A Quantum Information Perspective on Meson Melting

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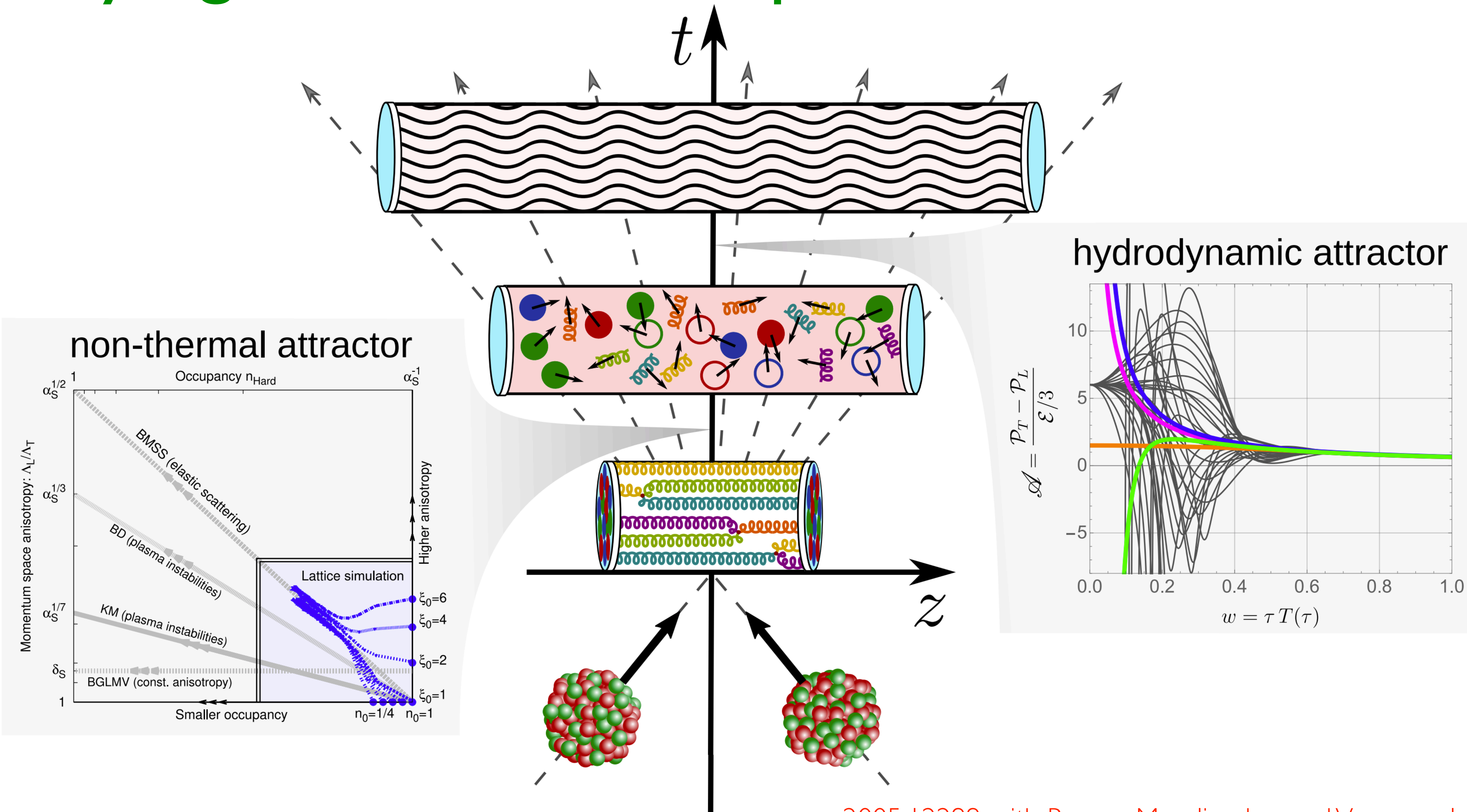
1912.08836
2206.10528 with Bañuls, Jansen, Knaute, Svensson

Introduction

2005.12299 with Berges, Mazeliauskas and Venugopalan

Why I got interested in quantum tech?



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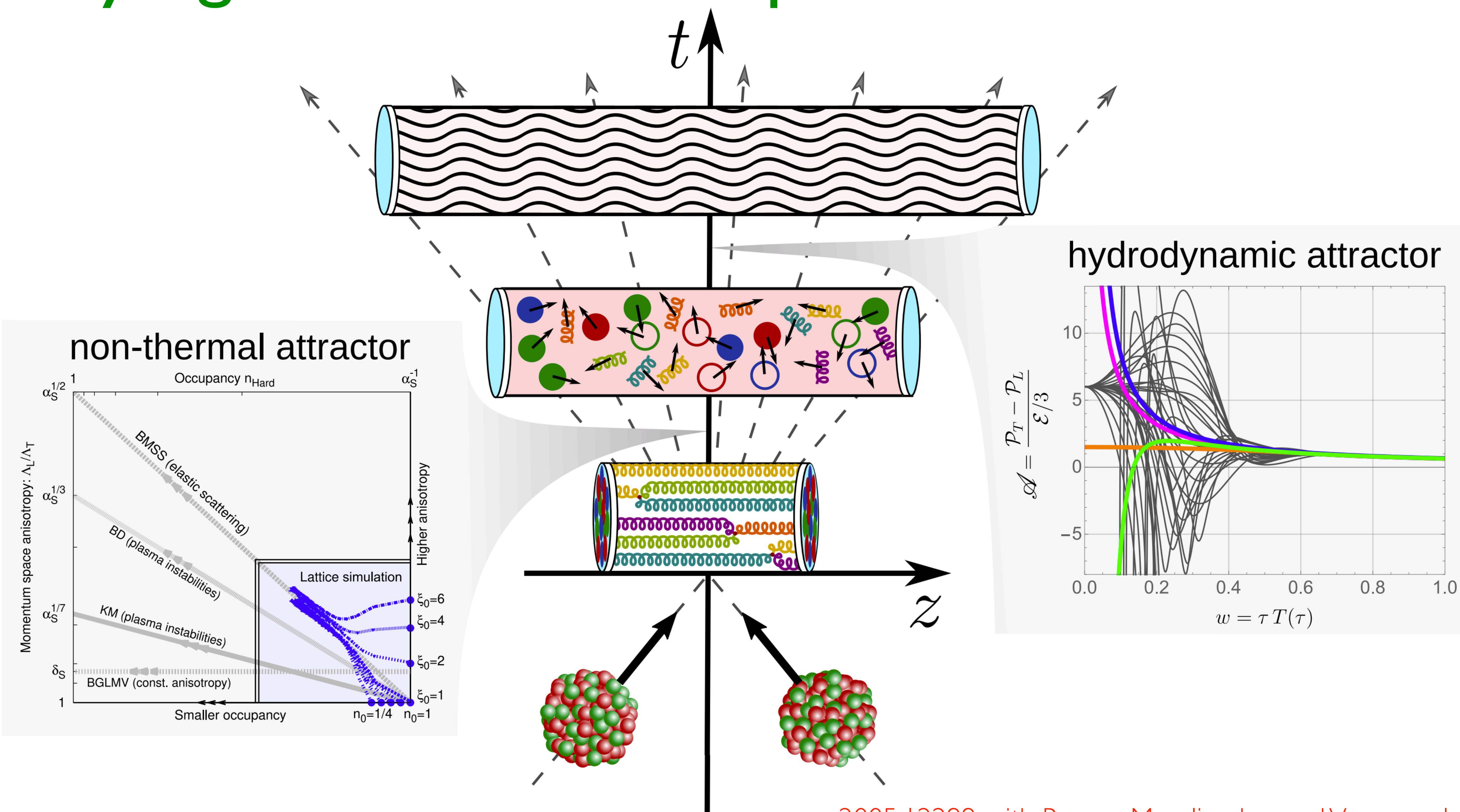
Why I got interested in quantum tech?



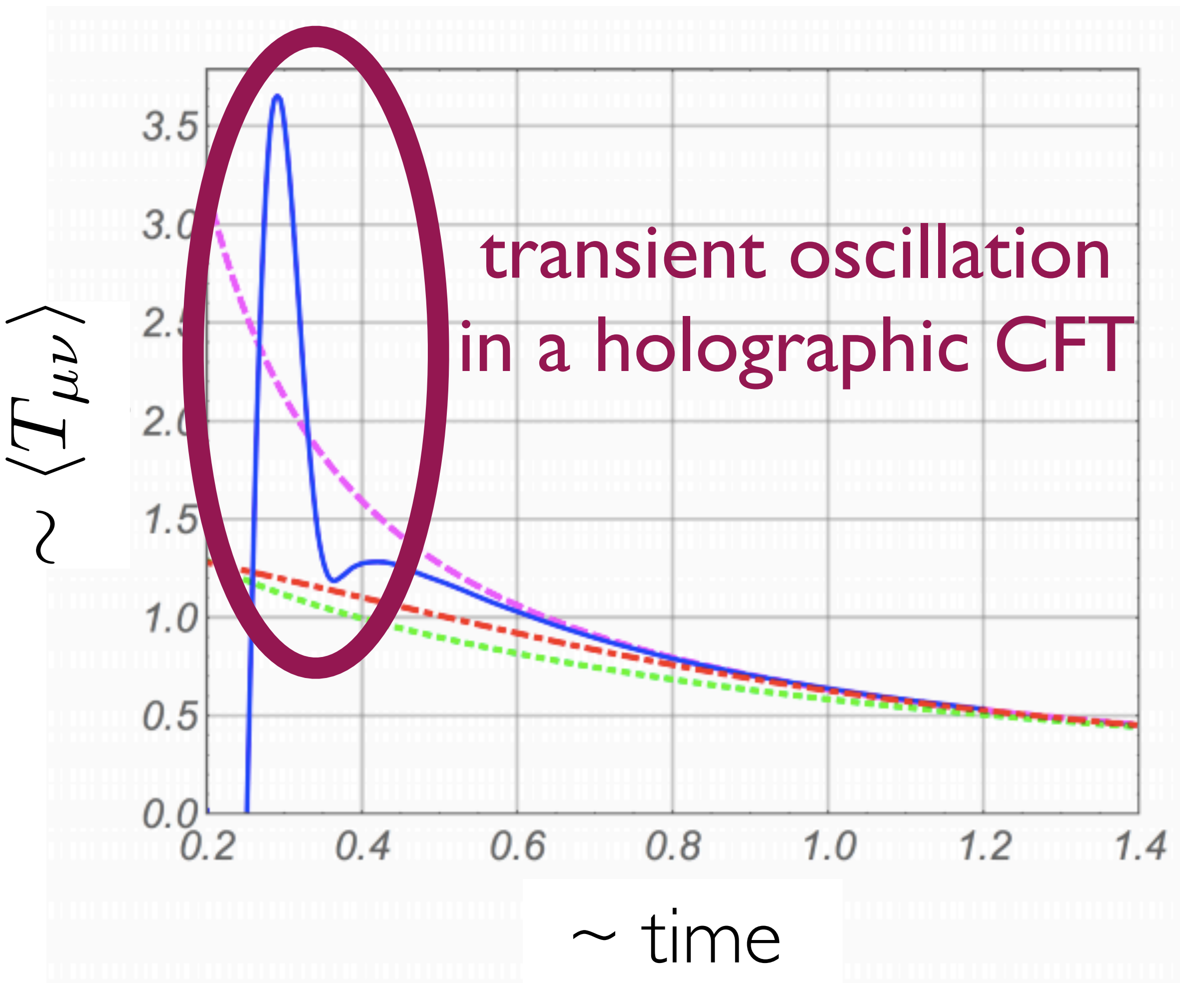
2005.12299 with Berges, Mazeliauskas and Venugopalan

No general purpose first principle tool for dynamics akin to what lattice is for QCD thermodynamics at low baryon chemical potential

One motivation: holographic heavy-ion collisions

1103.3452 with Janik and Witaszczyk

holographic studies of
heavy-ion collisions
=
studies of local thermalization in
(1+3)D strongly-coupled gauge
theories in the `t Hooft limit



The approach to hydrodynamics in strongly-coupled QFTs was empirically seen to be governed by a set of **exponentially decaying oscillatory modes**

The key question behind 1912.08836

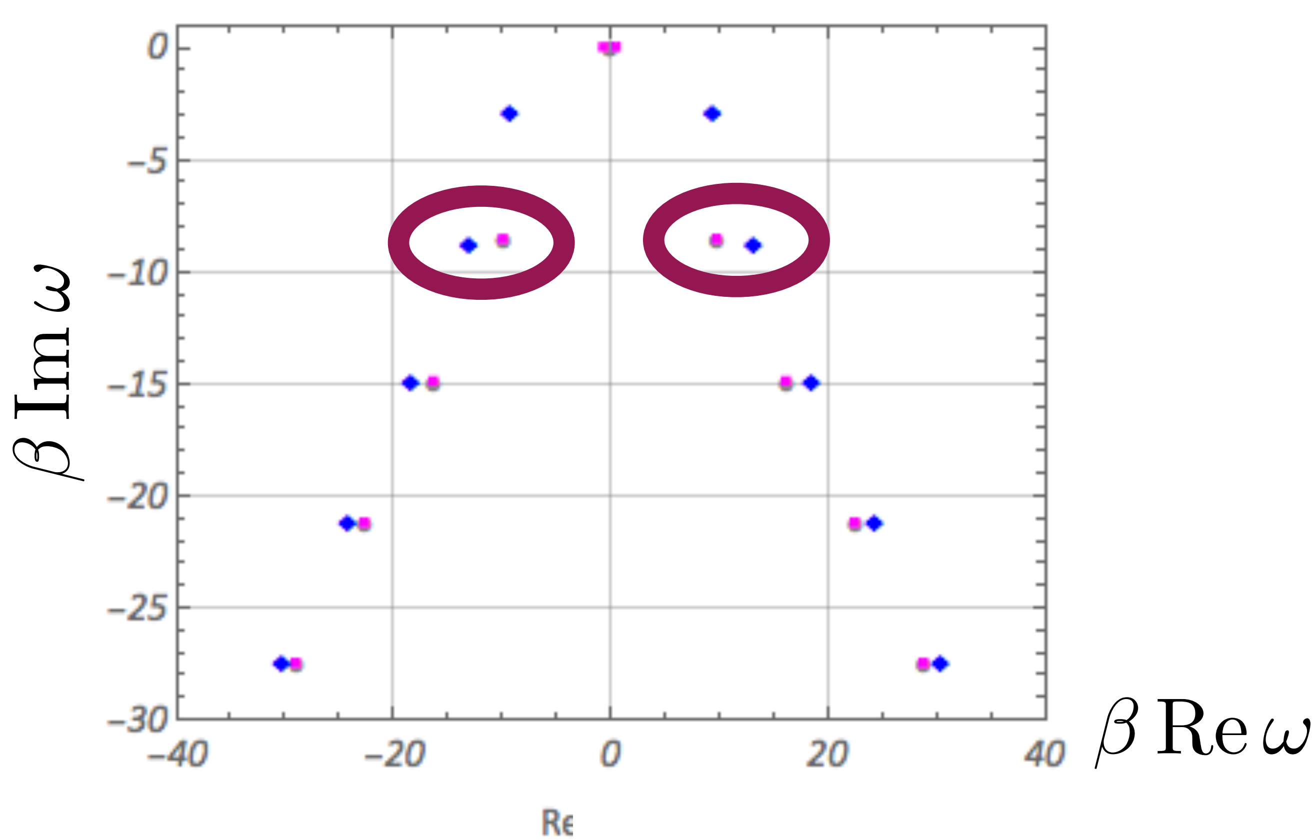
Such modes arise in linear response theory on top of $\rho_\beta \sim e^{-\beta H}$:

$$\delta\langle \mathcal{O}(t,p)\rangle = \int_{-\infty}^{\infty} d\omega\, e^{-i\,\omega\,t}\, G_R^{\mathcal{O}}(\omega,p)\, \mathcal{J}(-\omega,-p)$$

with $G_R^{\mathcal{O}}(t,x) = i\,\theta(t)\,\mathrm{tr}\,(\rho_\beta[\mathcal{O}(t,x),\mathcal{O}(0,0)])$

$\mathcal{O} = T_{\mu\nu}$

in a hCFT*:



$\beta\,p = 0.1$

$\beta\,p = 1$

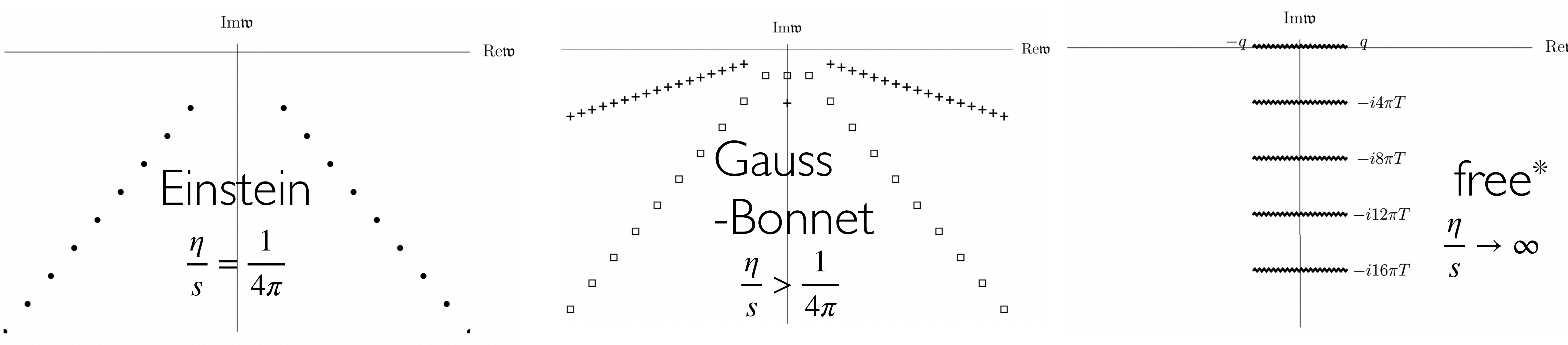
$\beta\,p = 10$

Motivating question: what is the singularity structure of $G_R^{\mathcal{O}}(\omega,p)$ in genuine interacting QFTs that do not have a large number of dofs & strong coupling?

What does it have to do with gravity?

The Xmas tree structure is sensitive to higher curvature corrections

1605.02173 by Grozdanov, Kaplis & Starinets



QNMs are not only smoking guns for black holes at LIGO, but in the future maybe also in quantum simulators of holographic spacetimes

2303.09974 by Biggs and Maldacena, 2303.11534 by Maldacena

$$\frac{T}{\lambda^{1/3}} = 0.3\ , \qquad \frac{\omega}{T} = 0.8\ , \qquad N = 16 \tag{15}$$

they found a result that only has a 13% difference with (14), within the numerical error of the computation. Notice that (14) is a prediction with no free parameters. They also found that as $T/\lambda^{1/3}$ gets larger there are larger deviations from gravity, as expected. For these values of ω/T, the corrections to the solution (11) are small [12].

The parameters in (15) suggest the following very rough counting for the number of qubits for a quantum simulation of the model in a regime where we start getting agreement with gravity. We have $8N^2$ qubits from the $16N^2$ Majorana fermions. For the bosons we have an infinite dimensional Hilbert space, but the important excitation levels are expected to be those up to $n \sim \lambda^{1/3}/\omega$. Therefore, we expect a number of qubits of the form

$$n_q \sim N^2 \left[8 + 9 \log_2 \left(\frac{\lambda^{1/3}}{\omega} \right) \right] \sim 7,000\ , \qquad \text{for} \quad N = 16\ , \qquad \frac{\lambda^{1/3}}{\omega} \sim 4 \tag{16}$$

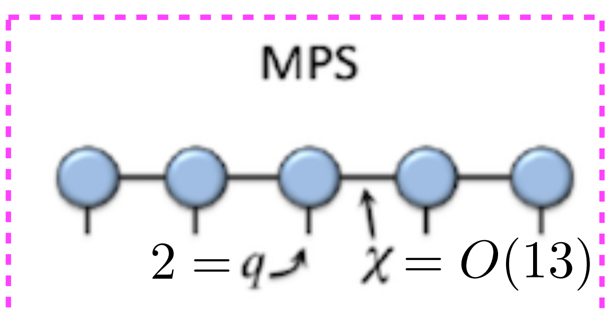
Setup

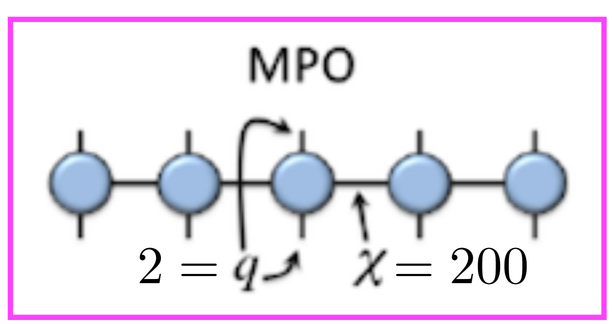
Tensor networks see, e.g., 1306.2164 by Orus

Hilbert space of quantum systems with dimension e^V is beyond capacities of classical computers, but interesting physics lies mostly* in its tiny corner

Ground states, low lying excited states and high-β thermal states of local Hamiltonians are generically characterized by area-law entanglement entropy

Tensor networks: building many-body states from smaller building blocks (tensors) in a way dictated by anticipated or proven entanglement structure:





PEPS, MERA, cMPS, cMPO, cMERA

+ TEBD

see J. Zakrzewski talk from Thu

Tensor networks are actual tools for numerical modelling and our ability to simulate depends on sizes of tensors needed to reach convergence and D

System

We will look at the quantum Ising model:

$$H = -J\left(\sum_{j=1}^{L-1}\sigma_z^j\sigma_z^{j+1} + h\sum_{j=1}^{L}\sigma_x^j + g\sum_{j=1}^{L}\sigma_z^j\right)$$

For $L\to\infty$, $h\approx 1$ and $g\approx 0$, the IR is described by a QFT:

fermion mass

$$H = \int_{-\infty}^{\infty} dx\,\{\frac{i}{2\pi}\left[\frac{1}{2}\left(\psi\partial_x\psi - \bar{\psi}\partial_x\bar{\psi}\right) - M_h\bar{\psi}\psi\right] + 0.062\,M_g^{15/8}\,\sigma(x)\}$$

Ising CFT (free Majorana fermions) interaction (fermionic string)

where $M_h \equiv 2J|1-h|$ and $\sigma_x^j \sim i\bar{\psi}\psi = \mathcal{O}_{\Delta=1}$

$M_g \equiv 5.416\,J\,|g|^{8/15}$ and $\sigma_z^j \sim \sigma = \mathcal{O}_{\Delta=\frac{1}{8}}$

More precisely, the QFT limit is $M_h/J\to 0,\ \beta\,J\gg 1$ with M_h/M_g fixed

More on Ising QFT

fermion mass

$$H = \int_{-\infty}^{\infty} dx \left\{ \frac{i}{2\pi} \left[\frac{1}{2} \left(\psi \partial_x \psi - \bar{\psi} \partial_x \bar{\psi} \right) - M_h \bar{\psi} \psi \right] + \mathcal{C} M_g^{15/8} \, \sigma(x) \right\}$$

Ising CFT (free Majorana fermions) interaction (fermionic string)

Integrable: ■ = CFT or ■ + ■ = free QFT or ■ + ■ = interacting QFT

Non-integrable: ■ + ■ + ■

Adding arbitrarily small g introduces a confining potential between fermions

McCoy & Wu, Zamolodchikov, …

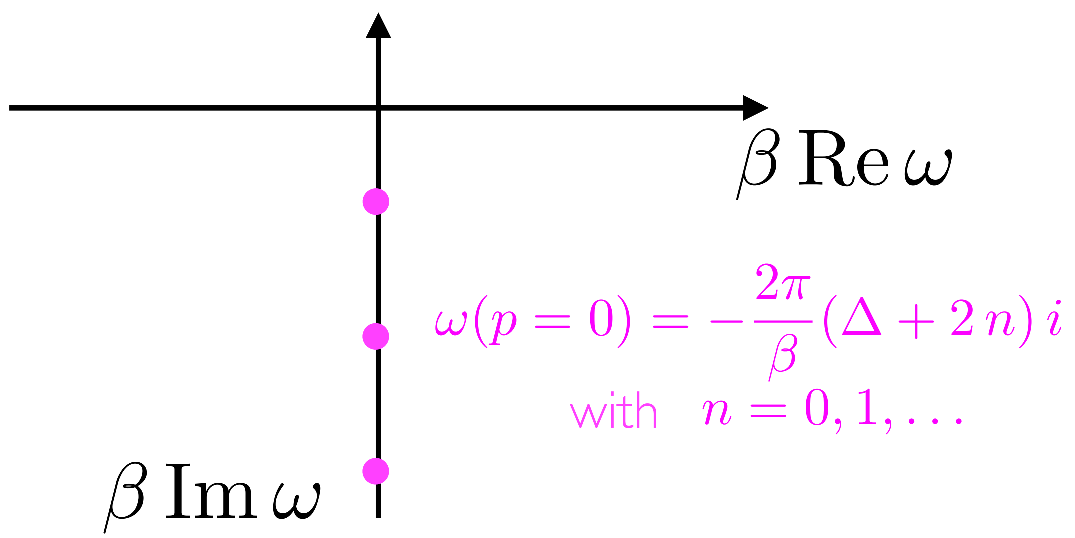
Bound states: mesons → integrable case: 8 stable mesons

Bound states: mesons → non-integrable case: a few stable mesons + unstable ones

Retarded thermal correlator in a (1+1)D CFT

From here onwards we focus for simiplicity on $p = 0$

In 1+1 dimensional CFT, the retarded correlator of a primary operator $\mathcal{O}_\Delta$ at any β is fixed by conformal invariance; its frequency singularities are



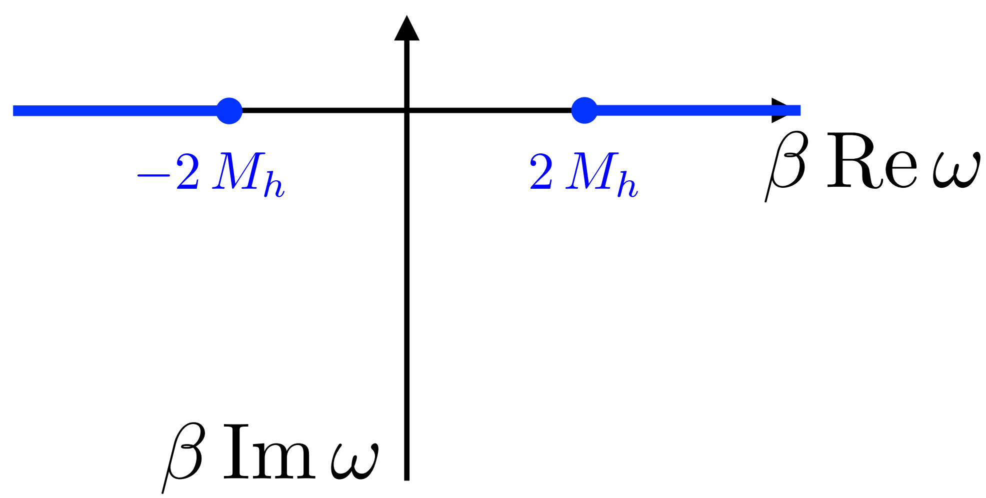
Since characteristic decay scale is $\sim 1/\Delta$ and TN can only evolve for limited time due to the growth of entanglement, we focus on $i\bar{\psi}\psi$ over σ :

$$G_R^{\mathcal{O}_{\Delta=1}}(t > 0, p = 0) = -\frac{4\pi}{\beta} e^{-\frac{2\pi}{\beta} t} \left(1 - e^{-\frac{4\pi}{\beta} t}\right)^{-1}$$

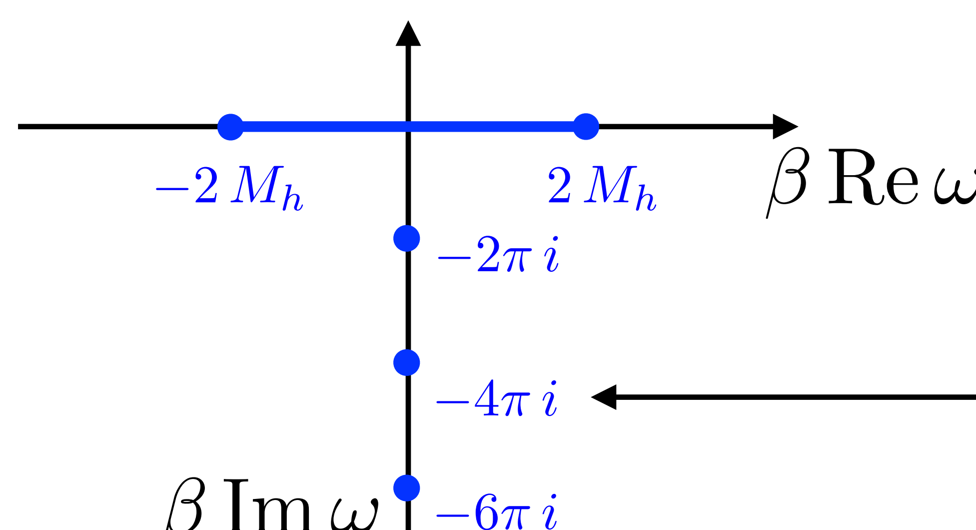
Retarded thermal correlator for free fermion QFT

$G_R^{i\bar{\psi}\psi}(\omega, p=0)$ can be thought of as a sum of exchanges of fermionic pairs of vanishing net momentum, but arbitrary relative momentum within each pair

This leads to the following structure of singularities:



But branch-cut is a choice how we connect branch points; other choice:



one can think of them as arising from Fermi-Dirac distribution as single poles (Matsubara frequencies):

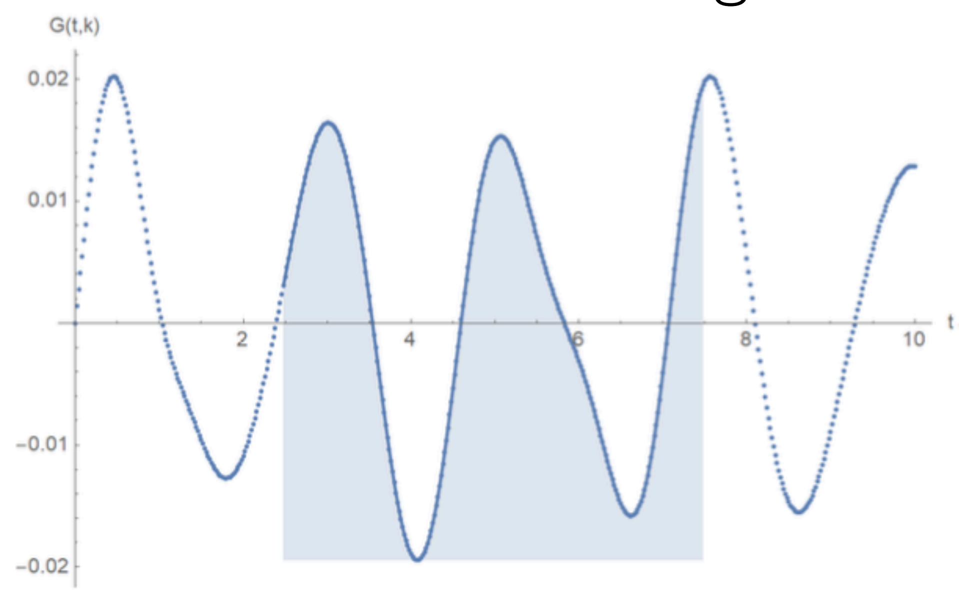
$$\int \mathrm{d}k\, e^{-i\,2\,\omega(k)\,t}\, \frac{1}{1+e^{\beta\,\omega(k)}}\,\cdots$$

Signal analysis

TNs are going to give us a correlator over a finite range of time bounded by the growth of entanglement and the signal is not periodic

Perfect tool: Prony analysis, which assumes the signal has the form $\sum_n c_n \, e^{\omega_n \, t}$

Uniformly sampled signal:



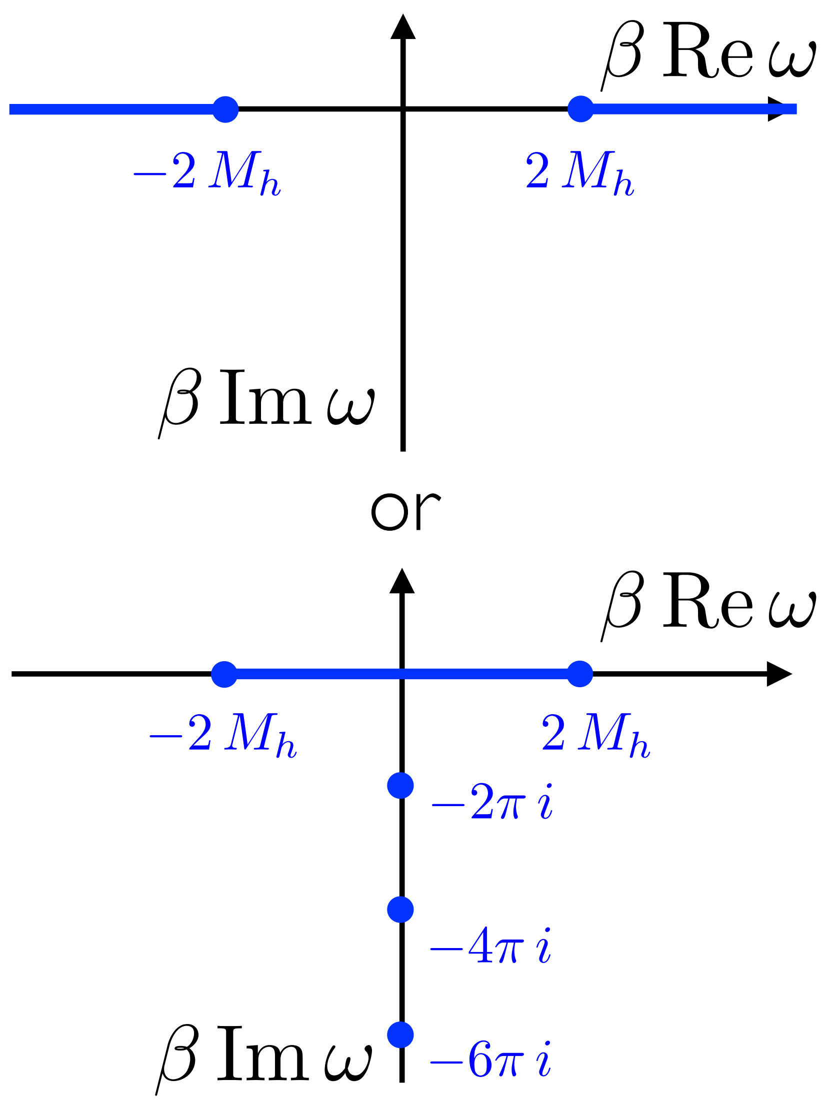
We first find frequencies ω_n and then fit amplitudes

Features that stay stable on many windows are not noise → QFT physics / artefacts

Benchmarks

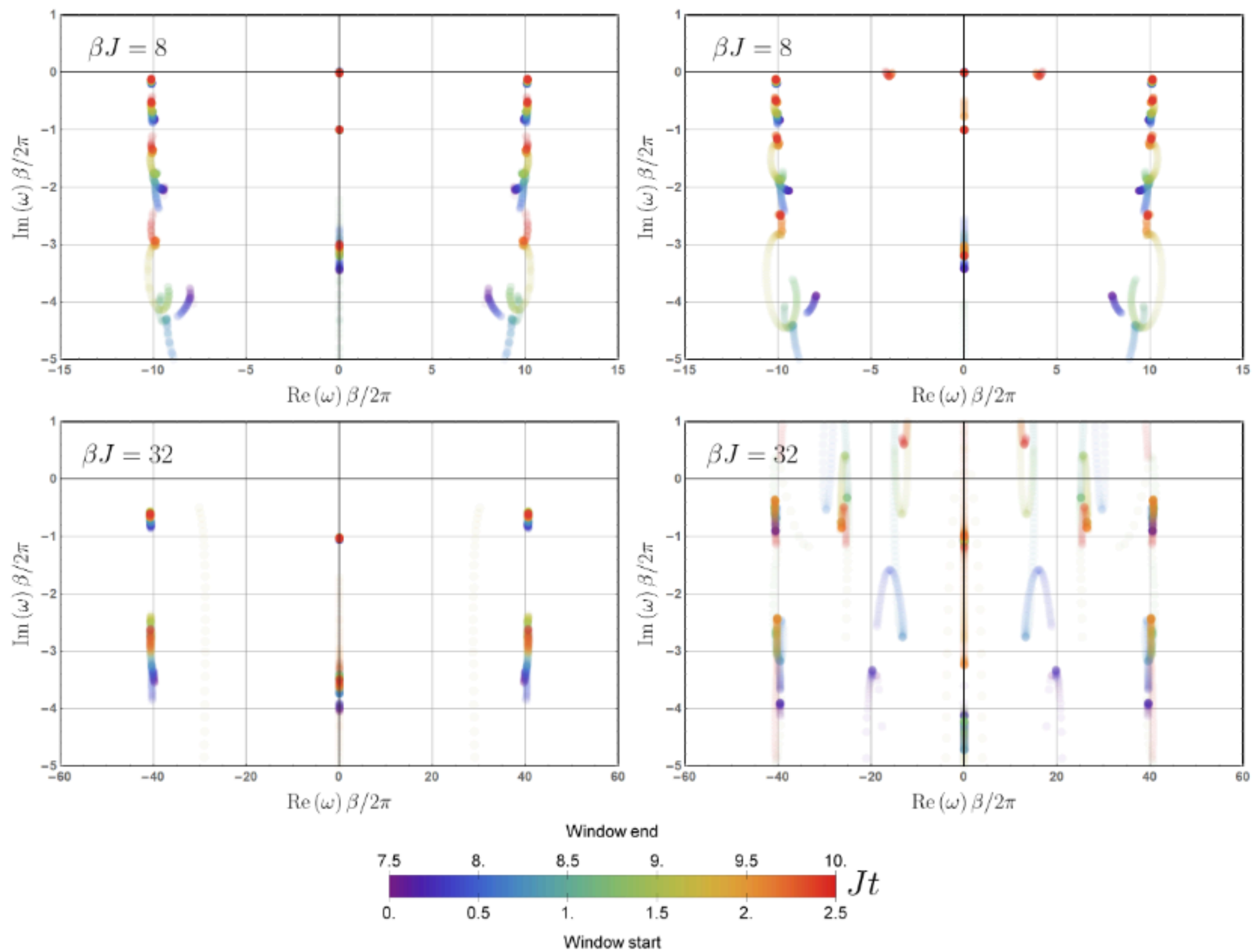
Benchmark for $\beta\, M_h = 0.2$ and $M_g = 0$

QFT:



free fermions via Jordan-Wigner:

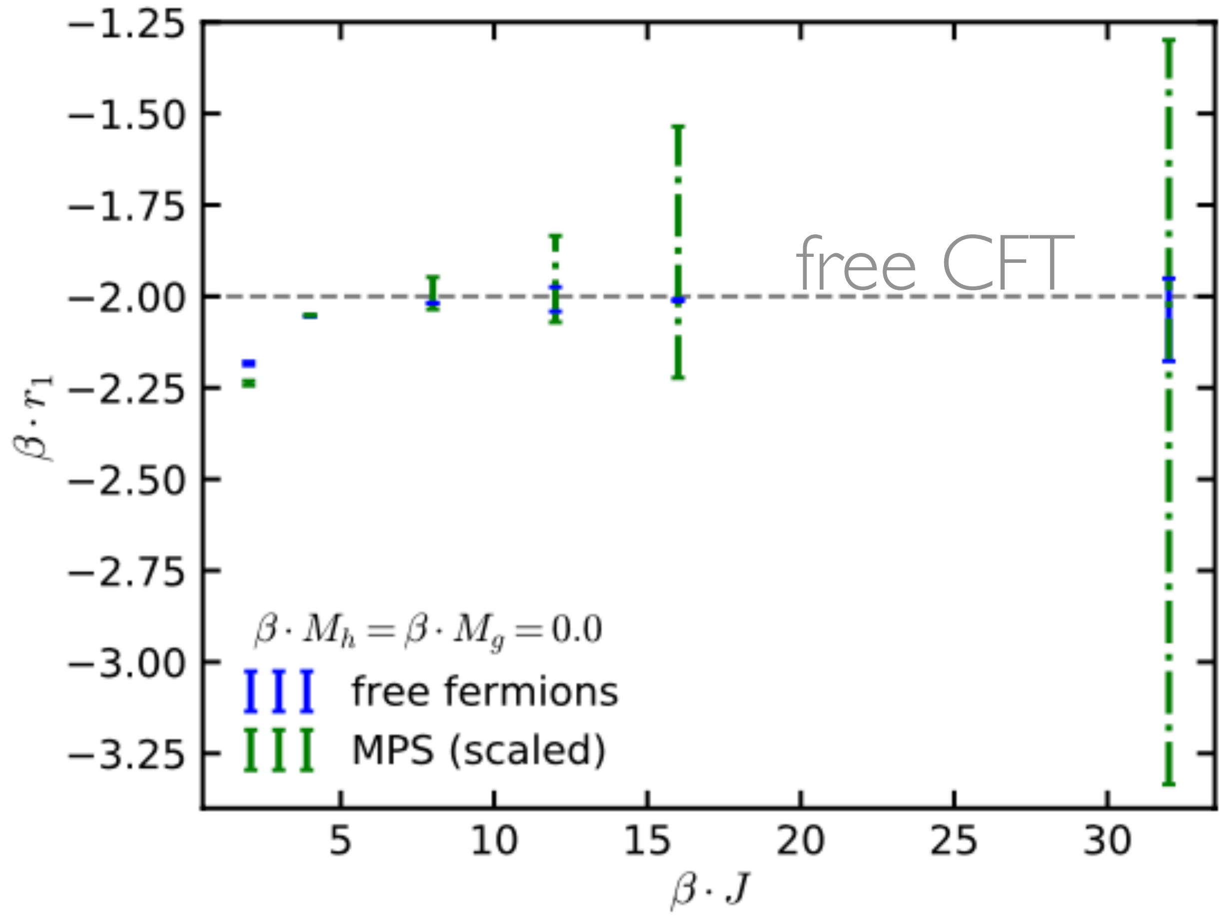
MPO:

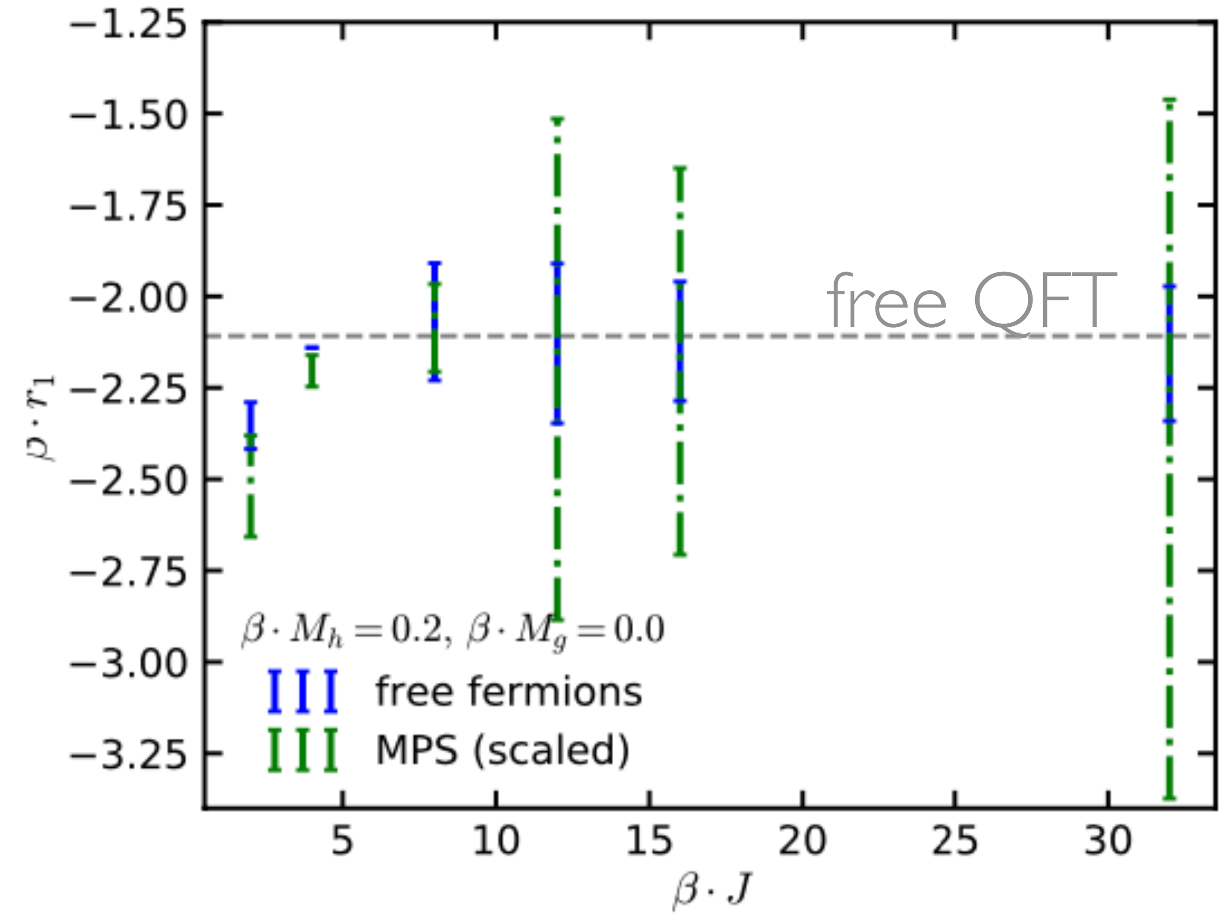


Can one claim victory in this case?

Benchmark: transients

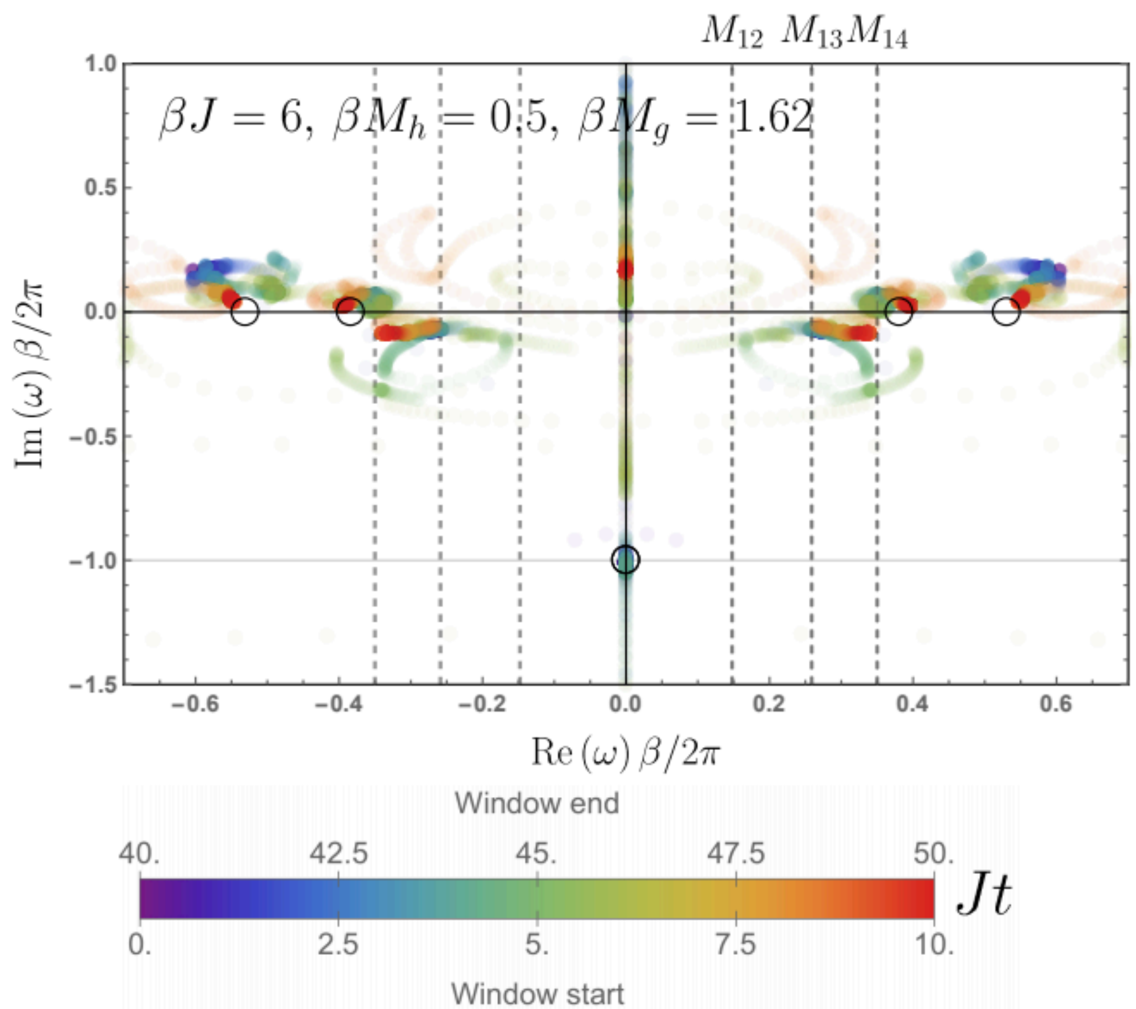
The frequencies of transients come okay, but if they represent QFT or the lattice model can be checked using the value of associated residue:

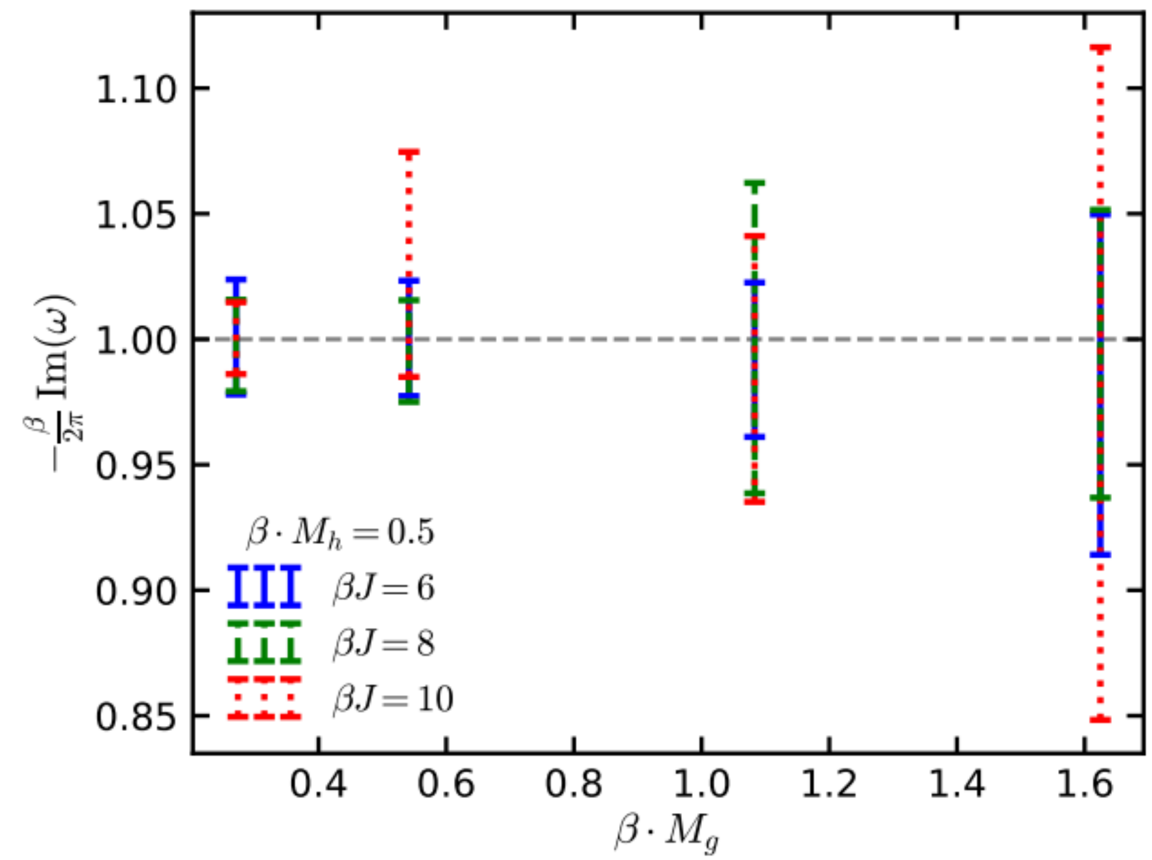




Predictions in interacting cases

Transients





Quite puzzlingly, despite trying really hard, we have never seen the leading transient frequency to move; we know in (1+3)D RG flows they do change

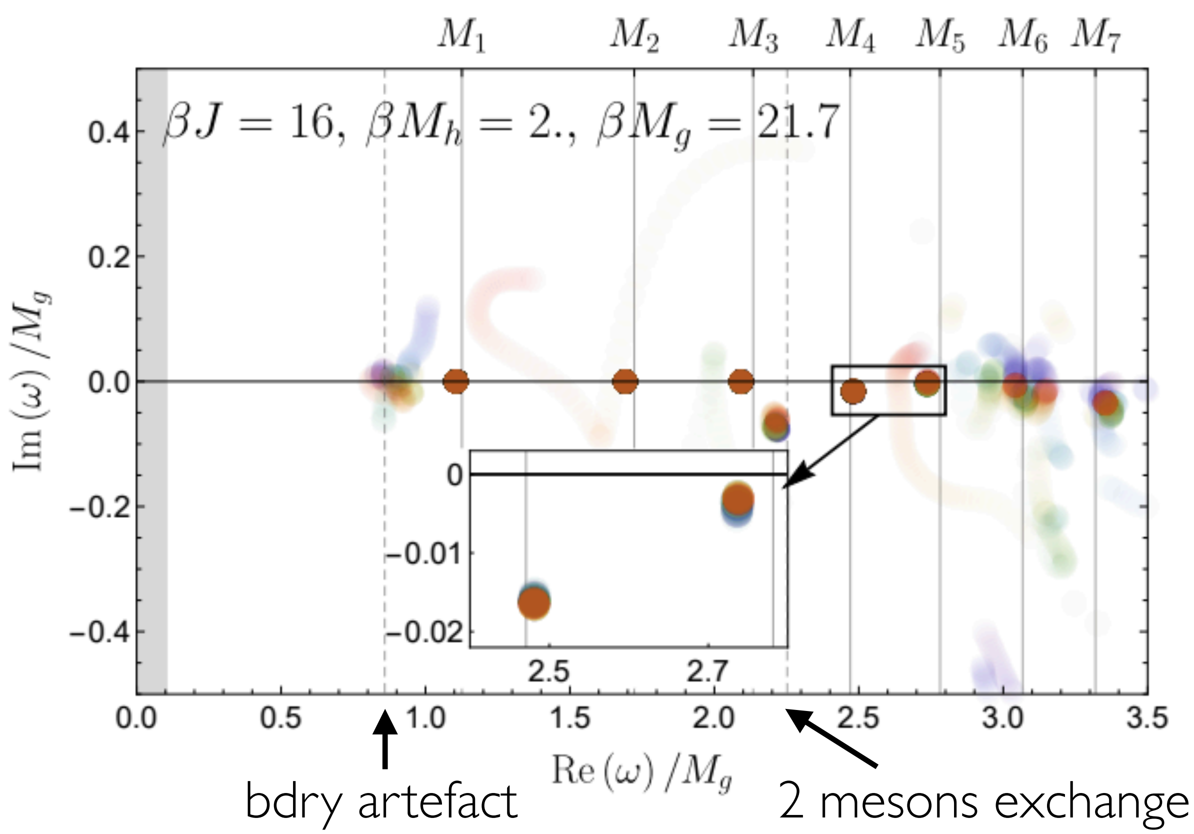
1503.07114 with Buchel & Myers

Simple argument: consider RG flow interpolating between two (1+1)D CFT; $\mathcal{O}$ has $\Delta_{UV} \to \Delta_{IR}$; then the leading transient $-\frac{2\pi\Delta_{UV}}{\beta} i \to -\frac{2\pi\Delta_{IR}}{\beta} i$

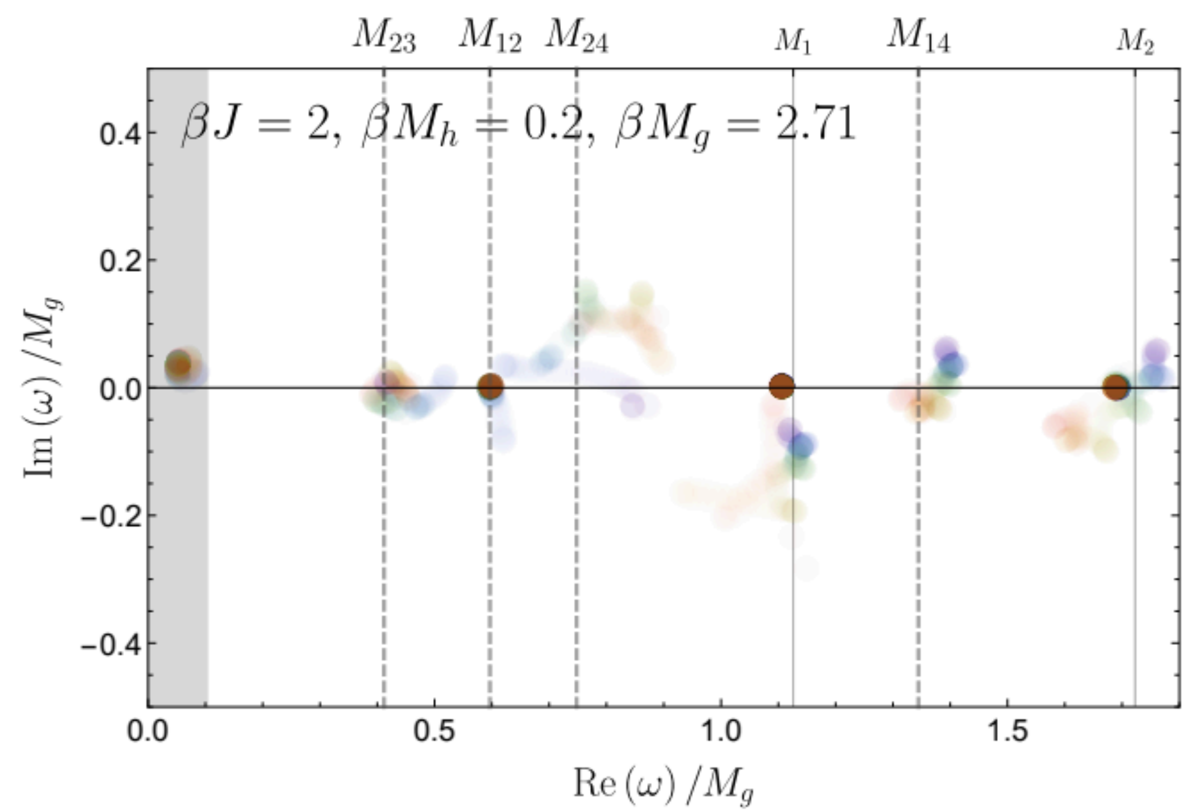
Mesons: positions

Upon turning on M_g, non-perturbative meson states appear; in the integrable case ($M_h = 0$) their masses are known exactly; otherwise approximately:

low temperature, ferro



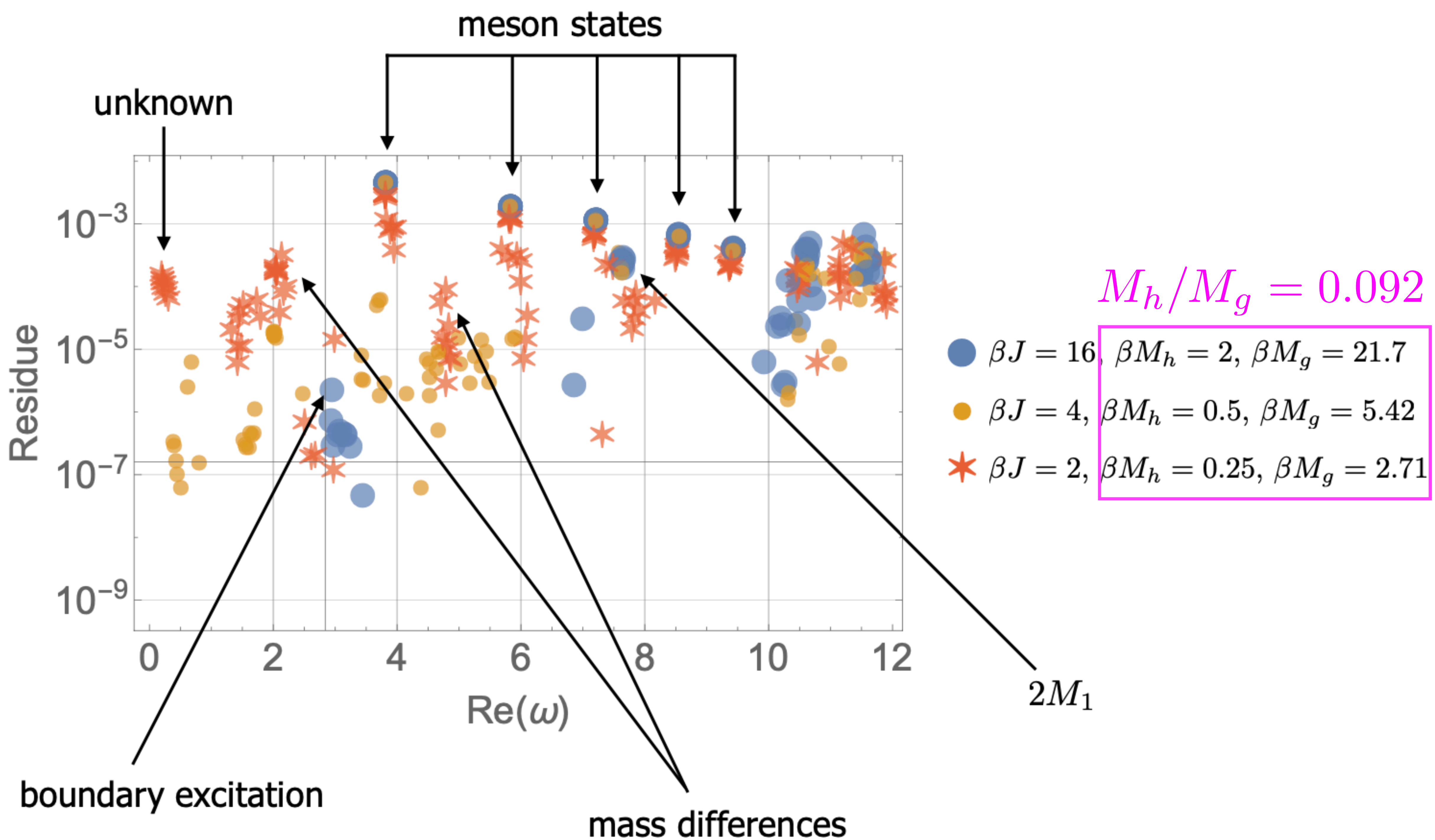
high temperature, ferro



In the integrable ~vacuum case we get:

	M_2/M_1	M_3/M_1	M_4/M_1	M_5/M_1	M_6/M_1	M_7/M_1
TN	1.6147(7)	1.962(1)	2.413(2)	2.936(3)	3.165(6)	3.52(3)
E_8	1.6180	1.9890	2.4049	2.9563	3.2183	3.891

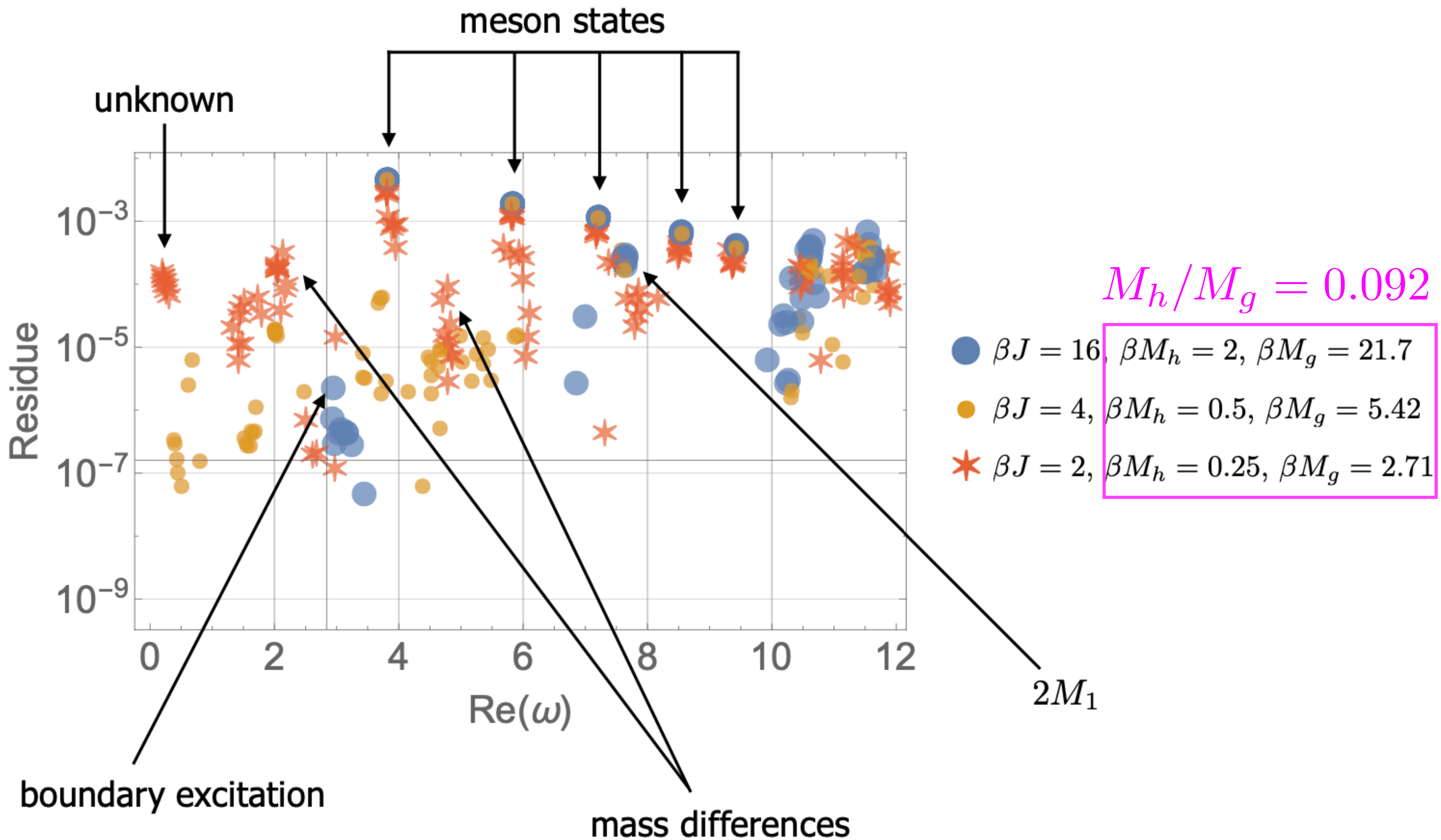
Masses: residues



Do Ising mesons melt?

2206.10528 with Bañuls, Jansen, Knaute, Svensson

We saw a decay of residues of the correlator…



… however, these simulations become unsustainable at higher temperatures

The key idea of 2206.10528

While we cannot compute the correlator at high enough temperatures using TNs, we can use similar methods to study other quantities in such a regime

statics

$$s_2 = -\frac{1}{L} \log \frac{\mathrm{Tr}\, \rho_\beta^2}{(\mathrm{Tr}\, \rho_\beta)^2}$$

dynamics

reflected entropy after a quench

Statics — free fermions

$$s_2(\beta) = \log 2 - \frac{1}{\pi}\int_0^\pi \mathrm{d}k \log\left[\frac{\cosh(\beta\epsilon_k)}{\cosh^2(\frac{\beta\epsilon_k}{2})}\right] \text{ with } \epsilon_k = 2J\sqrt{1+h^2-2h\cos(k)}$$

low temperature ↙ ↘ high temperature

$$s_2 \sim \frac{\mathrm{e}^{-2\beta J|h-1|}}{\sqrt{\frac{\pi\beta J h}{|h-1|}}}$$

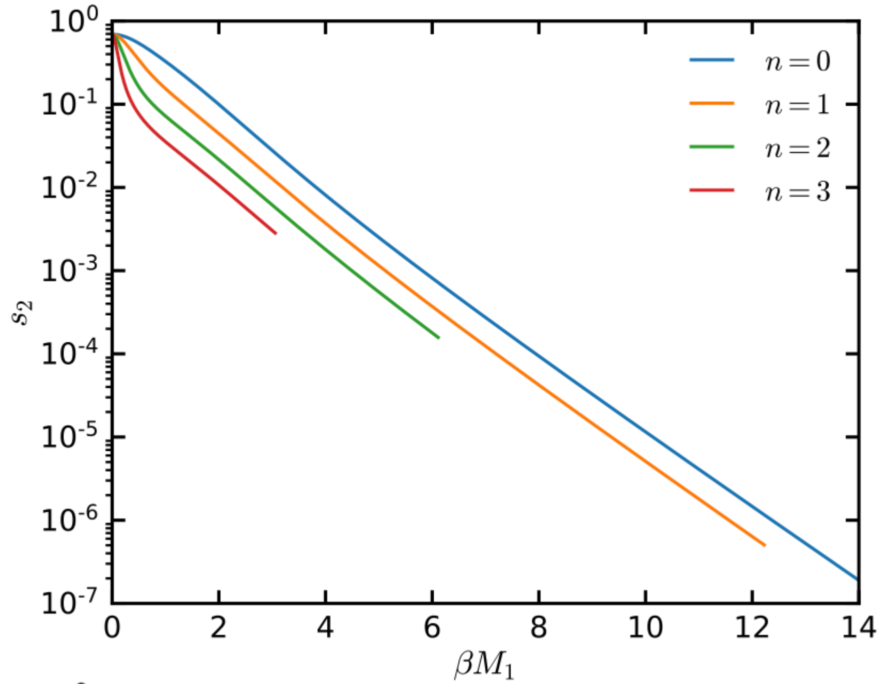
$$s_2 \sim \frac{\pi}{16}\left(\frac{1}{\beta J} + \frac{1}{16(\beta J)^3} + \dots\right)$$

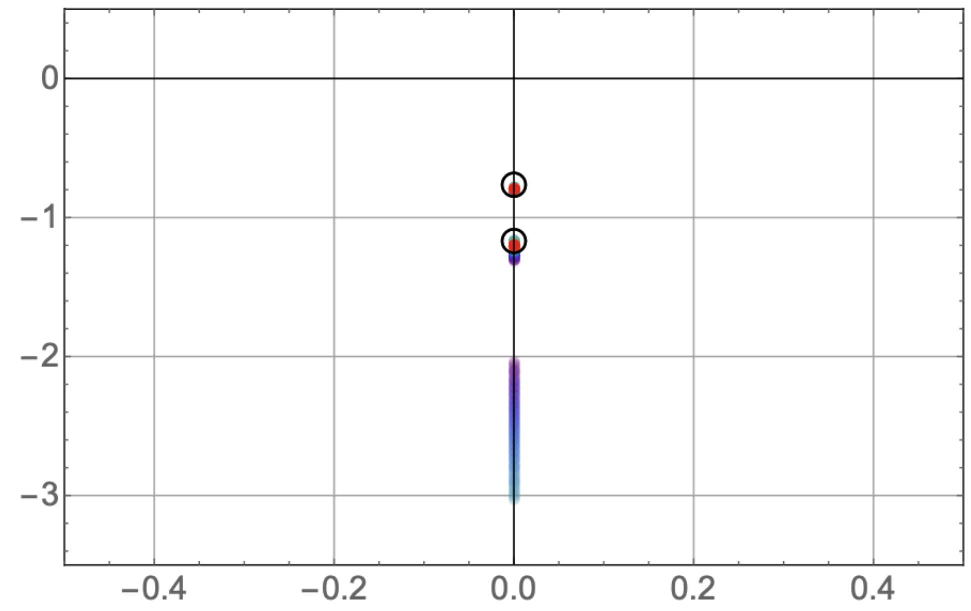
There are no mesons here, but we this builds for us an intuition what to expect in the interacting case

Statics with mesons

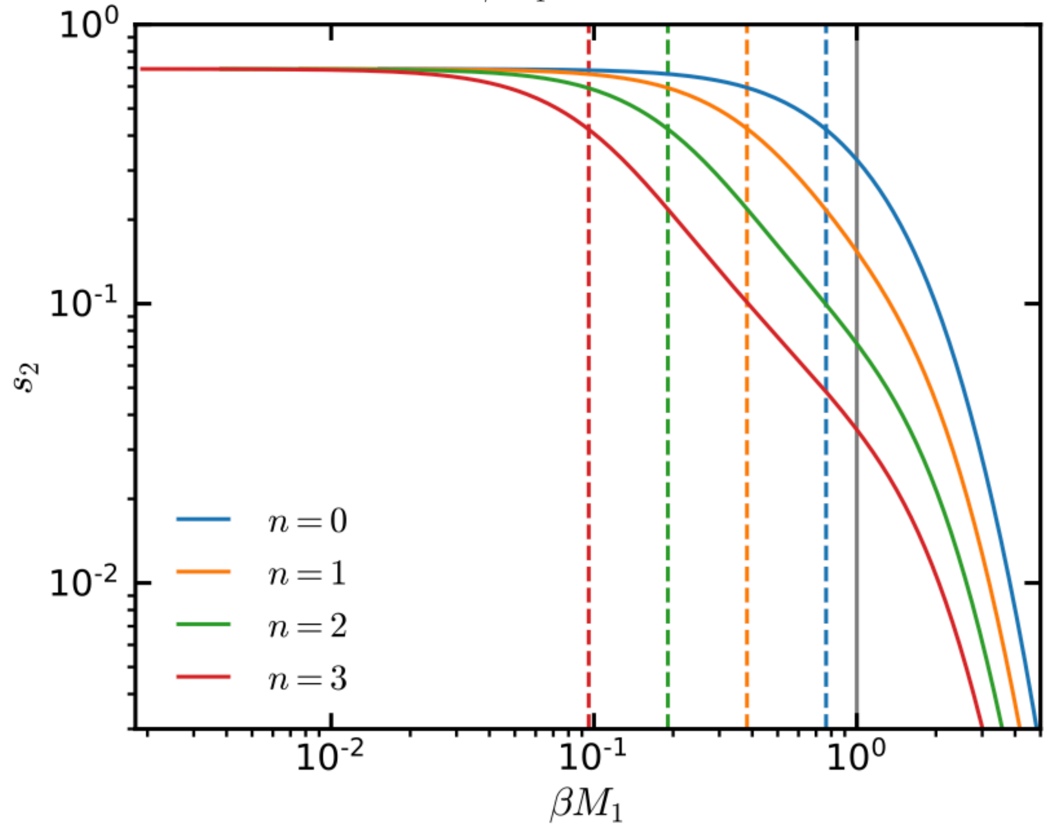
$$M_h^{(0)}/J = 0.125,\ M_g^{(0)}/J \approx 1.356 \quad \text{with} \quad M_{h,g}^{(n)} = \frac{M_{h,g}^{(0)}}{2^n}$$

low temperature:

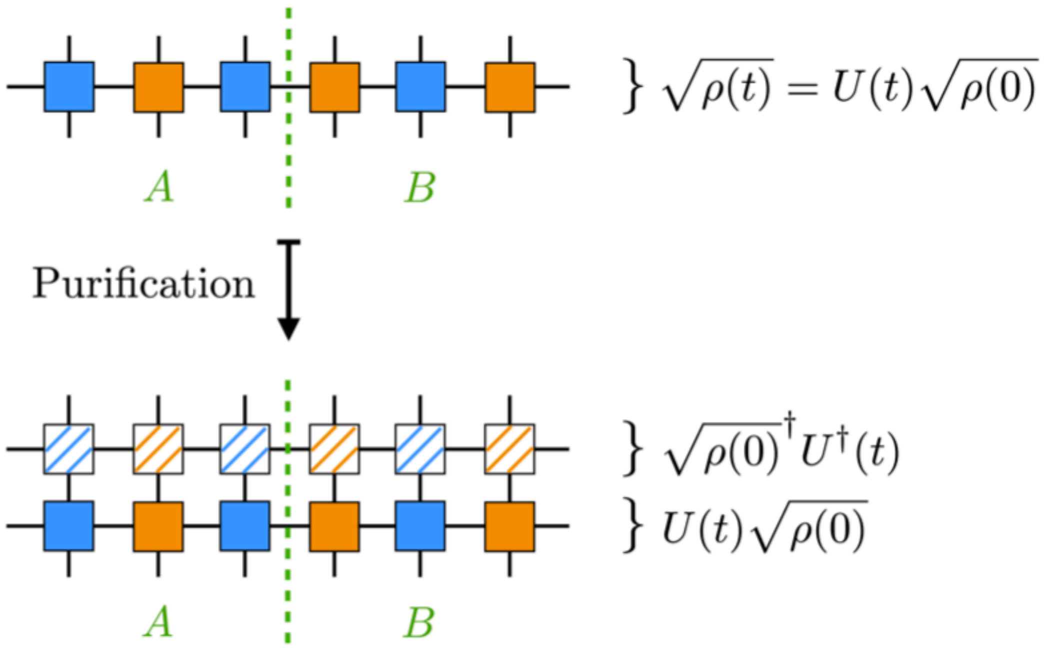




high temperature:

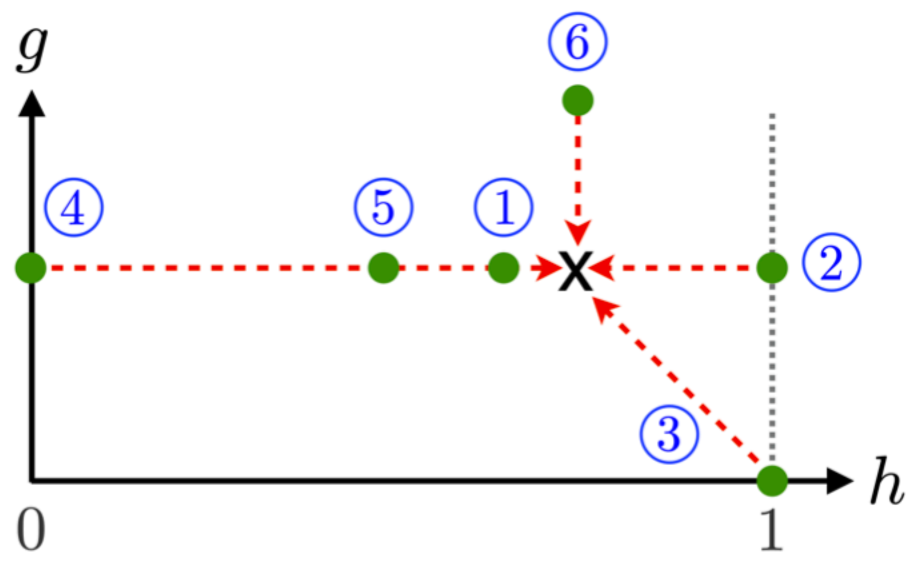


Dynamics: setup

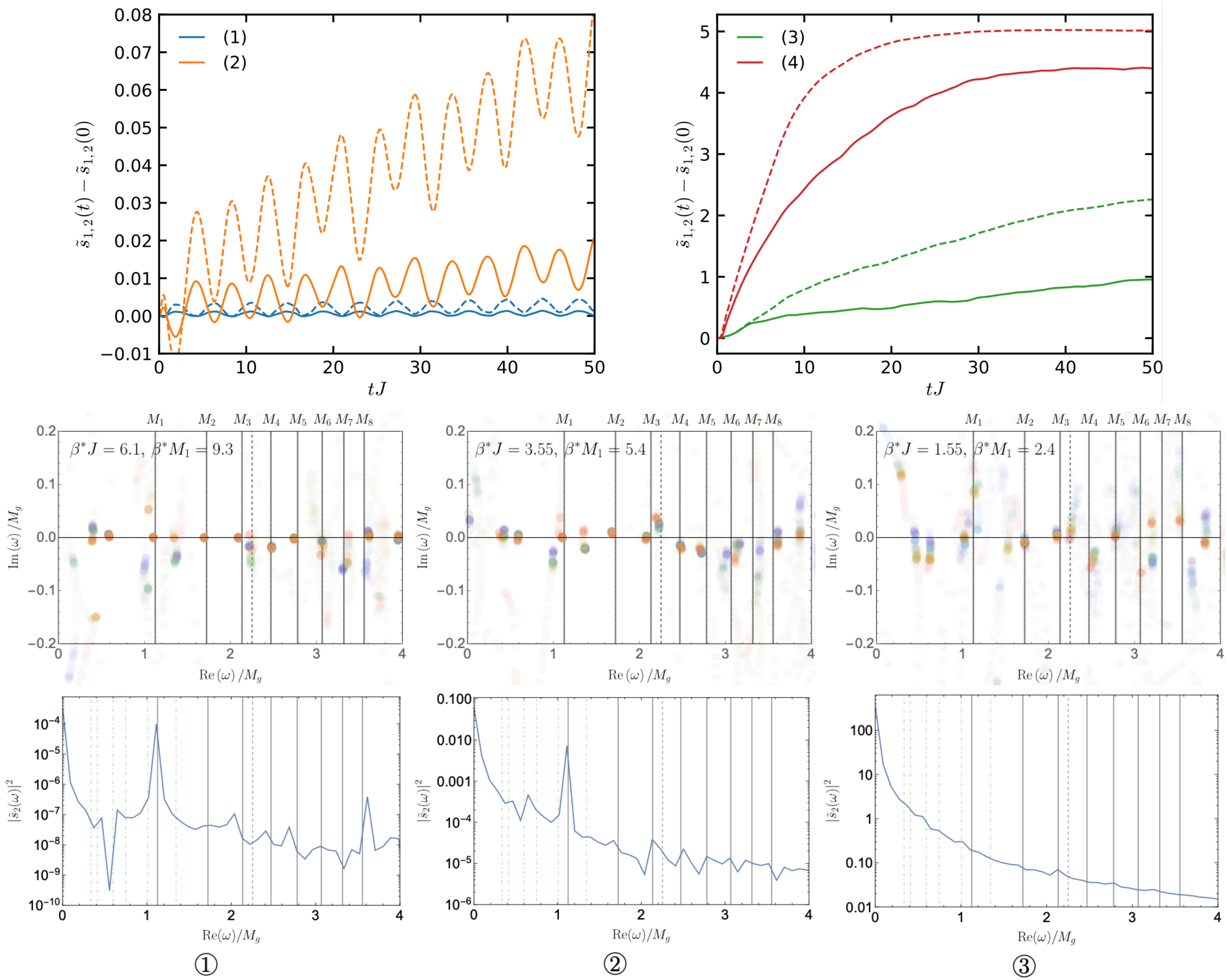


reflected entropy: S(A)

types of quenches:



Dynamics: results ⟶ yes, mesons do melt



Summary

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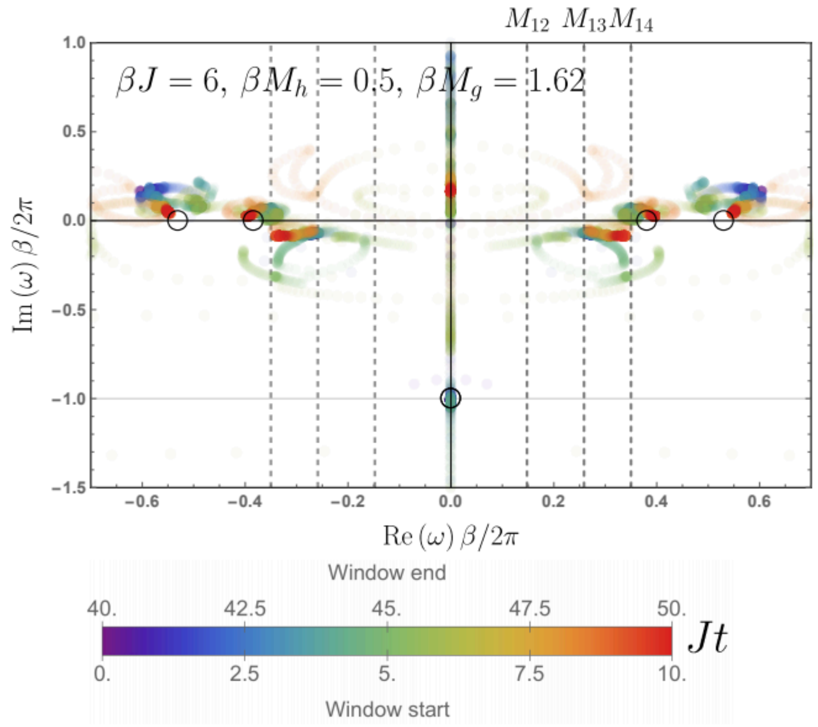

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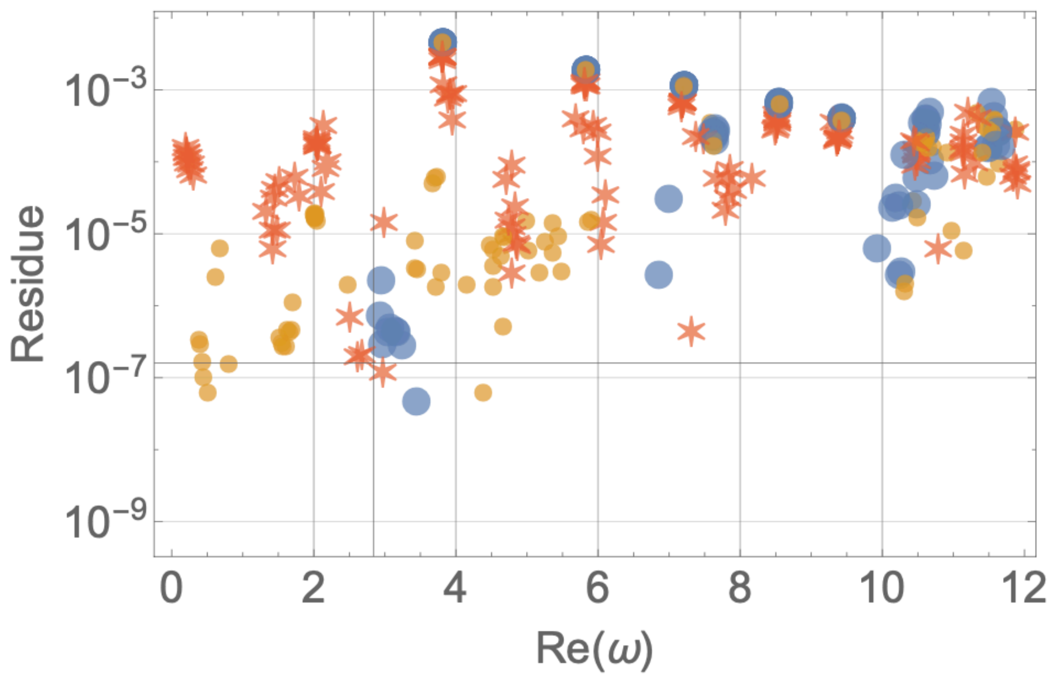

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