

*Towards real-time dynamics of QCD on
quantum computers – examples from
hot-QCD studies in heavy-ion collisions*

Quantum Entanglement in High Energy Physics 2023

**Wibe A. de Jong (LBNL), Kyle Lee (MIT), Mekena Metcalf (LBNL), James Mulligan (LBNL/UC Berkeley),
Mateusz Ploskon (LBNL), Felix Ringer (ODU/JLAB), Xiaojun Yao (U. Washington)**

*... proof-of-principle simulations
(some) on real quantum computers*

Quantum Entanglement in High Energy Physics 2023

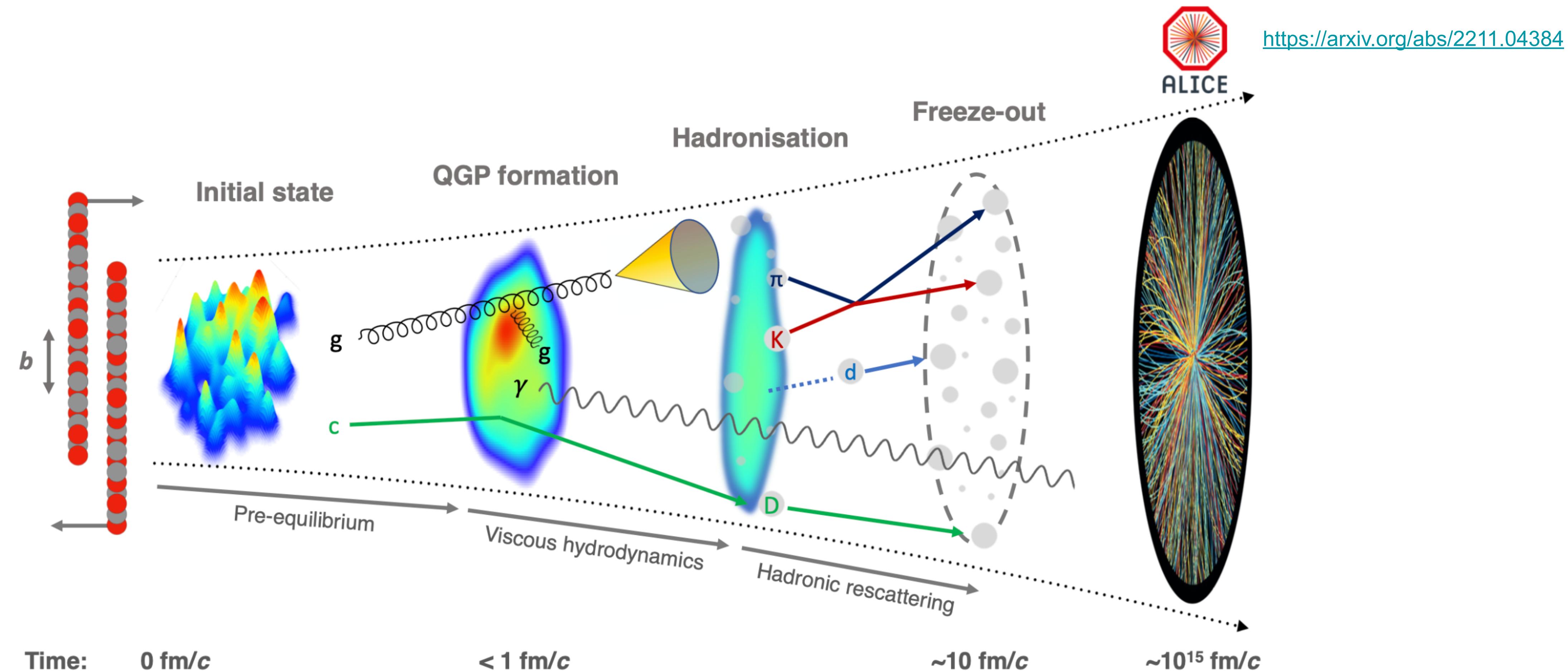
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Outline

- Few words on hot-QCD phenomena - selected topics
- ... and need for quantum computing simulations
- Quarkonia dissociation in hot deconfined medium (quark-gluon plasma)
- Non-equilibrium dynamics and thermalization - Schwinger model
- Outlook

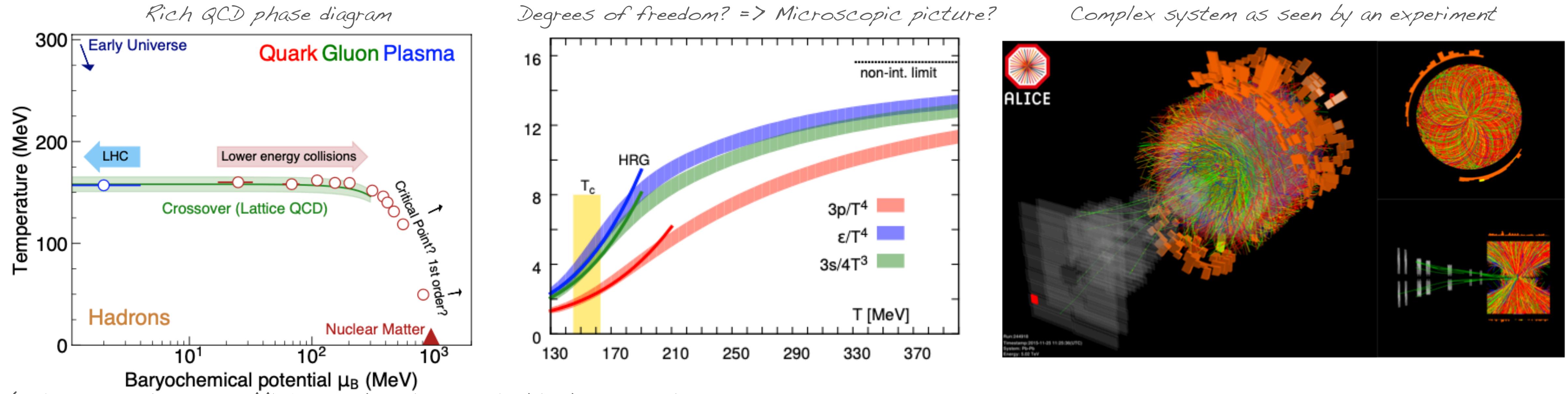
Hot QCD - Quark Gluon Plasma

"Standard model" of HIC - spell out the phases / time / T etc?



Heavy-ion collisions – emergent phenomena in QCD

Large phase-space for the use of quantum computing



Notes on ion collider physics at high energies

hot nearly perfect liquid quark-gluon plasma - deconfined partons - collective phenomena; energetic partons (jets) interact with the medium - jet quenching; bound states melt depending on temperature - dissociation and re-formation of quarkonia at phase transition; parton-hadron transition - connection to early universe at high temperature; complex structure at finite baryon density - 1st order phase transition, critical point, ...

Understanding of the emergent phenomena in HIC: microscopic description - real-time/[non]-equilibrium dynamics
 => direct need for quantum computations (Hamiltonian formulation of QCD, treatment of coherence effects, ...)

Heavy-ion collisions – emergent phenomena in QCD

Large phase-space for the use of quantum computing

AA collisions - rich QCD structure and emergent phenomena - selected topics:

- Deconfined QCD (color charged) medium - quark gluon plasma
- Parton-hadron phase transition (cross-over, critical point, 1st order)

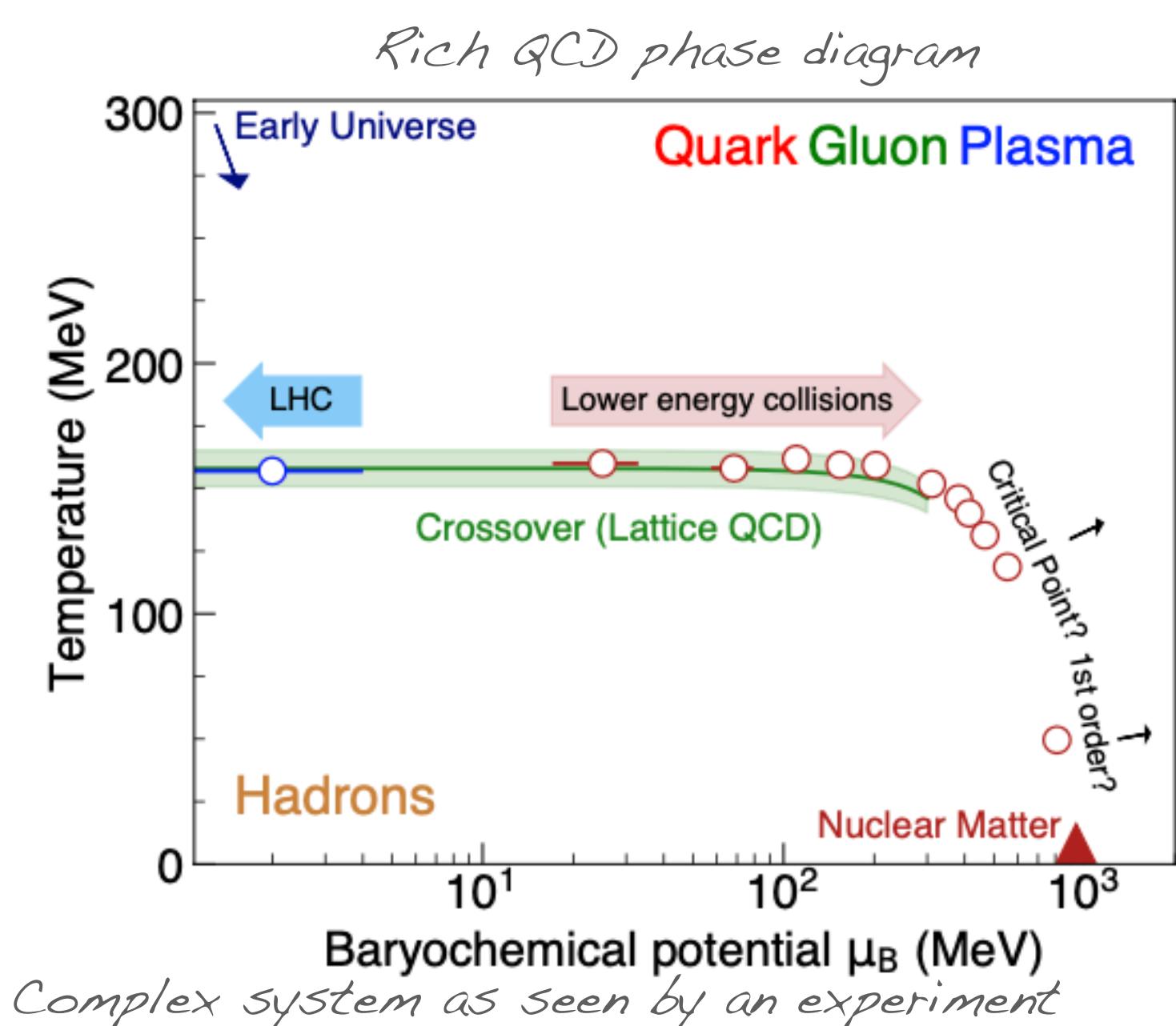
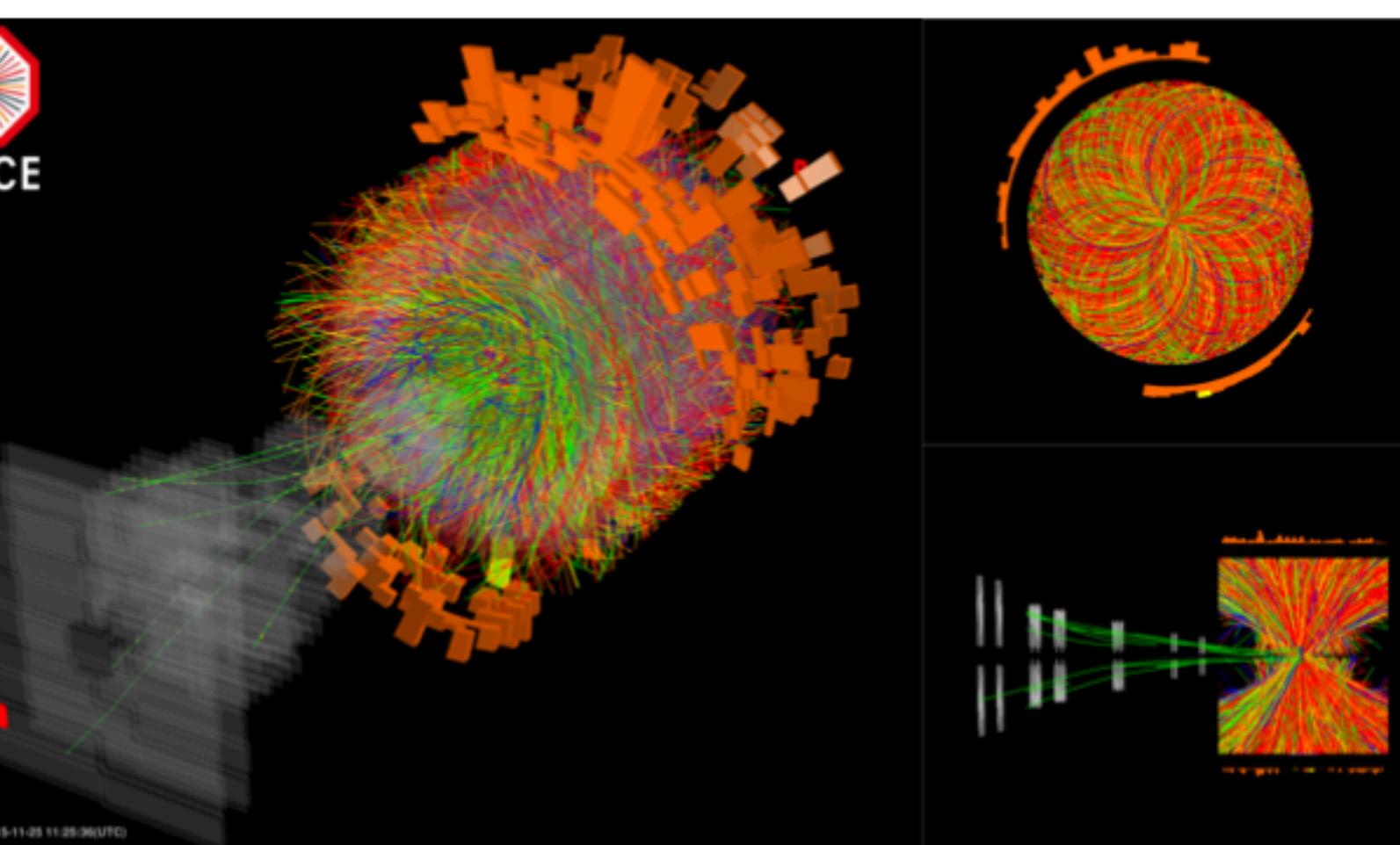
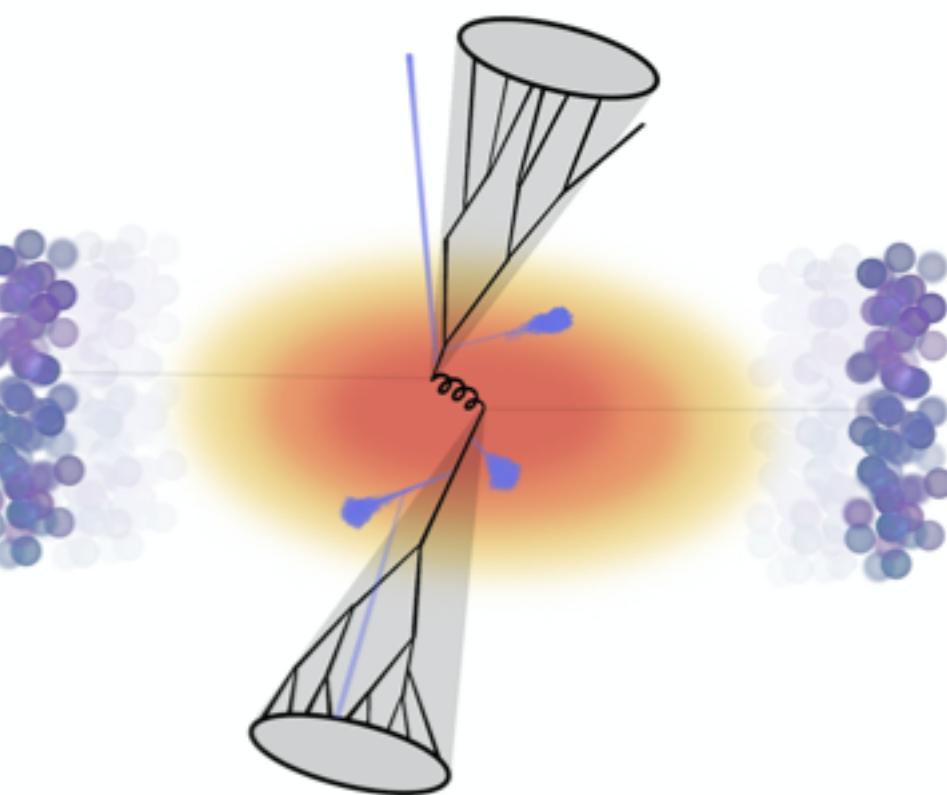
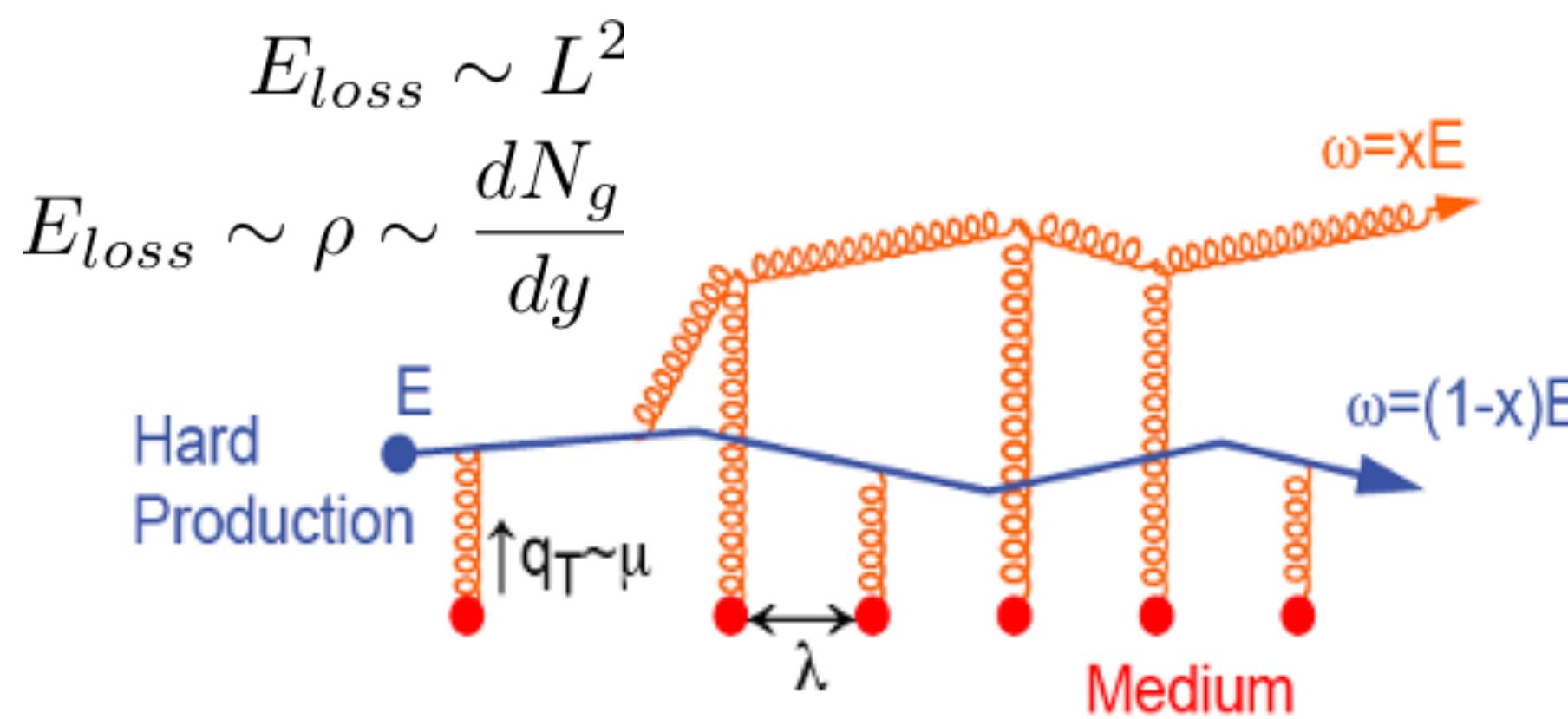
Primary goal at high energies: understand properties of QGP and its microscopic structure

Primary experimental tools:

- Bulk properties - collective flow (relativistic viscous hydrodynamics)
- Hard processes - probe medium with jets, heavy-quarks, quarkonia, ...

=> Unique need / central focus: real-time dynamics (non-equilibrium, expanding medium)

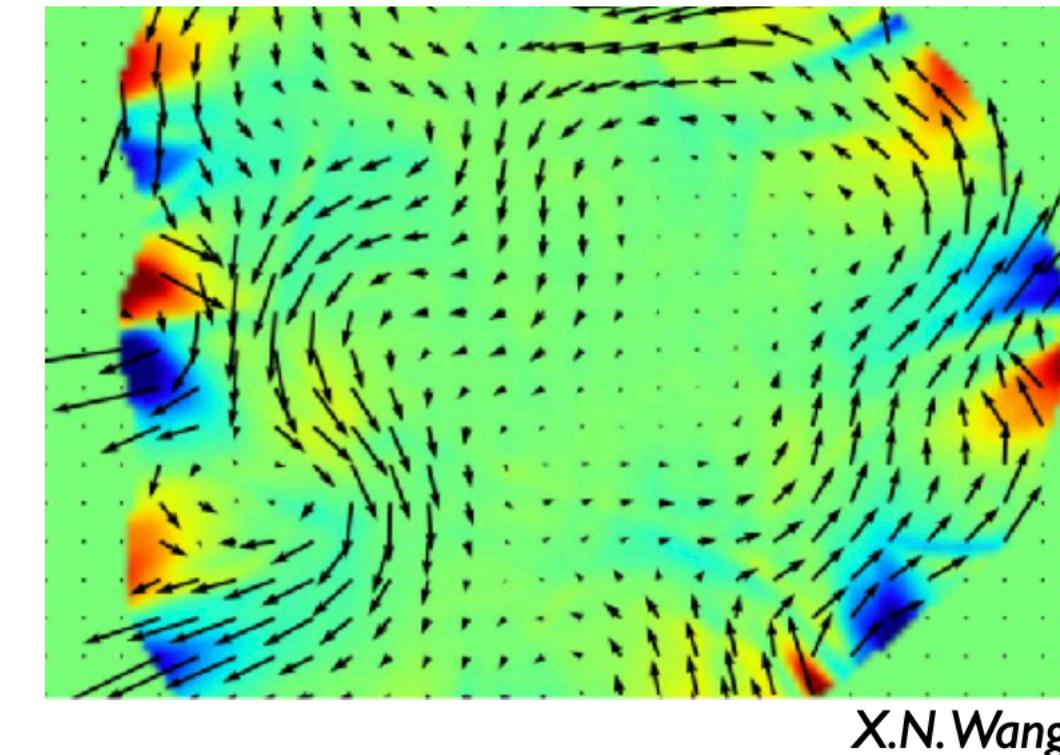
Problem? Real-time dynamics of lattice field theories can be simulated; however, due to the exponential size of the required Hilbert space, classical simulations are generally considered to be intractable... => quantum computers



10k feet view - possibly somewhat unjust to some theorist

Hard probes — theory

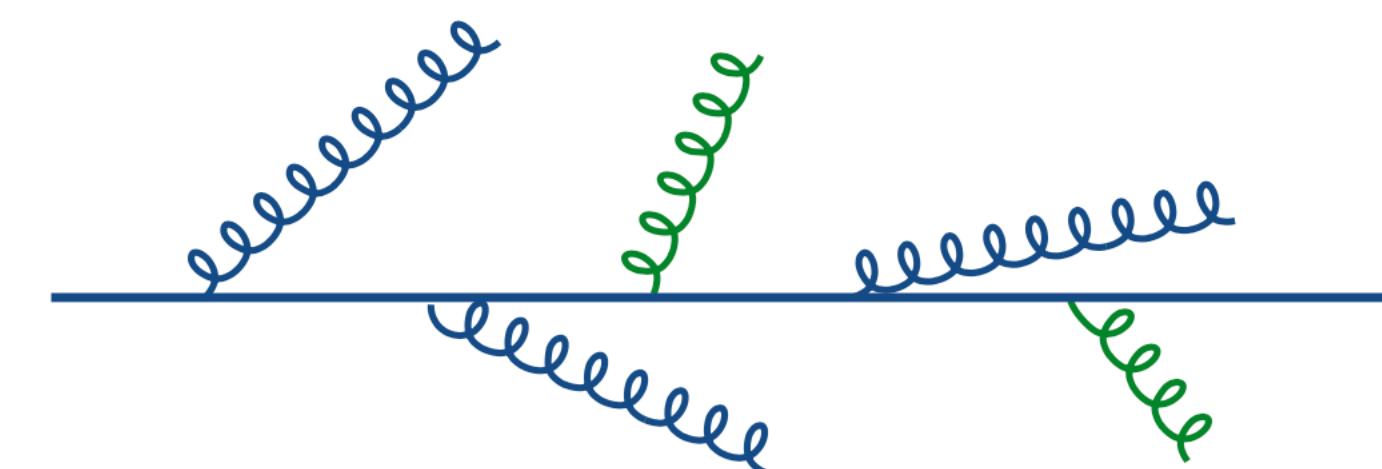
- In vacuum: calculate scattering of asymptotic states using perturbative QCD
Note that there is no sense of “time evolution”
- In medium: must combine probe evolution with hydrodynamic evolution of the QGP



In heavy-ion collisions, the modifications of the probe due to its evolution through the QGP are typically put in “by hand”, rather than a true real-time evolution

Medium-modified parton shower

Majumder PRC 88 (2013)
Caucal, Iancu, Mueller, Soyez PRL 120 (2018)
...



Notes on quantum computing...

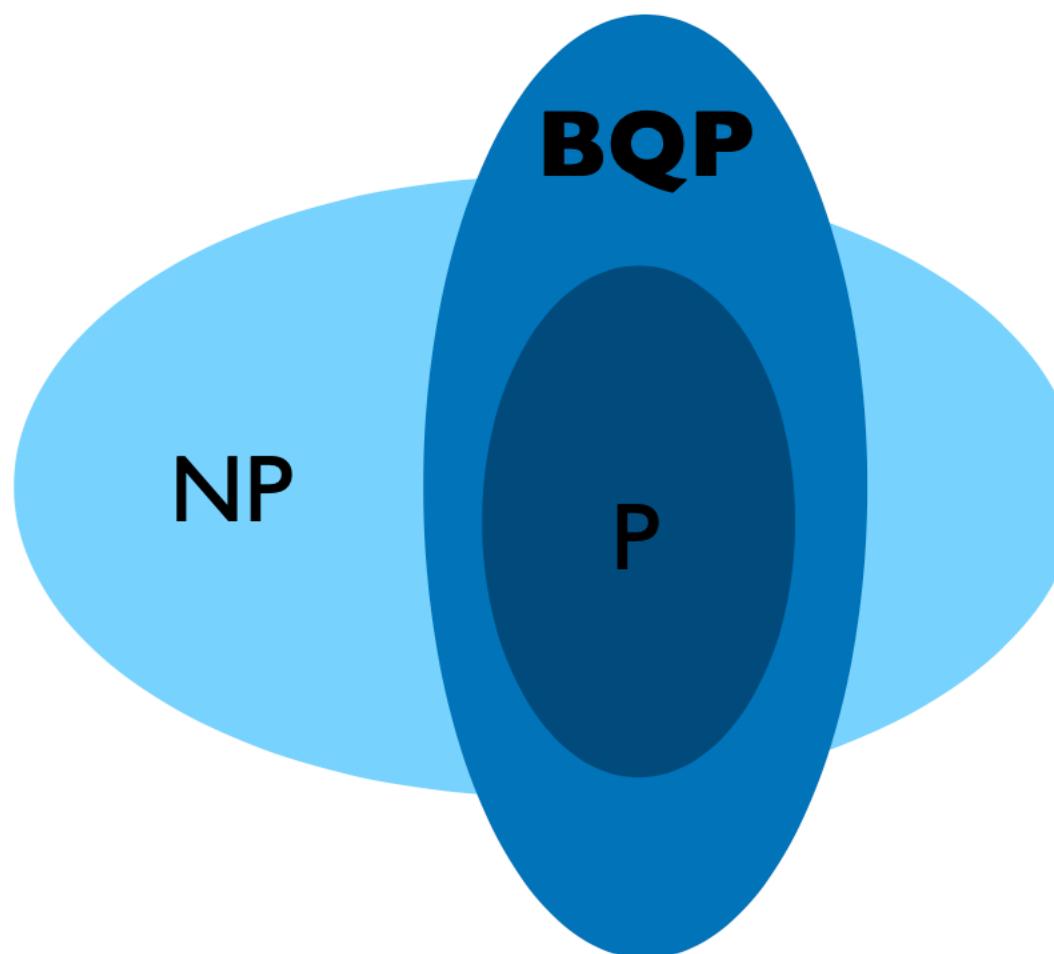
Superposition and entanglement

$$|\psi\rangle = \sum_{i=1}^{2^N} a_i |\psi_i\rangle$$

For N qubits, there are 2^N amplitudes

e.g. $|\psi\rangle = a_1|000\rangle + a_2|001\rangle + a_3|010\rangle + a_4|011\rangle + a_5|100\rangle + a_6|101\rangle + a_7|110\rangle + a_8|111\rangle$

If one can control this high-dimensional space, e.g. with appropriate interference of amplitudes, then one can potentially achieve **exponential speedup** of certain computations



Computational complexity

P: Polynomial-time solution on classical computer

NP: Polynomial-time verification on classical computer

BQP: Polynomial-time solution on quantum computer

QC can solve *some* classically hard problems

Notes on quantum computing...

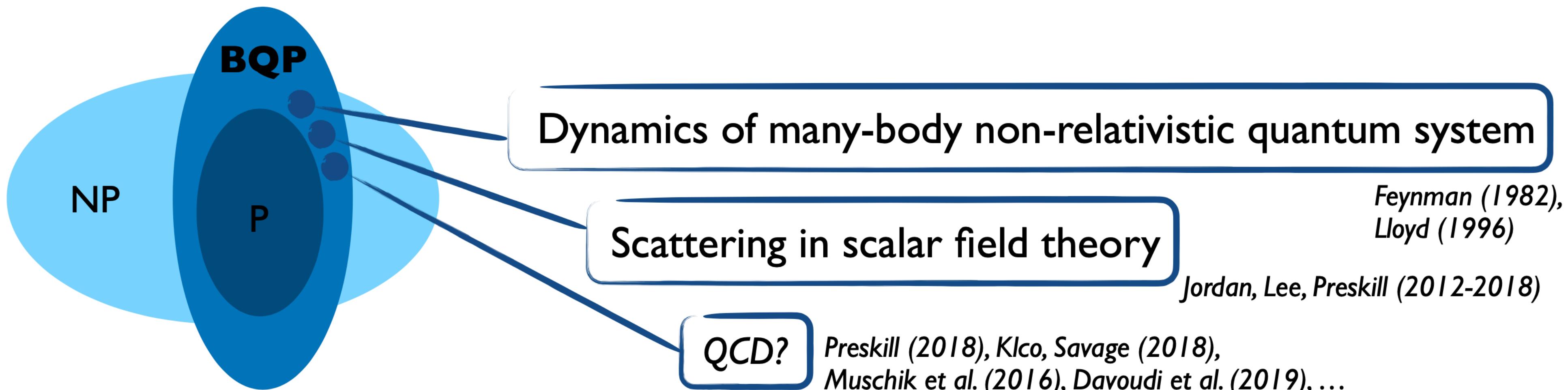
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Quantum devices

Strong improvements

Superconducting circuits



And a variety of others:



Things to look for:

- superconducting qubit coherence times
- long enough to perform many two-qubit operations

The dream: universal, fault-tolerant digital quantum computer

- Shor's and Grover's algorithm
- Quantum error correction

Shor, Preskill, Kitaev, Zoller ...

Noisy Intermediate Scale Quantum (NISQ) era

- Decoherence, limited number of qubits, imperfect gates
- Aim: achieve quantum advantage without full quantum error correction

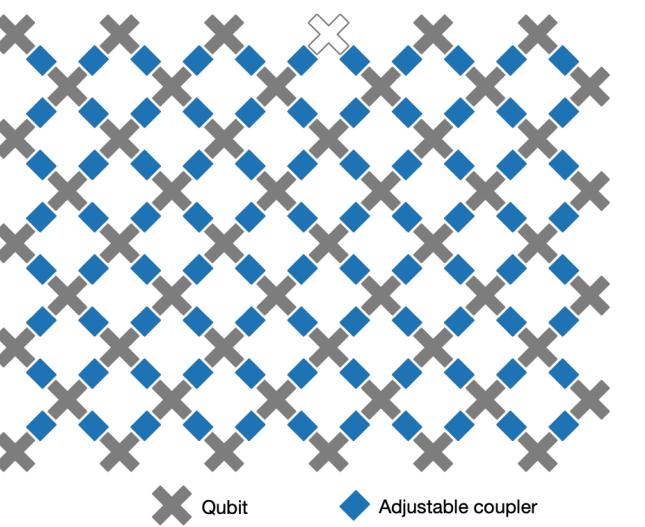
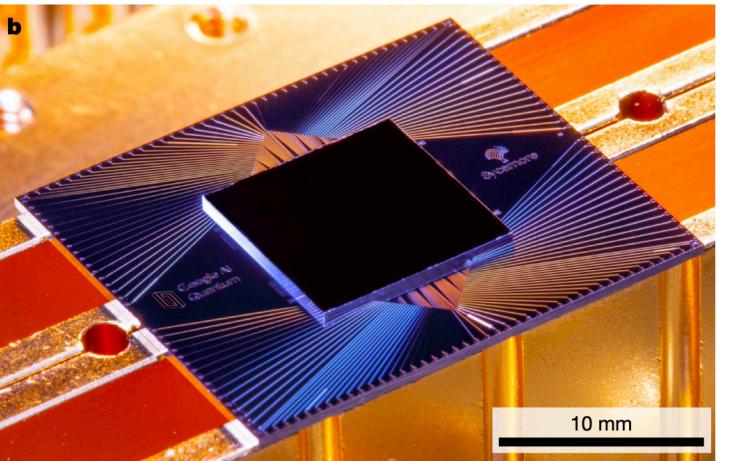
A note on hardware... strong progress...

For our first paper we used IBM Q Vigo - 5 qubits (2021)

In the mean time we used Montreal - 27 qubits (2022)

Article
Quantum supremacy using a programmable superconducting processor

Google
Martinis et al. '19

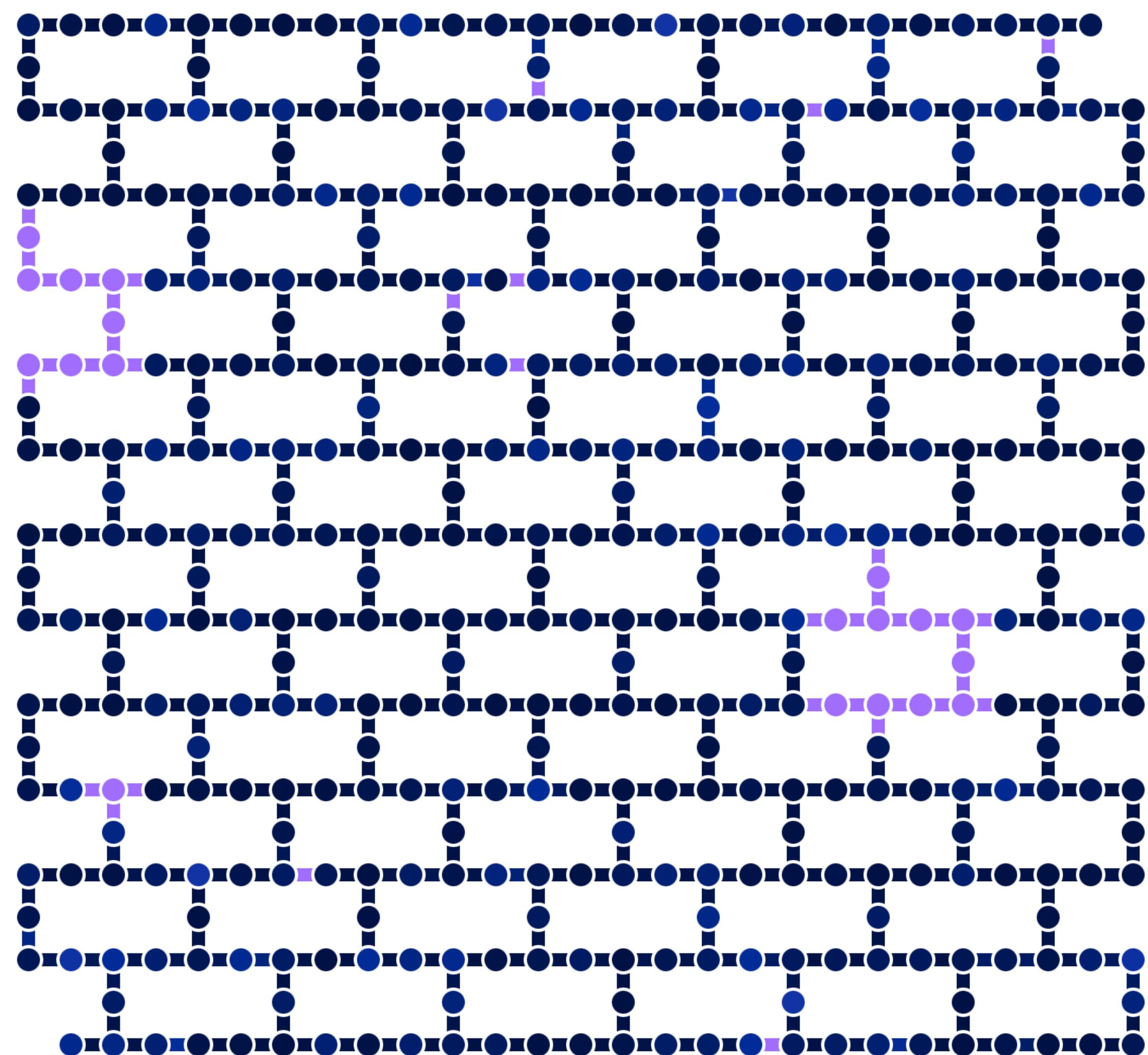
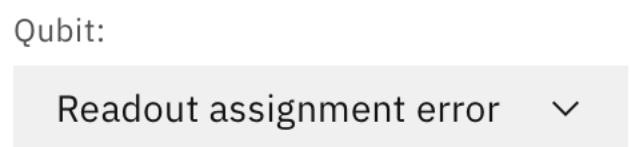


53-qubit sycamore device
99%+ gate fidelities

Algorithm: sampling of random circuits

$\mathcal{O}(10^3)$ times faster than best classical supercomputers

Just May 9th got notified via ORNL a New 433Q
IBM system Seattle is available!



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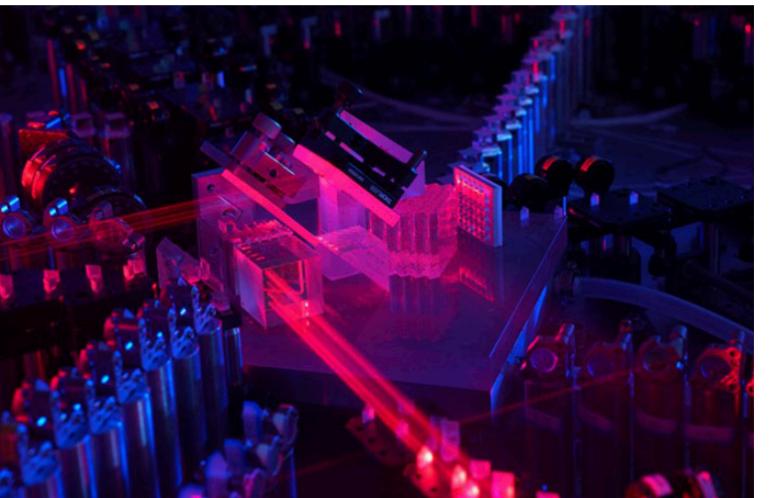
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Quantum computational advantage using photons

Han-Sen Zhong^{1,2*}, Hui Wang^{1,2*}, Yu-Hao Deng^{1,2*}, Ming-Cheng Chen^{1,2*}, Li-Chao Peng^{1,2},
Yi-Han Luo^{1,2}, Jian Qin^{1,2}, Dian Wu^{1,2}, Xing Ding^{1,2}, Yi Hu^{1,2}, Peng Hu³, Xiao-Yan Yang³,
Wei-Jun Zhang³, Hao Li³, Yuxuan Li⁴, Xiao Jiang^{1,2}, Lin Gan⁴, Guangwen Yang⁴, Lixing You³,
Zhen Wang³, Li Li^{1,2}, Nai-Le Liu^{1,2}, Chao-Yang Lu^{1,2}, Jian-Wei Pan^{1,2†}

Pan et al., Science (2020)

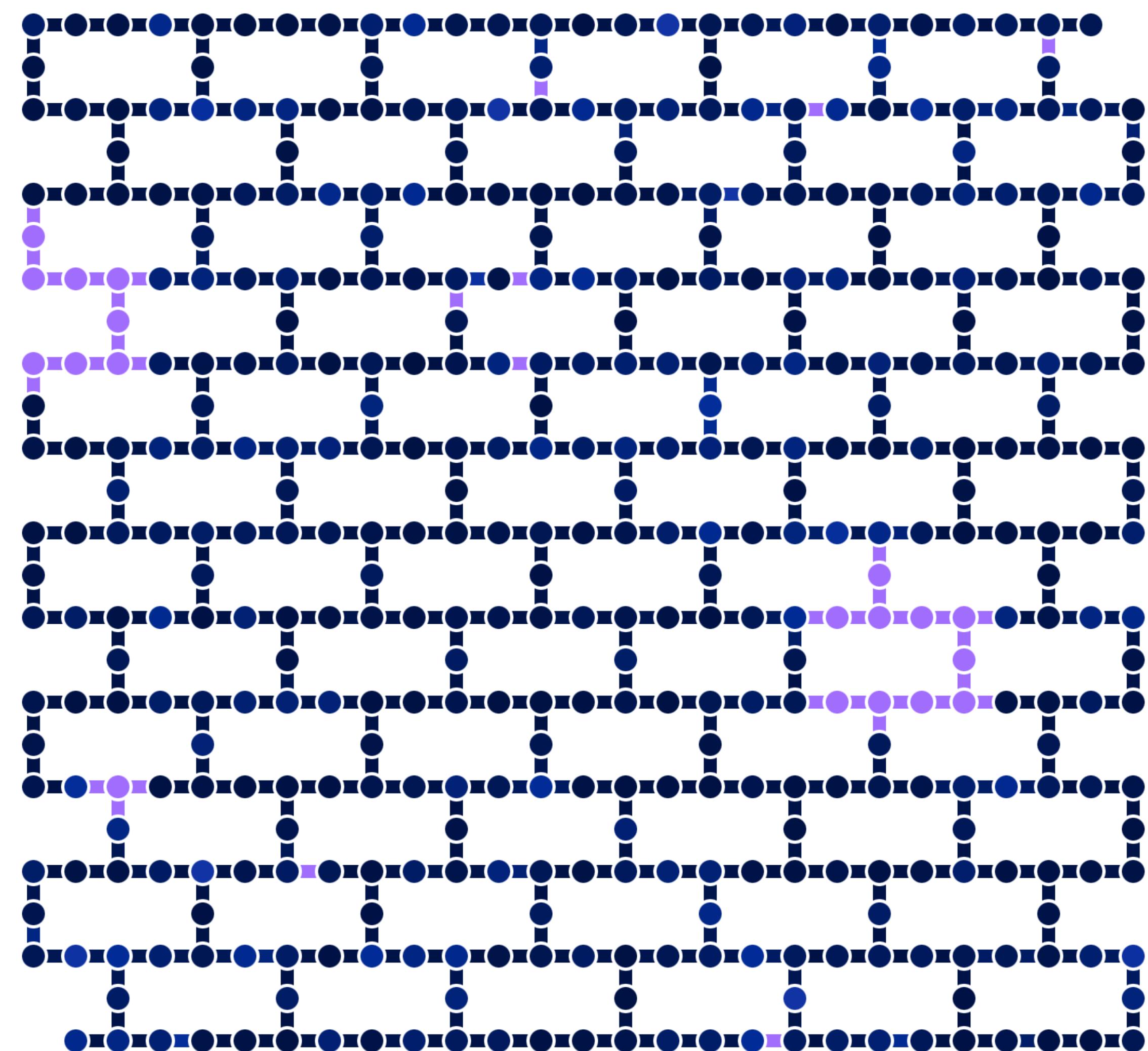
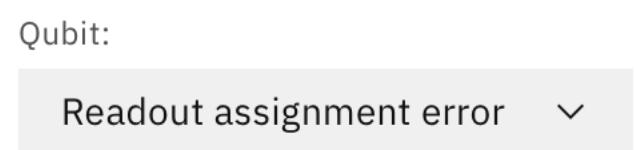


Photonic device — special-purpose

Algorithm: boson sampling

**Claim: $\mathcal{O}(10^{14})$ times faster than
best classical supercomputers**

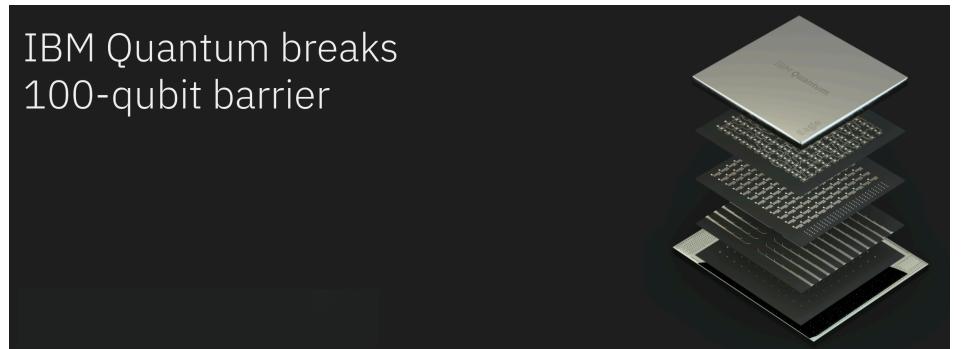
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'21



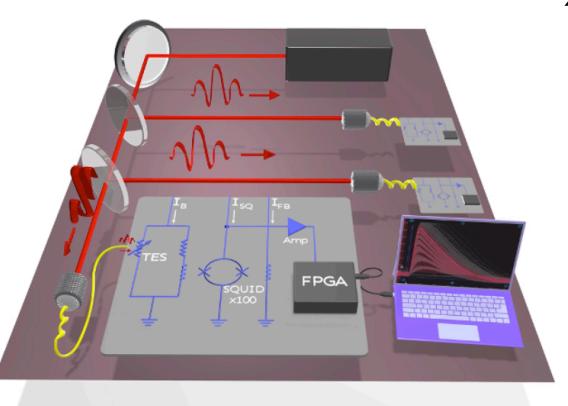
'22

topological qubits

Resolution of 100 photons and quantum generation of unbiased random numbers

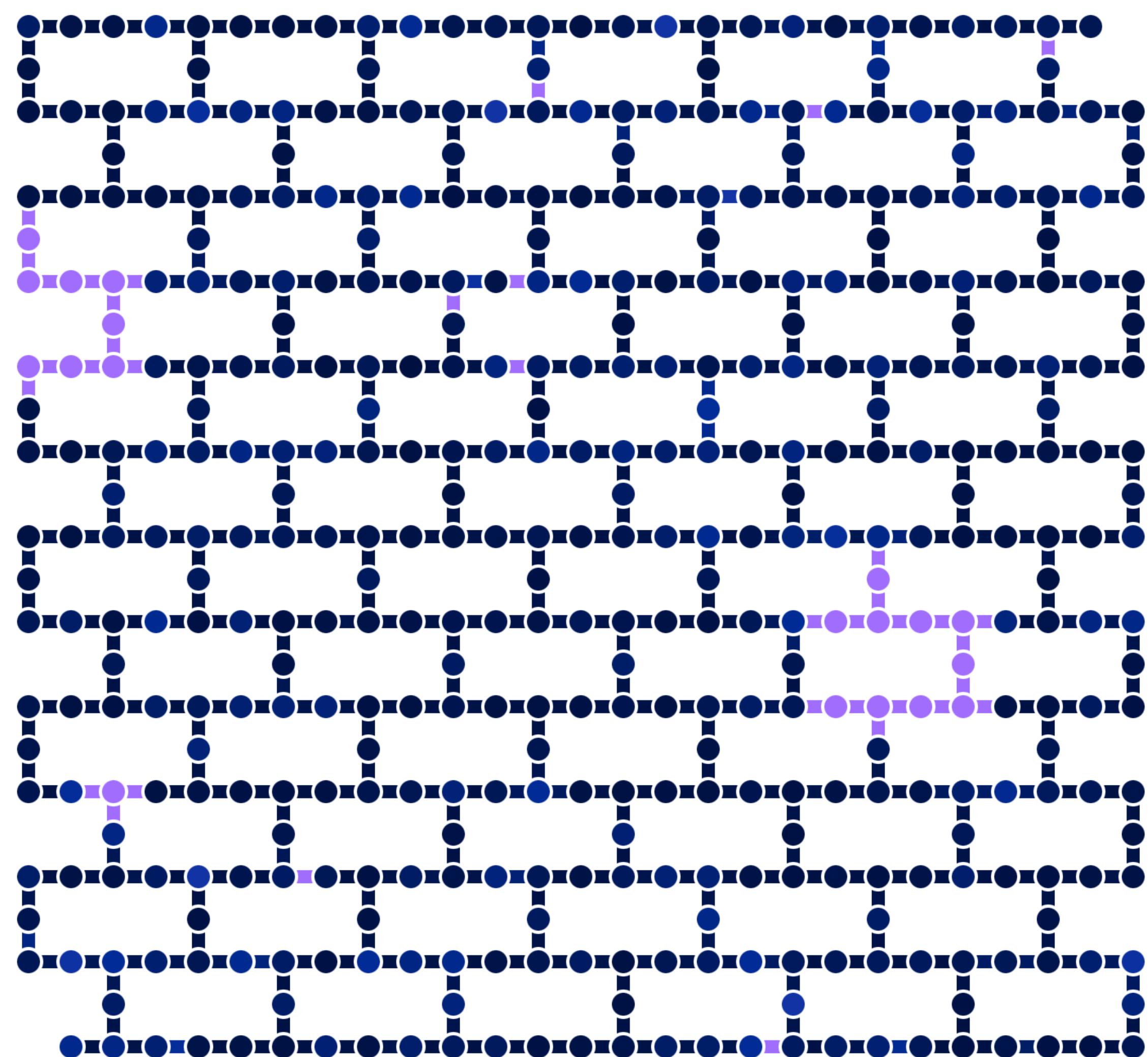
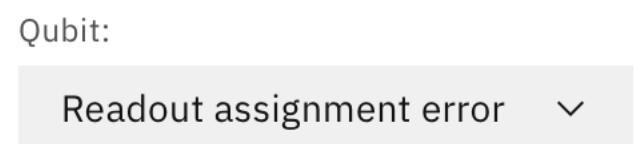
Miller Eaton [✉](#), Amr Hossameldin [✉](#), Richard J. Birrittella, Paul M. Alsing, Christopher C. Gerry, Hai Dong, Chris Cuevas & Olivier Pfister

Nature Photonics 17, 106–111 (2023) | [Cite this article](#)



'23

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Quantum simulations of QCD?

Problem 1: complexity in classical treatment \Rightarrow approximations

Problem 2: real-time dynamics in QCD on the lattice? - sign problem

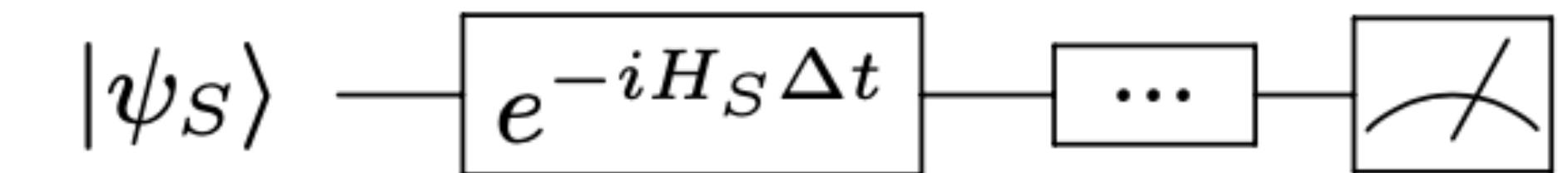
Feynman '81

Lloyd '96

Exponentially expensive to simulate an N -body quantum system on a classical computer: 2^N amplitudes!

But a quantum computer can naturally simulate a quantum system

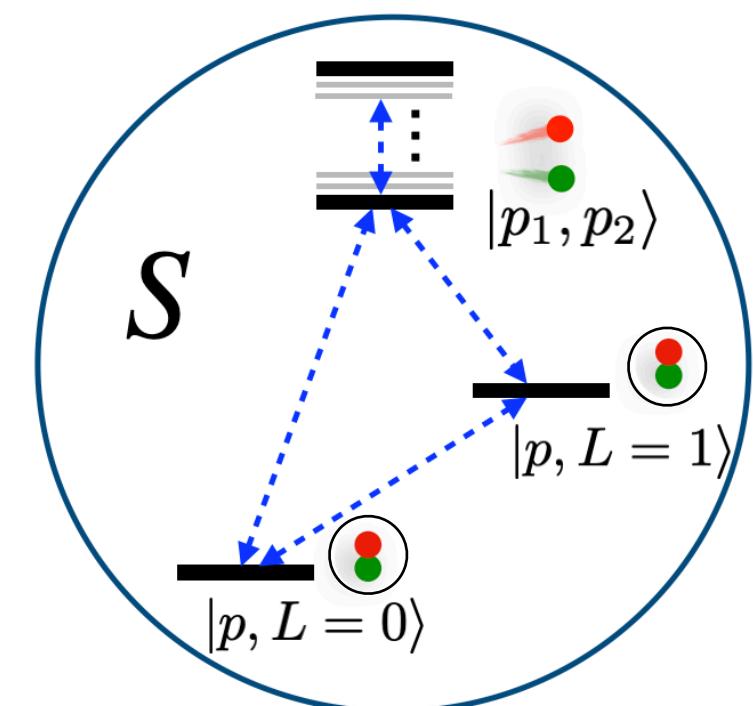
State preparation



Time evolution

Measurement

Evolution in time steps $\Delta t = t/N_{\text{cycle}}$



Time evolution of closed systems

- Quantum simulation of the Schrödinger equation
- The evolution is unitary and time reversible

Quantum simulations of QCD?

Problem 1: complexity in classical treatment => approximations

Problem 2: real-time dynamics in QCD on the lattice? - sign problem

Non-equilibrium dynamics are key to heavy-ion collisions

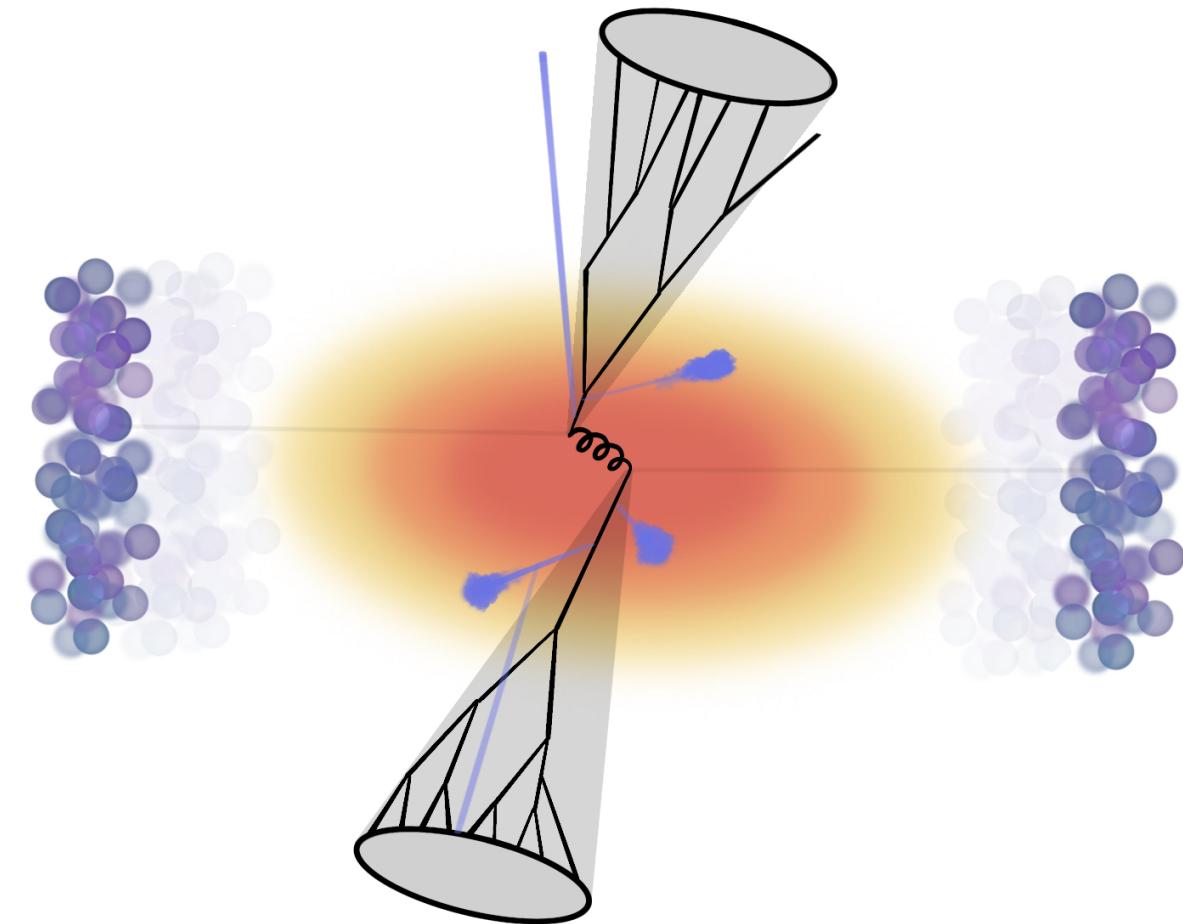
- Spacetime picture of jet fragmentation
- Transport coefficients
- Thermalization
- In vacuum: calculate scattering of asymptotic states using perturbative QCD
- In medium: must combine probe evolution with hydrodynamic evolution of the QGP
- In addition to real-time dynamics: sign problem in high-density systems

Real-time dynamics

Traditional Lattice QCD cannot simulate dynamics due to a sign problem

$$\int e^{i\mathcal{L}t} \quad t \rightarrow it$$

Quantum computers: directly simulate the Hamiltonian formulation of QCD



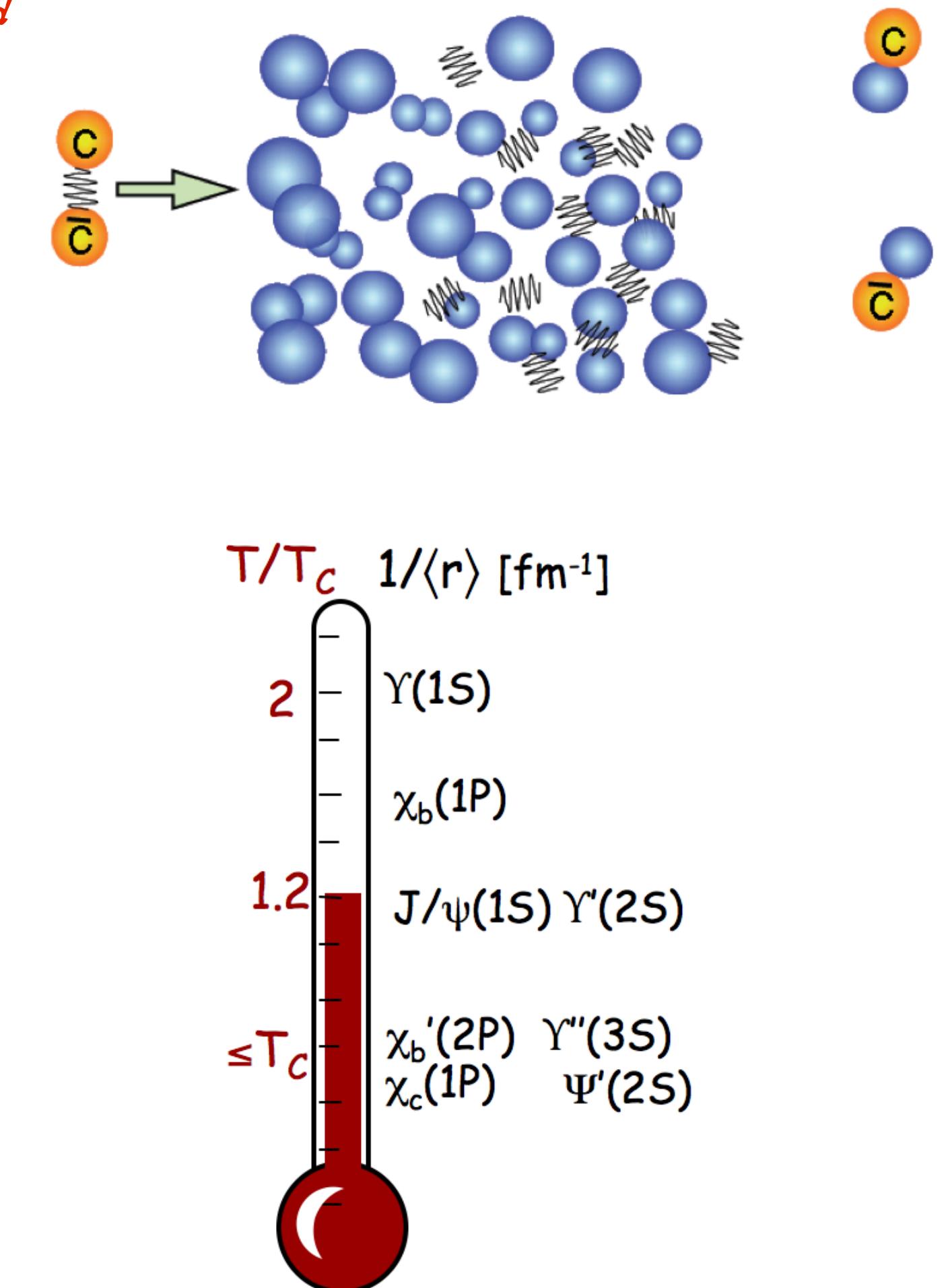
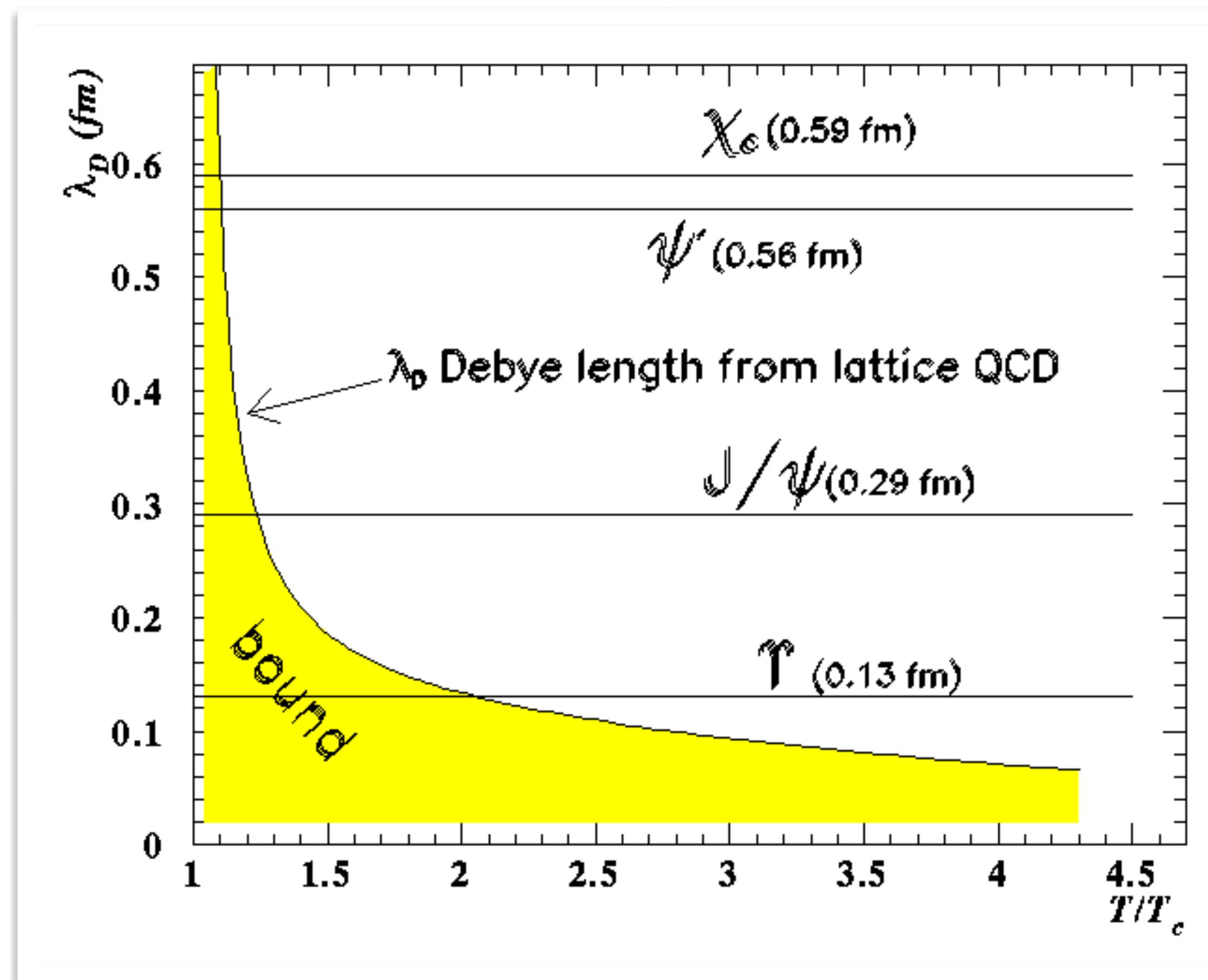
Jordan, Lee, Preskill '11, Preskill '18,
Klco, Savage et al. '18-'20
Davoudi et al.

Quarkonia in quark-gluon plasma - probe of temperature

QGP signature proposed by Matsui and Satz, 1986

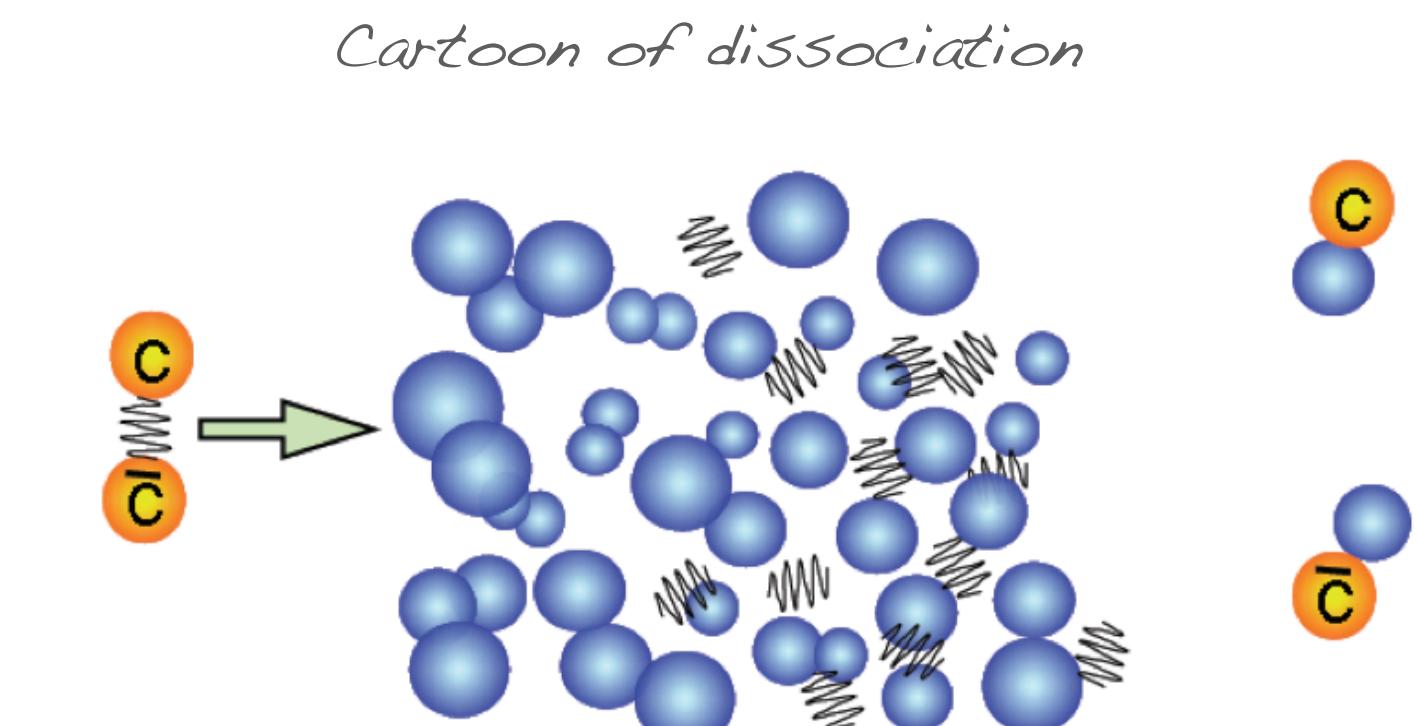
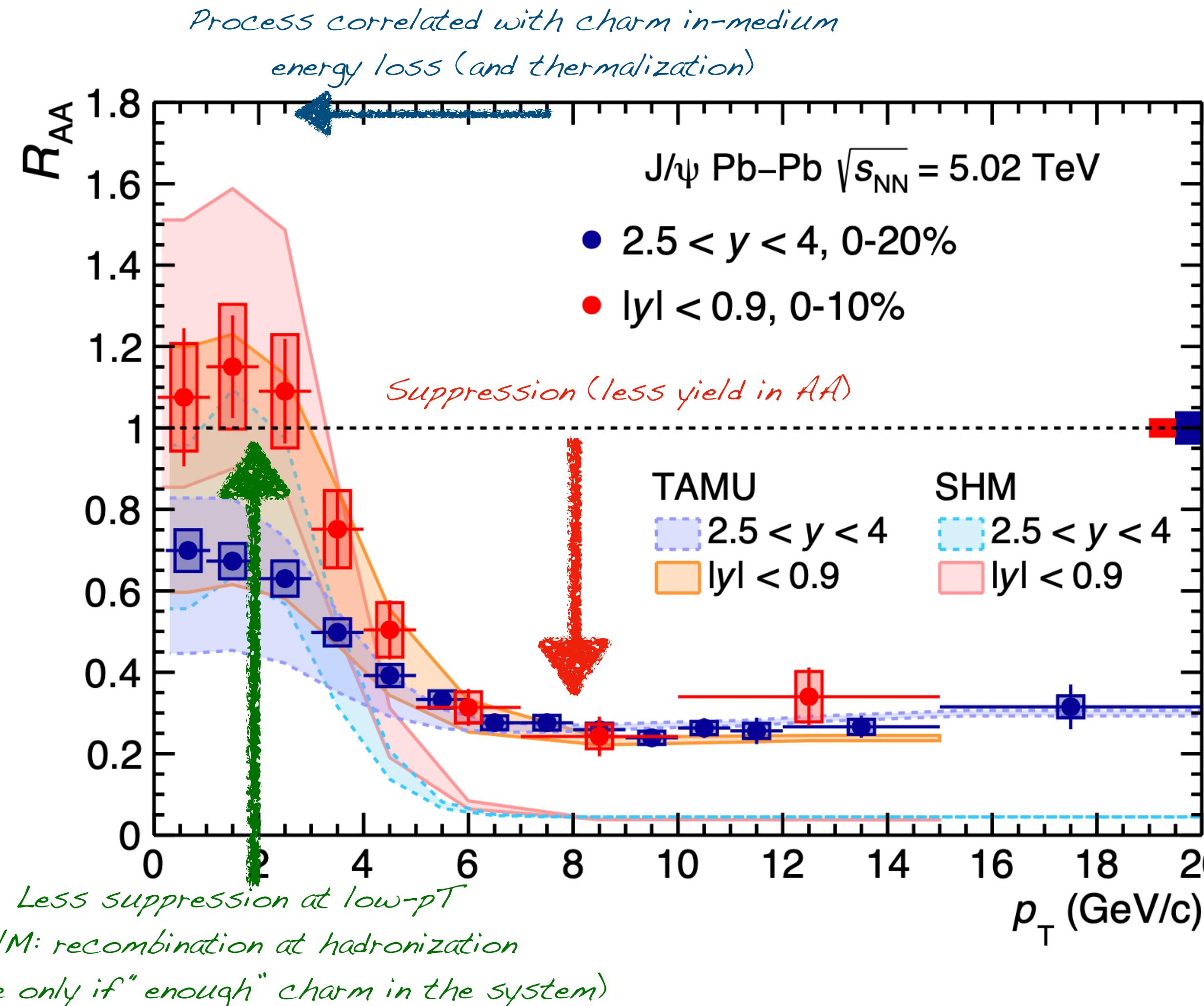
In the plasma phase the interaction potential is expected to be screened beyond the Debye length λ_D (analogous to e.m. Debye screening):

Charmonium(cc) and bottomonium(bb) states with $r > \lambda_D$ will not bind; their production will be suppressed ($gg\bar{b}$ states will "melt")



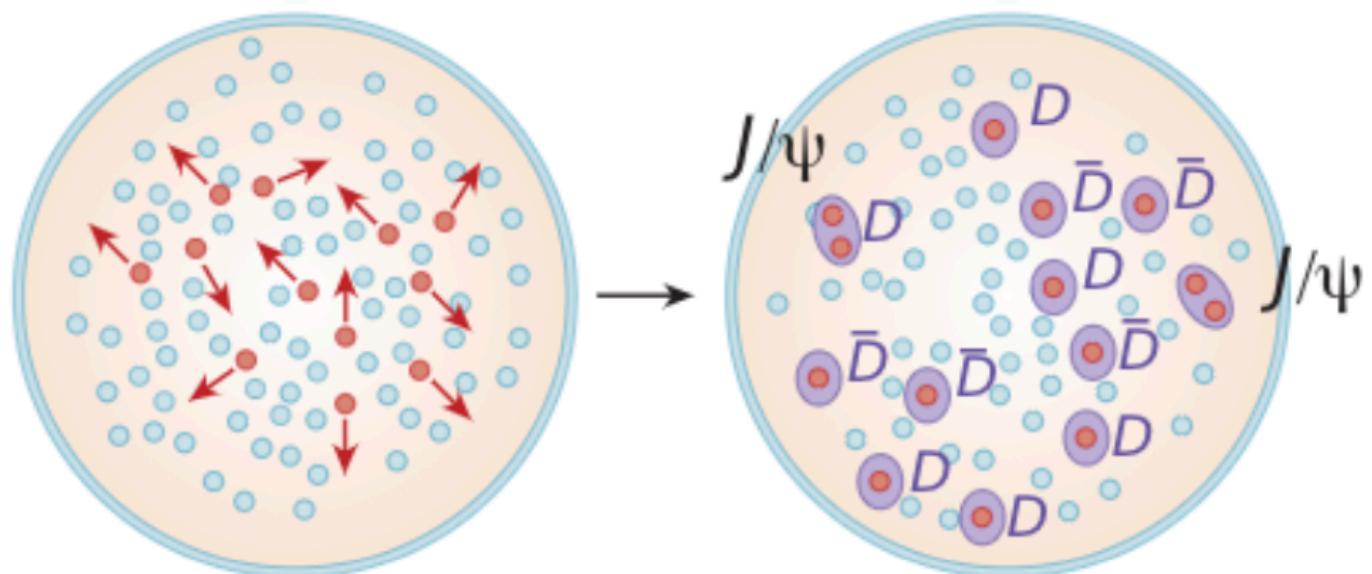
Quarkonia in QGP - experimental observations

Nuclear modification factor - suppression of yields in AA as compared to pp



Open quantum systems: Quarkonia

The evolution of quarkonia in the QGP can be described by the Lindblad equation



P. Braun-Munzinger and J. Stachel, Nature (2007)

“Simple” system:
reduces to quantum
mechanics (NRQCD)

Currently various approximations are considered

- Markovian limit
- Small coupling of system and environment
- Semi-classical transport

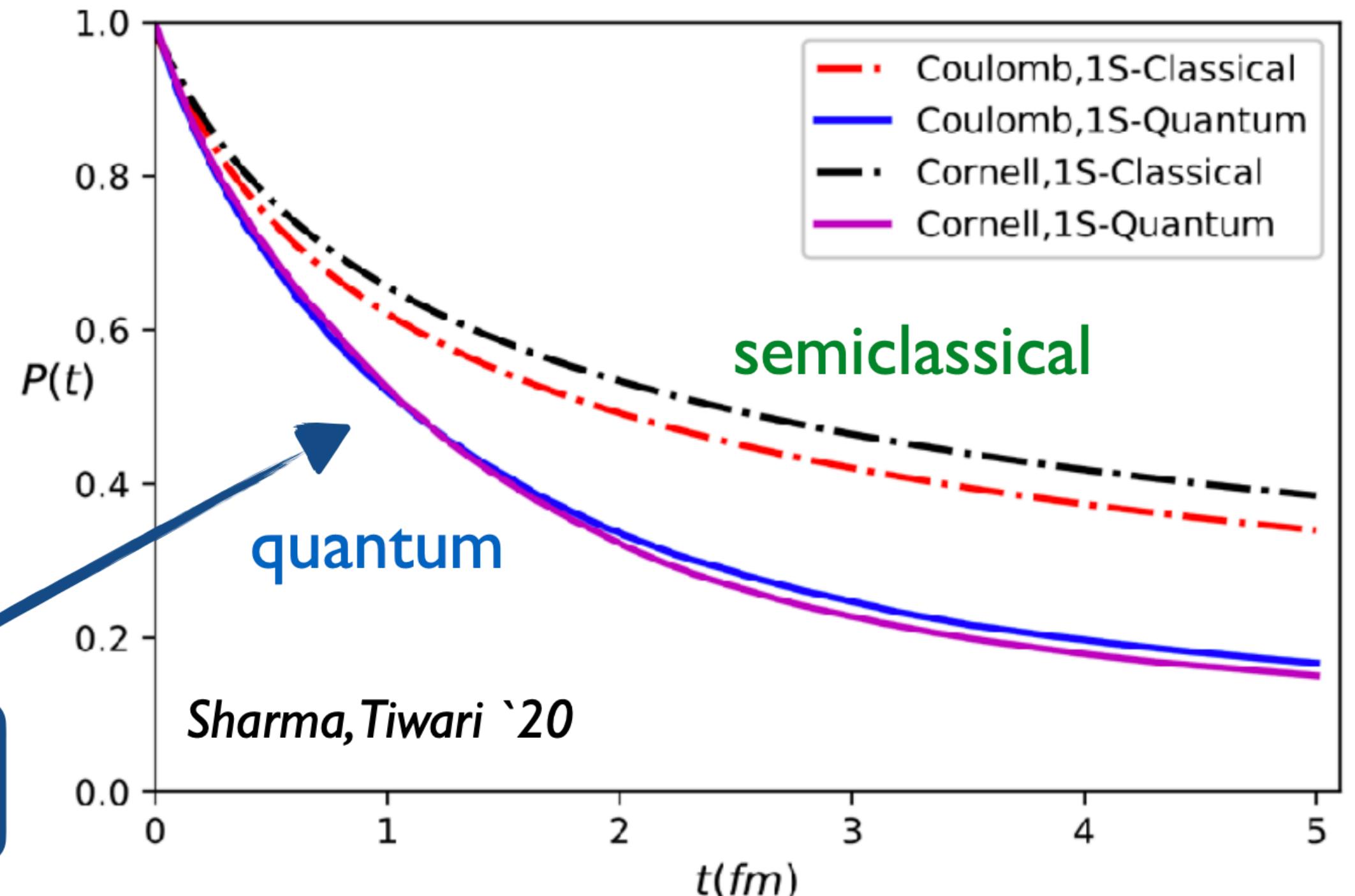
Blaizot, Escobedo '18, Yao, Mehen '18, '20

NRQCD + semiclassical approach vs. full quantum evolution

Quantum treatment has important phenomenological consequences

Akamatsu, Rothkopf et al. '12-'20, Brambilla et al. '17-'20
Yao, Mueller, Mehen '18-'20, Sharma, Tiwari '20

Survival probability of the vacuum state



Bjorken expanding QGP $T_0 = 475$ MeV

Quarkonia in QGP - Open Quantum System

Open-quantum-systems approach to in-medium heavy quarkonium dynamics

Alexander Rothkopf
<https://arxiv.org/abs/2211.10676>

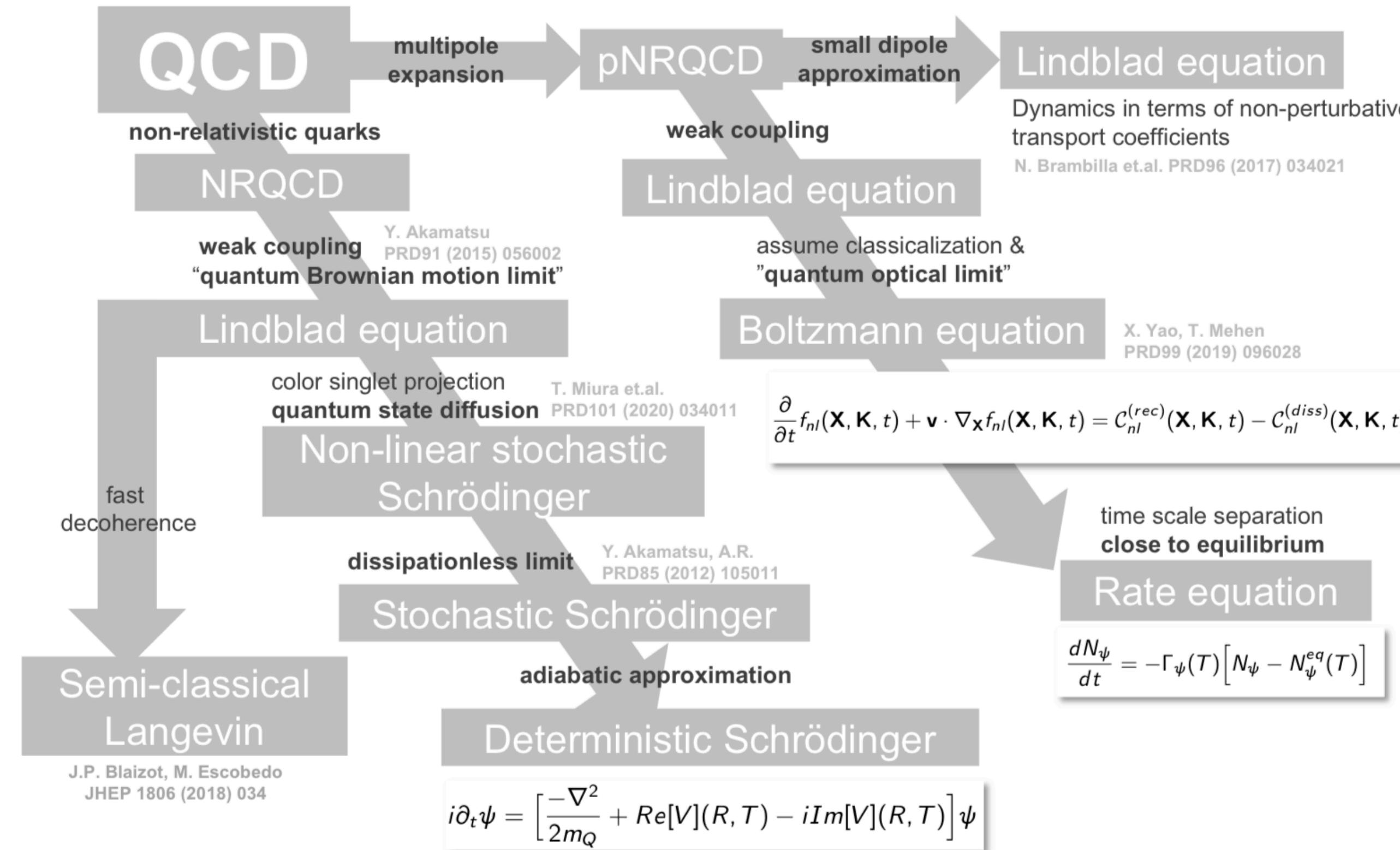
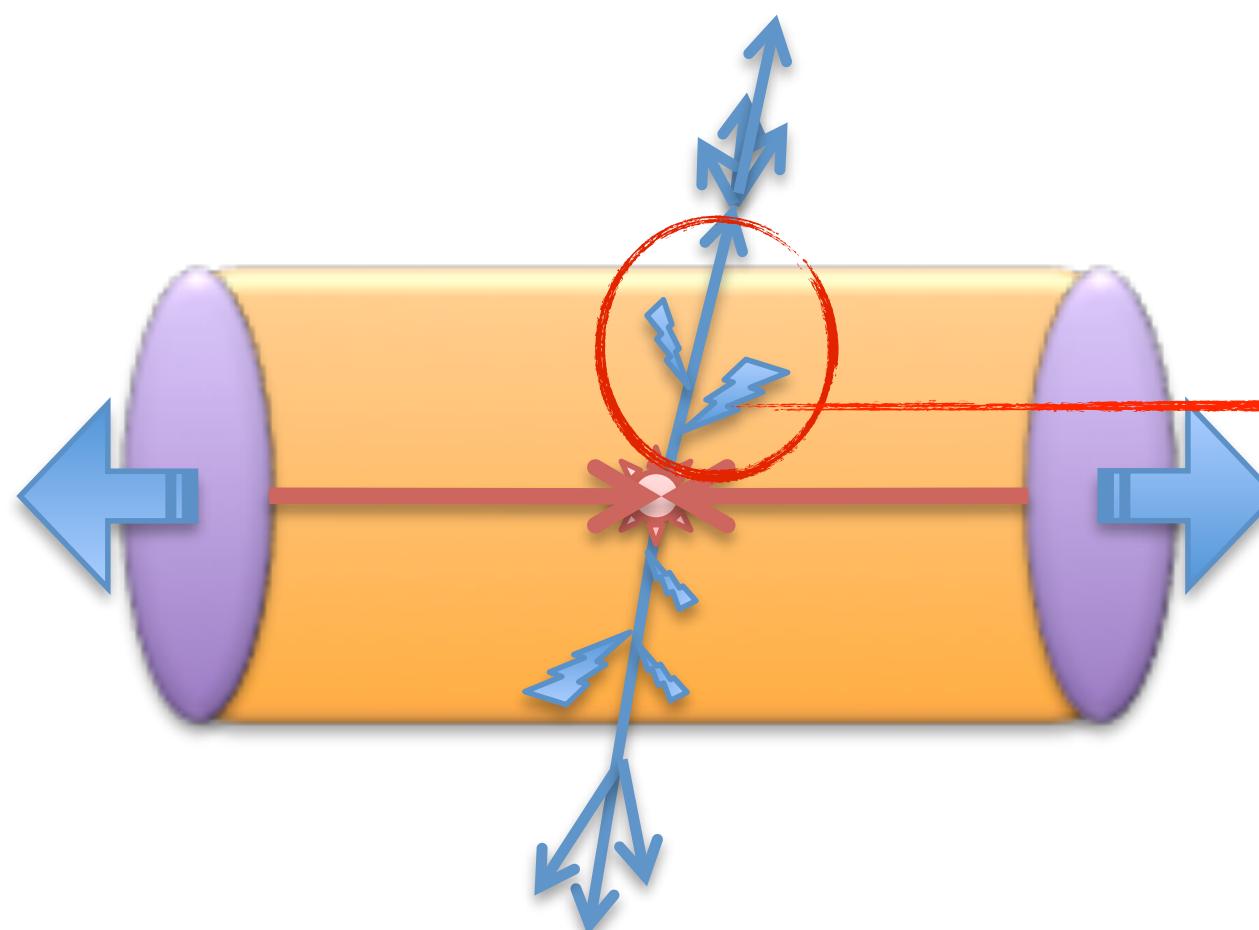
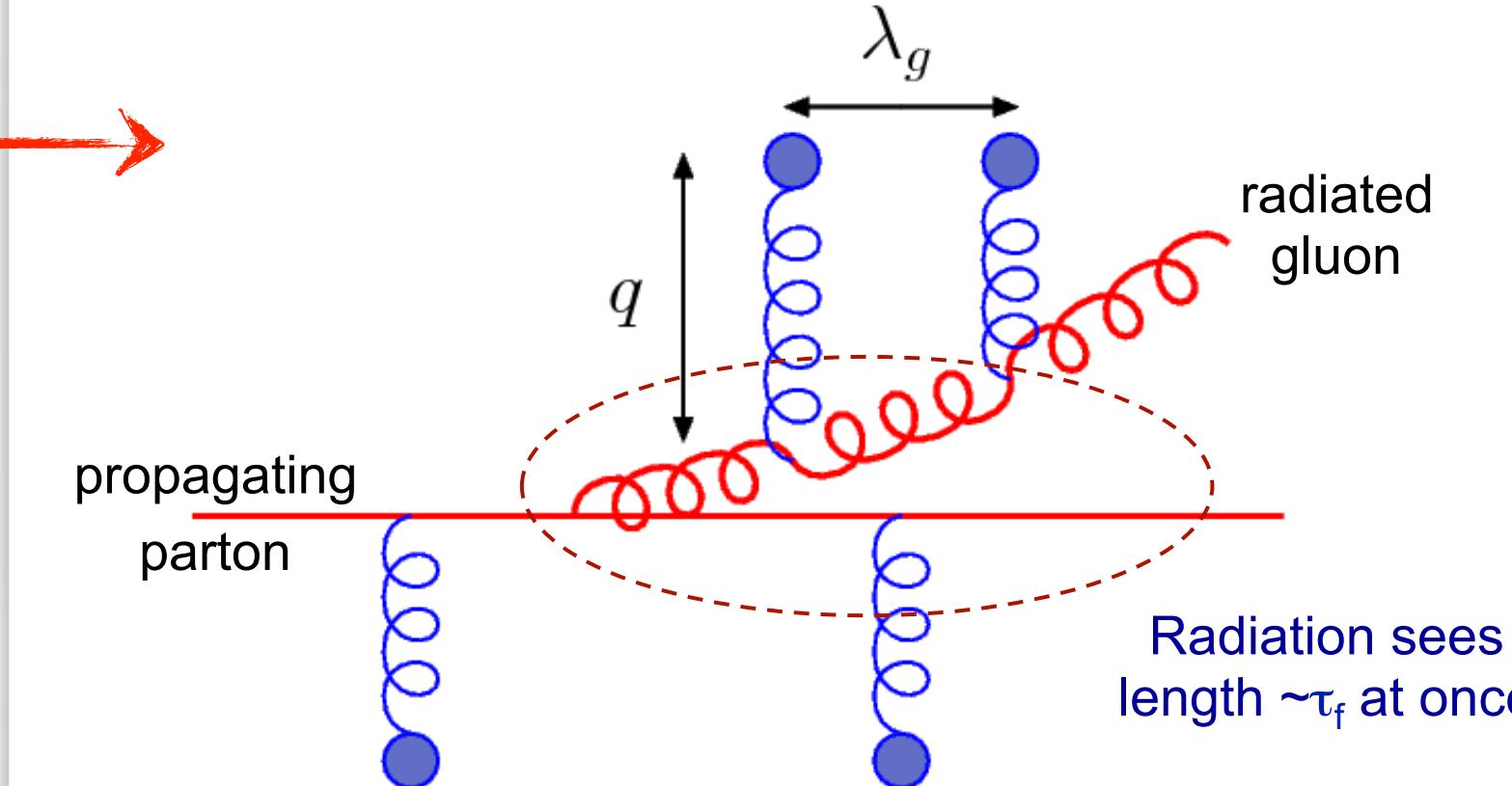


Figure 2. The systematic chain of approximations that the open quantum systems approach has built between ab-intio theory QCD, the effective field theories NRQCD and pNRQCD and the phenomenological approaches used in the past. At intermediate steps different genuine quantum descriptions of quarkonium, applicable in regimes with appropriate time-scale separation, have been established.

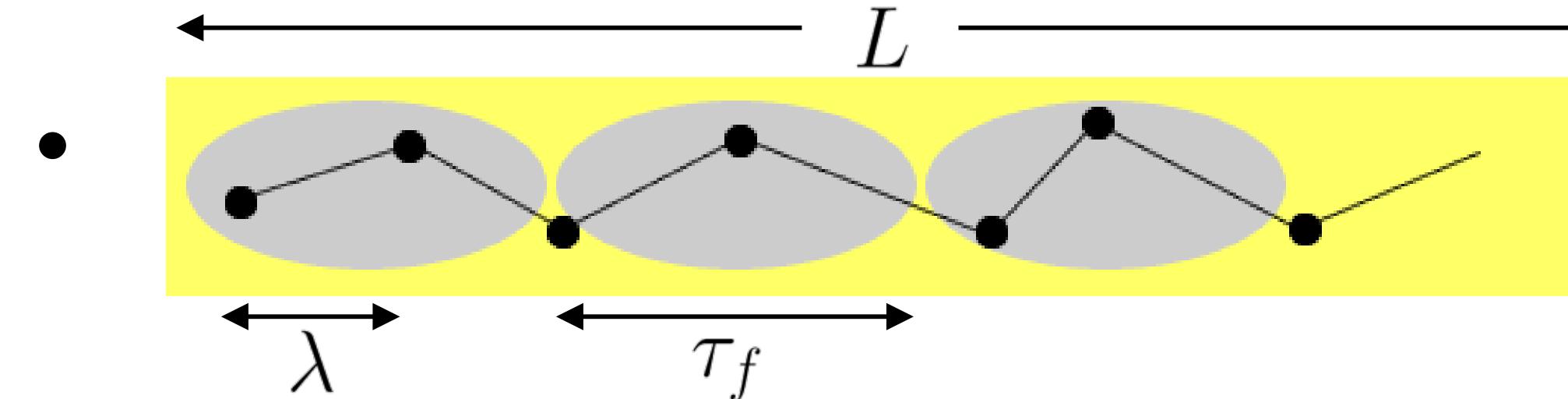
Bremsstrahlung in QCD: Formation time \leftrightarrow coherence effects



Landau-Pomeranchuk-Migdal effect
Formation time important



Formation time physics



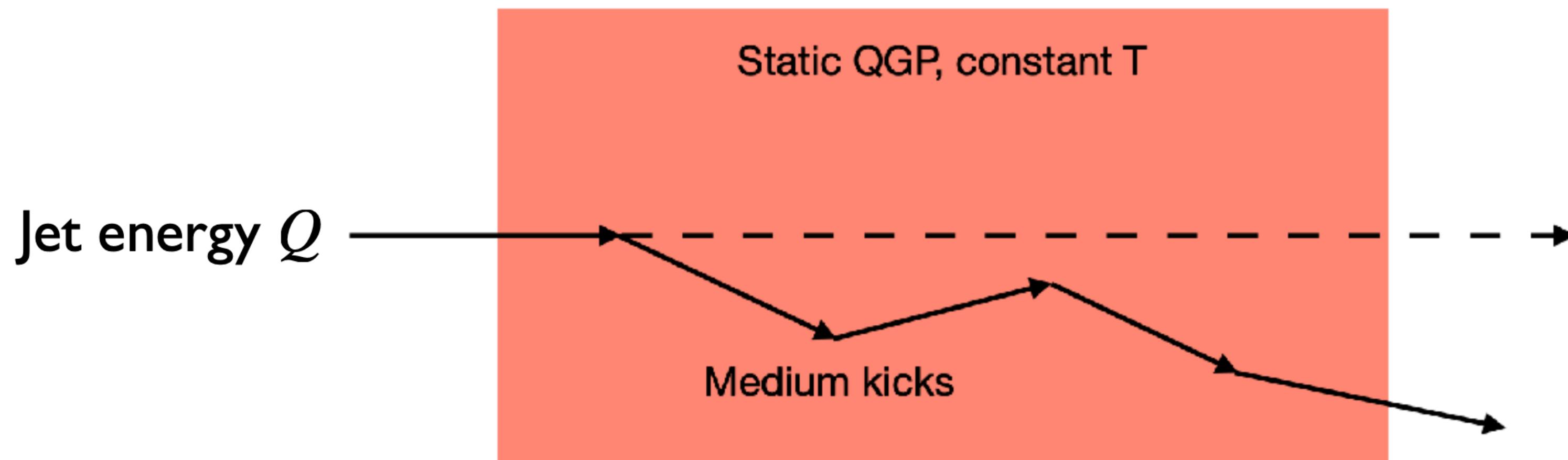
$$\tau_f \sim \frac{2\omega}{k_{\perp}^2}$$

- - $\tau_f < \lambda < L$ Incoherent multiple collisions
 - $\lambda < \tau_f < L$ LPM effect (radiation suppressed by multiple scatterings within one coherence length)
 - $\lambda < L < \tau_f$ Factorization limit (acts as one single scatterer)

Open quantum systems: Jet broadening

First steps in the direction of jet physics

Yao, Vaidya JHEP 10 (2020)
Vaidya 2010.00028



Soft Collinear Effective Theory

- Forward scattering, Glauber gluon exchange

Markovian master equation describes evolution of jet density matrix:

$$\partial_t P(Q, t) = -R(Q)P(Q, t) + \int \widetilde{dq} K(Q, q)P(q, t)$$

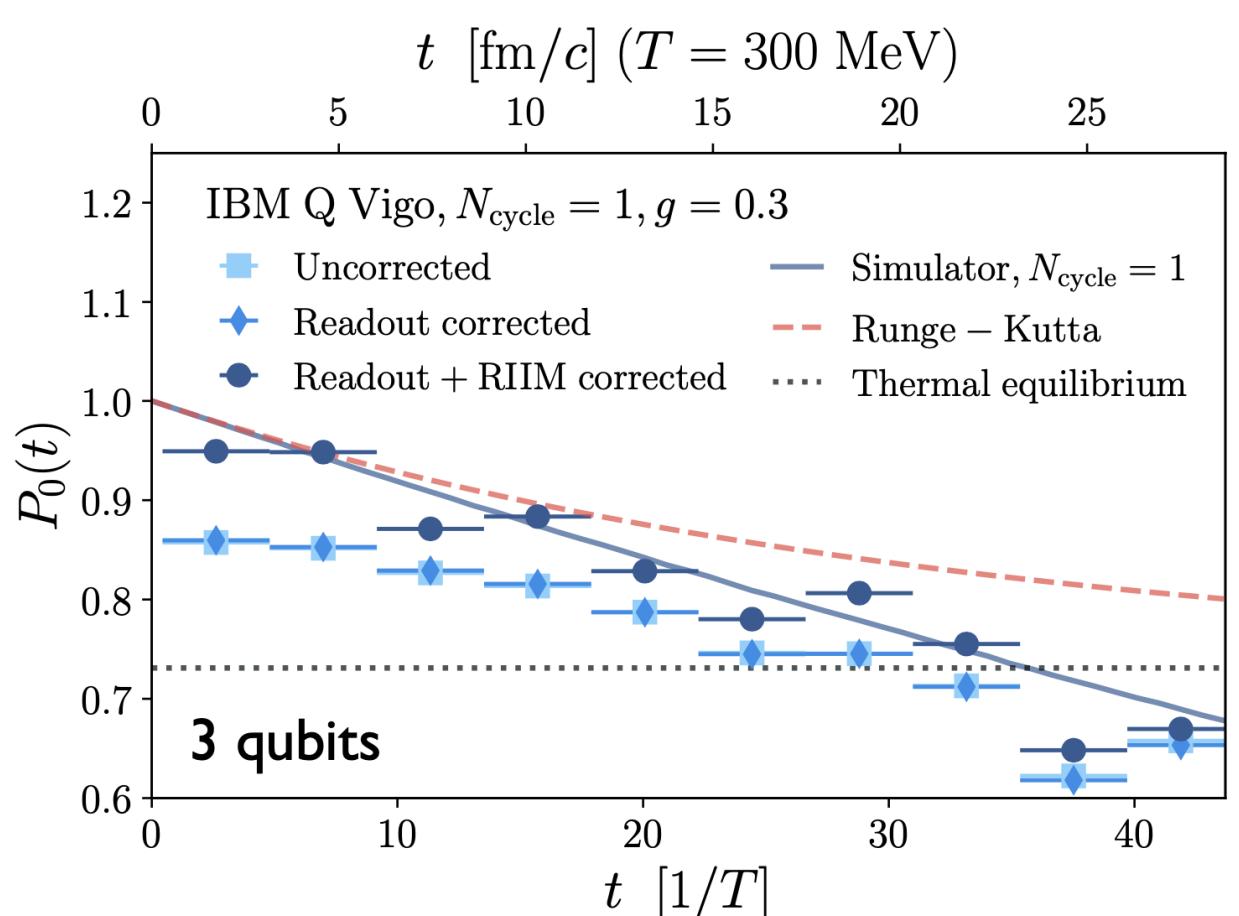
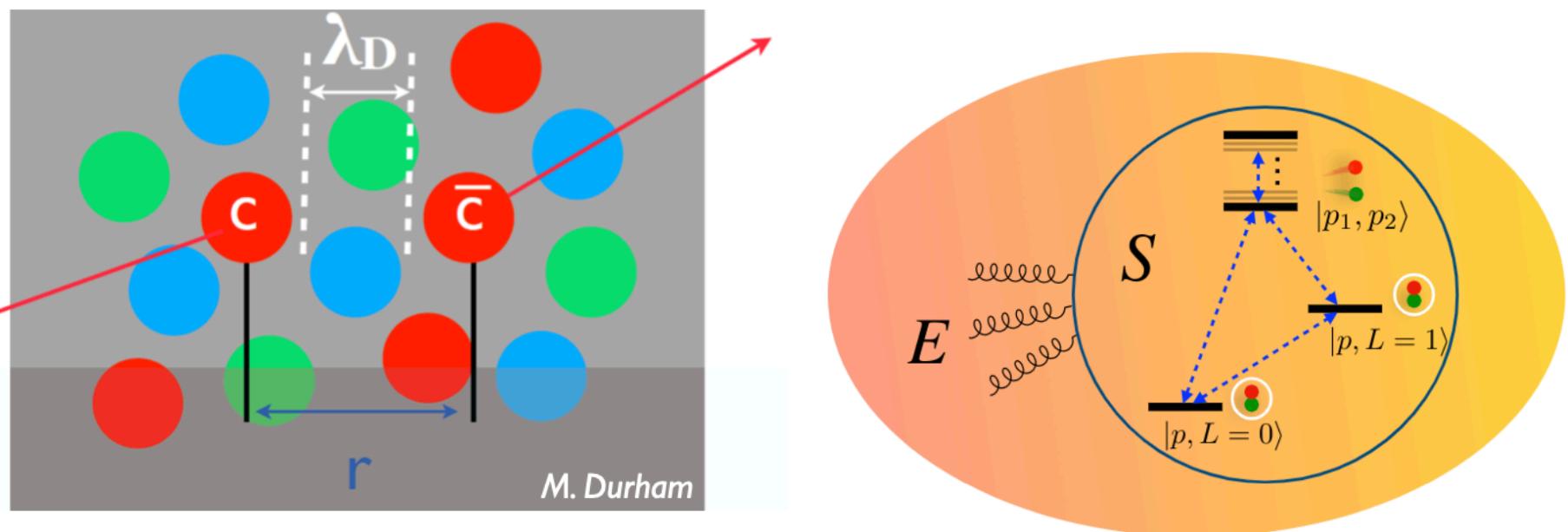
where the probability to be in a given momentum state is:

$$P(Q, t) = \langle Q | \rho_S(t) | Q \rangle$$

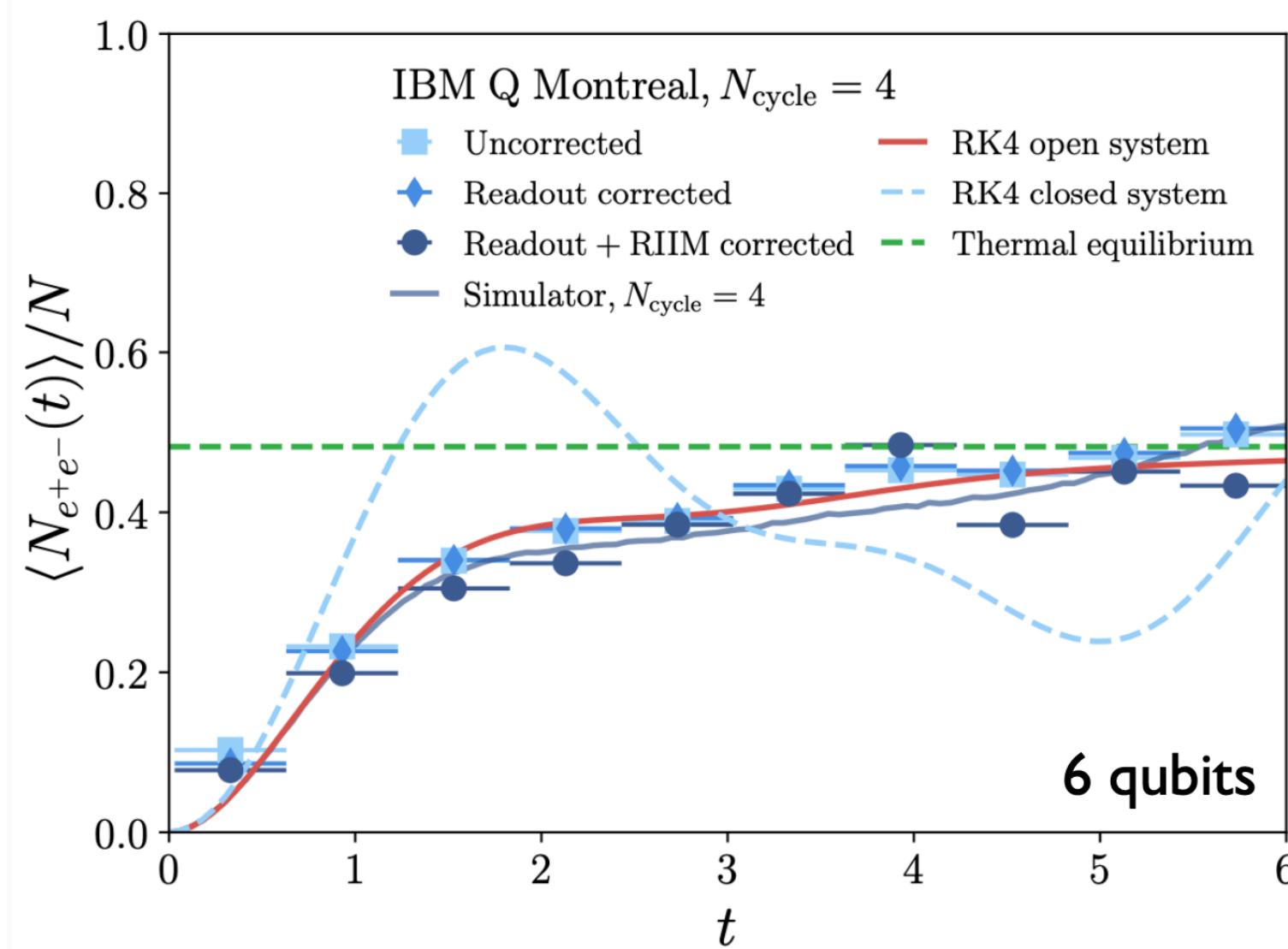
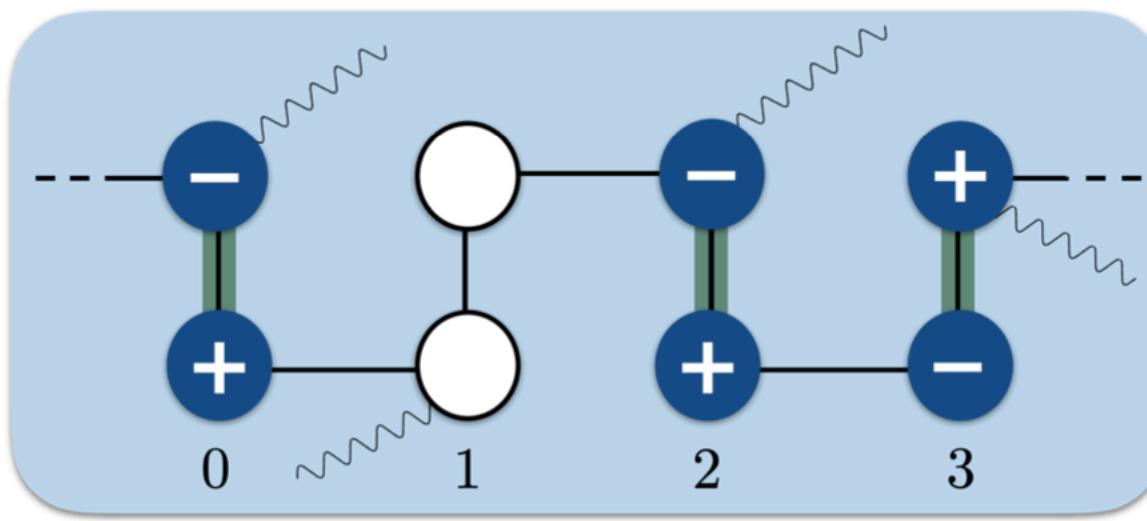
Outline

Main target: simulation of some relevant processes but mitigate classical computations shortcomings
 Non-relativistic QCD - many-body problems \Rightarrow quarkonia dissociation in hot colored medium
 Quantum Field Theory - thermalization (equilibration), hadronization, jet quenching, ...

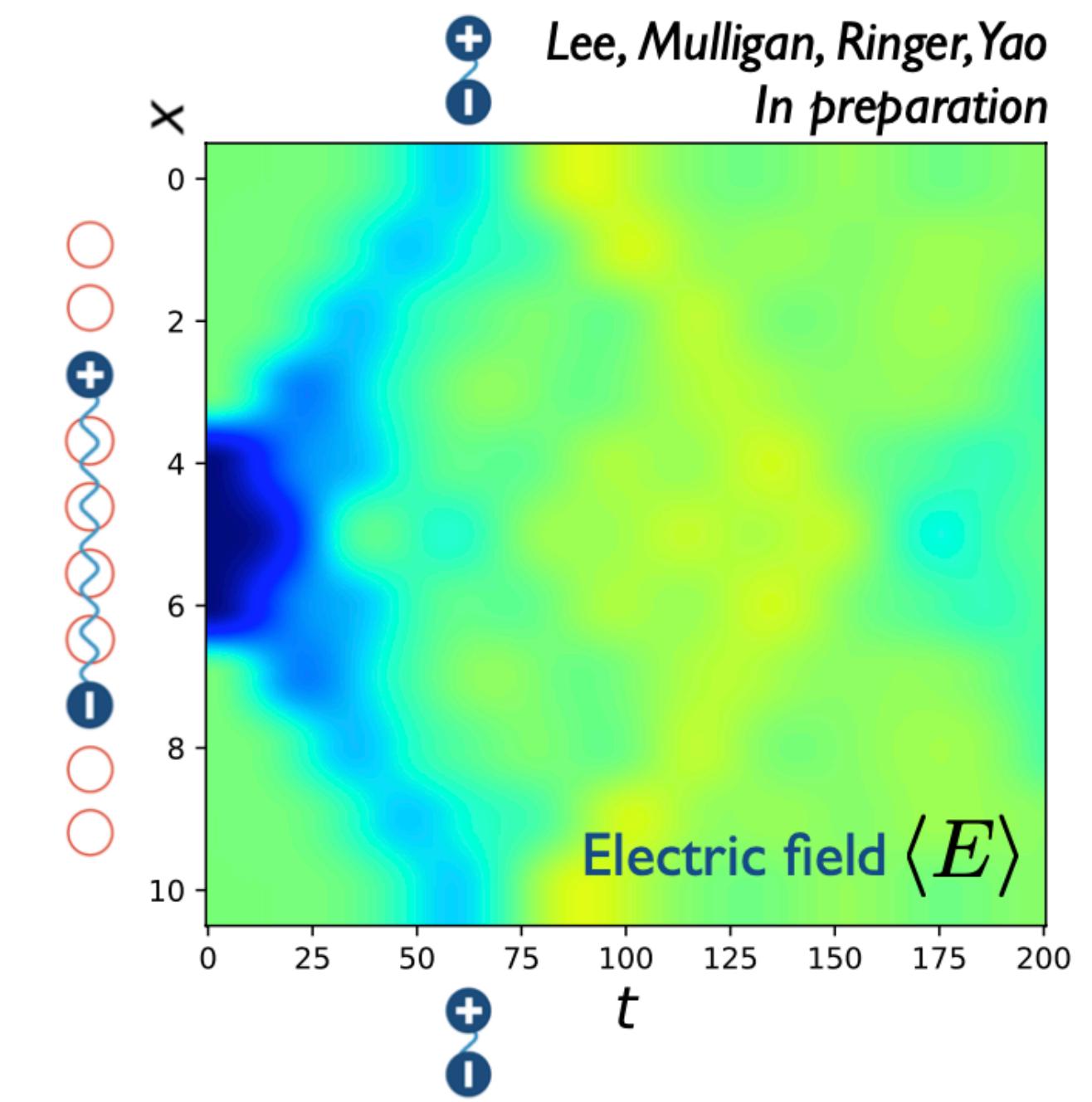
Towards quarkonia in hot QGP: open quantum system - proof of principle calc. -
[PhysRevD.104.L051501](#)



Thermalization/non-eq. dynamics
 Schwinger model: QED in 1+1D
[PhysRevD.106.054508](#)



Towards hadronization:
 String breaking mechanism in Schwinger model



Jet quenching in light front QCD
<https://arxiv.org/abs/2205.07902>

Open Quantum Systems

Study the real time dynamics of the quantum evolution of probes in the nuclear medium (LHC/RHIC/EIC)

Subsystem (\mathcal{H}_S) = Probe (jet or heavy quarks)

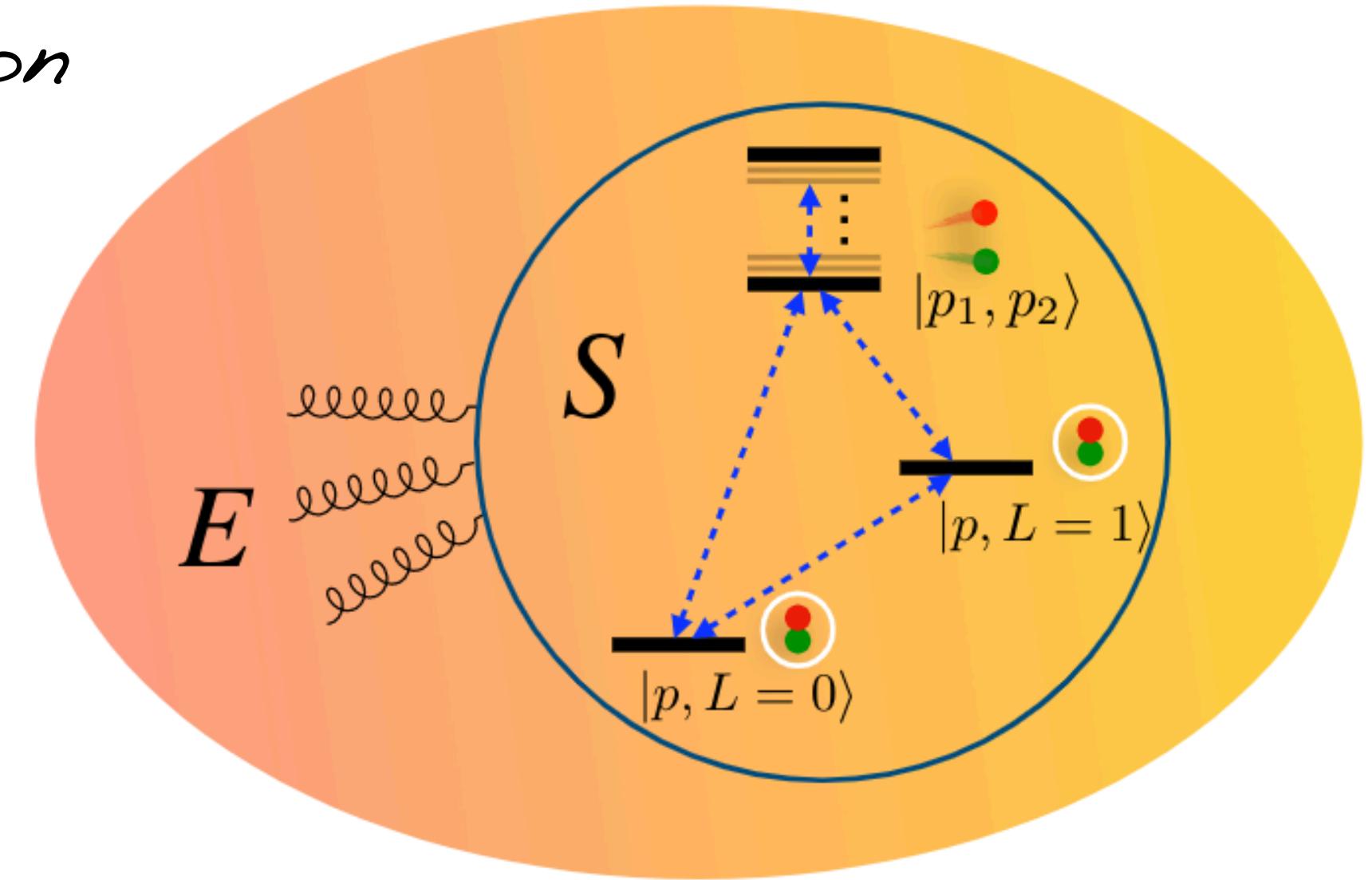
Environment (\mathcal{H}_E) = Nuclear matter

$$H(t) = H_S(t) + H_E(t) + H_I(t)$$

In the Markovian limit, the subsystem is

described by a *Lindblad equation*:

$$\frac{d}{dt}\rho_S = -i[H_S, \rho_S] + \sum_{j=1}^m \left(L_j \rho_S L_j^\dagger - \frac{1}{2} L_j^\dagger L_j \rho_S - \frac{1}{2} \rho_S L_j^\dagger L_j \right)$$
$$\rho_S = \text{tr}_E[\rho]$$



The time evolution governed by the von Neumann equation:

$$\frac{d}{dt}\rho^{(\text{int})}(t) = -i [H_I^{(\text{int})}(t), \rho^{(\text{int})}(t)]$$

Open Quantum Systems

Study the real time dynamics of the quantum evolution of probes in the nuclear medium (LHC/RHIC/EIC)

Subsystem (H_S) = Probe (jet or heavy quarks)

Environment (H_E) = Nuclear matter

Interactions

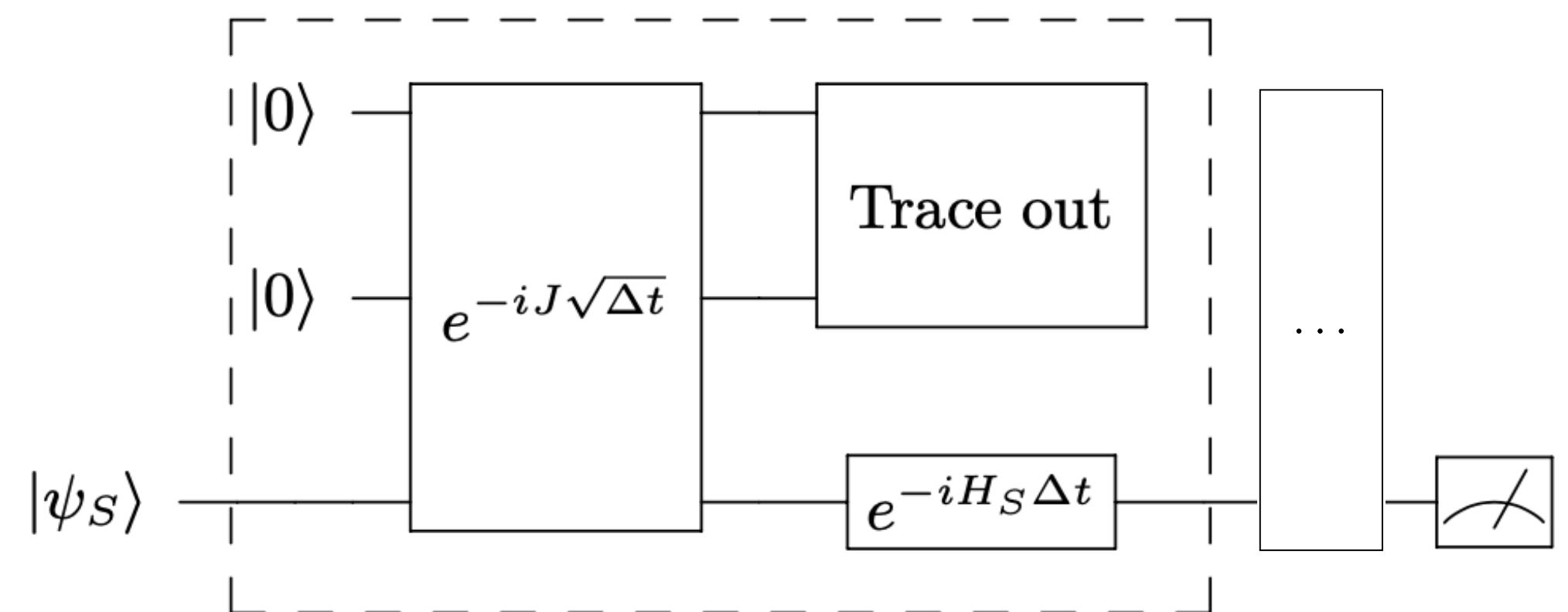
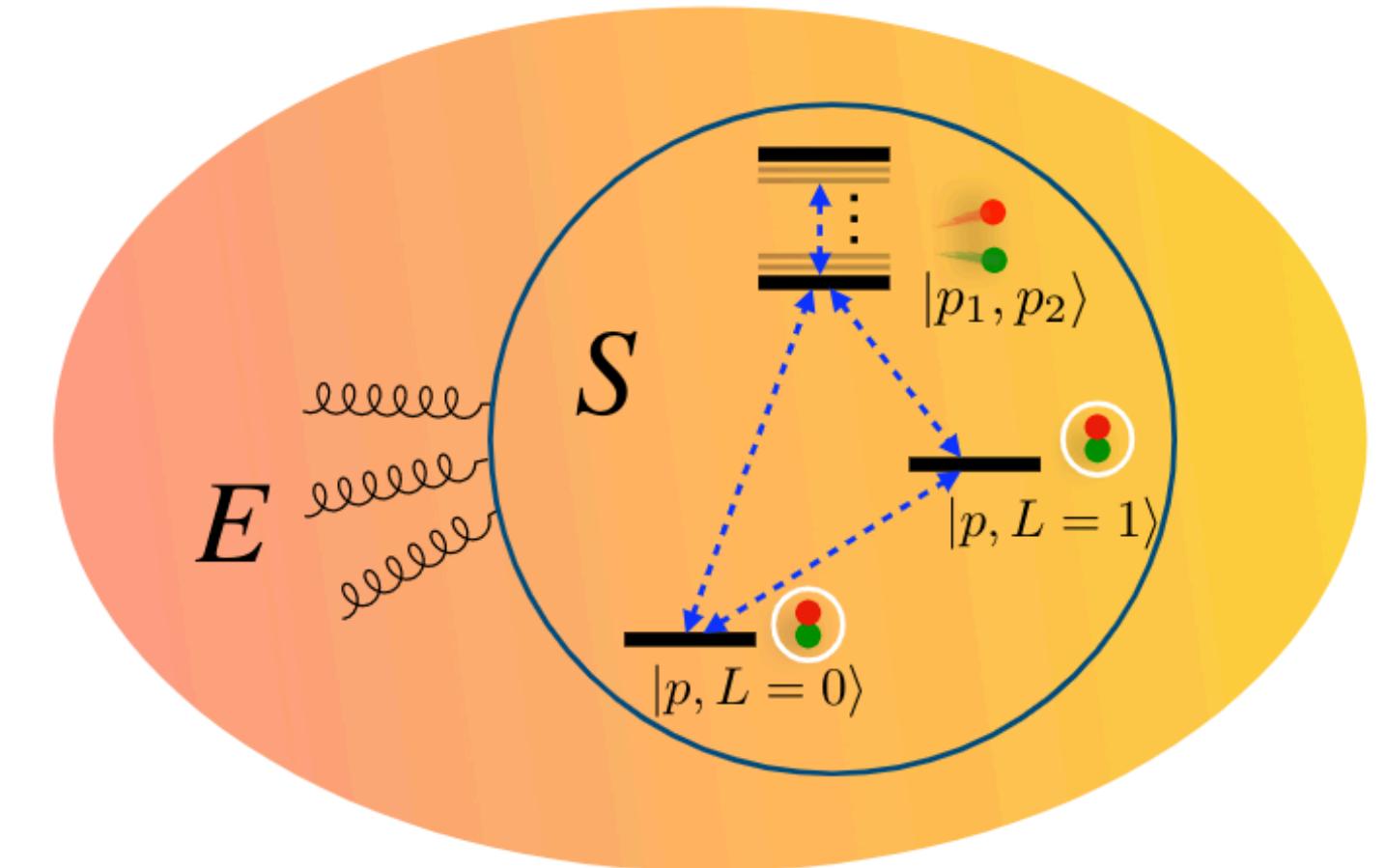
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$$\rho_S = \text{tr}_E[\rho]$$



A probe in the medium - open q. system

Full System Environment Interaction

Formulate the complete H of the system $H(t) = H_S(t) + H_E(t) + H_I(t)$

Split H s into free and interacting part

(interacting only within subsystem not environment)

H_{S1} example: collinear radiation off of a parton

$$H_S = H_{S0} + H_{S1}$$

Comment: instead of simulating the dynamics of fields, we focus on simulating the dynamics of particle states (valid for hard probes)

Use multi-particle states as the basis with momentum p and A all discrete quantum numbers then H_{S0} and H_{S1} are matrices (H_{S0} is diagonal)

$$|p_1, A_1\rangle \otimes \dots \otimes |p_n, A_n\rangle$$

Total density matrix of the subsystem and the environment evolves under the von Neumann equation in the interaction picture BUT for the environment system it is Heisenberg picture

$$\frac{d}{dt}\rho^{(\text{int})}(t) = -i[H_I^{(\text{int})}(t), \rho^{(\text{int})}(t)]$$

$$\rho^{(\text{int})}(t) \equiv e^{i(H_{S0}+H_E)t} \rho(t) e^{-i(H_{S0}+H_E)t}$$

$$H_{S1}^{(\text{int})}(t) \equiv e^{iH_{S0}t} H_{S1} e^{-iH_{S0}t}$$

$$H_I^{(\text{int})}(t) \equiv e^{i(H_{S0}+H_E)t} H_I e^{-i(H_{S0}+H_E)t}.$$

Assume that the initial density matrix factorizes and the environment density matrix is a thermal state

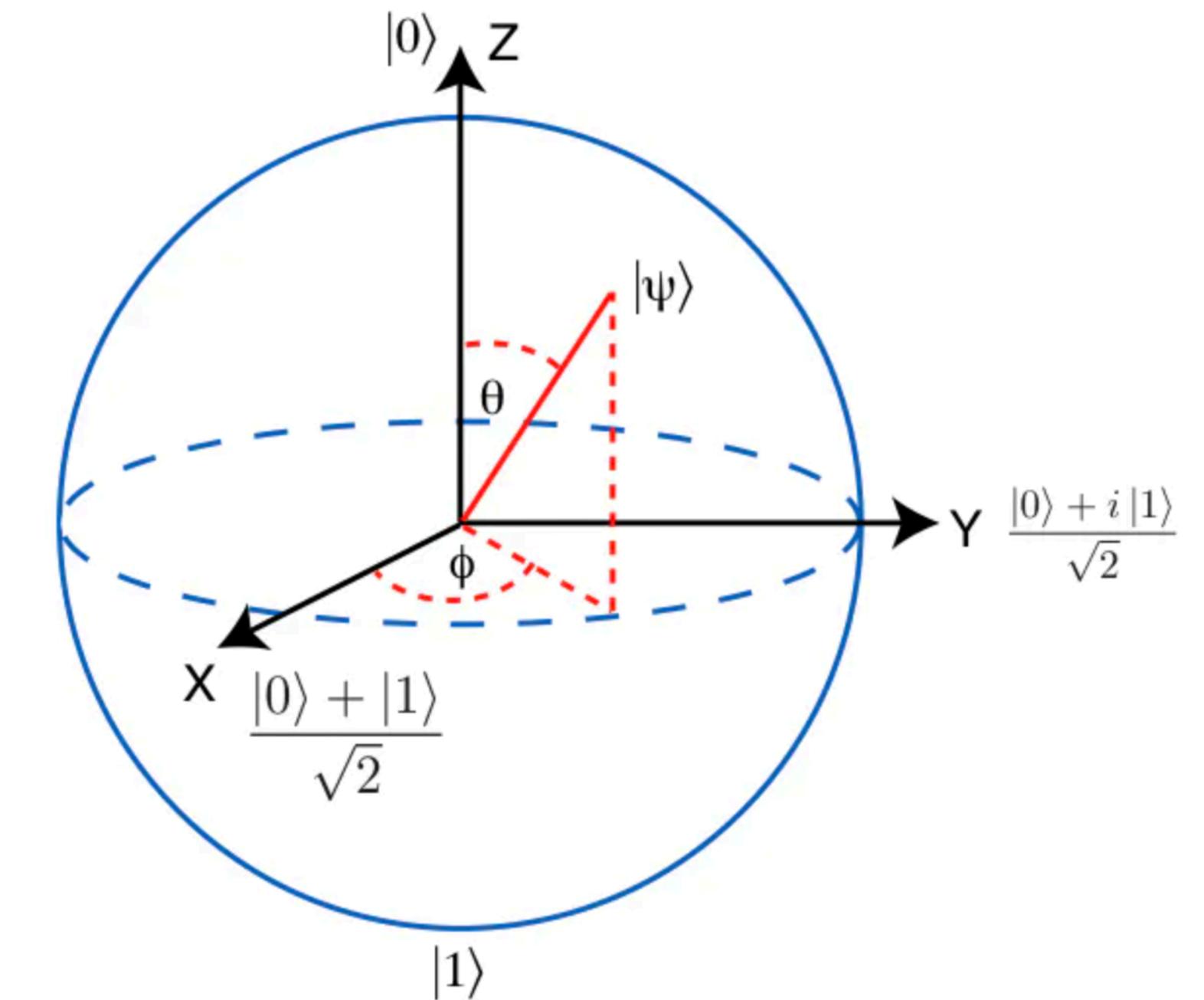
$$\rho(0) = \rho_S(0) \otimes \rho_E$$

$$\rho_E = \frac{e^{-\beta H_E}}{\text{Tr}(e^{-\beta H_E})}$$

$\beta = 1/T$ is the inverse of the QGP temperature

Quantum simulations

- Decompose a given unitary into elementary gates
- Here $|\psi(t)\rangle = U|\psi(0)\rangle = e^{-iH_S t}|\psi(0)\rangle$
- Trotter decomposition
- Optimization algorithms for a relatively small number of qubits



qsearch - optimization with a unitary loss function

Davis, Smith, Siddiqi et al. '20

Toy for quarkonia in a QGP on a QC

Aim at qualitative features; circuit depth limitation

Markovian Lindblad

$$\frac{d}{dt}\rho_S(t) = -i[H_{S1}(t) + H_L, \rho_S(t)] + \sum_{j=1}^m \left(L_j \rho_S(t) L_j^\dagger - \frac{1}{2} \{L_j^\dagger L_j, \rho_S(t)\} \right)$$

H_L denotes a thermal correction to H_S generated by loop effects of H_I , and the L_j are called Lindblad operators

Simulate the Lindblad equation \Rightarrow quantum algorithm based on the **Stinespring dilation theorem** - in terms of the evolution operators J and H_S

$$\rho(t) = |0\rangle_a\langle 0|_a \otimes \rho_S(t) = \begin{pmatrix} \rho_S(t) & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} \quad J = \begin{pmatrix} 0 & L_1^\dagger & \dots & L_m^\dagger \\ L_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_m & 0 & \dots & 0 \end{pmatrix}$$

$$H_S = H_{S0} = -\frac{\Delta E}{2} Z$$

$$H_E = \int d^3x \left[\frac{1}{2} \Pi^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} \lambda \phi^4 \right]$$

$$H_I = g X \otimes \phi(x=0),$$

X, Z to denote the single qubit Pauli gates (Pauli matrices)

H_S is a two level system with energy difference ΔE

environment H_E is a 3 + 1D scalar field theory

Π is the canonical momentum conjugate to ϕ

for simplicity, we set $m=\lambda=0$

interaction strength g between the subsystem and the environment is unitless

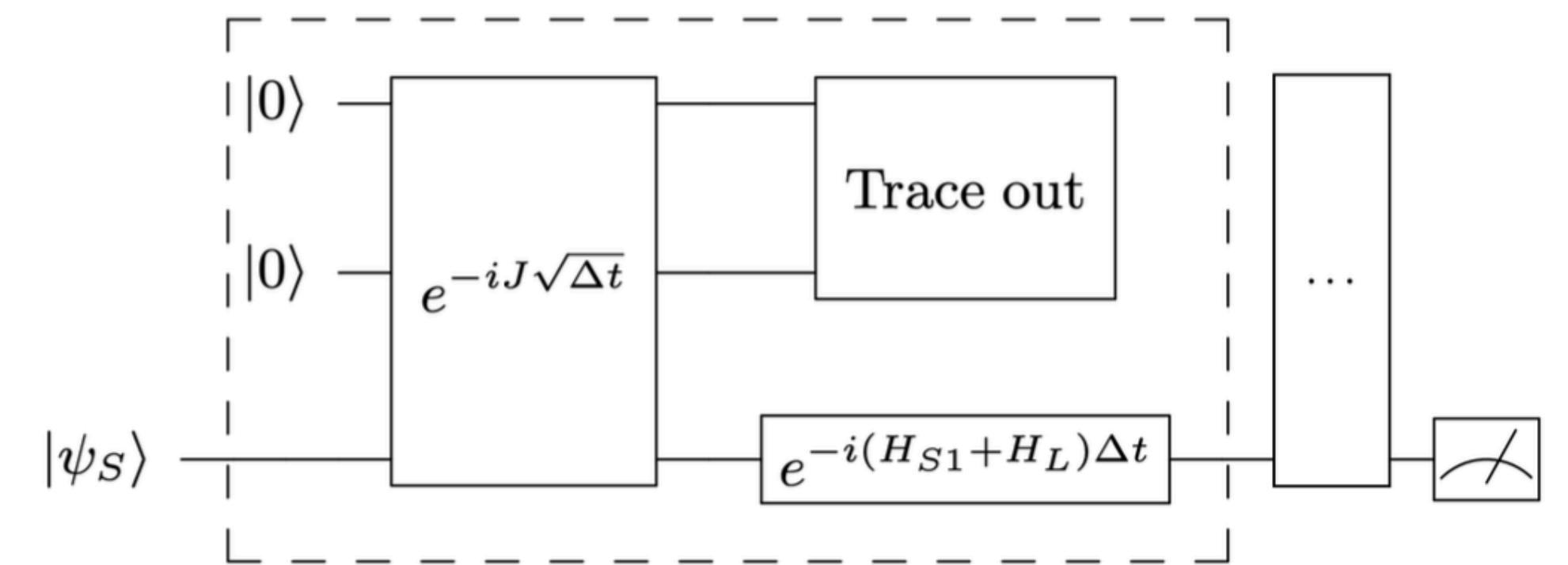


FIG. 2. Quantum algorithm to simulate Lindblad evolution based on the Stinespring dilation theorem. The portion enclosed by the dashed line corresponds to one “cycle” of time Δt . Each cycle is repeated multiple times as indicated by the ellipsis in the box on the right. The measurement of the subsystem is performed at the end.

OQS - non-unitarity

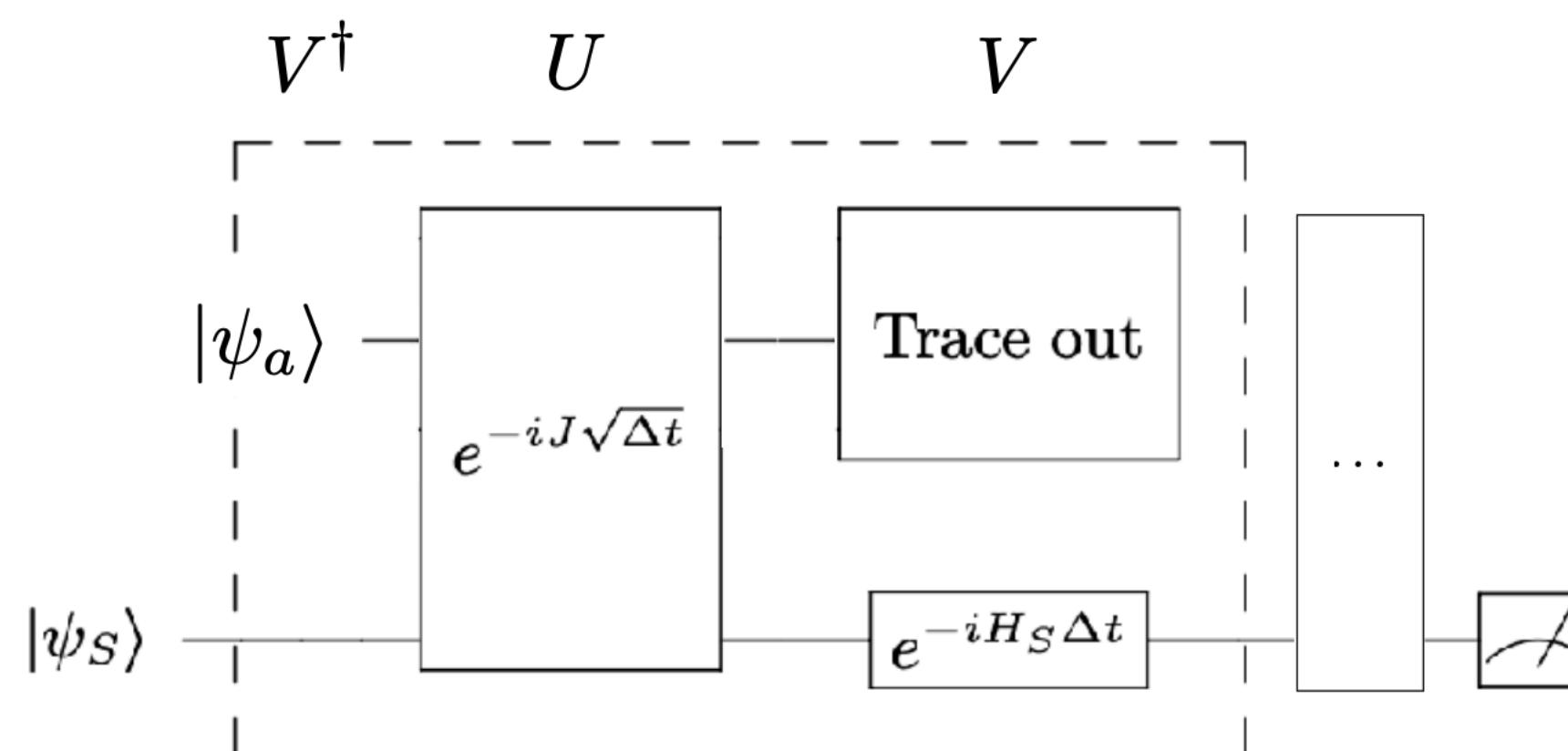
Intermezzo...

Issue: in OQS evolution of the subsystem is NOT unitary...

$$\frac{d}{dt}\rho_S = -i[H_S, \rho_S] + \sum_{j=1}^m \left(L_j \rho_S L_j^\dagger - \frac{1}{2} L_j^\dagger L_j \rho_S - \frac{1}{2} \rho_S L_j^\dagger L_j \right)$$

Stinespring dilation theorem

Any allowed quantum operation can be written as a unitary evolution acting on a larger space (after coupling to appropriate ancilla), and reducing back to the subsystem



$$V^\dagger V = 1 \quad VV^\dagger \neq 1$$

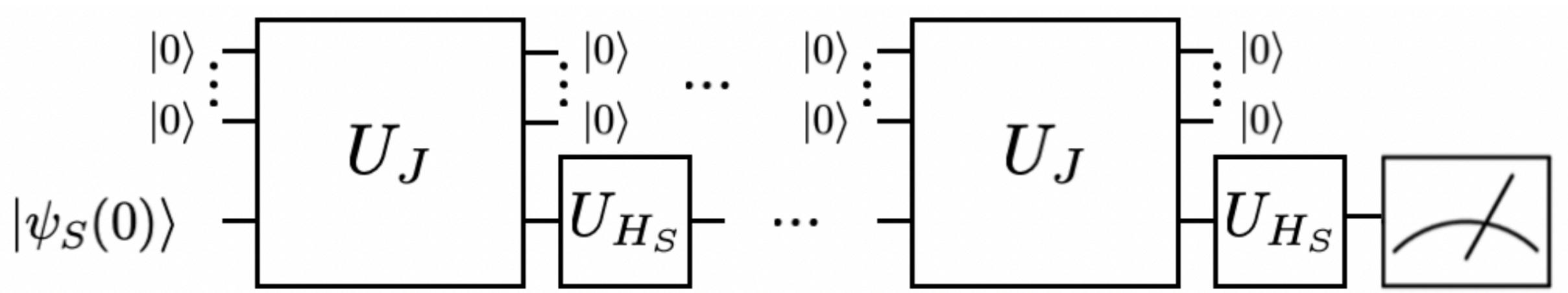
$$J = \begin{pmatrix} 0 & L_1^\dagger & \dots & L_m^\dagger \\ L_1 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ L_m & 0 & \dots & 0 \end{pmatrix}$$

Quantum algorithm for non-unitary evolution

- Time evolve the density matrix instead of pure states

$$\frac{d}{dt} \rho_S = -i [H_S, \rho_S] + \sum_{j=1}^m \left(L_j \rho_S L_j^\dagger - \frac{1}{2} \{ L_j^\dagger L_j, \rho_S \} \right)$$

- **Stinespring dilation theorem**



where

$$U_{H_S} = e^{-i H_S \sqrt{\Delta t}}$$

$$U_J = e^{-i J \sqrt{\Delta t}}$$

$$J = \begin{pmatrix} 0 & L_1^\dagger & \cdots & L_m^\dagger \\ L_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_m & 0 & \cdots & 0 \end{pmatrix}$$

- Time-irreversible

see also e.g. Cleve, Wang '16

- Evolve for N_{cycle} in small time steps Δt

Hu, Xia, Kais '20

Jong, Metcalf, Mulligan, Ploskon, FR, Yao '20

Metcalf, Kemper, Jong et al. '21

Trotter decomposition

- Need to map both U_{H_S} , U_J to quantum gates

$$H = \sum_j H_j = a_j P_j \quad \text{with Pauli strings} \quad P_j : \{\mathbb{1}, X, Y, Z\}^{\otimes n}$$

and coefficients $a_j = \frac{1}{2^n} \text{tr}[P_j H]$

- Trotter decomposition

$$U_1(t) = \prod_j e^{-iH_j t} \quad \text{each factor can be mapped to quantum gates}$$

→ evolve to t in r time steps with error $\|e^{-iHt} - U_1^r(t/r)\| \leq \frac{1}{2} \sum_{j>k} \| [H_j, H_k] \| \frac{t^2}{r}$

Toy for quarkonia in a QGP on a QC

Aim at qualitative features; circuit depth limitation

Markovian Lindblad

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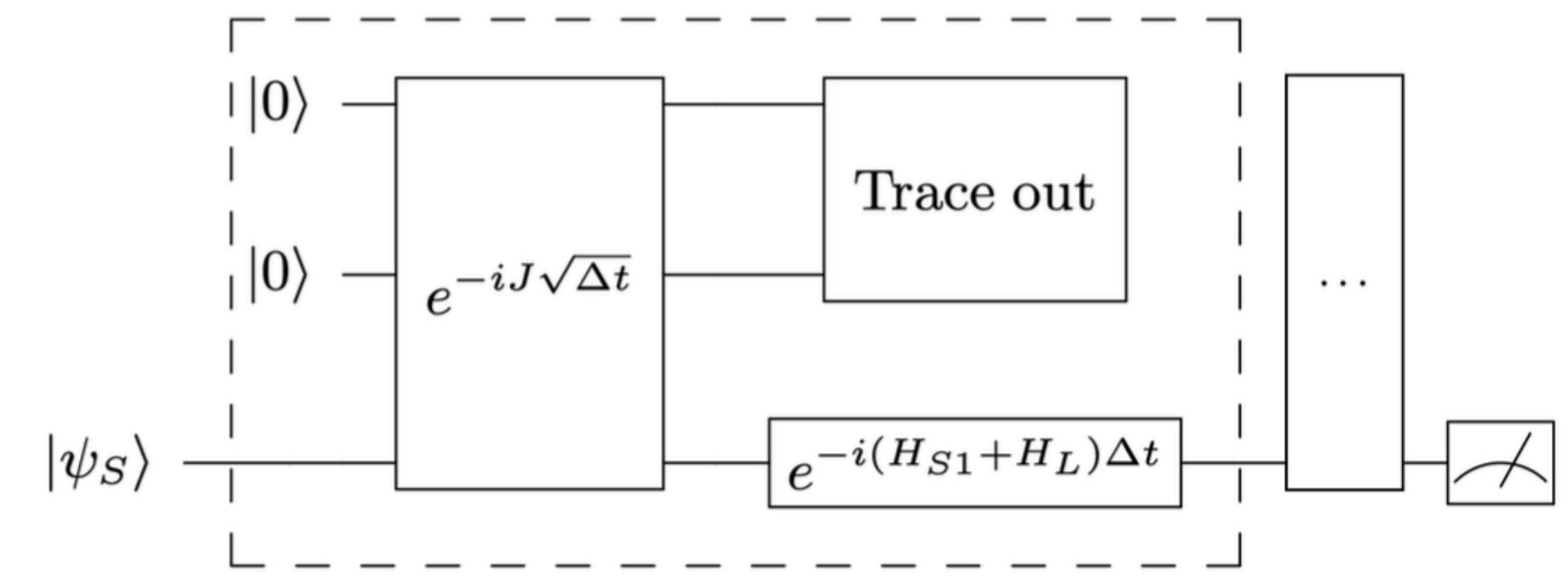


FIG. 2. Quantum algorithm to simulate Lindblad evolution based on the Stinespring dilation theorem. The portion enclosed by the dashed line corresponds to one “cycle” of time Δt . Each cycle is repeated multiple times as indicated by the ellipsis in the box on the right. The measurement of the subsystem is performed at the end.

In the Markovian limit, two Lindblad operators $j = 0, 1$ are relevant:

$$L_j = \frac{\sqrt{\Gamma_j}}{2} (X - (-1)^j iY), \quad (15)$$

where $\Gamma_0 = g^2 \Delta E n_B(\Delta E)/(2\pi)$, $\Gamma_1 = g^2 \Delta E/(2\pi) + \Gamma_0$ and $n_B(\Delta E) = 1/(\exp(\beta\Delta E) - 1)$ is the Bose-Einstein distribution. We will neglect H_L in this letter. For our numerical studies, we use a unit system where all quantities are counted in units of T , the temperature of the

Toy for quarkonia in a QGP on a QC

Aim at qualitative features; circuit depth limitation

$$H(t) = H_S(t) + H_E(t) + H_I(t)$$

$$H_S = -\frac{\Delta E}{2} Z$$

$$H_E = \int d^3x \left[\frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4 \right]$$

$$H_I = gX \otimes \phi(x=0)$$

$$L_j \sim g(X \mp iY) \quad j = 0, 1$$

Lindblad operators

$$J = \begin{pmatrix} 0 & L_0^\dagger & L_1^\dagger & 0 \\ L_0 & 0 & 0 & 0 \\ L_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Pauli matrices X, Y, Z

Interaction strength g

Two level system in a thermal environment
(Proxy for $c\bar{c}$ pair)

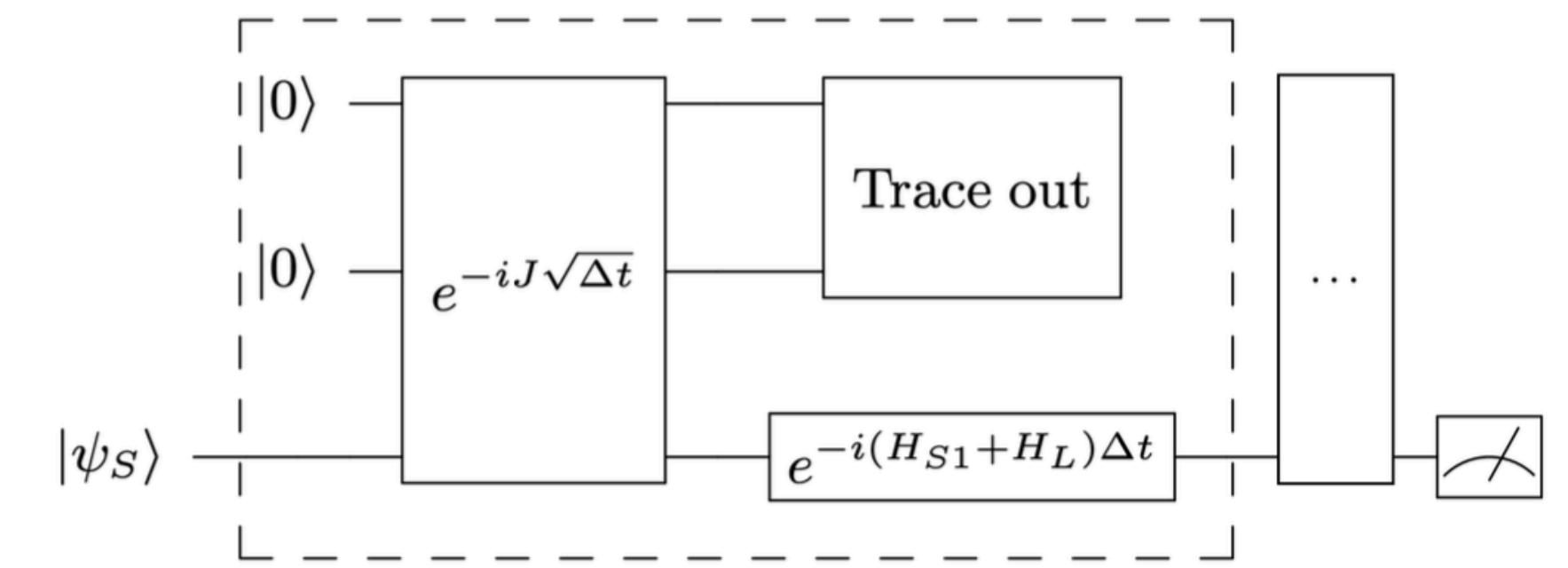
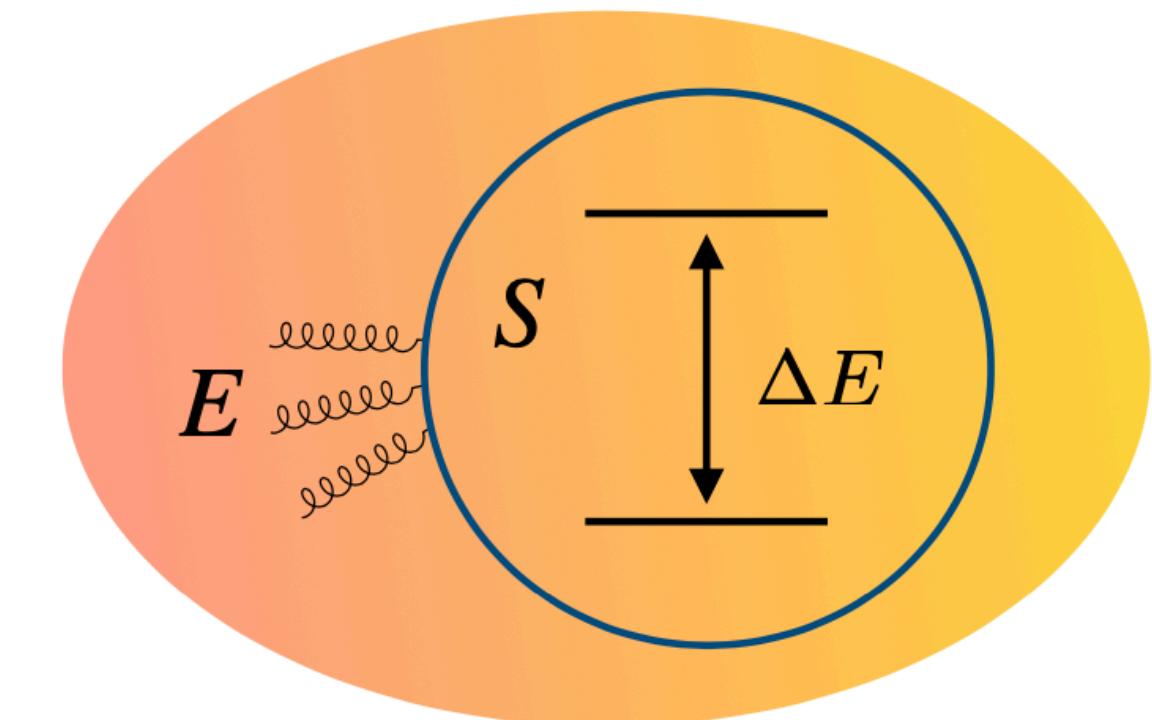


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Quantum Circuit Synthesis

Approximate unitary operations with a compiled circuit of one- and two-qubit gates

Optimization problem w/unitary loss function - *gsearch* Siddigi et al 2020 (LBNL)

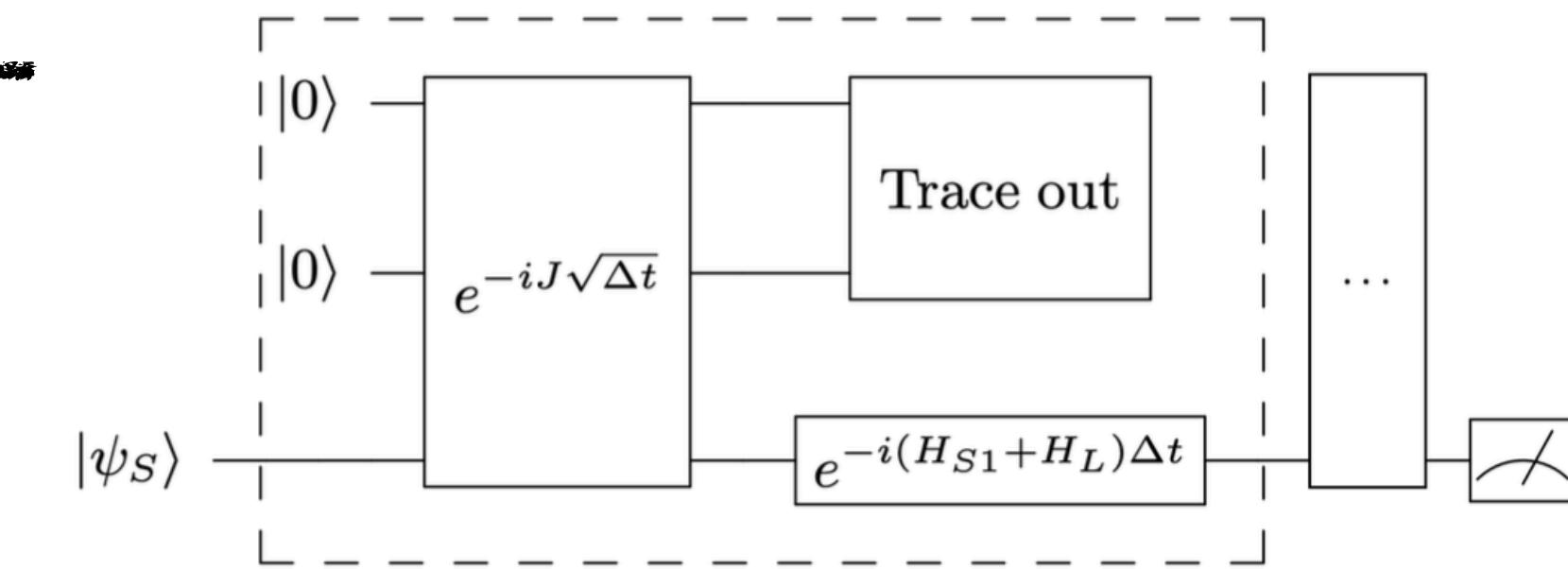
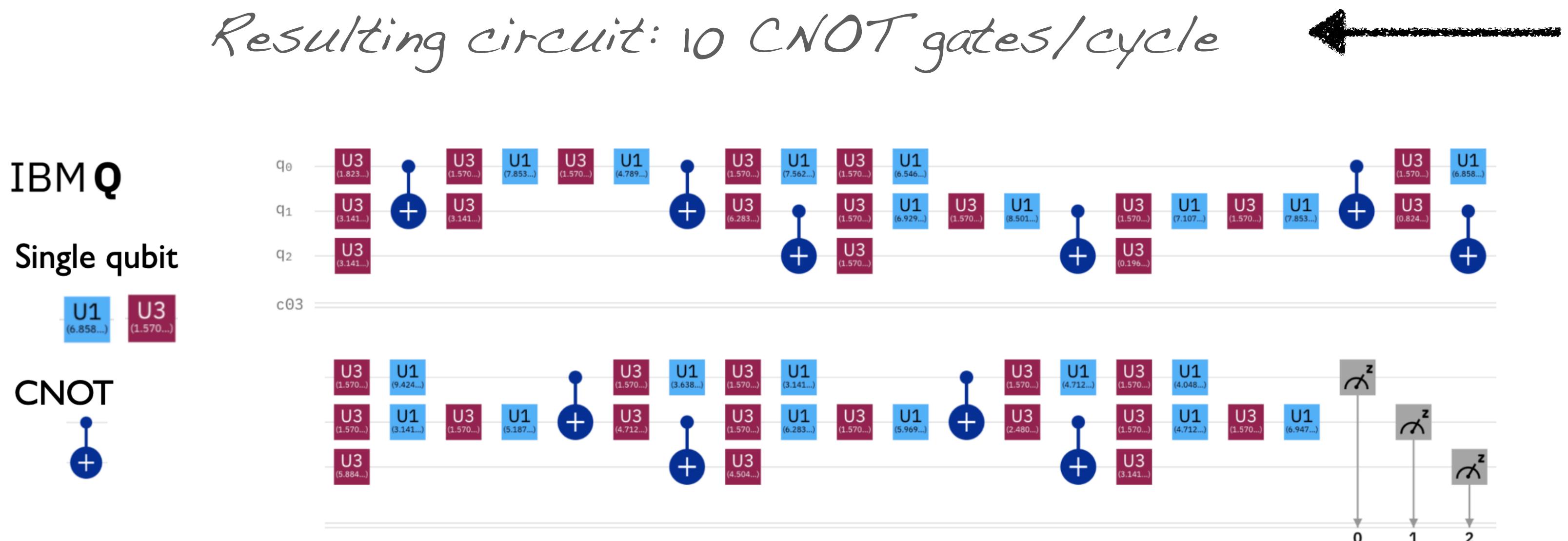


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Error mitigation

Readout error

Constrained matrix inversion

IBM Q qiskit-ignis

Gate error

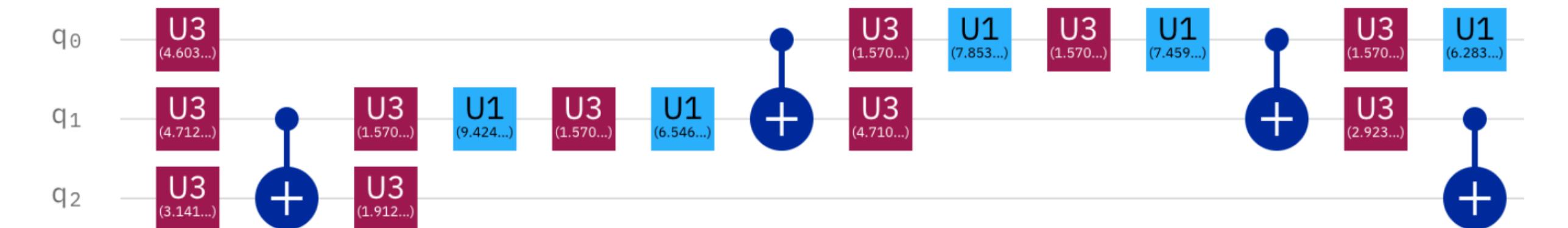
Zero-noise extrapolation of CNOT noise using Random Identity Insertions

Unfolding

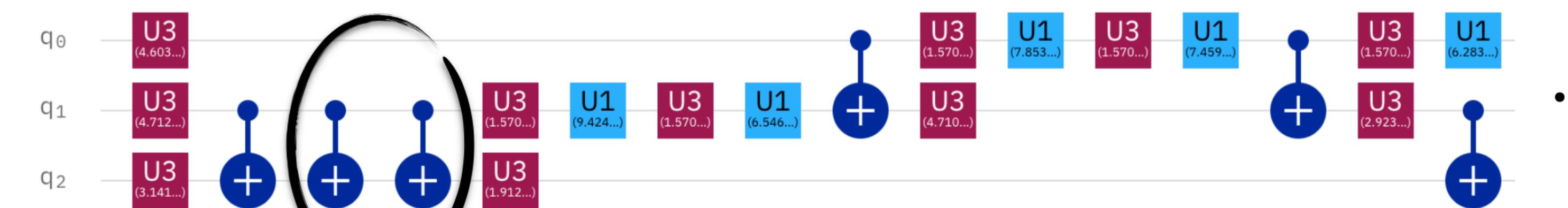
Nachman, Urbanek, de Jong, Bauer '19
@LBNL

He, Nachman, de Jong, Bauer '20
@LBNL

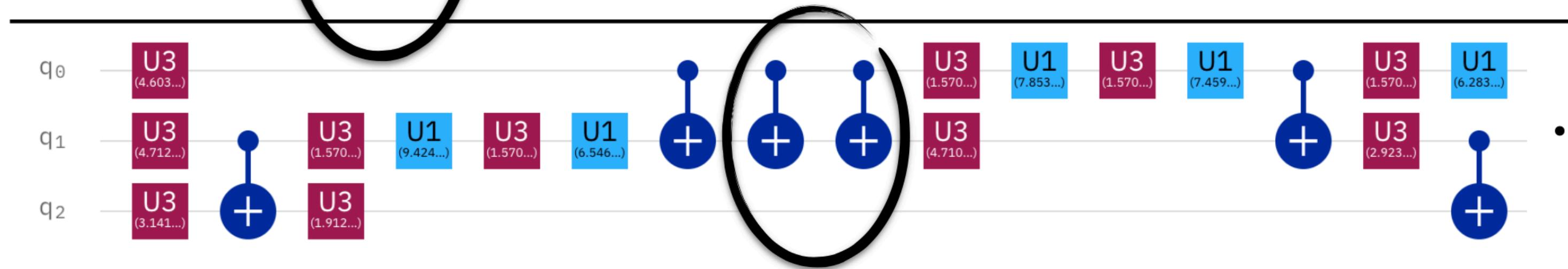
Circuit 1



Circuit 2

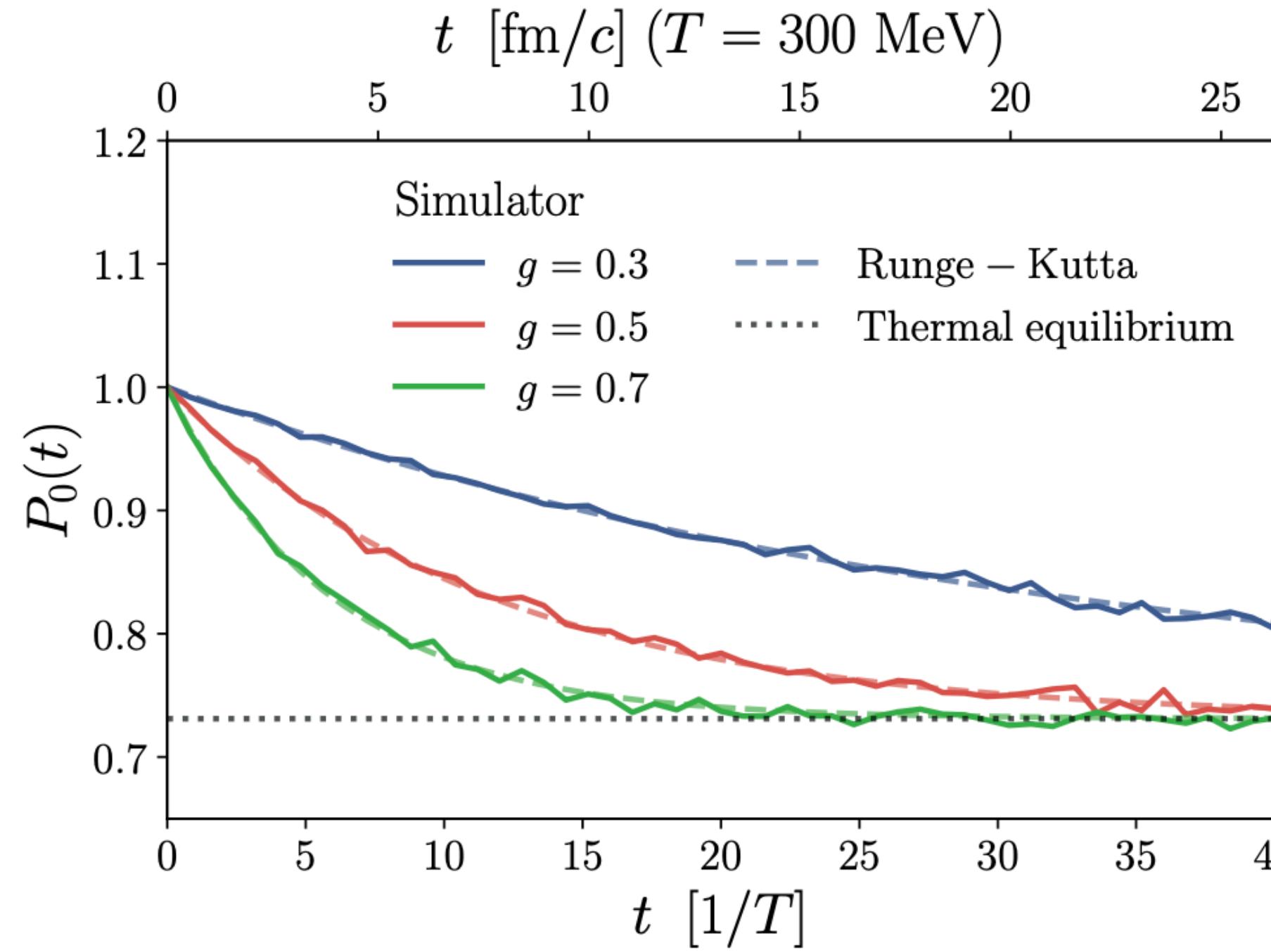


Circuit 3



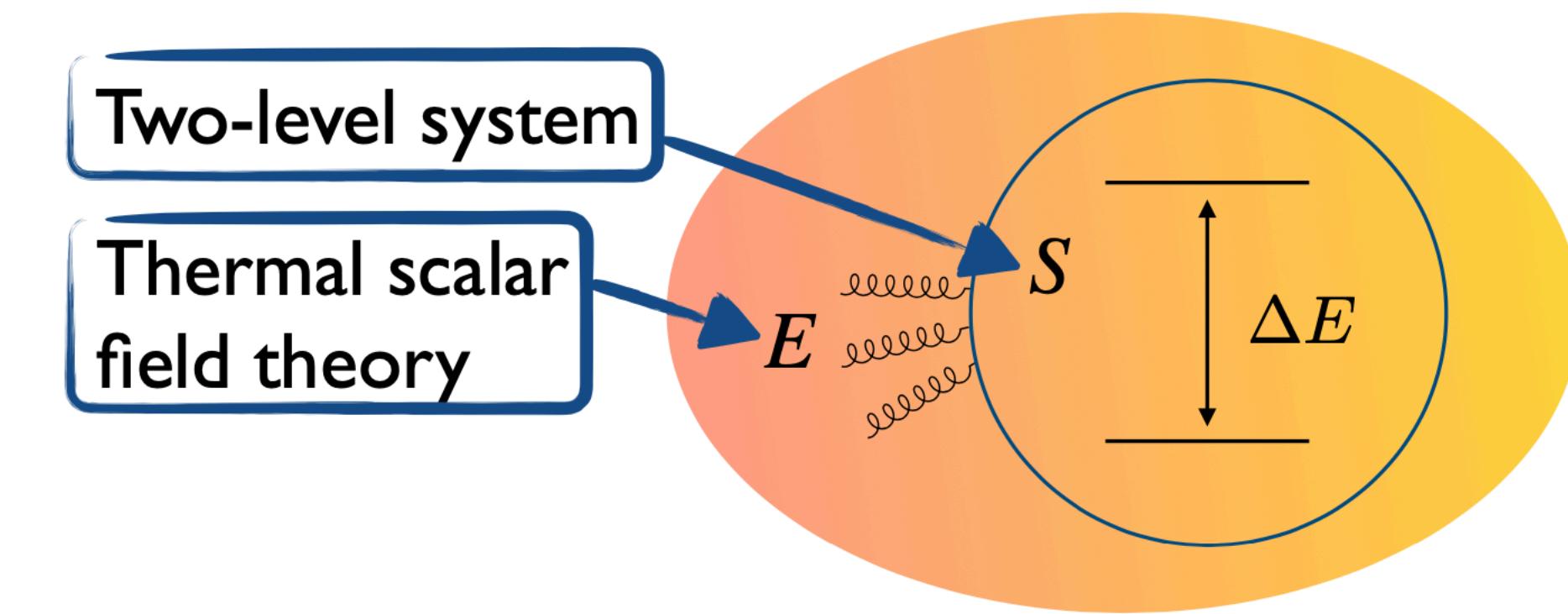
Toy for quarkonia in a QGP on a QC

Aim at qualitative features; circuit depth limitation



Note: stronger coupling - faster thermalization

FIG. 3. Simulation of the quantum circuit with $N_{\text{cycle}} = 100$ for various system-environment couplings, along with numerical solution using a 4th order Runge-Kutta method. The upper time axis corresponds to a medium with a temperature of $T = 300$ MeV. Each time point in the simulator result consists of 80192 shots (runs).



By exploring different environment density matrix, can vary medium properties

- Probe-medium coupling
- Initial temperature
- Microscopic structure

Toy for quarkonia in a QGP on a QC

Aim at qualitative features; circuit depth limitation

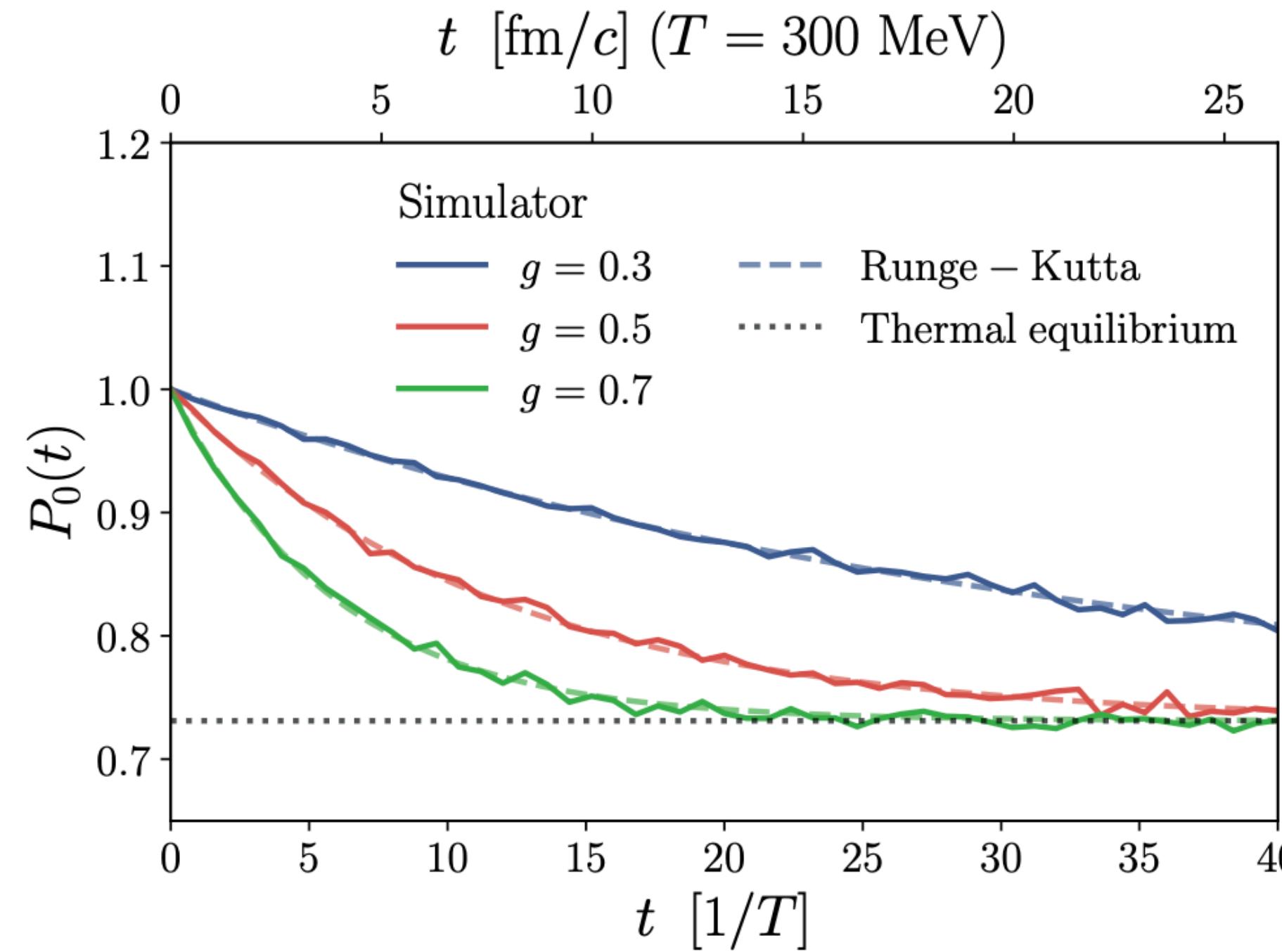


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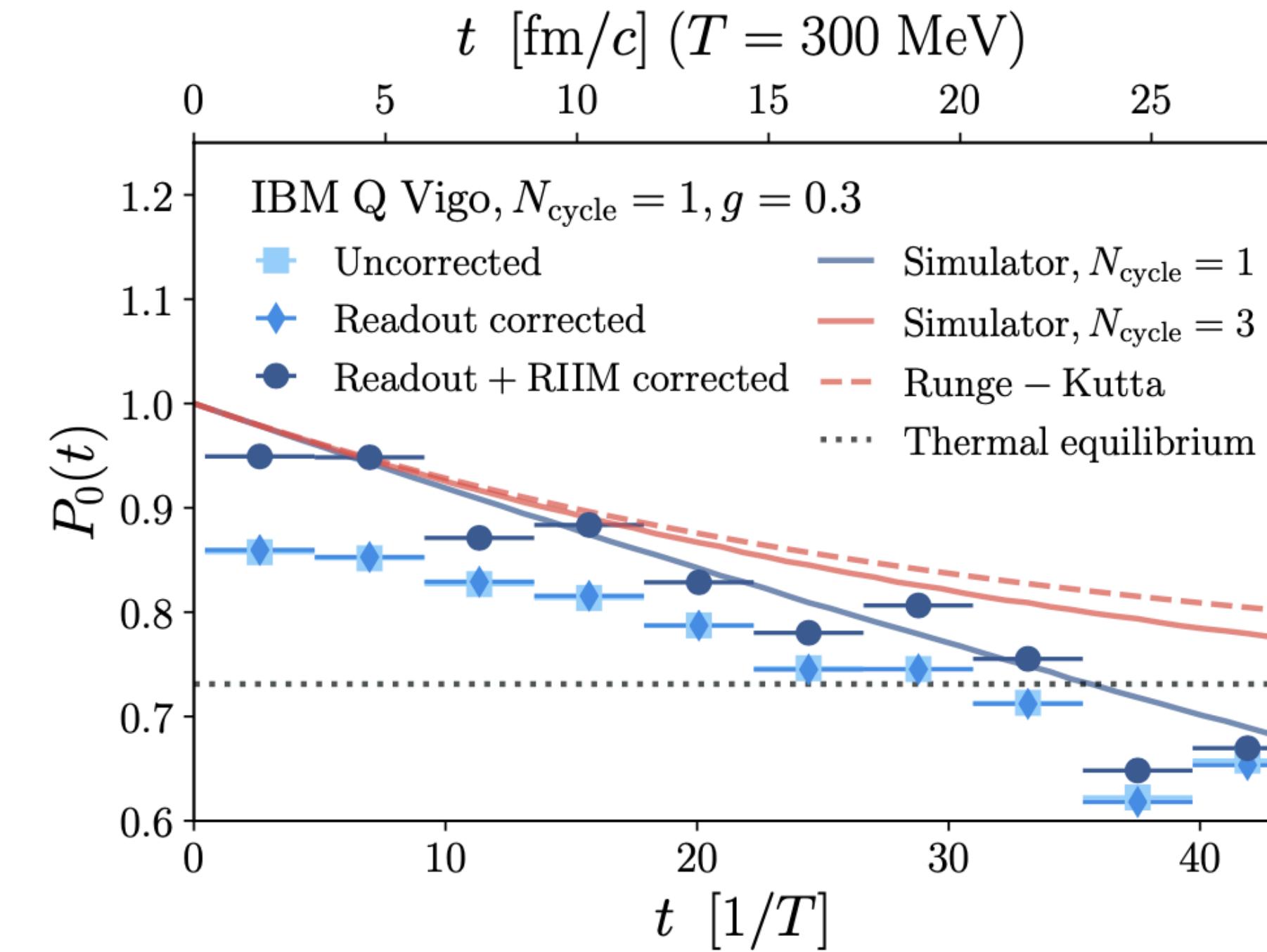


FIG. 4. Results from the IBM Q Vigo device including different error mitigations compared to results from the `qiskit` simulator for $N_{\text{cycle}} = 1$ and $N_{\text{cycle}} = 3$ and the Runge-Kutta method. Higher values of N_{cycle} quickly converge to the result using the Runge-Kutta method. Each time point in the simulator result consists of 800192 shots (runs).

Toy for quarkonia in a QGP on a QC

Aim at qualitative features; circuit depth limitation

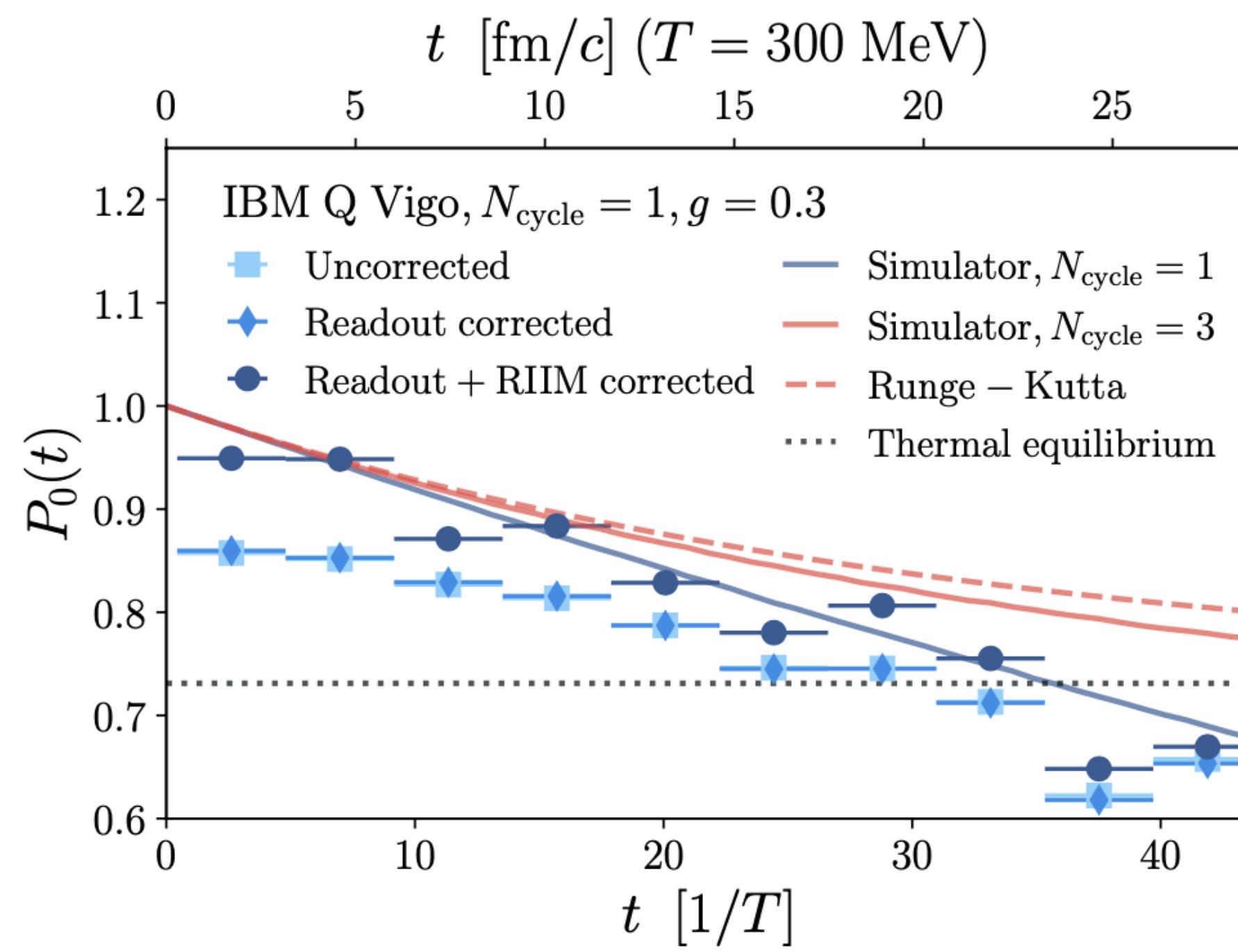
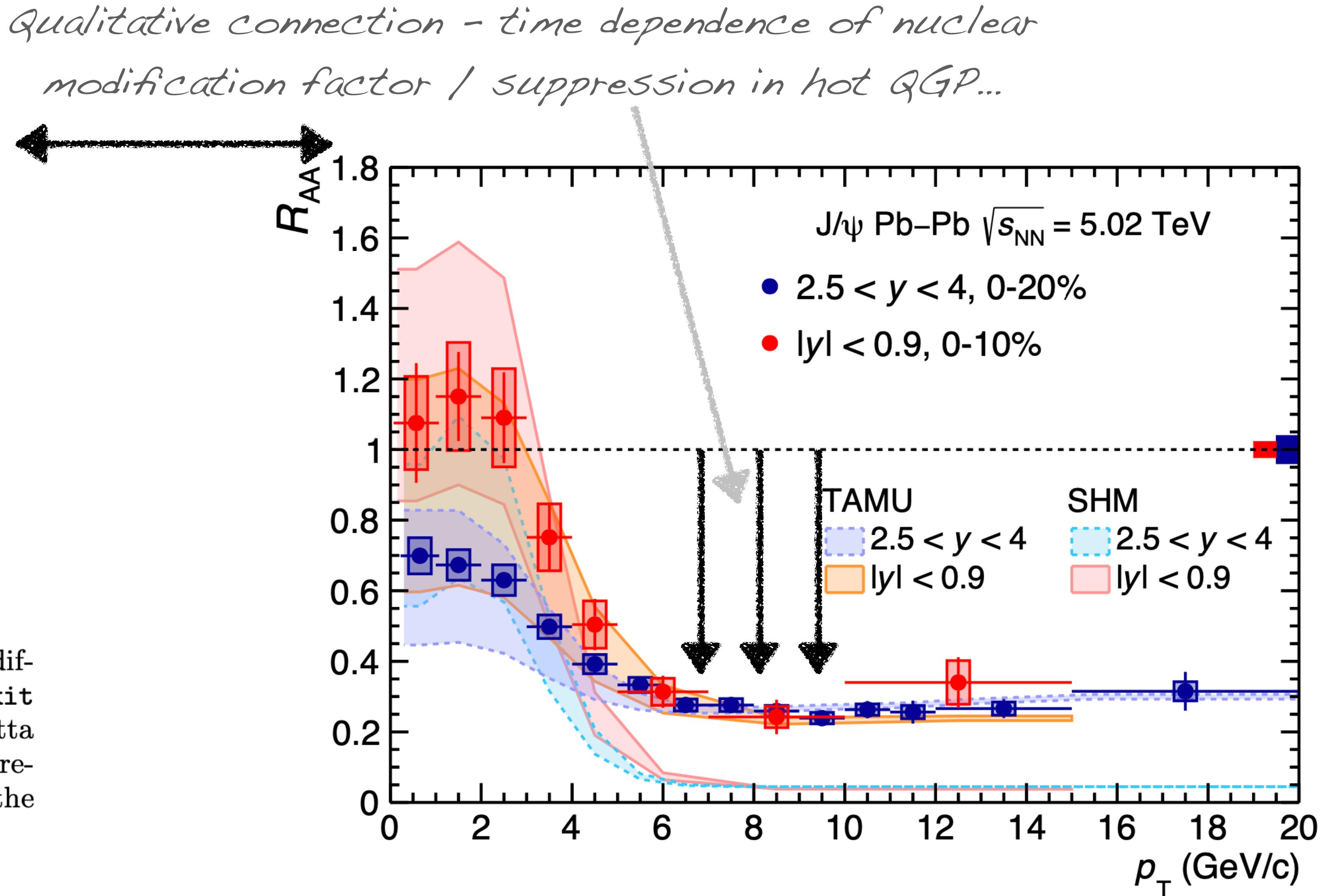


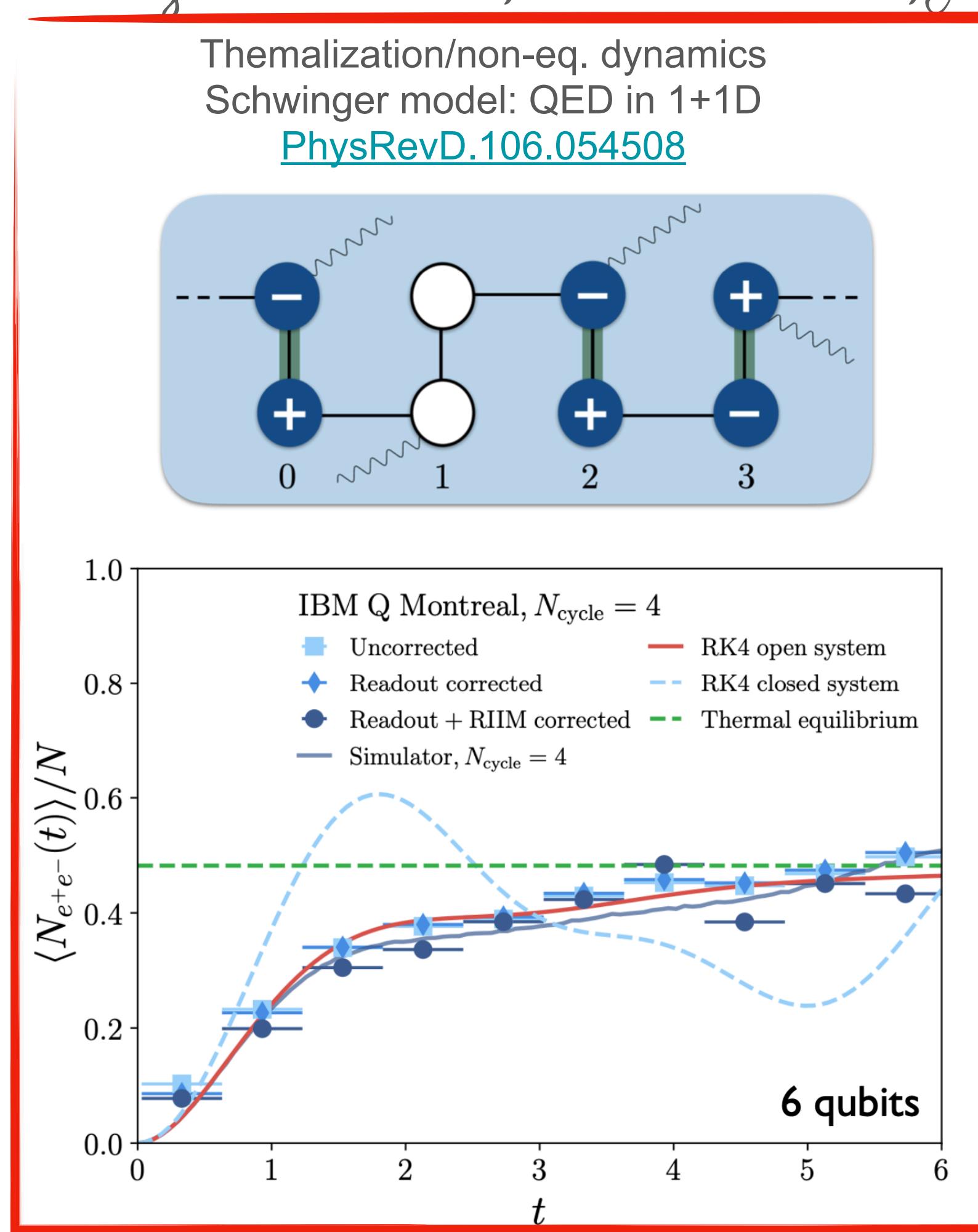
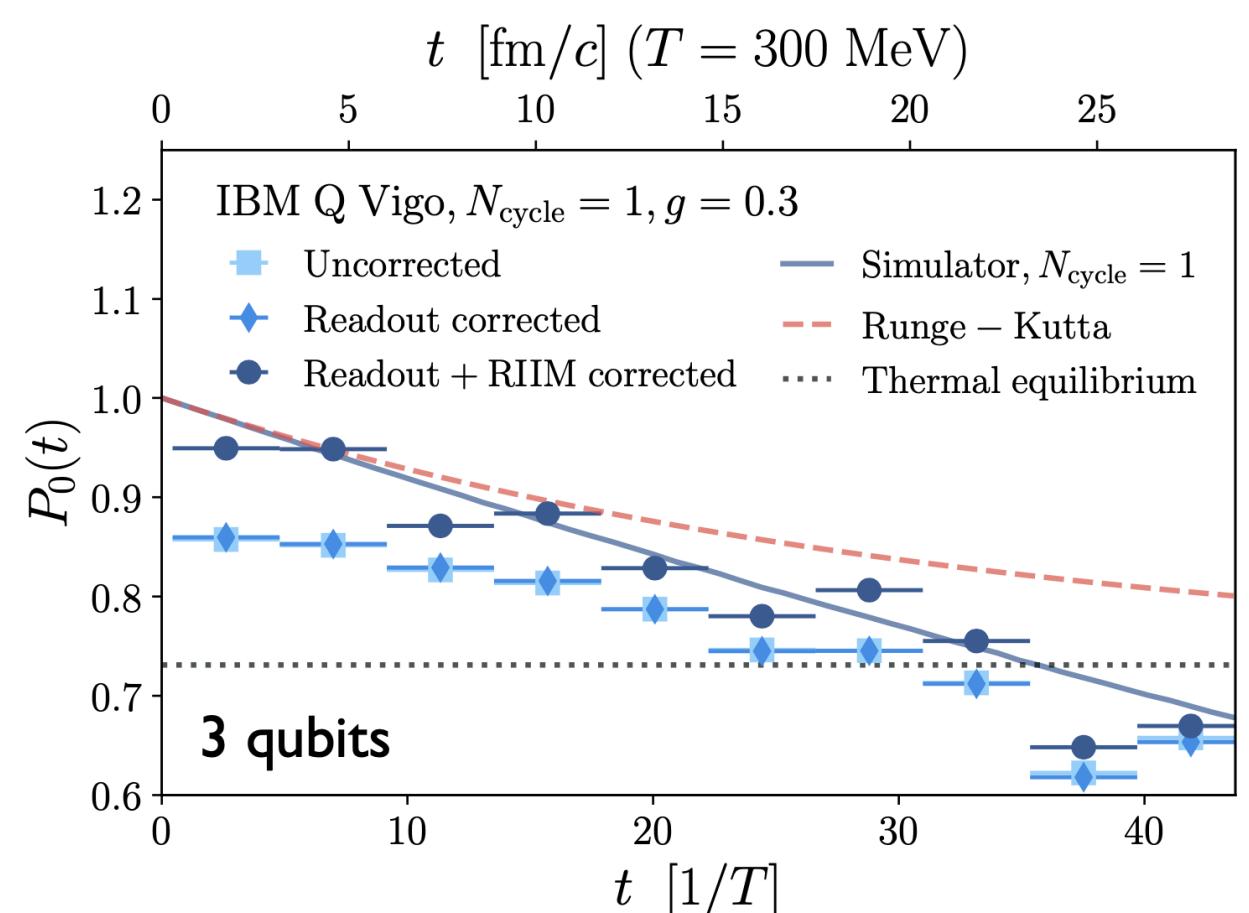
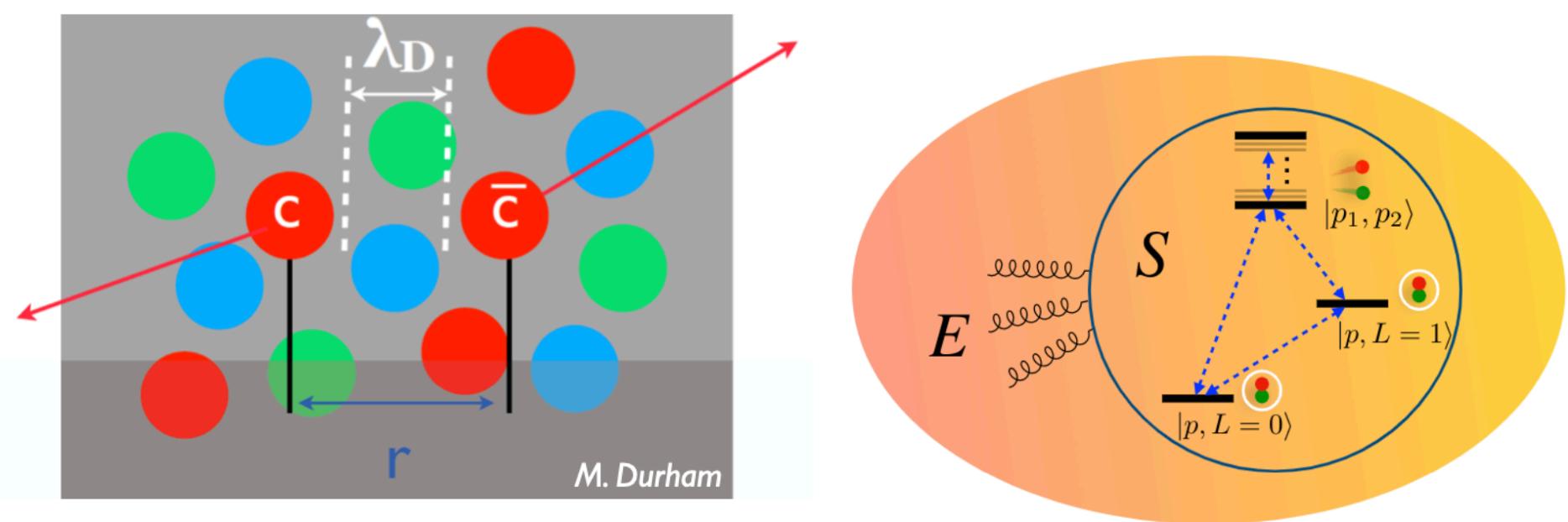
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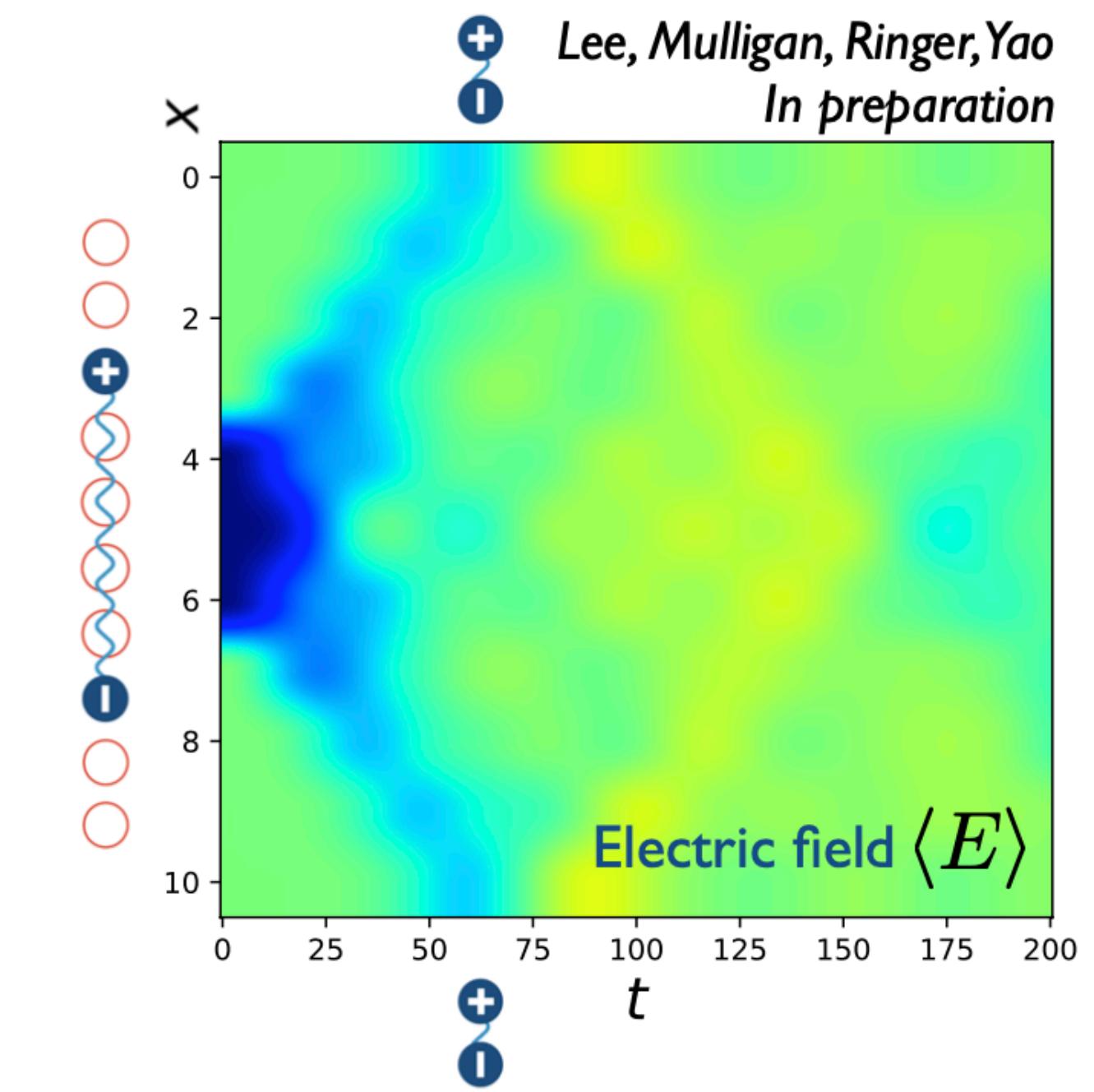
Outline

Main target: simulation of some relevant processes but mitigate classical computations shortcomings
 Non-relativistic QCD - many-body problems \Rightarrow quarkonia dissociation in hot colored medium
 Quantum Field Theory - thermalization (equilibration), hadronization, jet quenching, ...

Towards quarkonia in hot QGP: open quantum system - proof of principle calc. -
[PhysRevD.104.L051501](#)



Towards hadronization:
 String breaking mechanism in Schwinger model



Jet quenching in light front QCD
<https://arxiv.org/abs/2205.07902>

Towards QCD on quantum computers?

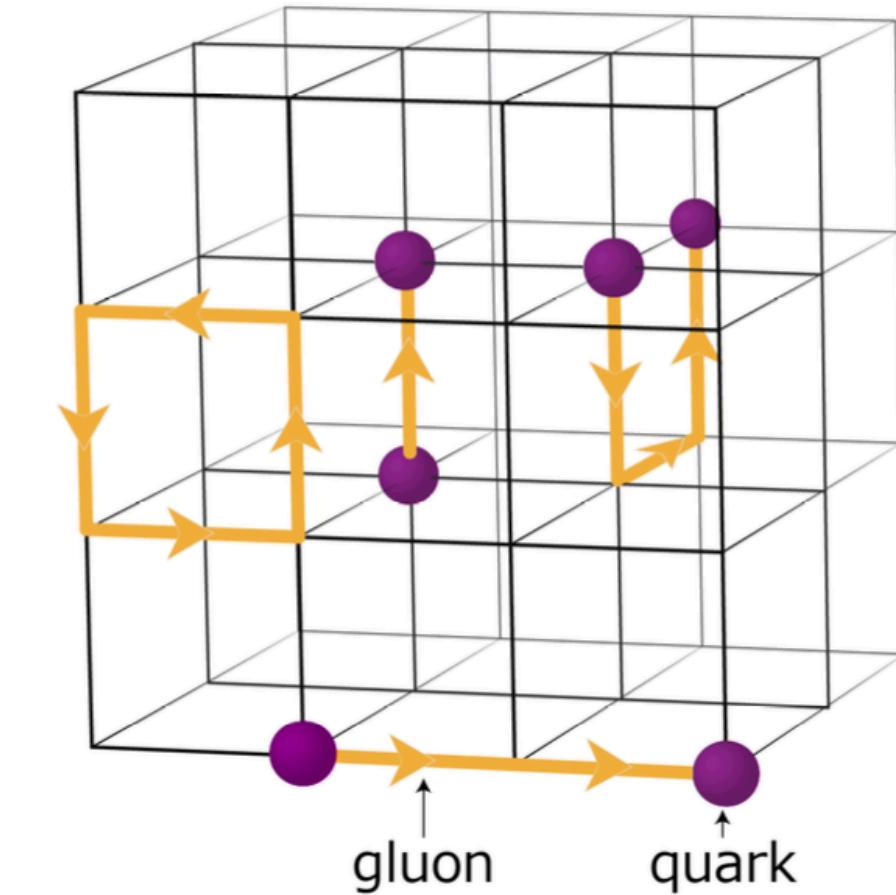
Discretize space on a lattice, and digitize fields

- No sign problem: real-time evolution and high density

Long-term goal: simulate limited energy range of QCD

- Good candidate: strongly-coupled regime
- Computing S -matrix at LHC not feasible: $\sim \mathcal{O}(10^{18})$ qubits

Bauer, Nachman, Freytsis (2021)



Near-term goals:

- Formulate how to efficiently digitize QCD
- Simulate simpler QFTs in order to gain insights about QCD

Klco et al. (2021)

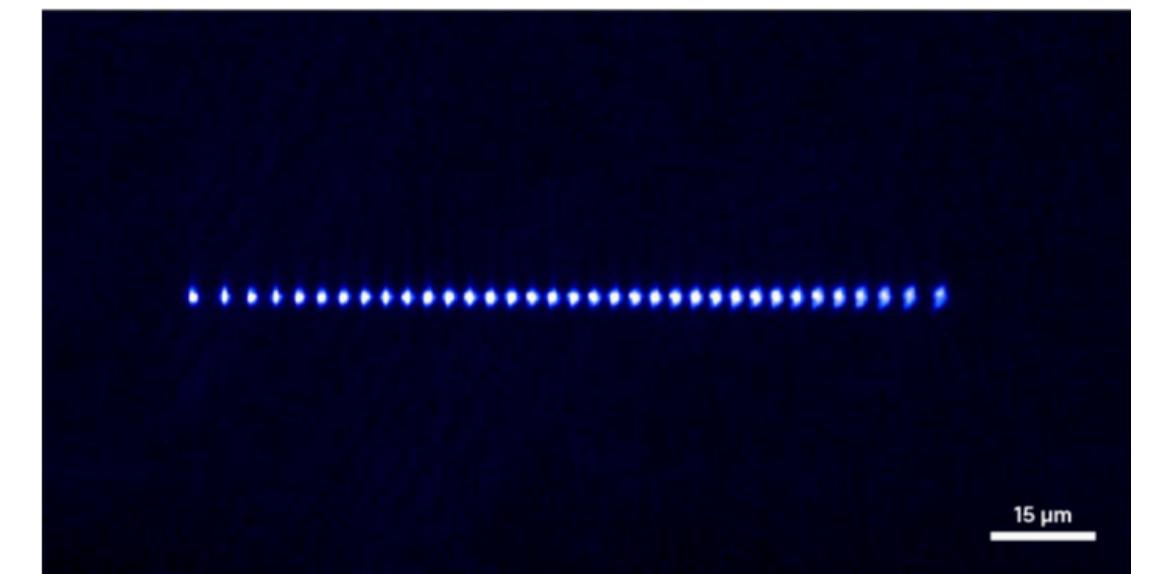
Raychowdhury, Stryker (2020)

Alexandru et al. (2019)

Davoudi et al. (2019)

Muschik et al. (2016)

...



Towards QCD... Schwinger model

$U(1)$ gauge theory in 1+1D

$$\mathcal{L} = \bar{\psi}(i\cancel{D} - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

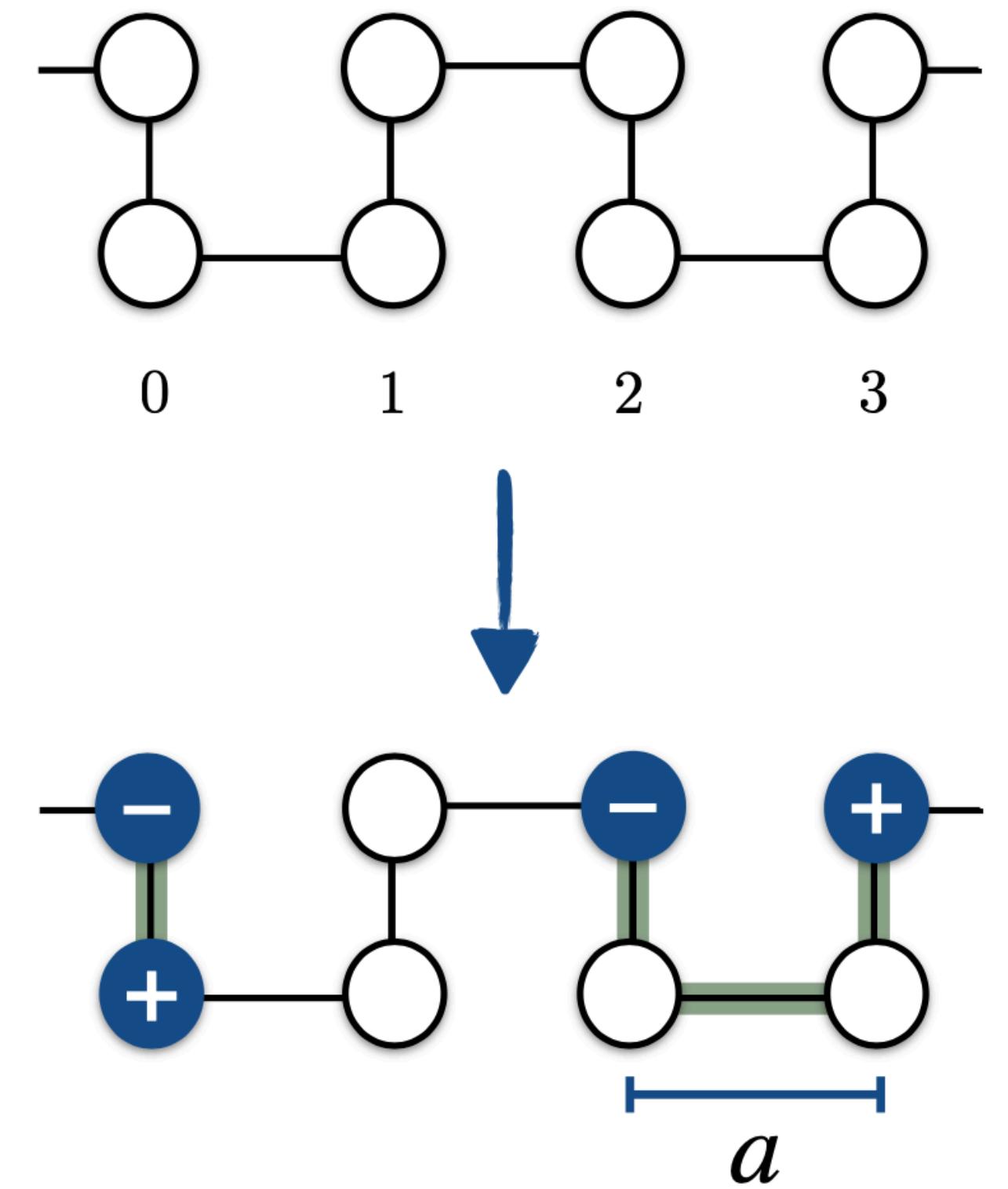
- Confinement
- Chiral symmetry breaking

Discretized Hamiltonian — staggered fermions

$$H_S = \frac{1}{2a} \sum_{n=0}^{N_f-1} (\sigma^+(n)L_n^-\sigma^-(n+1) + \sigma^+(n+1)L_n^+\sigma^-(n)) \\ + \sum_{n=0}^{N_f-1} \left(\frac{ae^2}{2} \ell_n^2 + m(-1)^n \frac{\sigma_z(n)+1}{2} \right)$$

- Time continuous, 1-dimensional spatial lattice $x=na$
- Continuum limit for $a \rightarrow 0$

Schwinger (1962)
Kogut, Susskind (1973-1977)



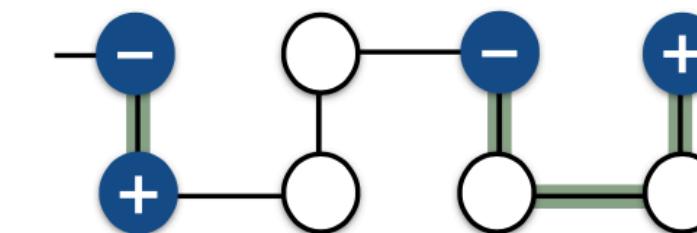
$N_f = 2N$ lattice sites
 σ^\pm create/destroy fermion
 L_n^\pm create/destroy gauge link

Towards QCD... Schwinger model

Klco, Savage (2018)

Jong, Lee, Mulligan, Ploskon, Ringer, Yao 2106.08394

Number of physical states



$$\sum_{M=1}^N \frac{2N}{M} \sum_{K=0}^{N-M} \binom{M-1+K}{M-1} \binom{2N-2K-M-1}{M-1} + 3$$

<u>N</u>	<u># states</u>
2	13
4	117
6	1186
100	$\approx 1.4 \cdot 10^{51}$

Truncate the Hilbert space: subgroups of definite momentum and parity

- $k = 0$, positive parity

$$N=2$$

$$H_S^{\mathbf{k}=\mathbf{0},+} = \begin{pmatrix} -2m & \frac{1}{a} & 0 & 0 & 0 \\ \frac{1}{a} & \frac{ae^2}{2} & \frac{1}{\sqrt{2}a} & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}a} & ae^2 + 2m & \frac{1}{\sqrt{2}a} & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}a} & \frac{3ae^2}{2} & \frac{1}{\sqrt{2}a} \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}a} & 2ae^2 - 2m \end{pmatrix}$$

$$N = 4$$

$$H_S^{\mathbf{k}=\mathbf{0},+} =$$

$$\left(\begin{array}{cccccccccccccccccc} -4m & \frac{\sqrt{2}}{a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{a} & \frac{ae^2}{2}-2m & \frac{1}{a} & \frac{1}{\sqrt{2}a} & \frac{1}{\sqrt{2}a} & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{a} & ae^2 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{a} & \frac{1}{2a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}a} & 0 & ae^2 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & ae^2 & 0 & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & ae^2 & 0 & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2a} & 0 & 0 & 0 & \frac{3}{2}ae^2-2m & 0 & 0 & \frac{1}{2a} & \frac{1}{2a} & \frac{1}{2a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{a} & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & \frac{3}{2}ae^2+2m & 0 & \frac{1}{2a} & 0 & \frac{1}{2a} & \frac{1}{a} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2a} & \frac{1}{\sqrt{2}a} & 0 & \frac{1}{\sqrt{2}a} & 0 & 0 & \frac{3}{2}ae^2+2m & 0 & \frac{1}{2a} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{2a} & 0 & 2ae^2 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{2a} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & 0 & \frac{1}{2a} & 0 & 2ae^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{2a} & 0 & 0 & 0 & 2ae^2 & 0 & \frac{1}{2a} & \frac{1}{2a} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a} & 0 & 0 & 0 & 0 & 2ae^2+4m & 0 & \frac{1}{a} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & 0 & \frac{1}{2a} & 0 & \frac{5}{2}ae^2-2m & 0 & \frac{1}{2a} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & 0 & \frac{1}{2a} & \frac{1}{a} & 0 & \frac{5}{2}ae^2+2m & \frac{1}{a} & \frac{1}{\sqrt{2}a} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2a} & \frac{1}{a} & 3ae^2 & 0 & \frac{1}{a} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}a} & 0 & 3ae^2 & \frac{1}{\sqrt{2}a} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a} & \frac{1}{\sqrt{2}a} & \frac{7}{2}ae^2-2m & \frac{1}{a} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{a} & 4ae^2-4m \end{array} \right)$$

Towards QCD... Schwinger model

# Lattice sites	# physical states
2	13
4	117
6	1186
8	12389
10	130338
12	1373466
14	14478659
16	152642789
18	1609284589
20	16966465802
22	178875298901
24	1885860059450
26	19882389467226
28	209617576827139
30	2209972232664381
32	23299464416598565
34	245643376943334578
36	2589787797664451205
38	27303813033527765202
40	287860730073528550874
42	3034880139881813717315
44	31996366649585363767573
46	337333743537756100879133
48	3556468013241491667189146
50	37495403206807318414369013
52	395309407087754112140758506
54	4167698276777054670723455386
56	43939528417026911122159567875
58	463249023632234177261405300845
60	4883977266653121669892843601621
62	51491169380469520010932588566978
64	542865042036754830902171123029413
66	5723359119852258403113870140242562
68	60340668634504203157034795212955546
70	636164220174432677391393885245144899
72	6707000836890367782408329702790282053
74	70711081823674035762289079915112854861
76	745498206168787335244532860509164924714
78	7859695553615600796141819438529000076949
80	82863799918438013669248087407553825812634
82	873622812751857344335782363756954898488282
84	9210497463448372721760862275682543209296131
86	97105137693198992443830116213599788185967005
88	1023767478774681068348344873972808077874359941
90	10793454141511128457710661202368593062740833234
92	113794054529196161165625459023712299579083440581
94	1199716668679035436321002120920649282974697423410
96	12648464729209871610324476298554289482677637882458
98	133351202148602605751037762196292594217046242903875
100	1405905261641056248331375526910312847554957270229877

Klco, Savage (2018)
Jong, Lee, Mulligan, Ploskon, Ringer, Yao 2106.08394

<u>N</u>	<u># states</u>
2	13
4	117
6	1186
100	$\approx 1.4 \cdot 10^{51}$

Towards QCD... Schwinger model - open system!

Jong, Lee, Mulligan, Ploskon, Ringer, Yao 2106.08394

Couple the Schwinger model to a thermal scalar field theory $\phi(x)$

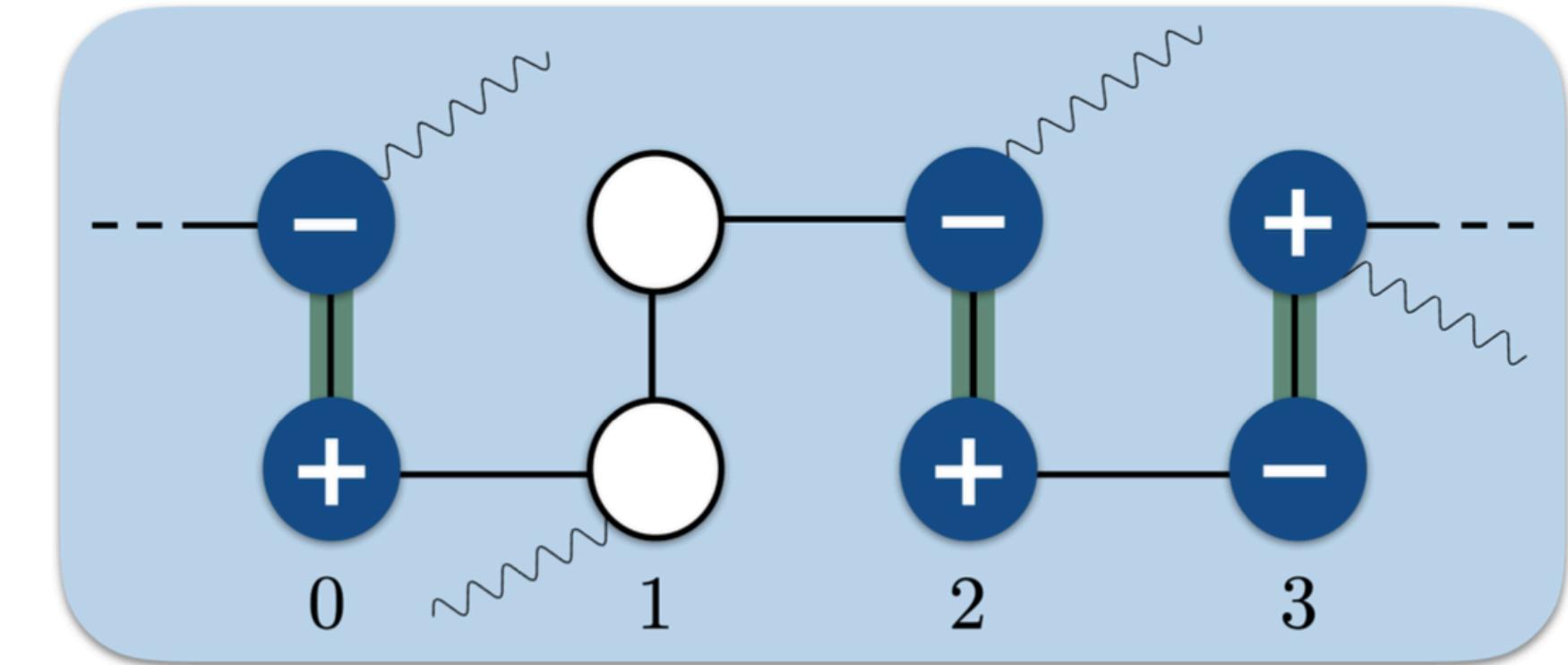
$$H_I = \lambda \int dx \phi(x) \bar{\psi}(x) \psi(x)$$

In the Quantum Brownian Motion limit,

$\tau_R \gg \tau_E$ — Markovian approximation

$\tau_S \gg \tau_E$ — valid if $T \gg \Delta E_S$

the subsystem is described by a Lindblad equation:



τ_S — subsystem intrinsic time scale

τ_R — subsystem relaxation time

τ_E — environment correlation time

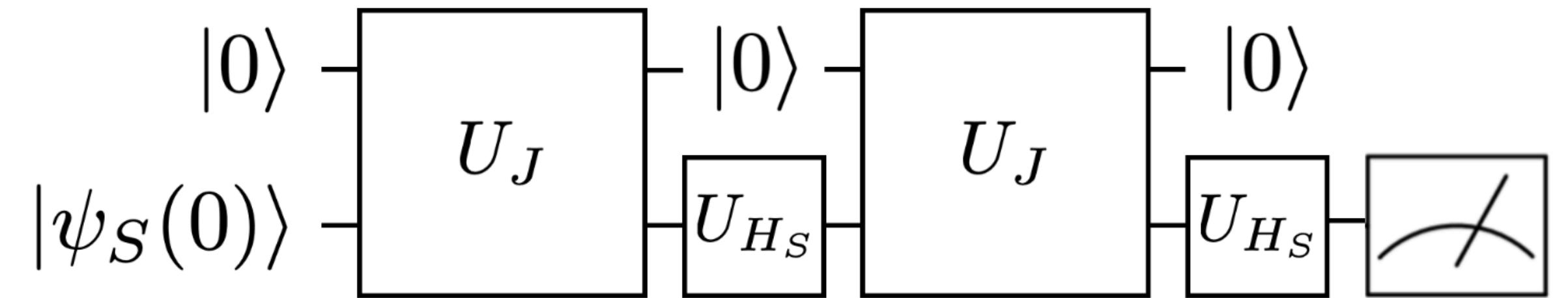
$$\frac{d\rho_S(t)}{dt} = -i[H_S, \rho_S(t)] + L\rho_S(t)L^\dagger - \frac{1}{2}\{L^\dagger L, \rho_S(t)\}$$

$$L = \sqrt{aN_f D} \left(O_S - \frac{1}{4T} [H_S, O_S] \right) \quad O_S(x) = \bar{\psi}(x) \psi(x) \quad D = \lambda^2 \int dt dx \text{Tr}_E (\rho_E \phi(t, x) \phi(0, 0))$$

Towards QCD... Schwinger model - open system!

Jong, Lee, Mulligan, Ploskon, Ringer, Yao 2106.08394

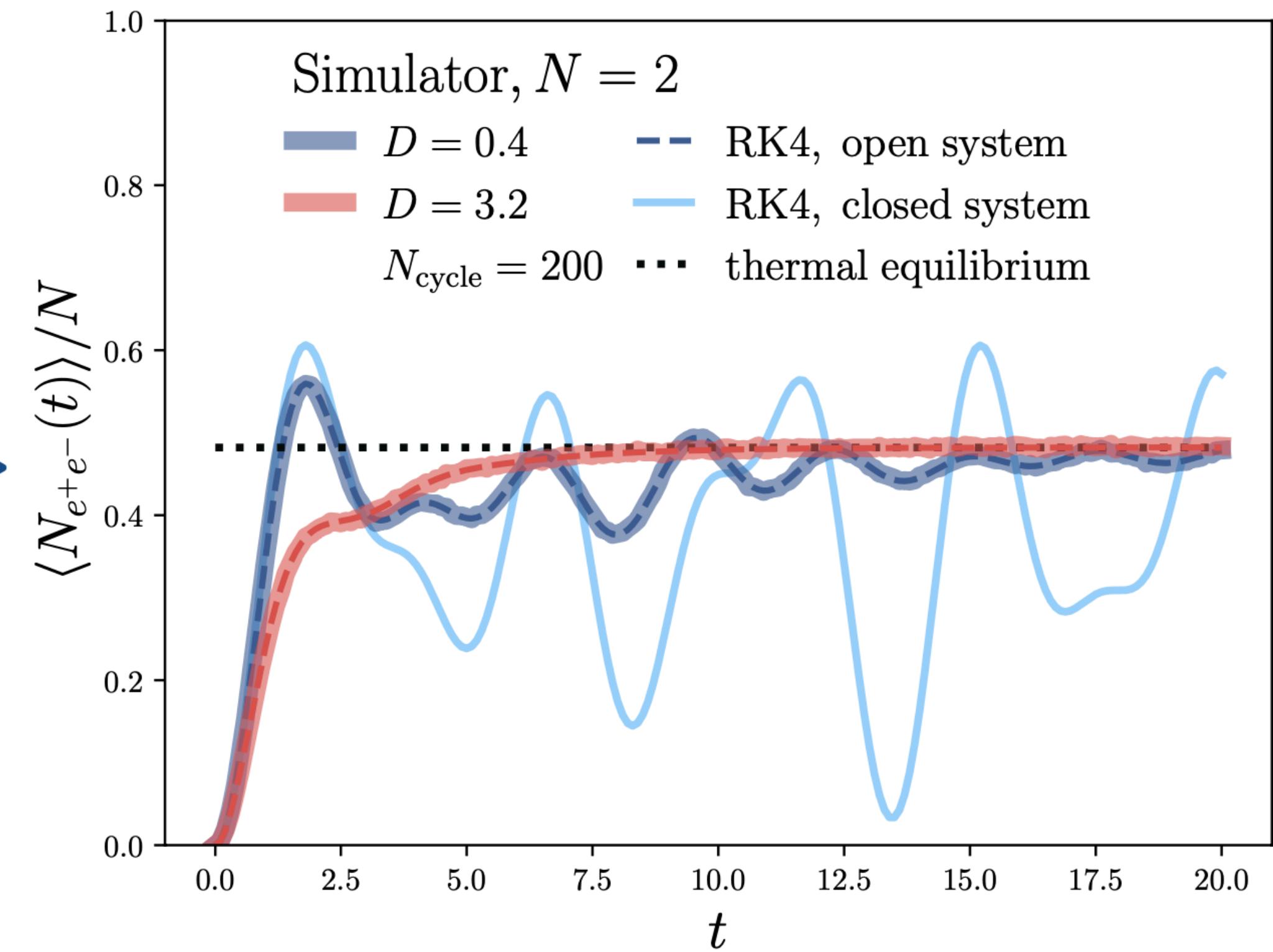
We construct a quantum circuit to solve the Lindblad evolution



Classical simulation of the quantum circuit reproduces the thermalization observed in numerical solution

Number of e^+e^- pairs

Note: The environment correlator $D \sim \lambda^2$ determines how fast the system equilibrates



Towards QCD... Schwinger model - open system!

Jong, Lee, Mulligan, Ploskon, Ringer, Yao 2106.08394

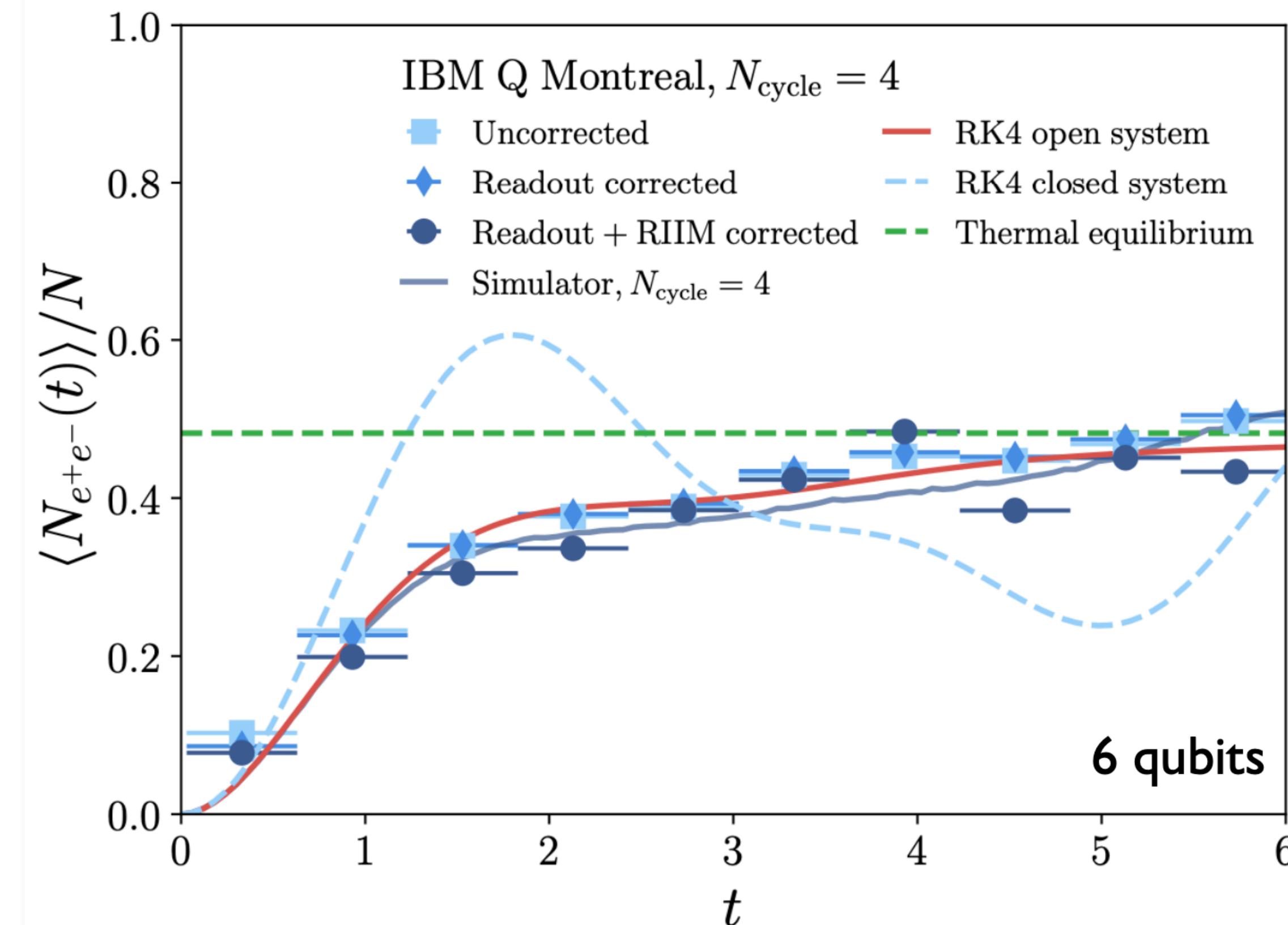
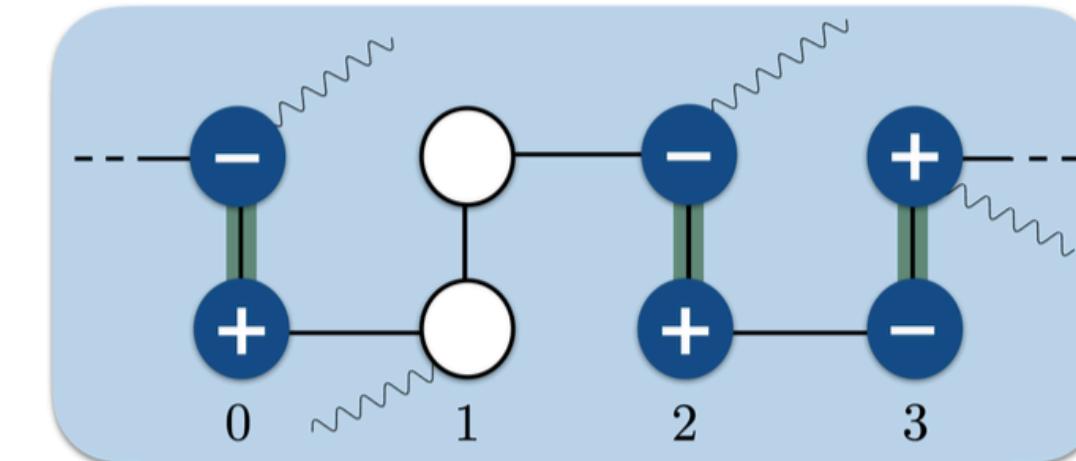
Simulation on IBM Q device

- Error mitigation applied

Number of e^+e^- pairs

First quantum simulation of open quantum systems described by quantum field theories

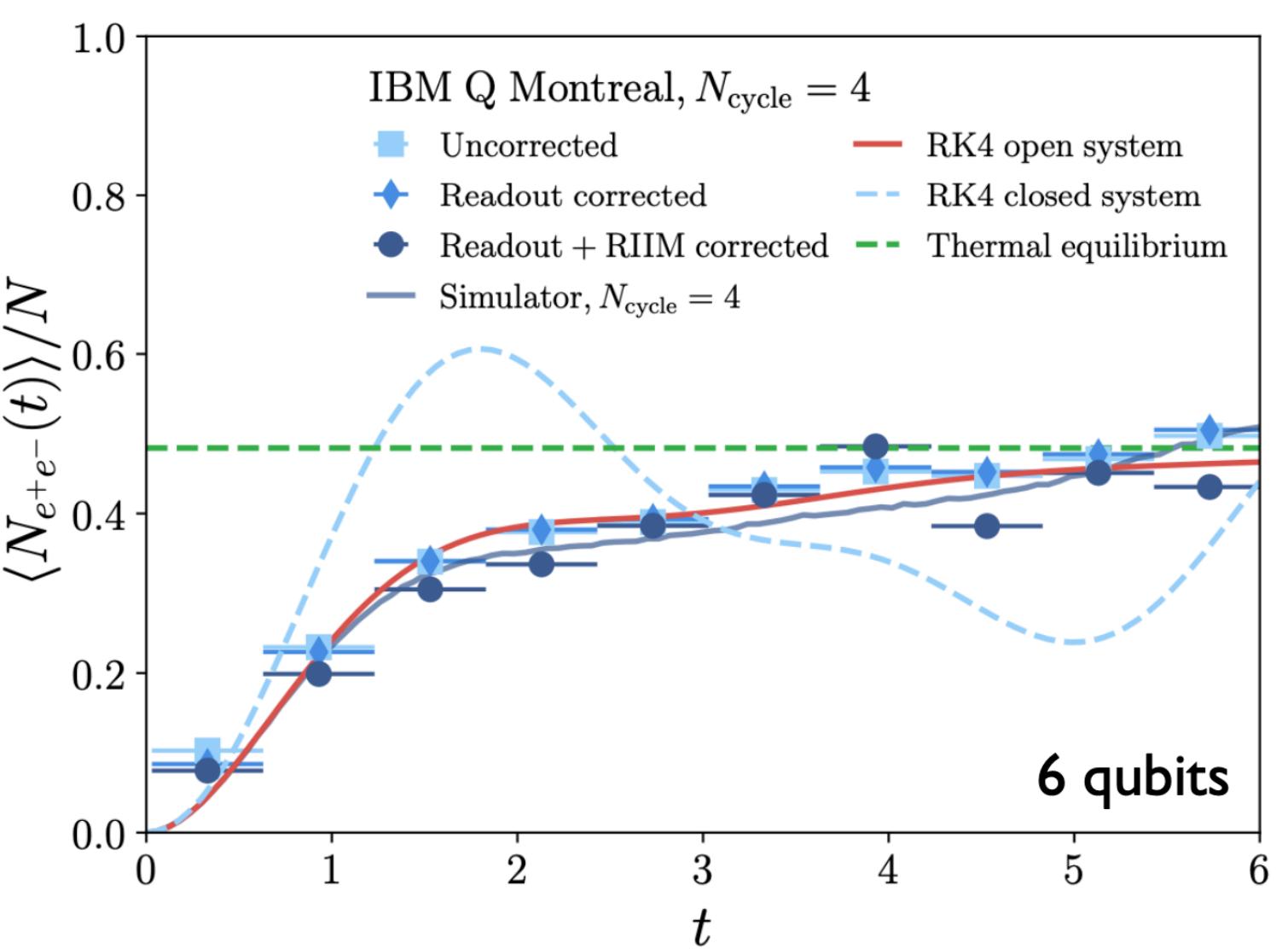
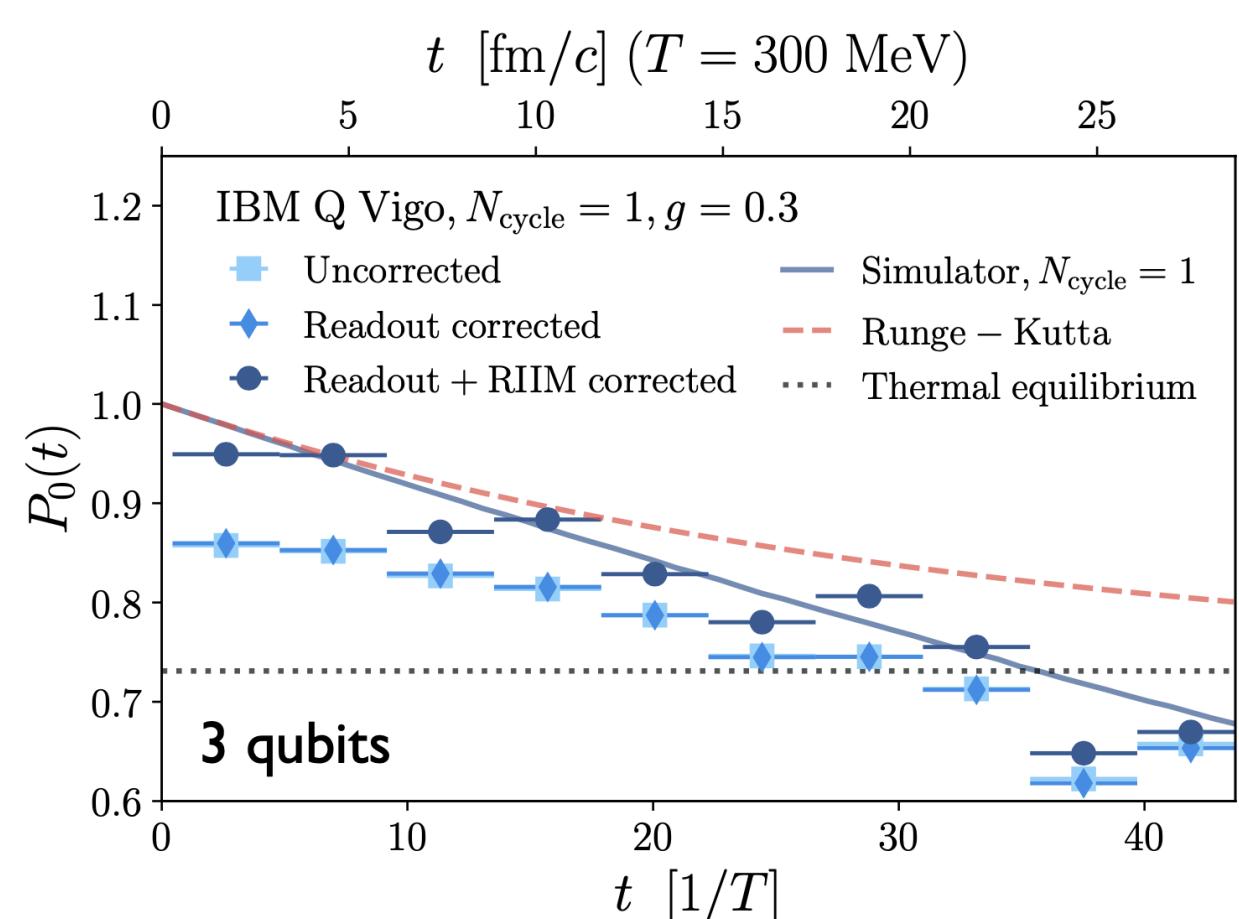
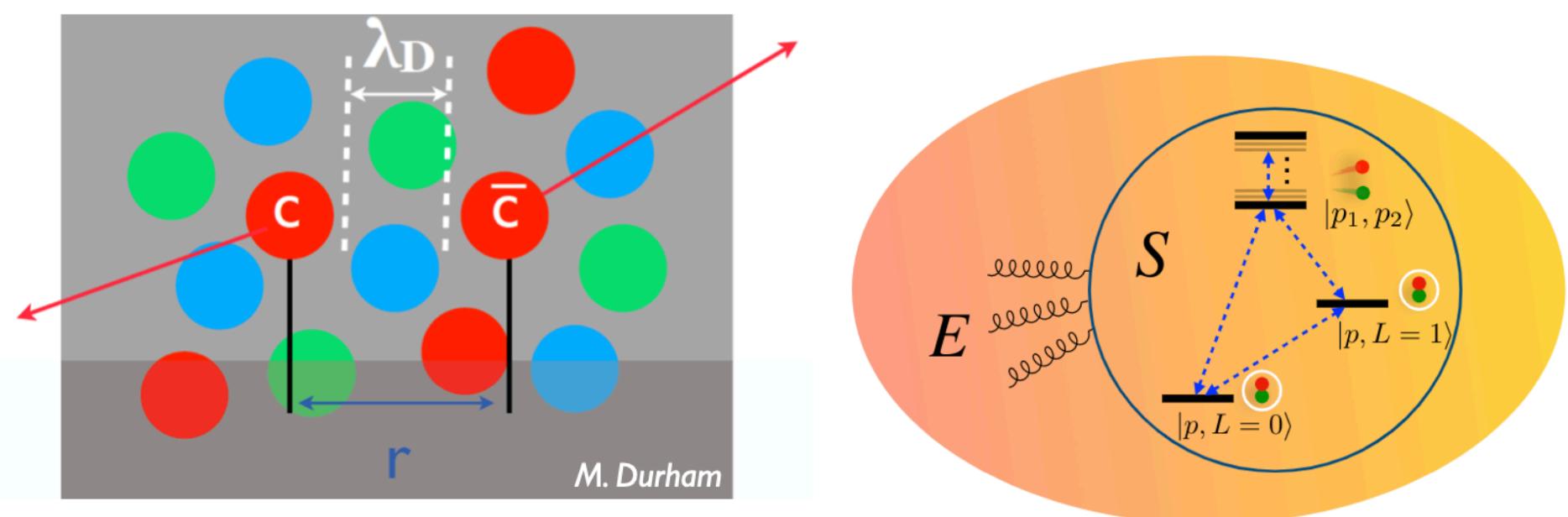
Real-time dynamics of thermalization process



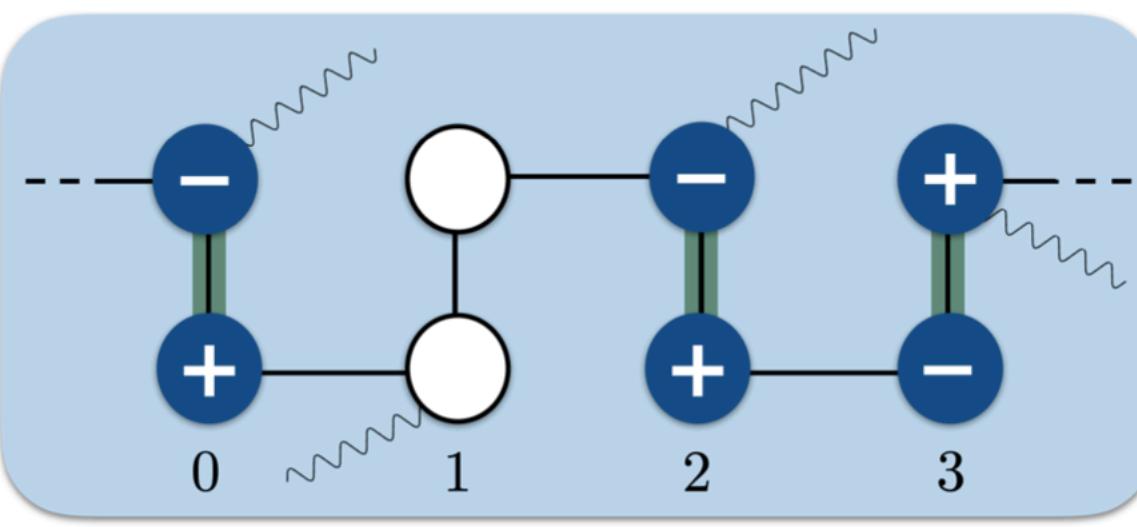
Outline

Main target: simulation of some relevant processes but mitigate classical computations shortcomings
 Non-relativistic QCD - many-body problems \Rightarrow quarkonia dissociation in hot colored medium
 Quantum Field Theory - thermalization (equilibration), hadronization, jet quenching, ...

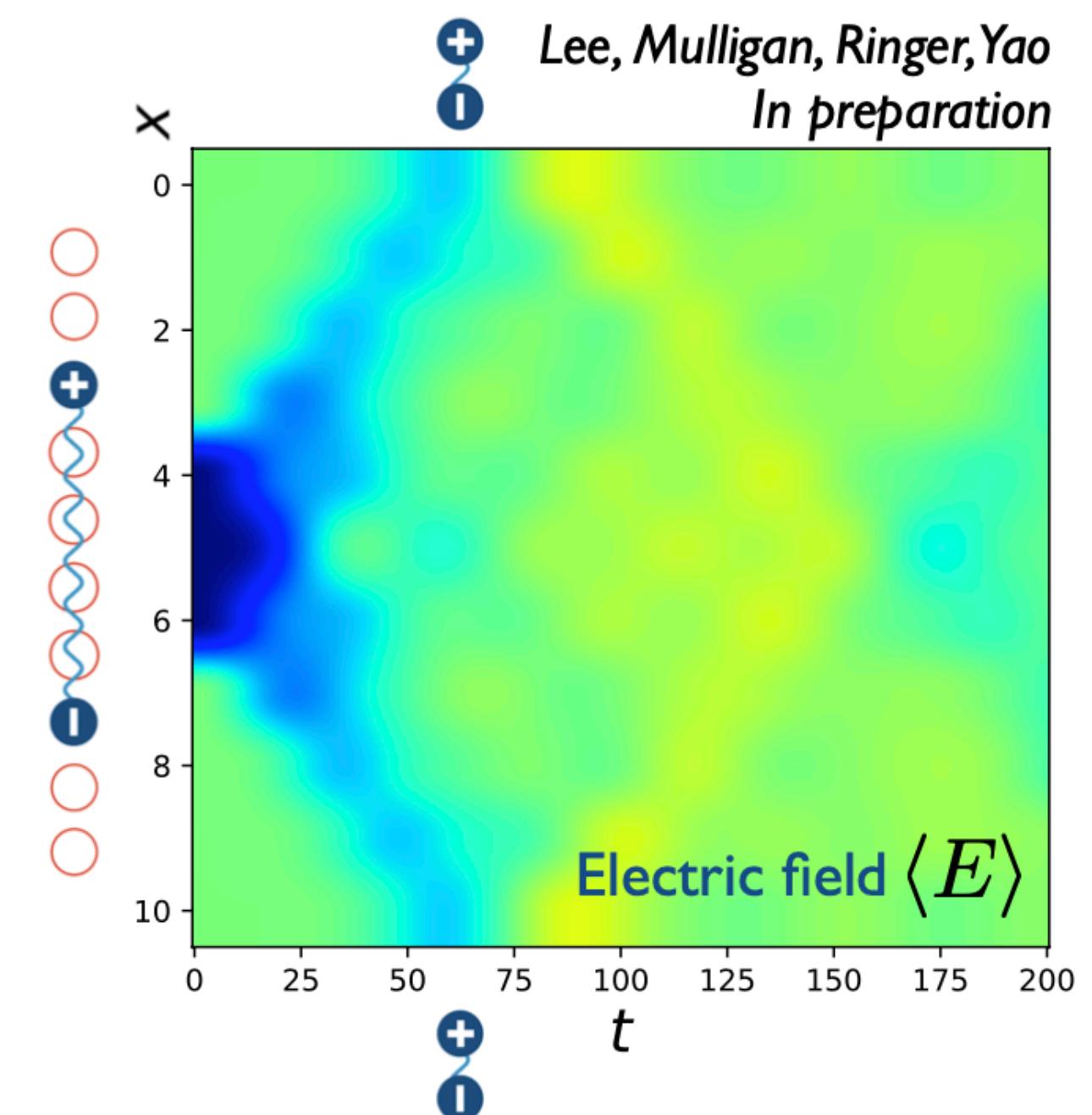
Towards quarkonia in hot QGP: open quantum system - proof of principle calc. -
[PhysRevD.104.L051501](#)



Thermalization/non-eq. dynamics
 Schwinger model: QED in 1+1D
[PhysRevD.106.054508](#)



Towards hadronization:
 String breaking mechanism in Schwinger model

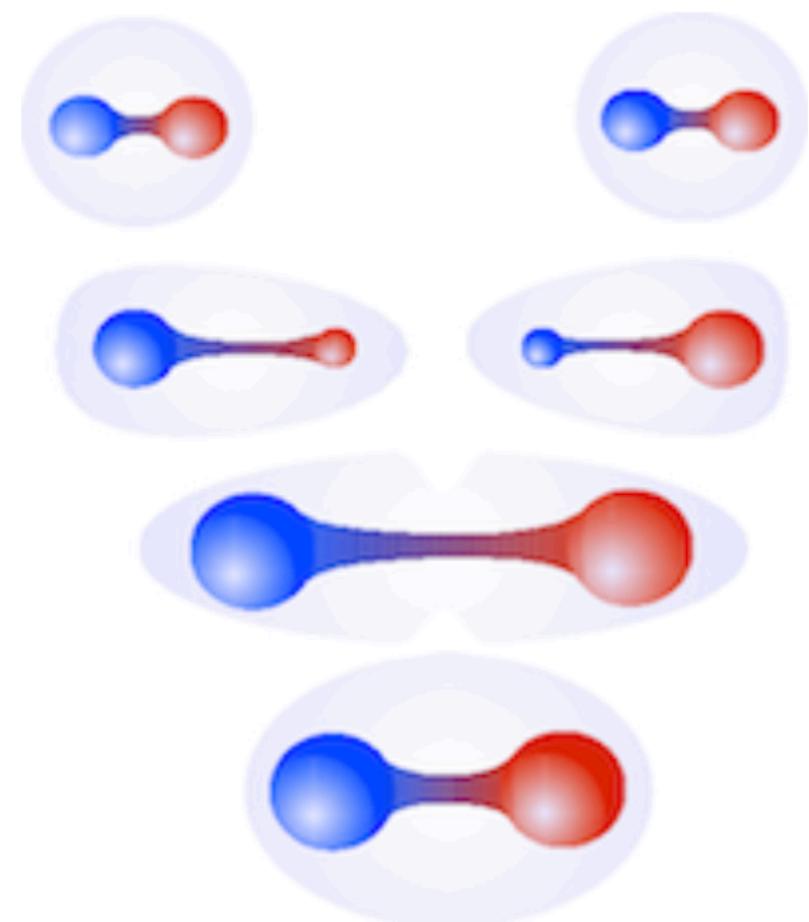


Jet quenching in light front QCD
<https://arxiv.org/abs/2205.07902>

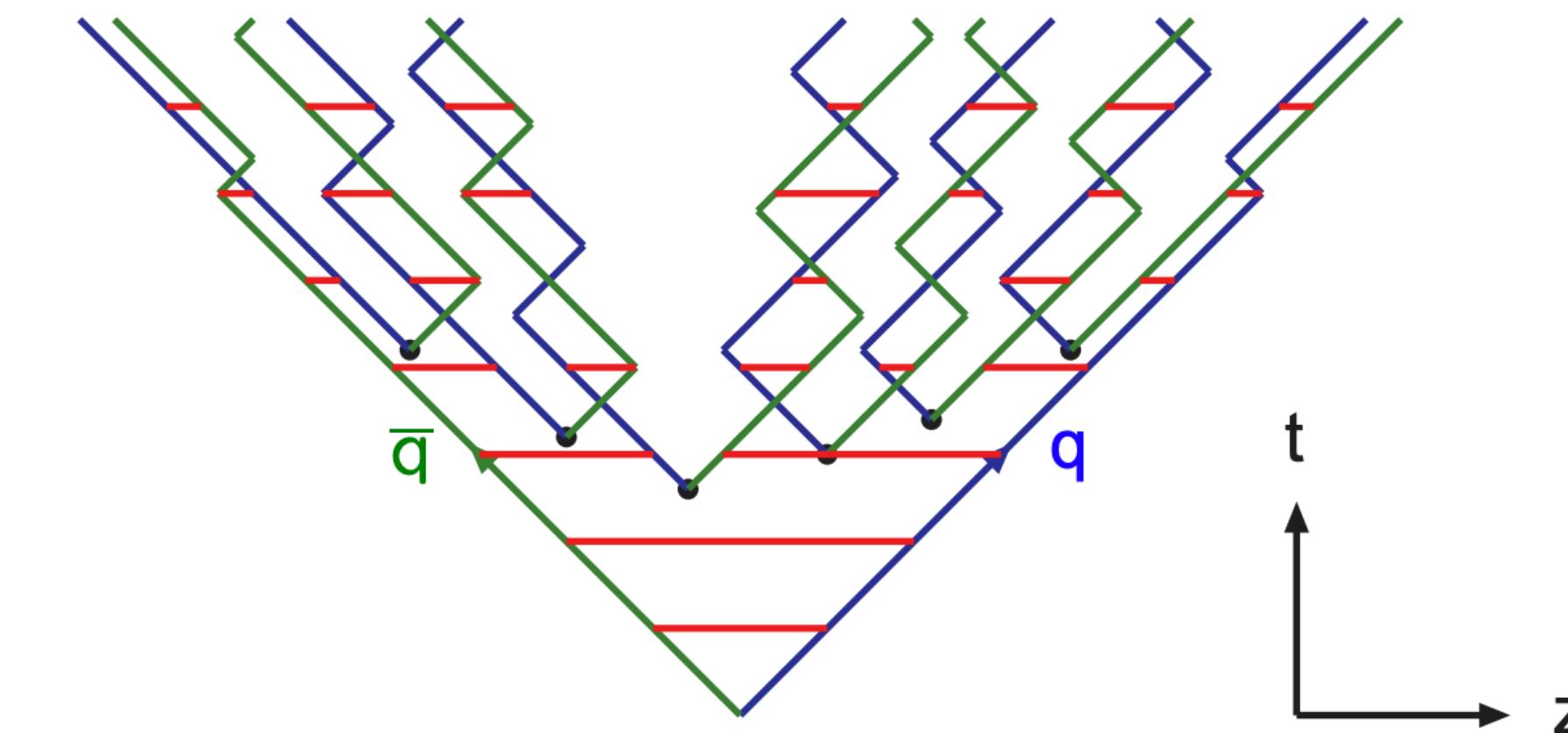
Quantum simulation of string breaking?

Remaining within **Schwinger model**

$$\mathcal{L} = \bar{\psi}(iD^\mu - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$



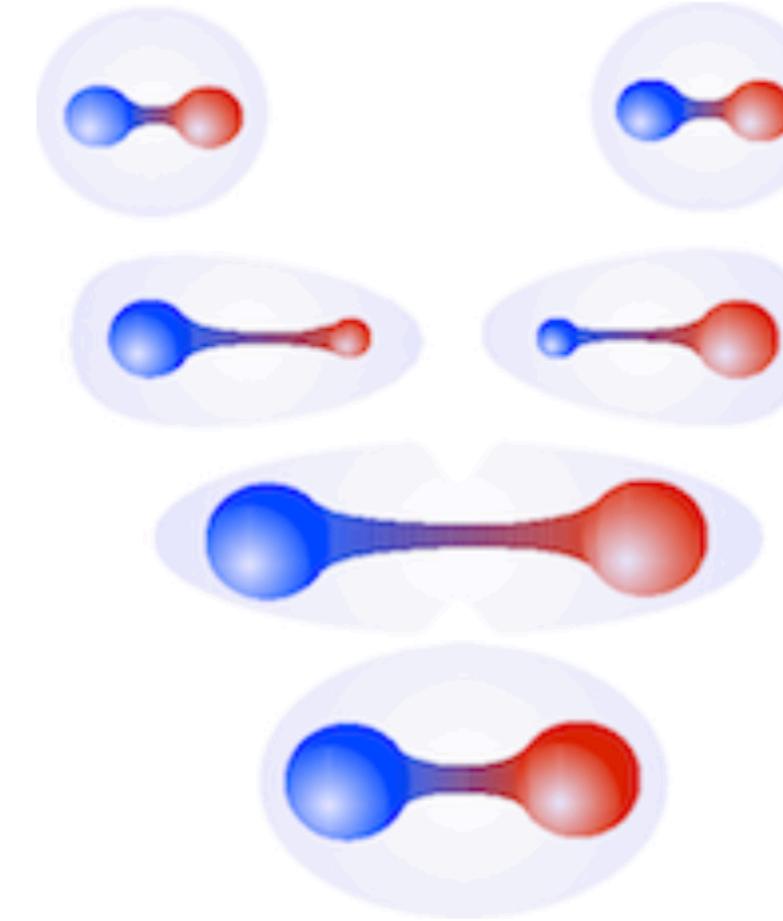
QCD string breaking



Pythia Lund string hadronization

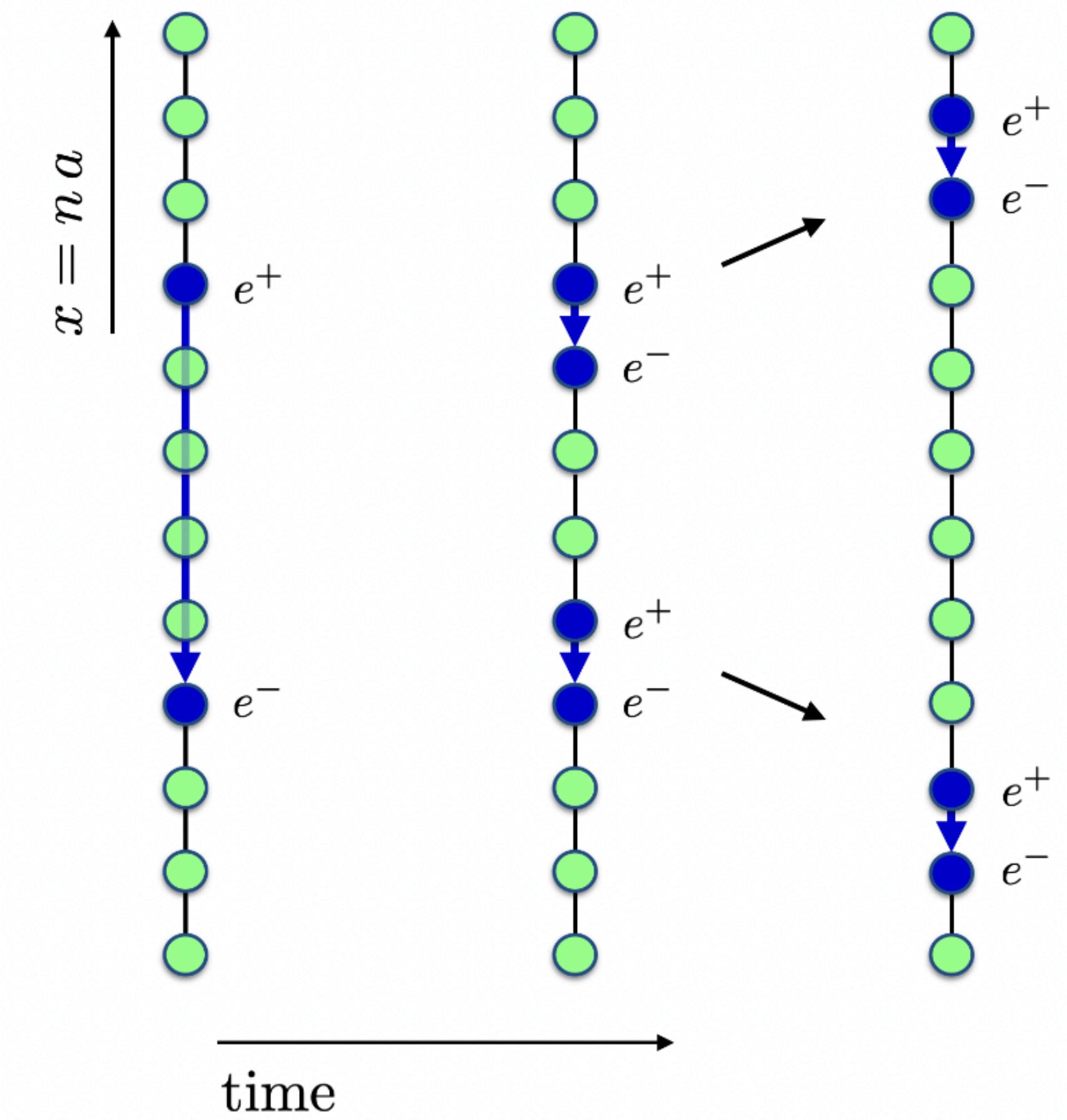
The string-breaking mechanism

- Related to hadronization in QCD
- Study real-time dynamics of hadronization



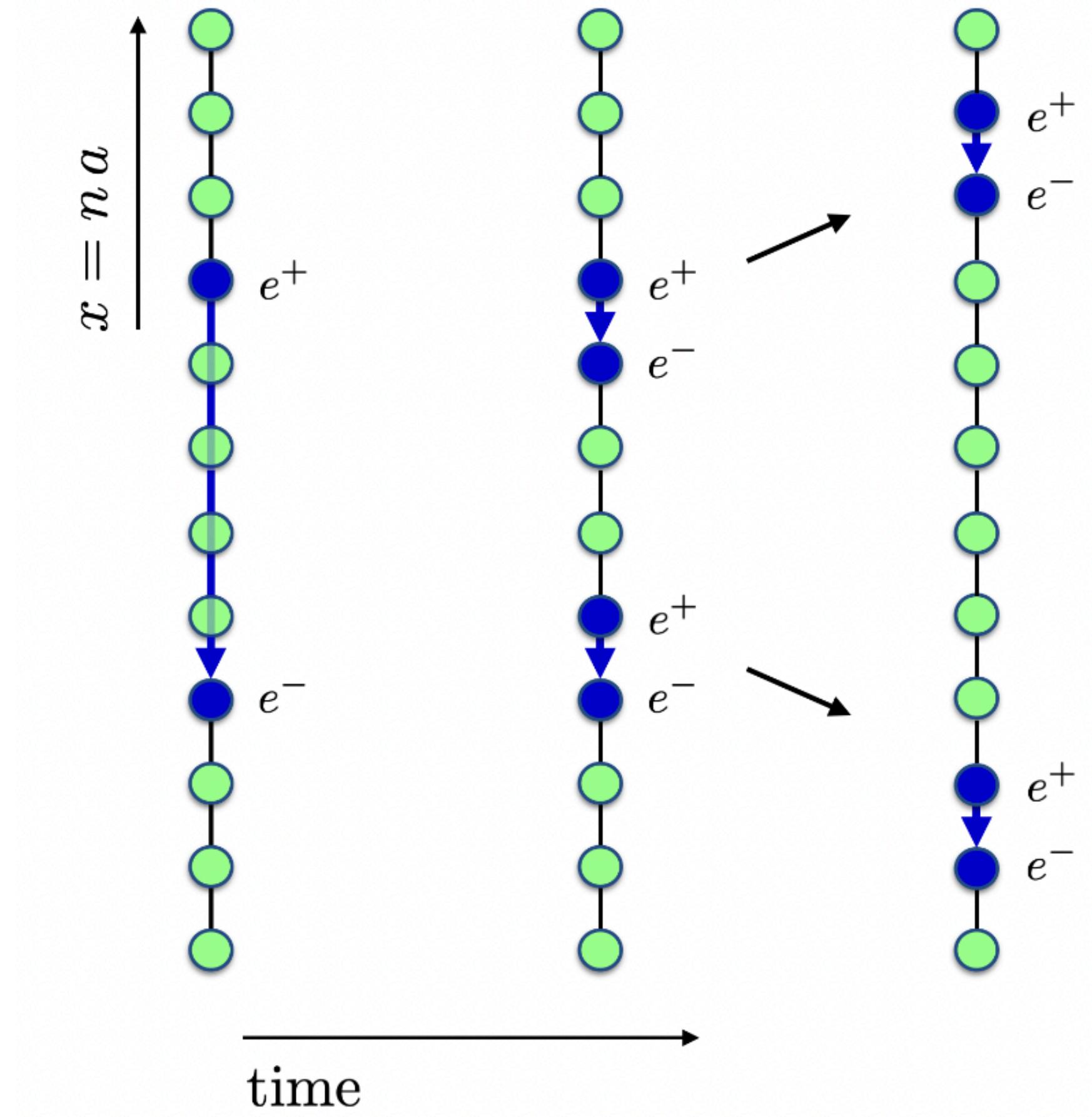
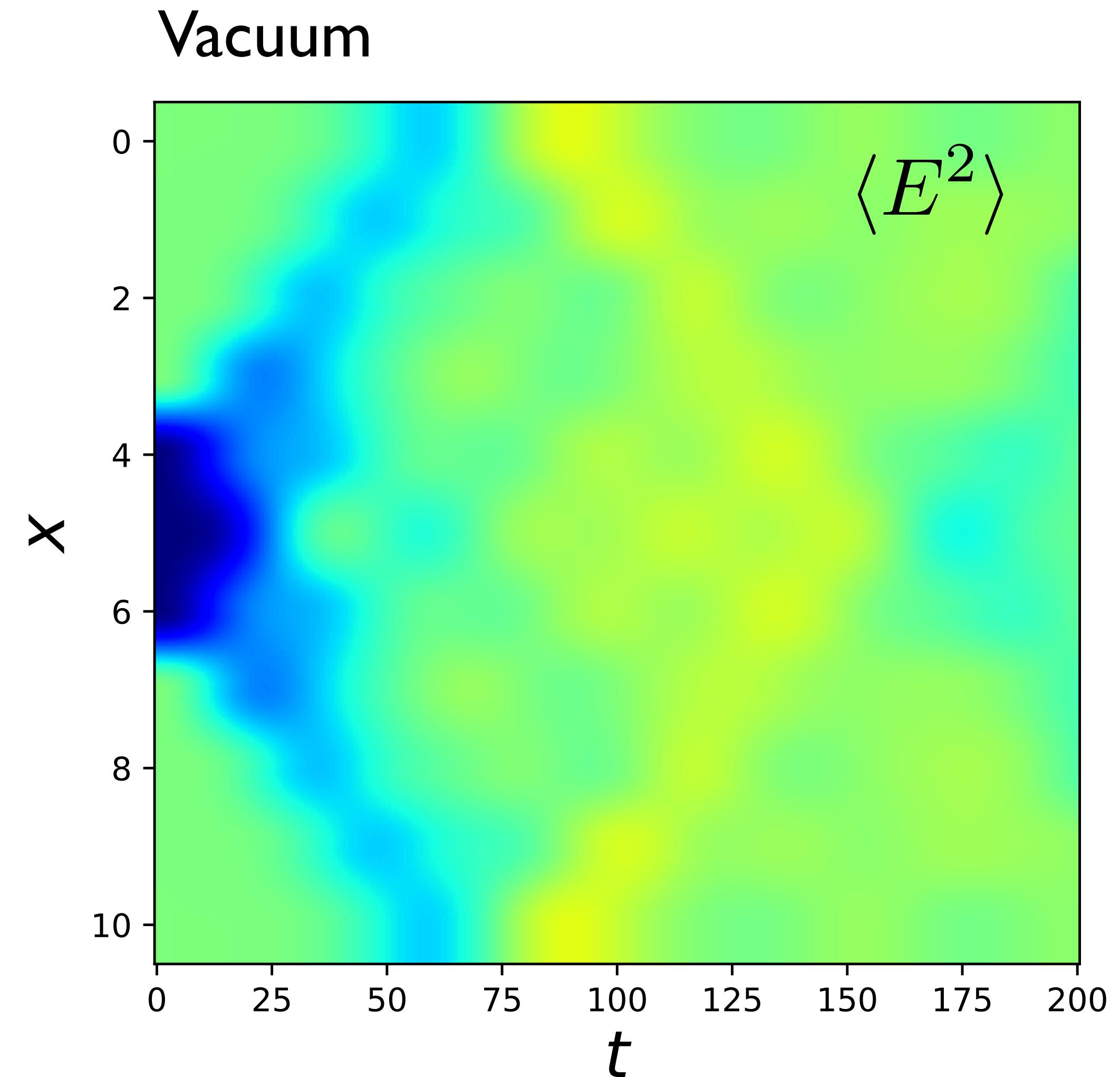
- Whether the string breaks or not depends on the fermion mass & charge (m, e)

- Time continuous, 1-dimensional spatial lattice $x=na$
- Continuum limit for $a \rightarrow 0$



see also Zoller et al, Magnifico et al

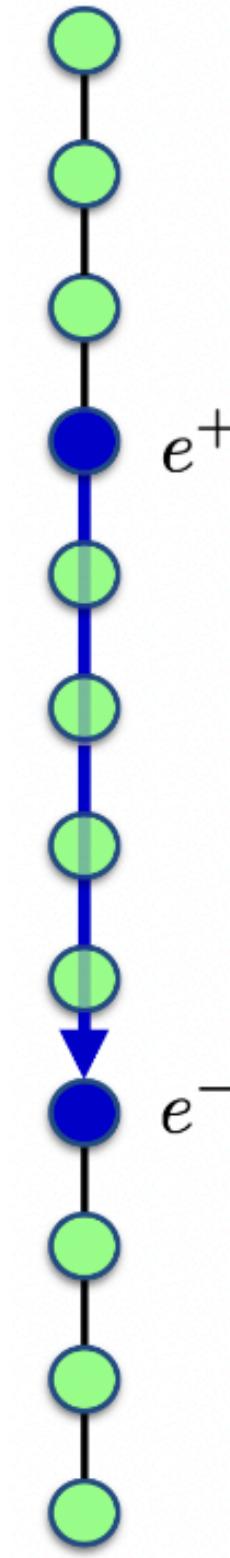
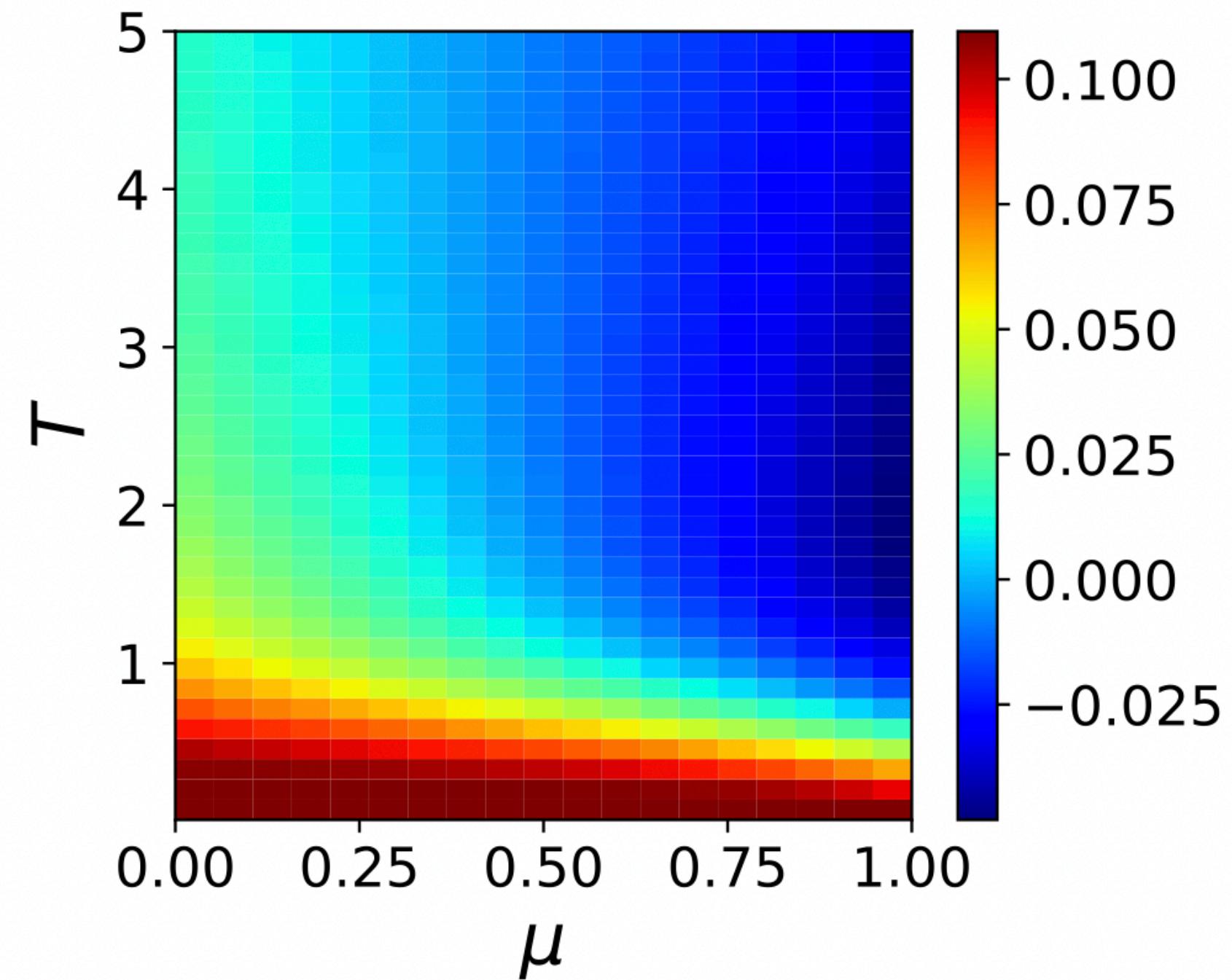
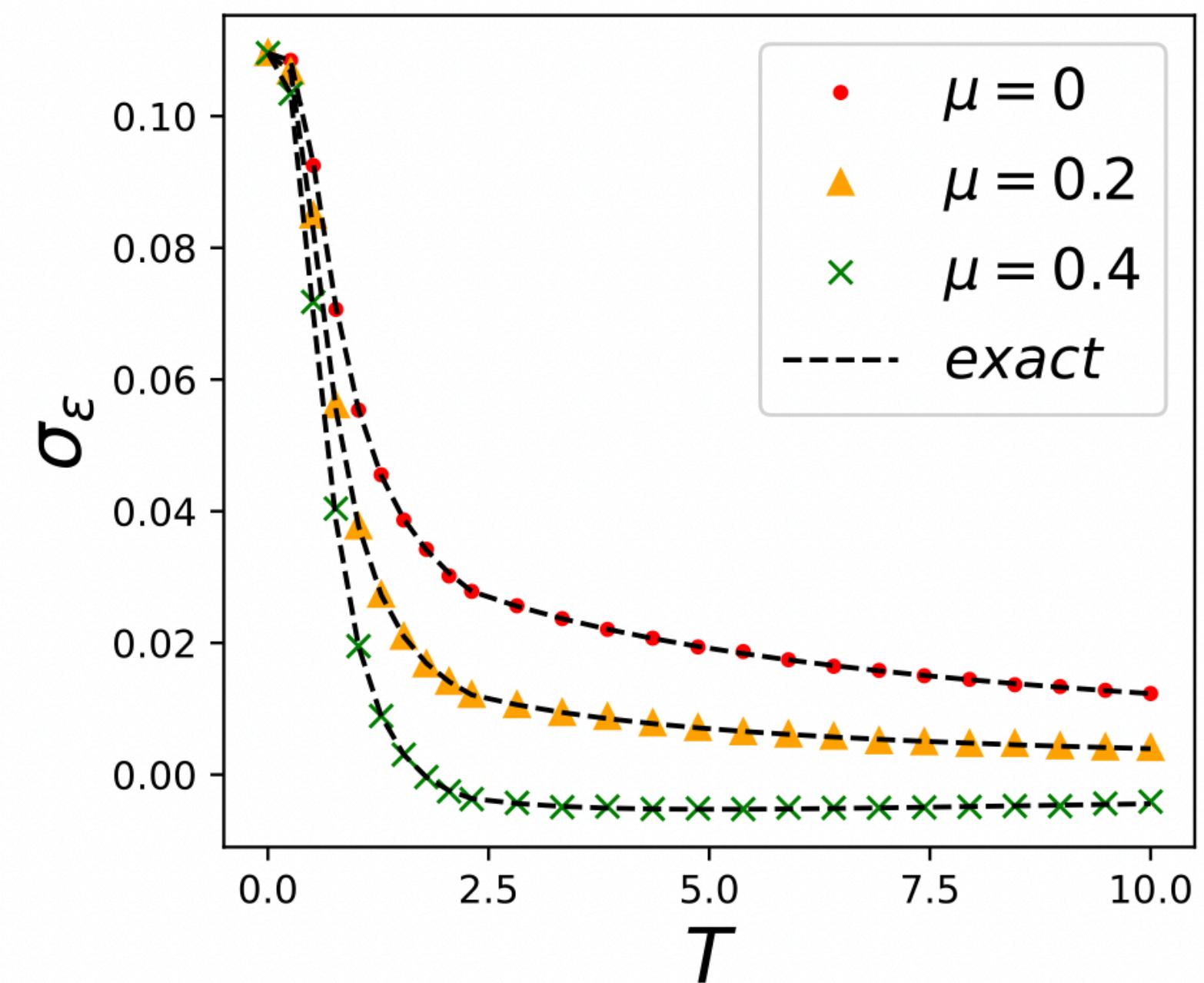
The string-breaking mechanism



trivial vacuum evolution subtracted, see also Zoller et al, Magnifico et al

String tension & medium effects

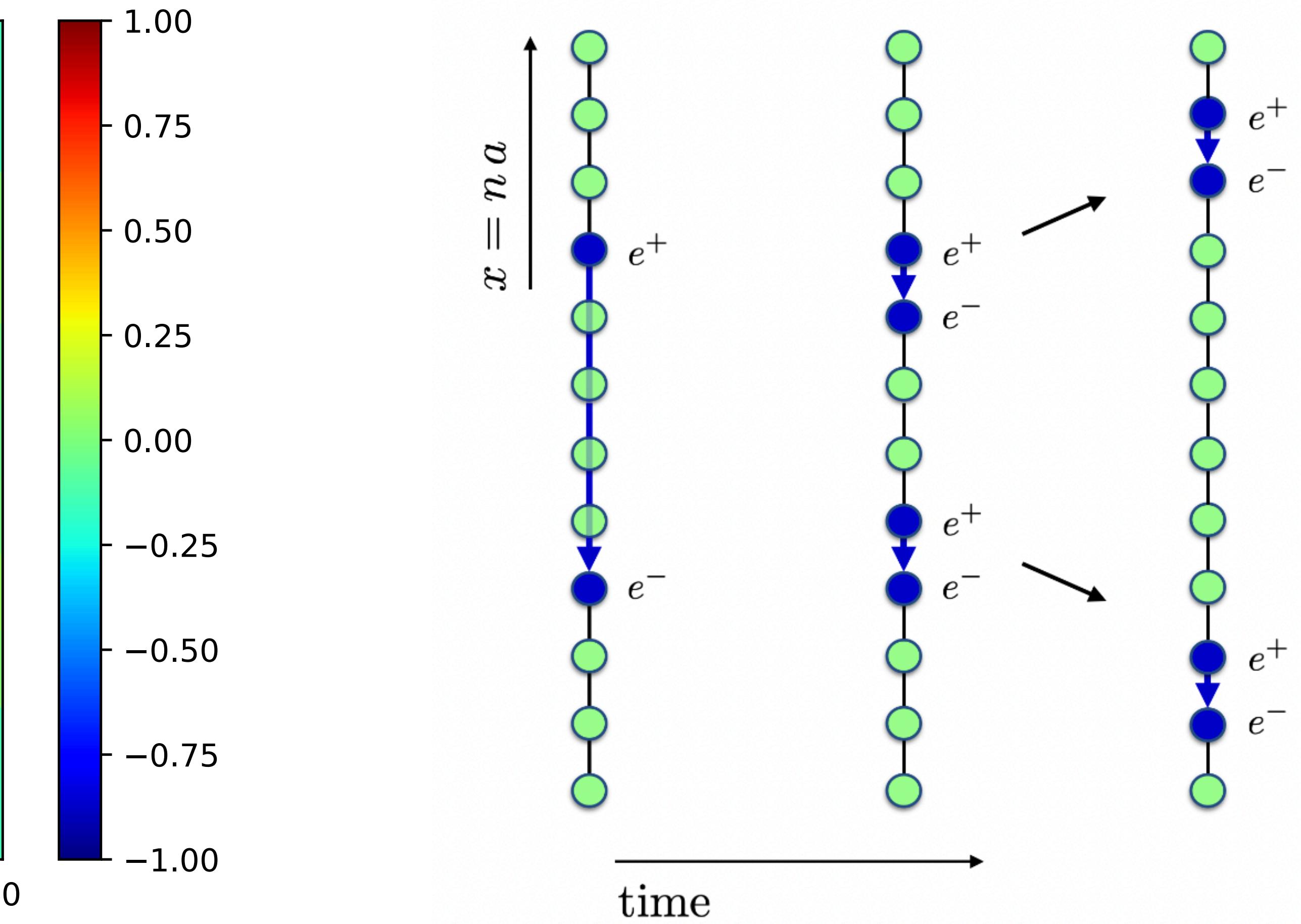
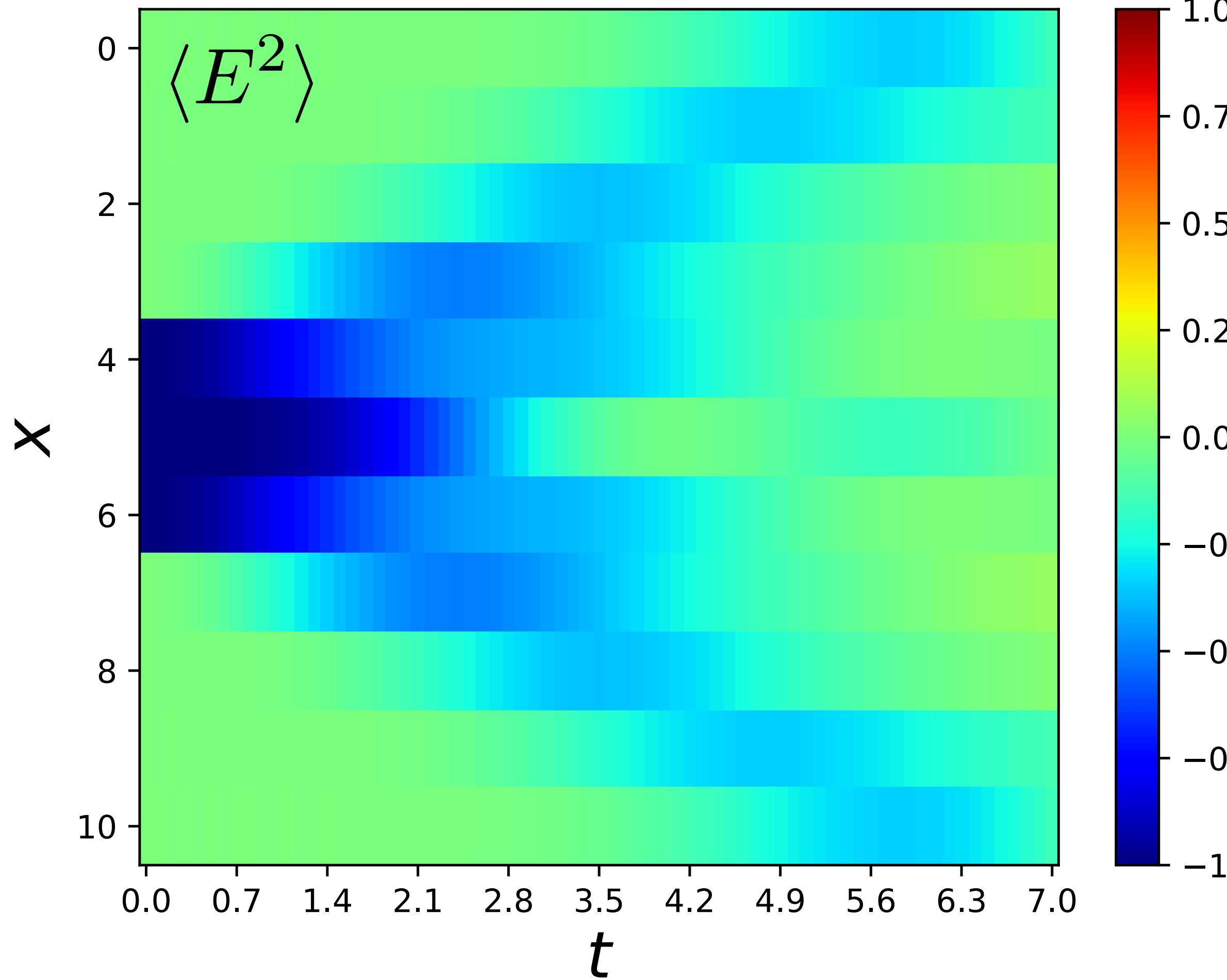
- Static case - finite temperature or chemical potential lead to a reduced string tension



Fischler, Kogut, Susskind '79, Pisarski, Alvarez '82,
Buyens, Verstraete, van Acoleyen '16
Xie, Guo, Xing, Xue, Zhang, Zue '22

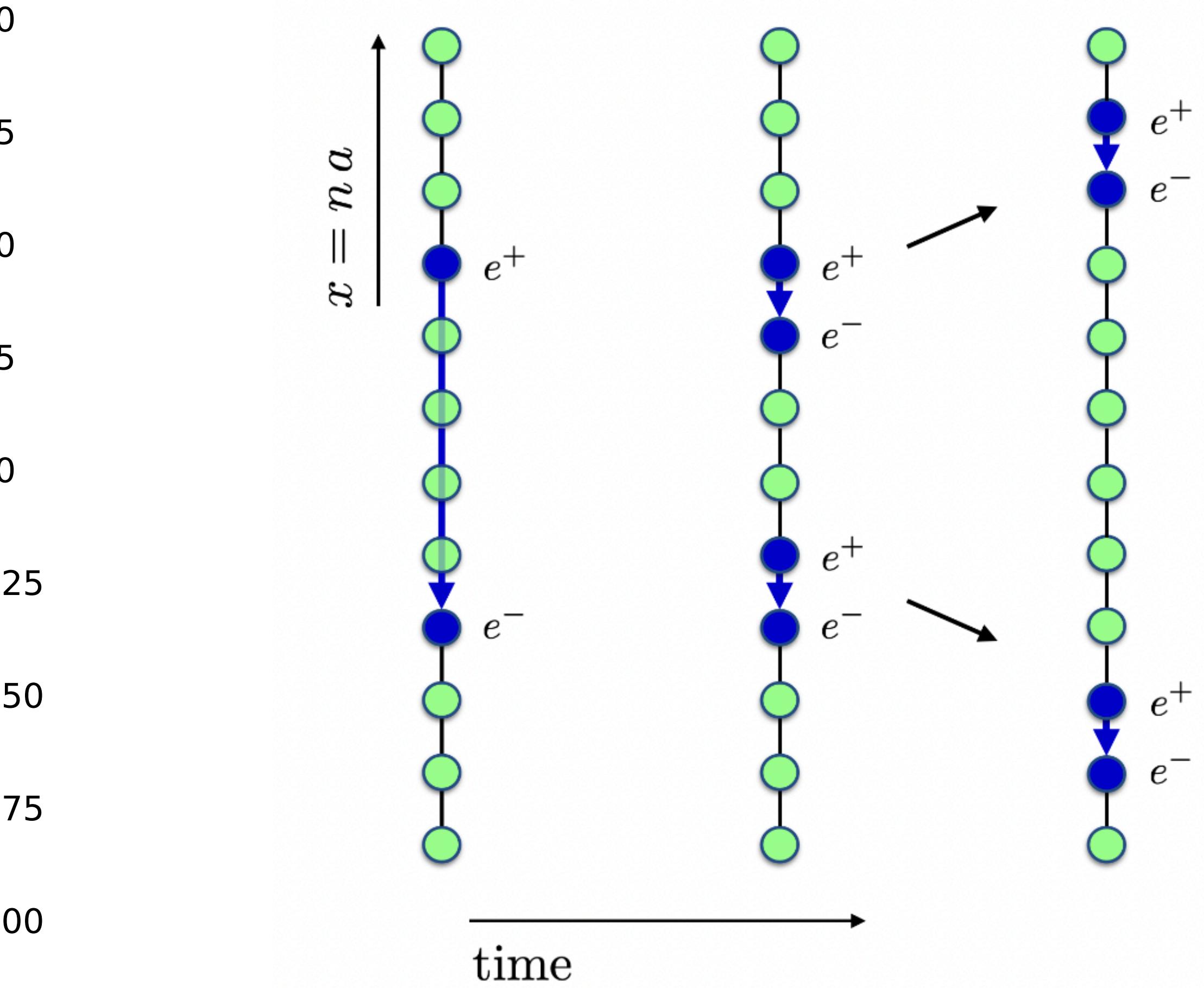
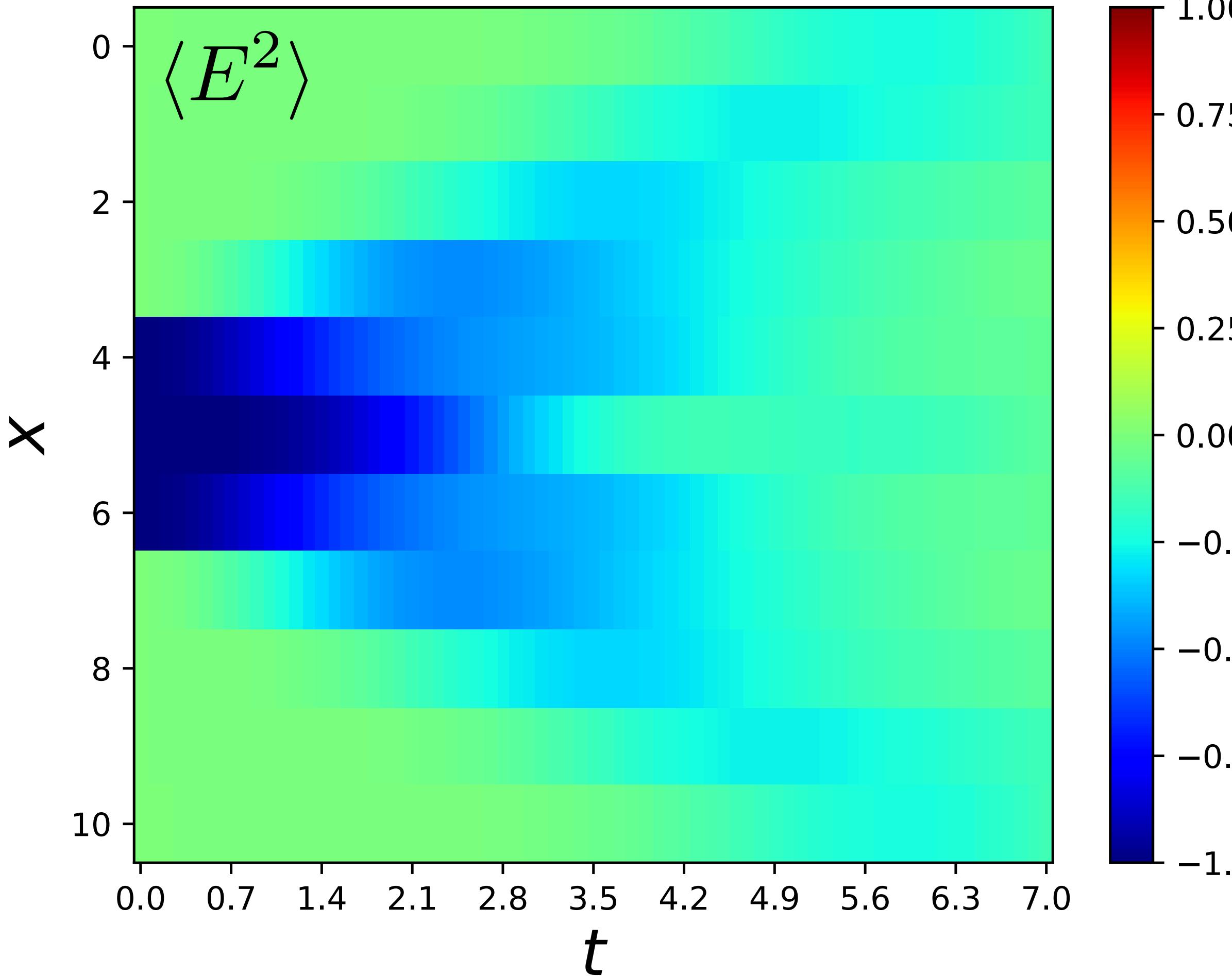
The string-breaking mechanism

Vacuum



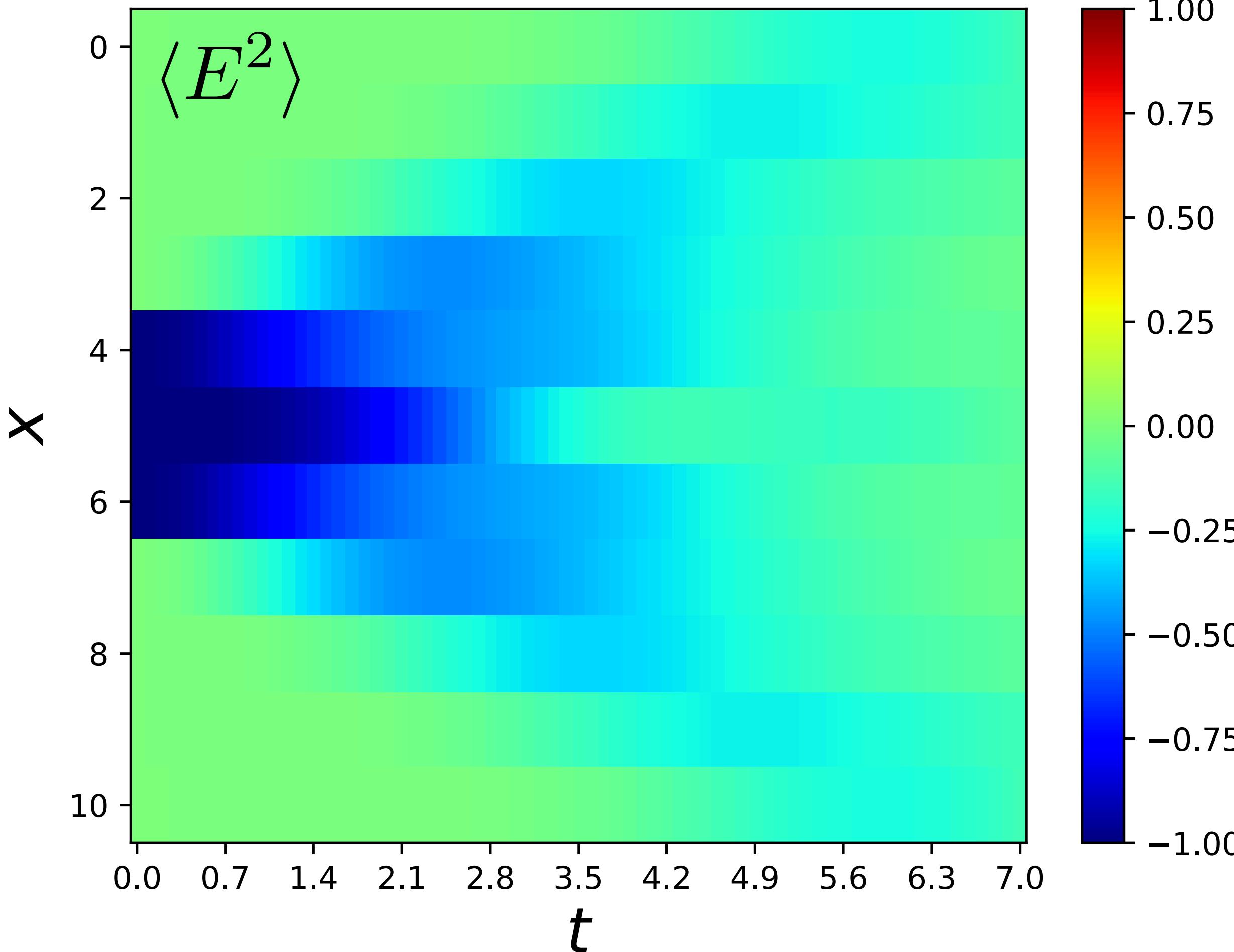
The string-breaking mechanism

Medium



The string-breaking mechanism

Medium



- Delay of the string breaking mechanism analogous to classical drag
- Melting because of high temperature
- Potential impact on simulations of heavy-ion collisions?

see e.g. Loshaj, Kharzeev '13

Instead of summary / outlook / challenges

Strong interest in QC for heavy-ion collisions - enormous potential for use and resolution of long standing questions - hand in hand with the experimental program: simulations of real-time dynamics, high-/low-density systems (synergy with some of the EIC physics)

Reasonable (?) shorter term goal: Need for quantum-classical hybrid simulations - complexity of heavy-ion collisions / medium (simulate critical elements processes with quantum computers) + need for classical computing resources (not only validation checks etc)

Challenges: Availability and evolution of the physics driven toolkits / investment in effort

Can we bring quantum computer simulation to the standards/high-level of known particle/event generators? - simplifying the problem of interfacing => quantum simulations as a mainstream tool? - need for substantial funding initiative / timing?

Development of (high level) tools for nuclear particle physics

Not discussed in detail: Non-equilibrium & relaxation dynamics; Decoherence & late-time dynamics; Non-equilibrium & relaxation dynamics; Decoherence & late-time dynamics

AI/ML on QC: ChatGPT answering...

Question: Main use cases for quantum computers?

Quantum computers have the potential to revolutionize various fields by providing computational capabilities beyond what is achievable with classical computers. While the technology is still in its early stages, there are several promising use cases for quantum computers:

1. Cryptography and Security: Quantum computers have the potential to break many of the cryptographic algorithms that currently secure our digital communications and transactions. On the flip side, quantum computing can also offer new cryptographic protocols that are resistant to attacks from classical computers.
2. Optimization and Simulation: Quantum computers can potentially solve optimization problems more efficiently than classical computers. Applications include optimizing complex systems such as supply chains, logistics, financial portfolios, and drug discovery. Quantum computers can also simulate quantum systems, providing insights into quantum chemistry, material science, and the behavior of quantum systems in general.
3. Machine Learning and AI: Quantum computers could enhance machine learning algorithms by processing and analyzing large datasets more efficiently. They can also help in developing new algorithms for solving complex machine learning problems, leading to advancements in AI.
4. Quantum Communication and Cryptography: Quantum computers can enable secure quantum communication protocols such as quantum key distribution (QKD), which allows for unbreakable encryption based on the laws of quantum physics. These technologies can enhance the privacy and security of data transmission.
5. Financial Modeling and Portfolio Optimization: Quantum computers can potentially analyze vast amounts of financial data and optimize investment portfolios, risk management, and portfolio hedging strategies more effectively. This can help financial institutions make better-informed decisions and improve their financial models.
6. Particle Physics and Quantum Simulation: Quantum computers can simulate the behavior of particles and quantum systems, allowing for advancements in fundamental physics research, quantum field theory, and understanding the properties of complex physical systems.

It's important to note that while these potential use cases are promising, the development and realization of practical quantum applications are still ongoing. Quantum computing is a rapidly evolving field, and more applications are likely to emerge as the technology advances and quantum algorithms and error correction techniques improve.

AI/ML on QC: ChatGPT answering...

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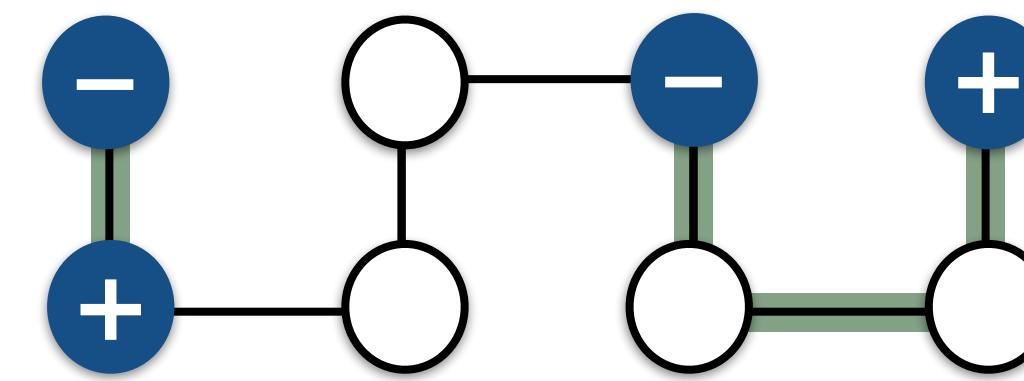
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Additional slides

Schwinger model - Hamiltonian formulation

$$H_S = \frac{1}{2a} \sum_{n=0}^{N_f-2} (\sigma^+(n)L_n^- \sigma^-(n+1) + \sigma^+(n+1)L_n^+ \sigma^-(n)) \\ + \frac{1}{2}ae^2 \sum_{n=1}^{N_f-1} \ell_n^2 + \frac{1}{2}m \sum_{n=0}^{N_f-1} (-1)^n \sigma_z(n)$$

Kogut, Susskind '70s



1. Staggered fermions $\psi = (\psi_u, \psi_d)^T$

$\psi_u(x)$ n : even

$\psi_d(x)$ n : odd

$N \rightarrow 2N = N_f$

+ Jordan-Wigner transformation

2. Electric flux $E(n)$, gauge link $U(n+1, n)$

Solve like a harmonic oscillator

$$E(n)|\ell_n\rangle = e\ell_n|\ell_n\rangle$$

$$L_n^\pm |\ell_n\rangle = |\ell_n \pm 1\rangle$$

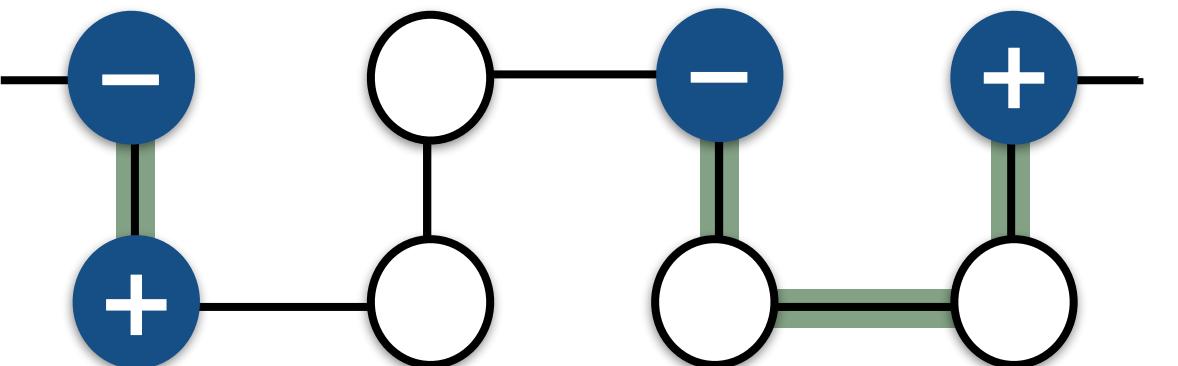
Local Hilbert space requires truncation

Schwinger model - Hamiltonian formulation

- Enforce Gauss's law directly *Klco, Savage et al. '18*

physical states

$$\partial_1 E = -e\psi^\dagger \psi$$



see e.g. *Shaw, Lougovski, Stryker, Wiebe '20*

- Number of physical states (periodic boundary conditions)

Combinatorial result for $|\ell_n| \leq 1$, spatial sites N

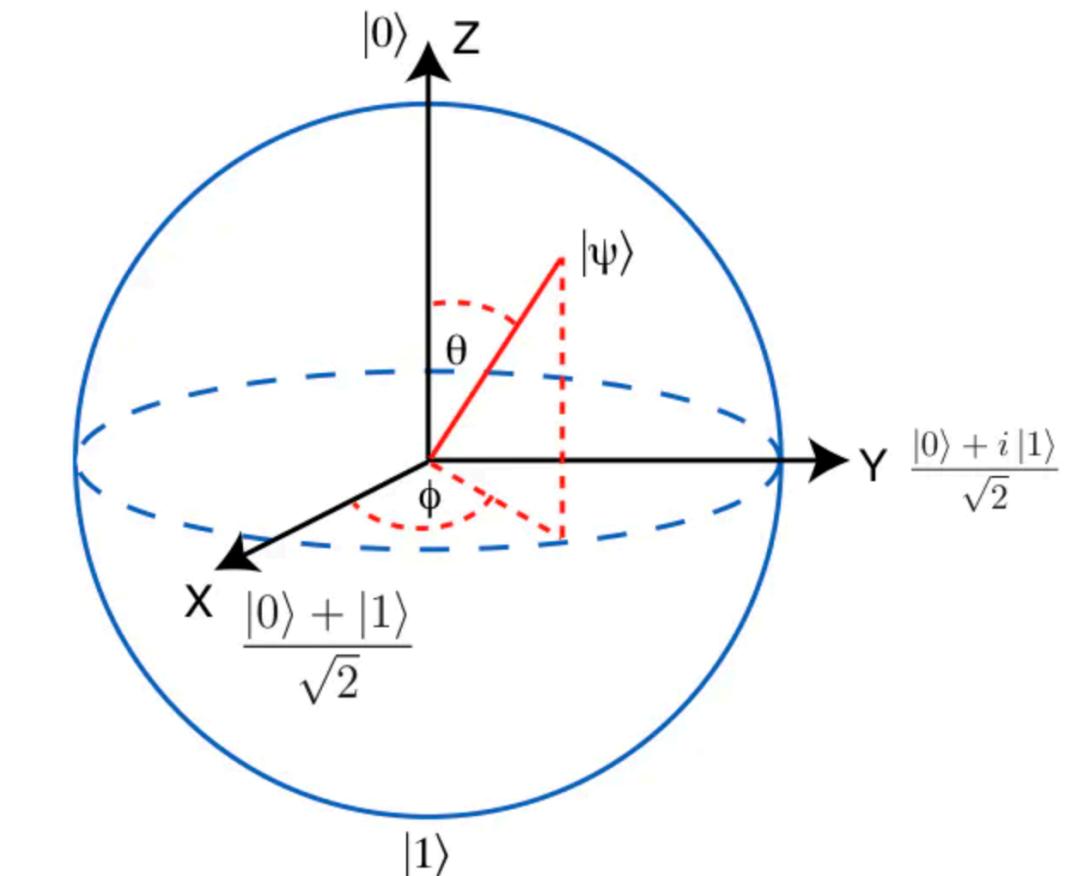
$$\sum_{M=1}^N \frac{2N}{M} \sum_{K=0}^{N-M} \binom{M-1+K}{M-1} \binom{2N-2K-M-1}{M-1} + 3$$

Lattice sites N

2	13
4	117
6	1186
8	12389
10	130338
12	1373466
14	14478659
16	152642789
18	1609284589
20	16966465802
22	178875298901
24	1885860059450
26	19882389467226
28	209617576827139
30	2209972232664381
32	23299464416598565
34	245643376943334578
36	2589787797664451205
38	27303813033527765202
40	287860730073528550874
42	3034880139881813717315
44	31996366649585363767573
46	337333743537756100879133
48	3556468013241491667189146
50	37495403206807318414369013
52	395309407087754112140758506
54	416769827677054670723455386
56	43939528417026911122159567875
58	4632490236322341177261405300845
60	4883977266653121669892843601621
62	51491169380469520010932588566978
64	542865042036754830902171123029413
66	5723359119852258403113870140242562
68	60340668634504203157034795212955546
70	636164220174432677391393885245144899
72	6707000836890367782408329702790282053
74	70711081823674035762289079915112854861
76	745498206168787335244532860509164924714
78	7859695553615600796141819438529000076949
80	82863799918438013669248087407553825812634
82	873622812751857344335782363756954898488282
84	9210497463448372721760862275682543209296131
86	97105137693198992443830116213599788185967005
88	1023767478774681068348344873972808077874359941
90	10793454141511128457710661202368593062740833234
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94	1199716668679035436321002120920649282974697423410
96	12648464729209871610324476298554289482677637882458
98	133351202148602605751037762196292594217046242903875
100	1405905261641056248331375526910312847554957270229877

Quantum simulations

- Decompose a given unitary into elementary gates
- Here $|\psi(t)\rangle = U|\psi(0)\rangle = e^{-iH_S t}|\psi(0)\rangle$
- Trotter decomposition
- Optimization algorithms for a relatively small number of qubits



Qsearch - optimization with a unitary loss function

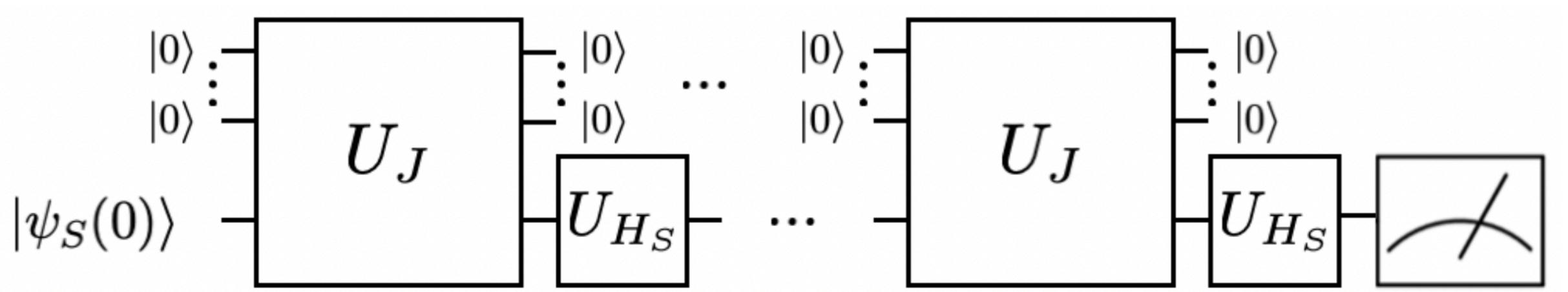
Davis, Smith, Siddiqi et al. '20

Quantum algorithm for non-unitary evolution

- Time evolve the density matrix instead of pure states

$$\frac{d}{dt} \rho_S = -i [H_S, \rho_S] + \sum_{j=1}^m \left(L_j \rho_S L_j^\dagger - \frac{1}{2} \{ L_j^\dagger L_j, \rho_S \} \right)$$

- Stinespring dilation theorem



- Time-irreversible
- Evolve for N_{cycle} in small time steps Δt

where

$$U_{H_S} = e^{-i H_S \sqrt{\Delta t}}$$
$$U_J = e^{-i J \sqrt{\Delta t}}$$

$$J = \begin{pmatrix} 0 & L_1^\dagger & \cdots & L_m^\dagger \\ L_1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L_m & 0 & \cdots & 0 \end{pmatrix}$$

see also e.g. Cleve, Wang '16
Hu, Xia, Kais '20
Jong, Metcalf, Mulligan, Ploskon, FR, Yao '20
Metcalf, Kemper, Jong et al. '21

Trotter decomposition

- Need to map both U_{H_S} , U_J to quantum gates

$$H = \sum_j H_j = a_j P_j \quad \begin{matrix} \text{with Pauli strings} \\ \text{and coefficients} \end{matrix} \quad P_j : \{\mathbb{1}, X, Y, Z\}^{\otimes n}$$
$$a_j = \frac{1}{2^n} \text{tr}[P_j H]$$

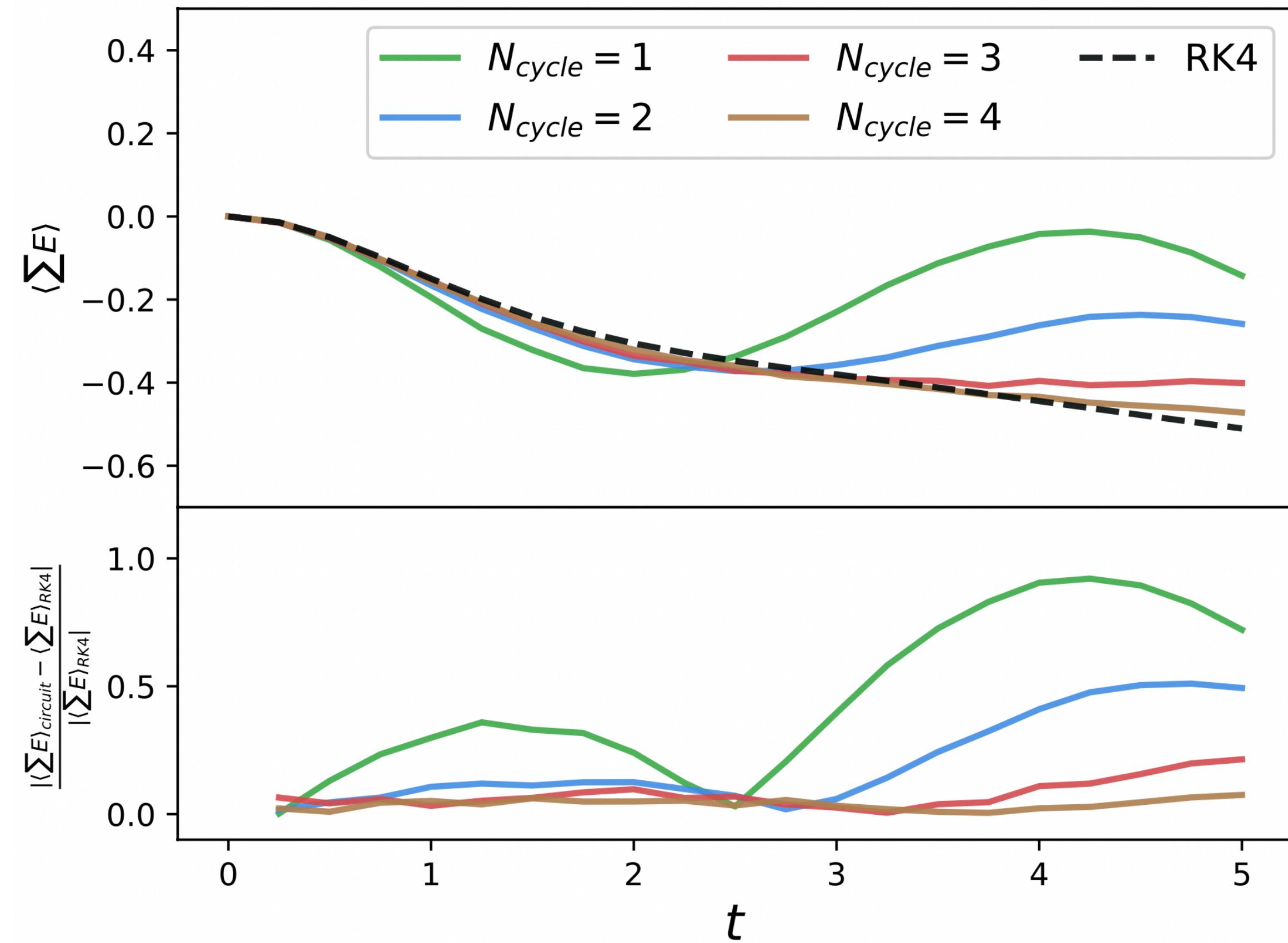
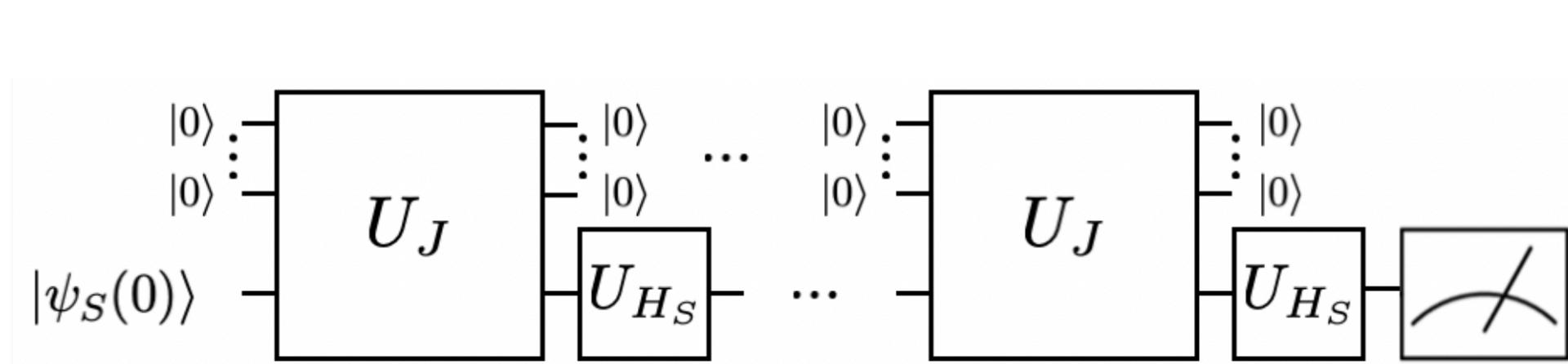
- Trotter decomposition

$$U_1(t) = \prod_j e^{-iH_j t} \quad \text{each factor can be mapped to quantum gates}$$

→ evolve to t in r time steps with error $\|e^{-iHt} - U_1^r(t/r)\| \leq \frac{1}{2} \sum_{j>k} \| [H_j, H_k] \| \frac{t^2}{r}$

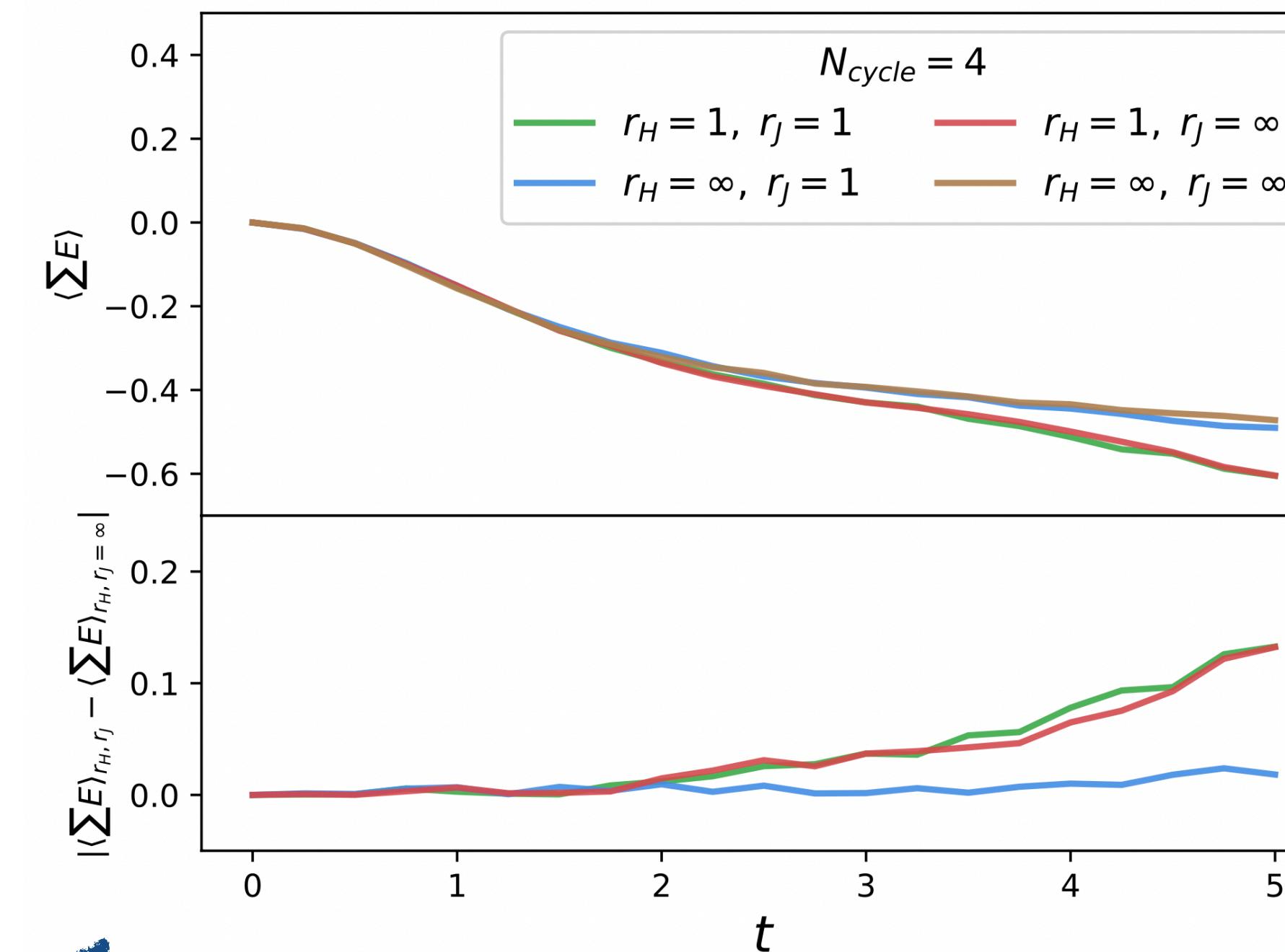
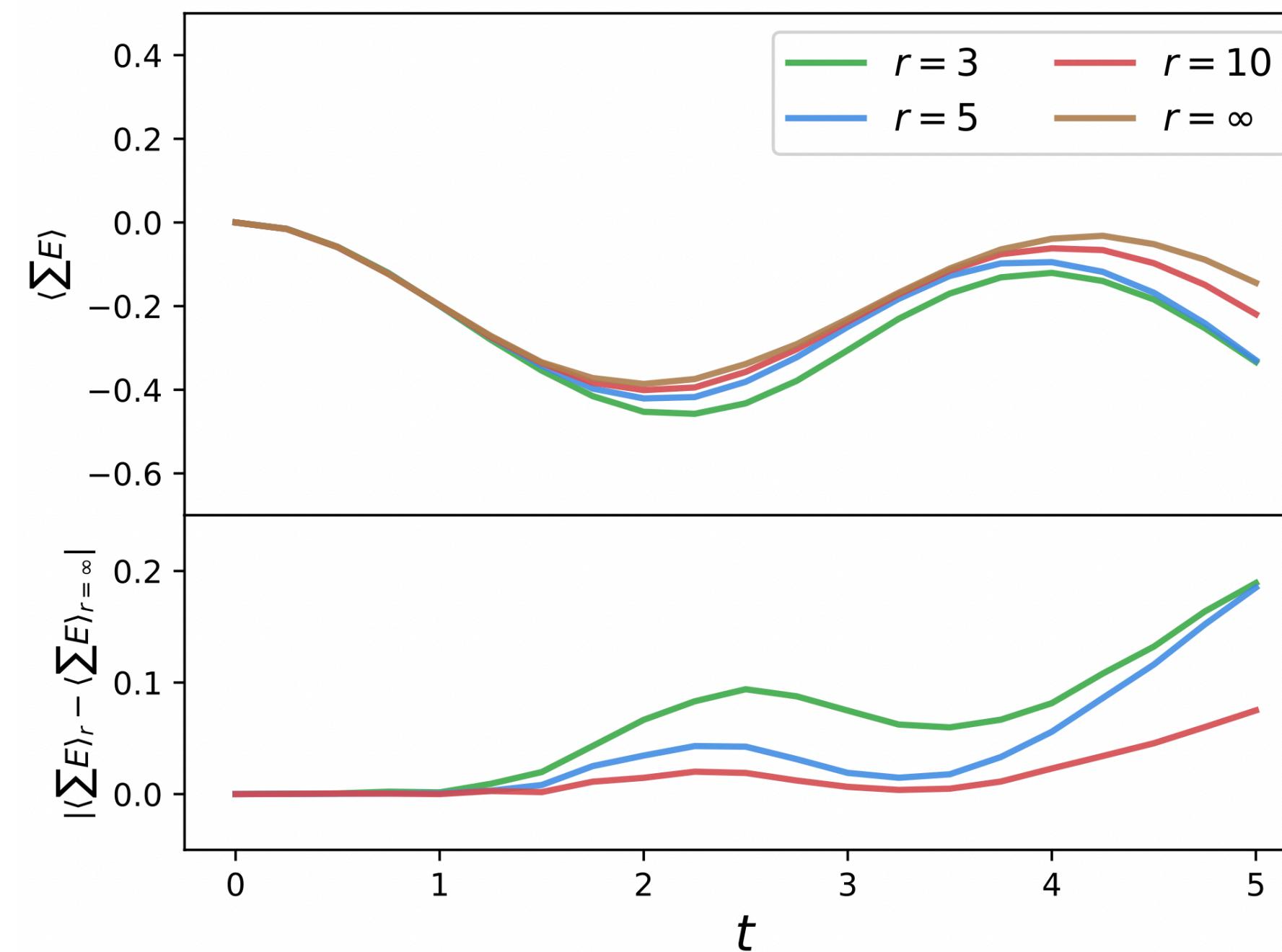
Non-unitary evolution & Trotter errors

I. Necessary number of cycles



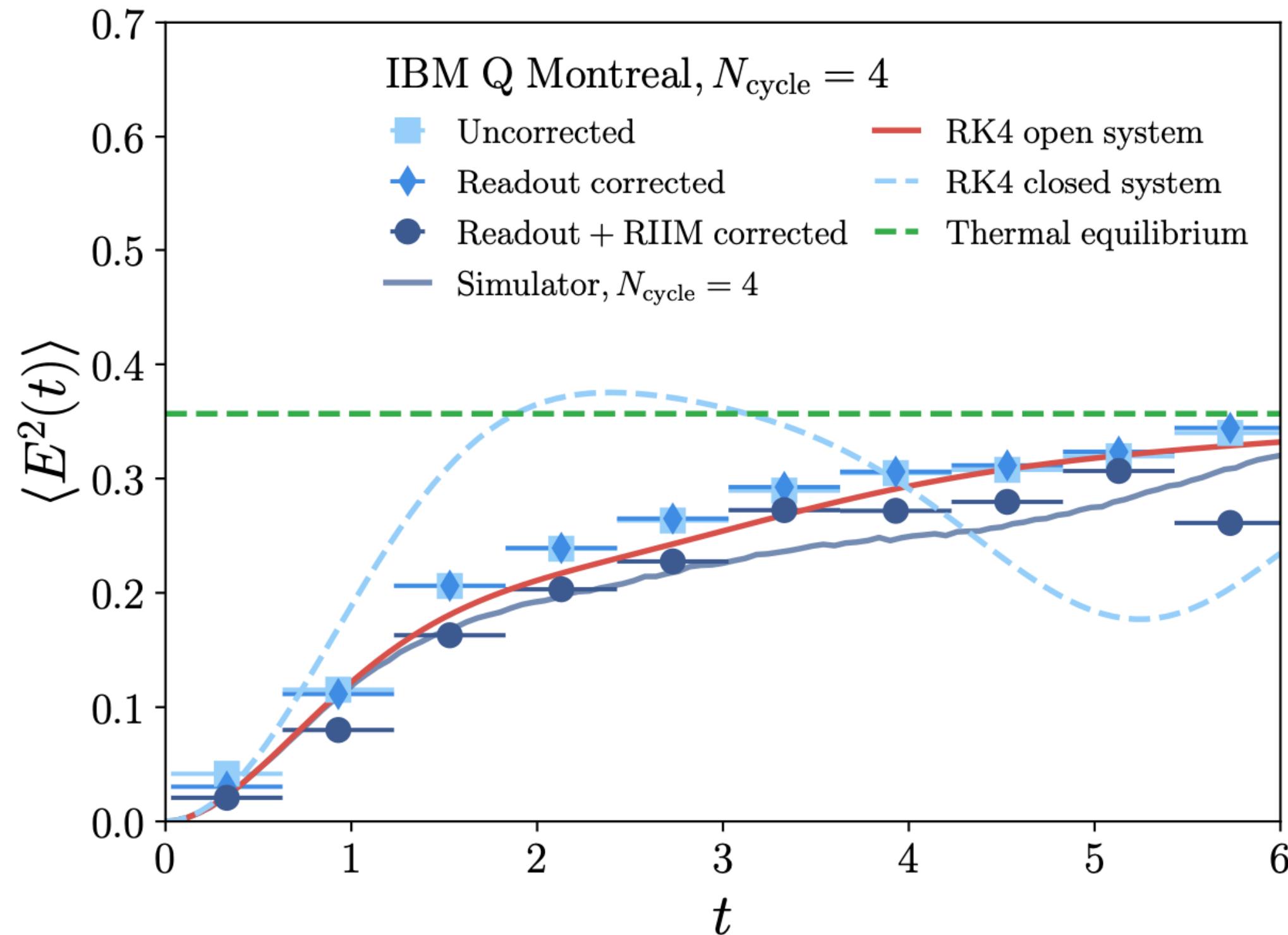
Non-unitary evolution & Trotter errors

2. Trotter errors for the closed & open system



More gates & qubits but Trotter error not necessarily larger!

Small-scale simulation on IBMQ



- Up to 4 cycles with each 50 CNOT and 500 single-qubit gates
- Approximate preparation of thermal state from non-equilibrium dynamics

