}}(-\boldsymbol{q})}{2\mu^{2}(\boldsymbol{q})}}$$

The entropy of the whole ensemble in the weak field limit (expansion in powers of μ^2)

$$\sigma^{\mathbf{E}} = -\delta_1 \log \delta_1 \qquad\qquad \delta_1 = \int_{\mathbf{q}} \langle\, \mathbf{M}^{\mathbf{P}}_{\rho}(\mathbf{q})\,\rangle_{(\rho,\mathbf{S})} \neq \delta_1(\mathbf{T})$$

The eigenvalue and, therefore, the entropy of the density matrix averaged over the event ensemble does not depend on time (in the weak field limit).

Time dependence of the entropy for the ensemble of events is much weaker than for a single event. We believe that the reason for this lies in the "monogamy of entanglement"

It is known that if a system A is maximally entangled with system B, then none of those systems can be entangled with the third system C. for very weak fields the soft gluon degrees of freedom are maximally entangled with the valence charges. By the monogamy of entanglement it then means that coupling to another degree of freedom (ξ) does not change the entropy of the soft gluons.

Energy evolution in CGC

The energy evolution in CGC is governed by the JIMWLK Hamiltonian

$$\frac{\mathbf{d}}{\mathbf{dy}}\mathbf{W}[\mathbf{S}] \;=\; \mathbf{H}_{\mathrm{JIMWLK}}\mathbf{W}[\mathbf{S}]$$

$$\mathbf{H}_{\mathrm{JIMWLK}} = \int \frac{\mathbf{d}^2\mathbf{z}_\perp}{2\pi}\mathbf{Q}^{\mathbf{a}}_{\mathbf{i}}[\mathbf{z}_\perp,\mathbf{j}]\mathbf{Q}^{\mathbf{a}}_{\mathbf{i}}[\mathbf{z}_\perp,\mathbf{j}]$$

$$\mathbf{Q}^{\mathbf{a}}_{\mathbf{i}}[\mathbf{z}_\perp,\mathbf{S}] = \frac{\mathbf{g}}{2\pi}\int \mathbf{d}^2\mathbf{x}_\perp \frac{(\mathbf{x}_\perp-\mathbf{z}_\perp)_{\mathbf{i}}}{(\mathbf{x}_\perp-\mathbf{z}_\perp)^2}\Big[\mathbf{S}^{\mathbf{ab}}(\mathbf{z}_\perp)-\mathbf{S}^{\mathbf{ab}}(\mathbf{x}_\perp)\Big]\mathbf{J}^{\mathbf{b}}_{\mathbf{R}}(\mathbf{x}_\perp)$$

$$\mathbf{J}^{\mathbf{a}}_{\mathbf{R}}(\mathbf{x}_\perp) = -\mathrm{tr}_{\mathrm{c}}\left\{\mathbf{S}(\mathbf{x}_\perp)\mathbf{T}^{\mathbf{a}}\frac{\delta}{\delta\mathbf{S}^\dagger(\mathbf{x}_\perp)}\right\}\;,$$

CGC density matrix

N. Armesto, F. Dominguez, A. Kovner, ML, and V. Skokov, 1901.08080 [hep-ph] (JHEP)

Diagonal element of the target density matrix

$$\mathbf{W[S]} = \langle \mathbf{S} | \, \hat{\rho}_\mathbf{t} \, | \mathbf{S} \rangle$$

Off-diagonal matrix elements of the target density matrix:

$$\langle \mathbf{S} | \, \hat{\rho}_\mathbf{t} \, | \mathbf{S}' \rangle \equiv \mathbf{W[S, S']}$$

Lindbladian non-unitary evolution of the density matrix of an open system.

$$\frac{\mathbf{d}}{\mathbf{dy}} \hat{\rho}_\mathbf{t} = \int \frac{\mathbf{d^2 z_\perp}}{\mathbf{2}\pi} \left[\hat{\mathbf{Q}}^\mathbf{a}_\mathbf{i}[\mathbf{z}_\perp], \left[\hat{\mathbf{Q}}^\mathbf{a}_\mathbf{i}[\mathbf{z}_\perp], \hat{\rho}_\mathbf{t} \right] \right]$$

$$\frac{\mathbf{d}}{\mathbf{dy}} \mathbf{W[S, S']} = \int \frac{\mathbf{d^2 z_\perp}}{\mathbf{2}\pi} \left[\mathbf{Q}^\mathbf{a}_\mathbf{i}[\mathbf{z}_\perp, \mathbf{S}] + \mathbf{Q}^\mathbf{a}_\mathbf{i}[\mathbf{z}_\perp, \mathbf{S}'] \right]^\mathbf{2} \mathbf{W[S, S']}$$

$\hat{Q}$ is a Lindblad operator

Energy evolution of Entanglement Entropy

$$\frac{\mathbf{d}}{\mathbf{dy}}\,\mathbf{tr}\,\hat{\rho}^{\mathbf{2}} < \mathbf{0}$$

$$\sigma_{\mathbf{R}} = -\ln(\mathbf{tr}\hat{\rho}^{\mathbf{2}}) \qquad\qquad \frac{\mathbf{d}}{\mathbf{dy}}\sigma_{\mathbf{R}} > \mathbf{0}$$

In the dilute limit at large rapidity, the entanglement entropy grows linearly with rapidity according to $\frac{d}{dy}\sigma_e = \gamma$, where γ is the leading BFKL eigenvalue.

The evolution of $\hat{\rho}$ in the saturated regime and relate it to the Levin-Tuchin law and find that the entropy again grows linearly with rapidity, but at a slower rate.

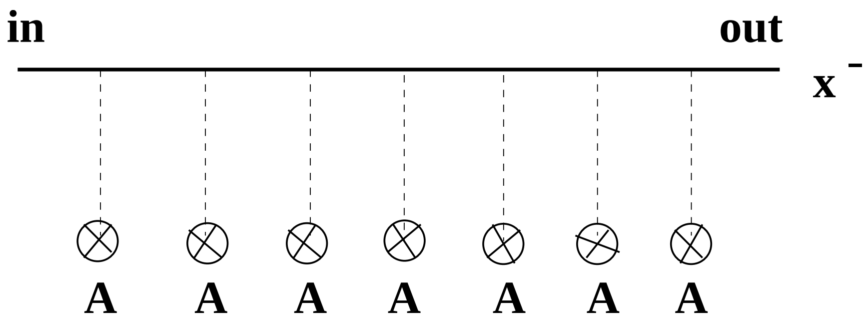
The energy evolution generates mixing/increases entropy even if we start with a pure state. Decoherence inside the parton cascade

Summary

What I have reported is a set of pilot ideas on "Quasi-Thermodynamics"

- **Momentum space entanglement between modes separated in rapidity**
- **Entropy release as a result of scattering process**
- **Time-dependent decoherence/entropy production as a result of finite resolution measurement**
- **Decoherence/entropy generation as a result of energy evolution**

Eikonal scattering approximation



Eikonal scattering is a color rotation
Eikonal factor does not depend on rapidity

In the light cone gauge ($A^+ = 0$) the large target field component is $A^- = \alpha^t$.

$$S(x) = \mathcal{P}\exp\left\{ i \int dx^+ \, T^a \, \alpha^a_t(x, x^+) \right\} .$$

$$\text{"}\Delta\text{"}\alpha^t = \rho^t \qquad (YM)$$

$$|in\rangle = |z, b\rangle \, ; \qquad |out\rangle = |z, a\rangle \, ; \qquad |out\rangle = S\,|in\rangle$$