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C2QA

Entanglement and spin correlations at high energies

11th May 2023, Quantum Entanglement in HEP 2023

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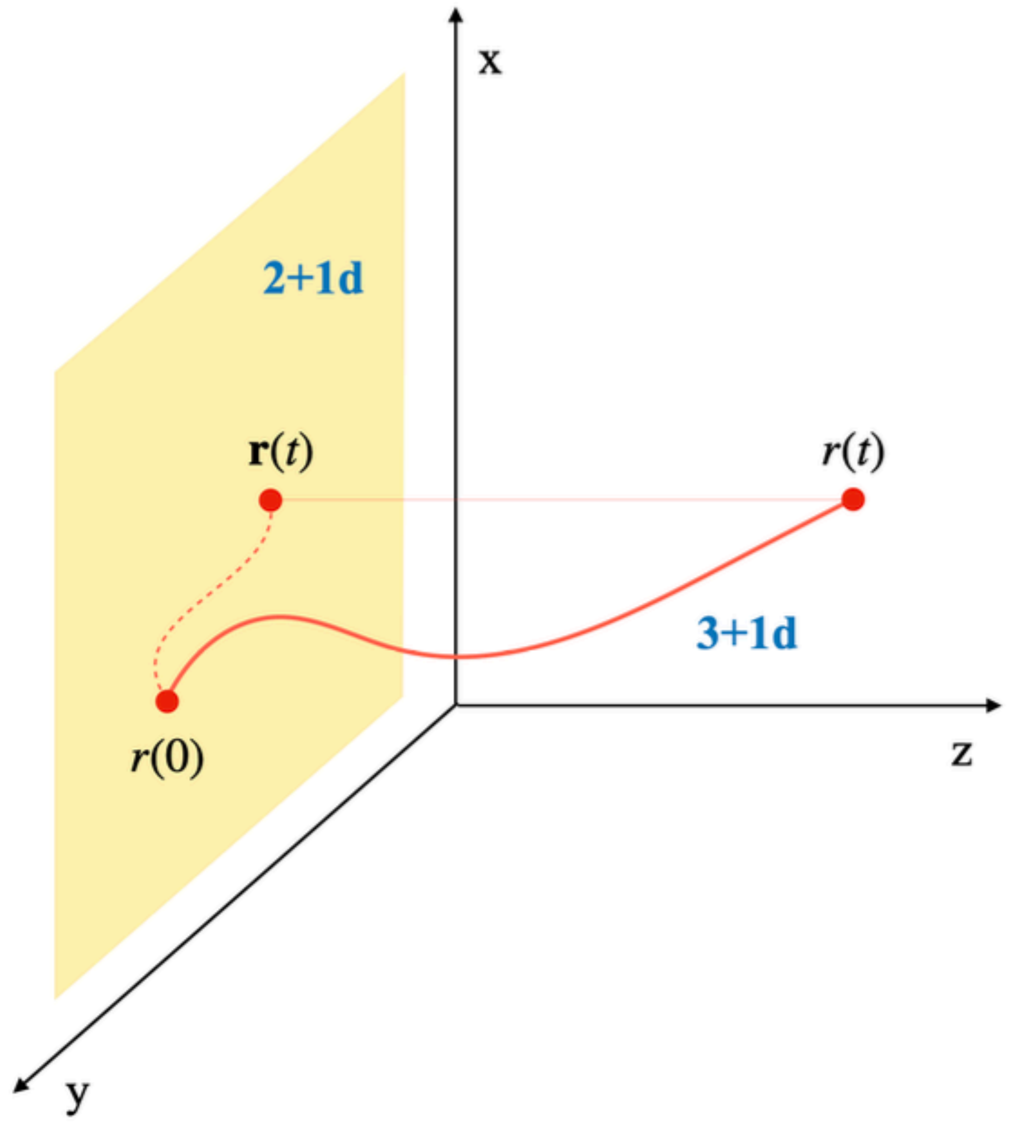
Mainly based on: 2305.XXXX, with J.-P. Blaizot, Y. Mehtar-Tani

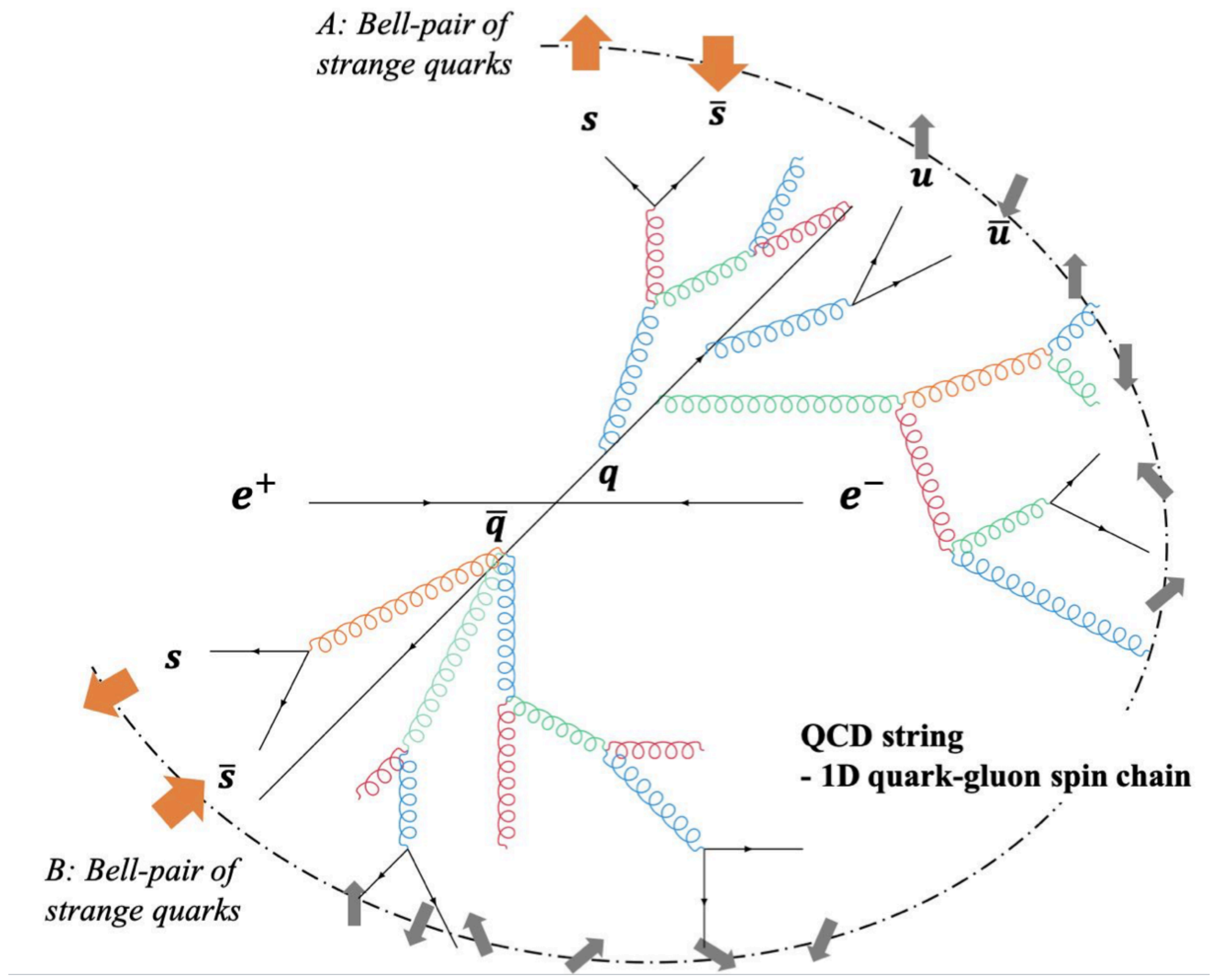
23XX.XXXX, with W. Gong, R. Venugopalan

Outline

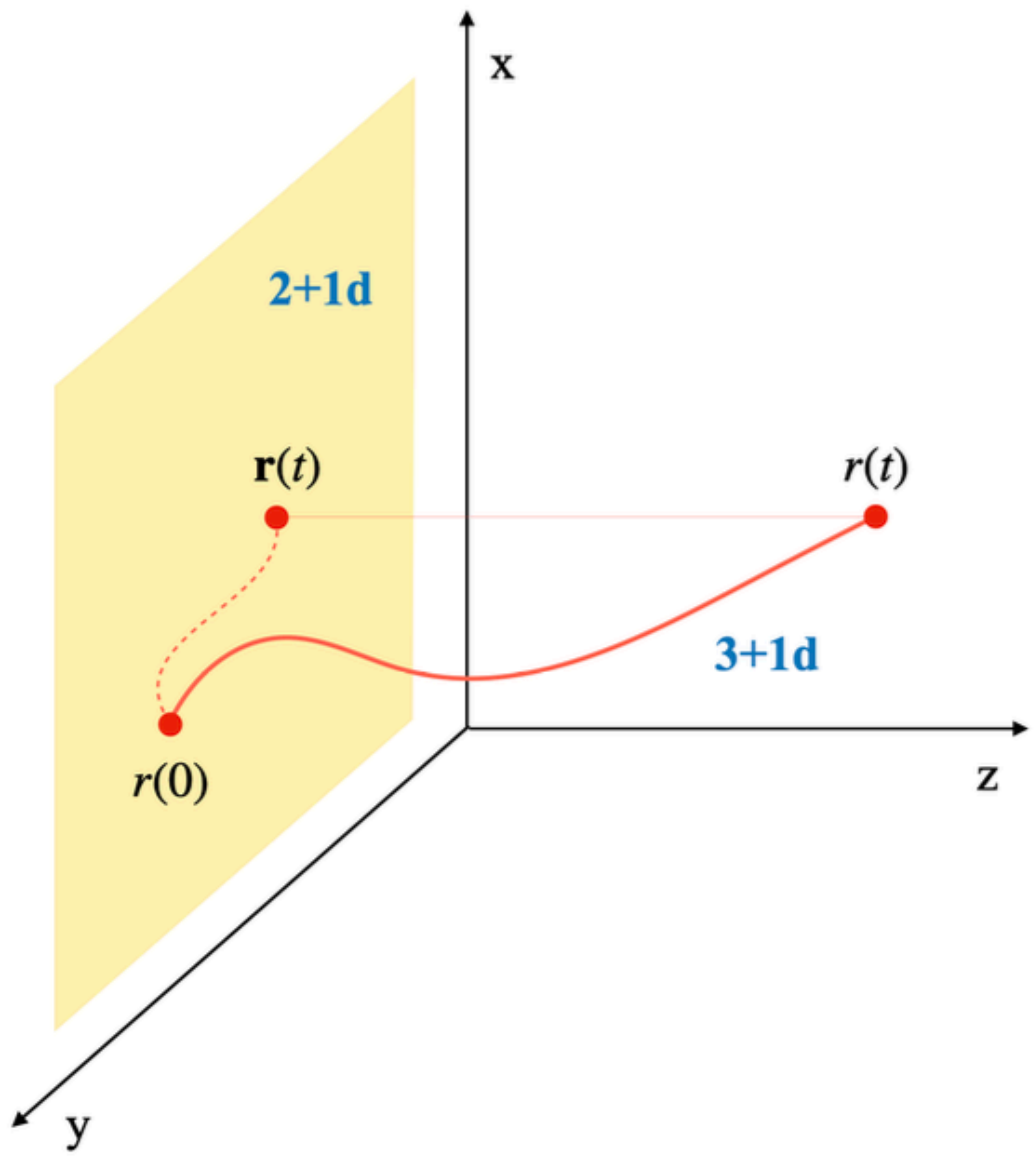
Quantum to classical description of parton evolution in QCD matter



Spin correlations and entanglement in 1+1d models

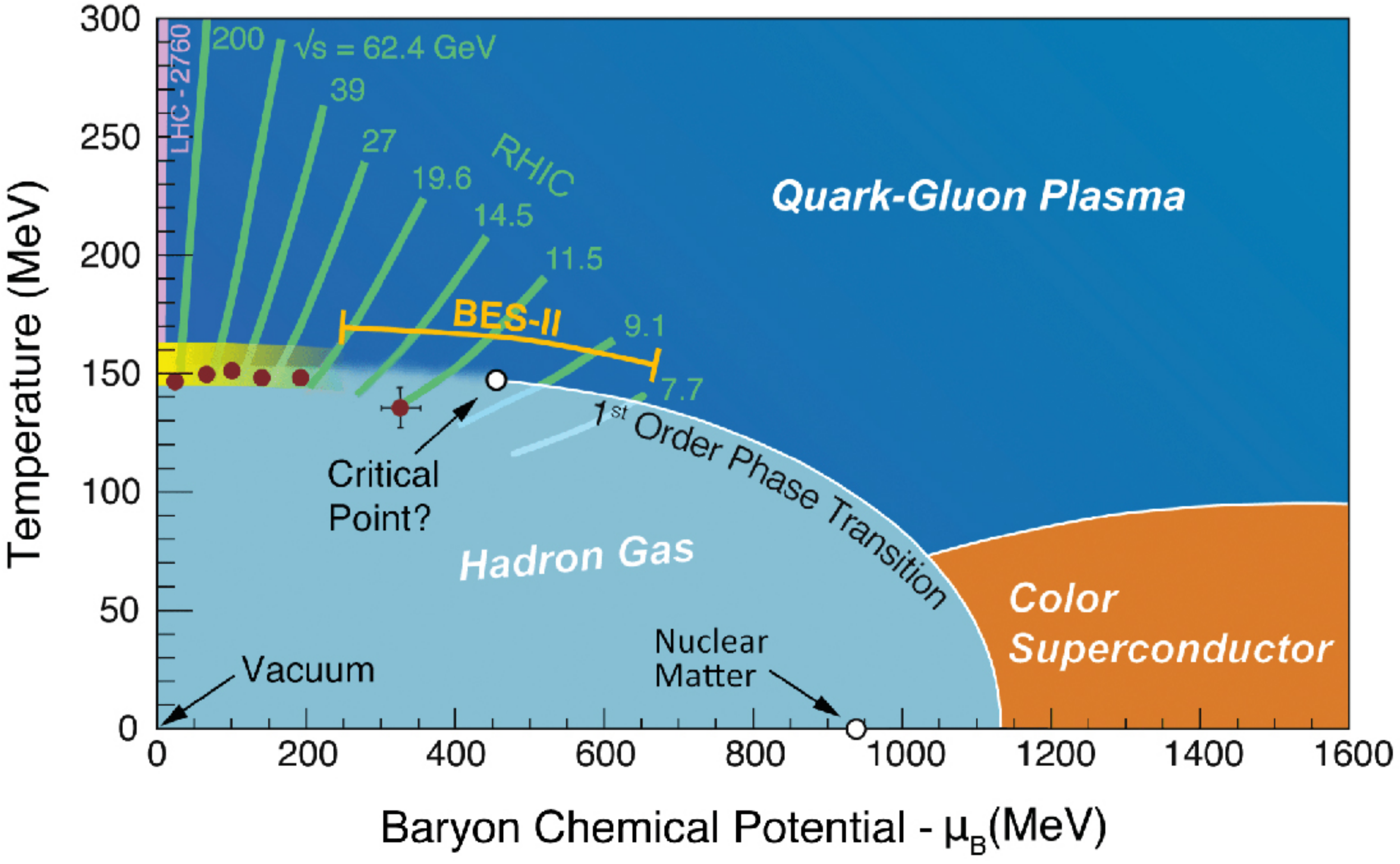


Quantum to classical description of partons in QCD matter

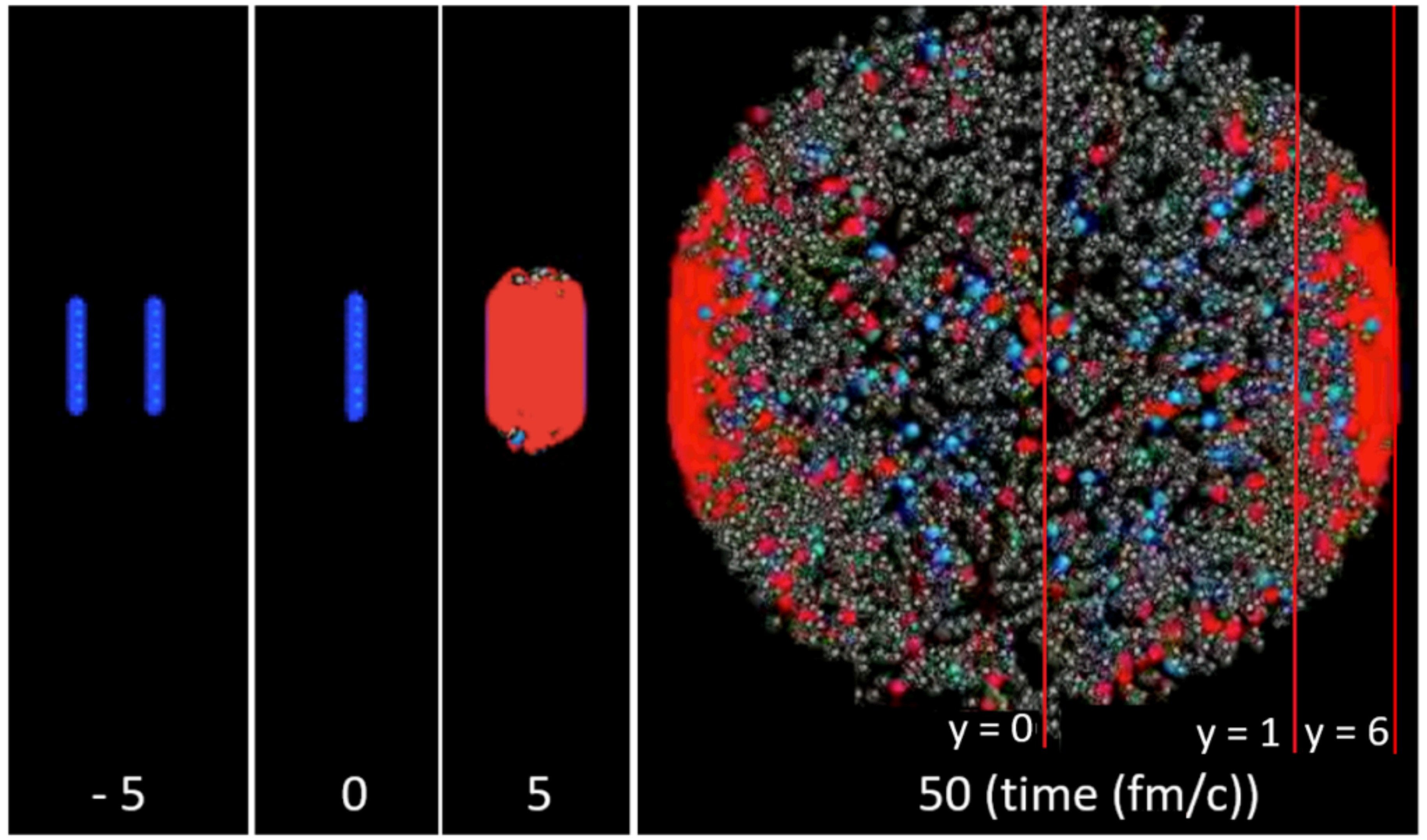


Parton evolution in QCD matter

Heavy ions collisions (HICs) allow to explore the QGP and the QCD phase diagram



1802.0480, Busza, Rajagopal, van der Schee



HICs can also be seen as ideal open quantum system set ups Blaizot, Escobedo, Yao, Vaydia, Akamatsu, …

How do small systems (quarkonia, heavy flavors, jets …) decohere ?

How do quantum systems thermalize ?

Which aspects of the evolution are truly quantum?

Parton evolution in QCD matter

Partonic evolution in matter is commonly studied using effective kinetic descriptions Arnold, Moore, Yaffe, …

$$\left(\partial_t + \frac{\boldsymbol{p}}{|\boldsymbol{p}|} \cdot \nabla_x\right) \underbrace{f_a(\boldsymbol{p}, \mathbf{x}, t)}_{\text{(Classical) phase space distribution for partons flavor a}} = \underbrace{-C_a^{2\leftrightarrow 2}[\{f_i\}] - C_a^{1\leftrightarrow 2}[\{f_i\}]}_{\text{Computed in QCD}}$$

Example: integrating out spatial dependence and assuming small angle scattering, one obtains

$$\left(\partial_L - \frac{\hat{q}}{4}\partial_{\boldsymbol{p}}^2\right)\mathcal{P}(\boldsymbol{p}, L) = 0$$

$\hat{q}$ = diffusion constant

This describes the broadening of the momentum distribution due to matter and is a classical result.

Why and how does this description emerge ?

Quark reduced density matrix in matter

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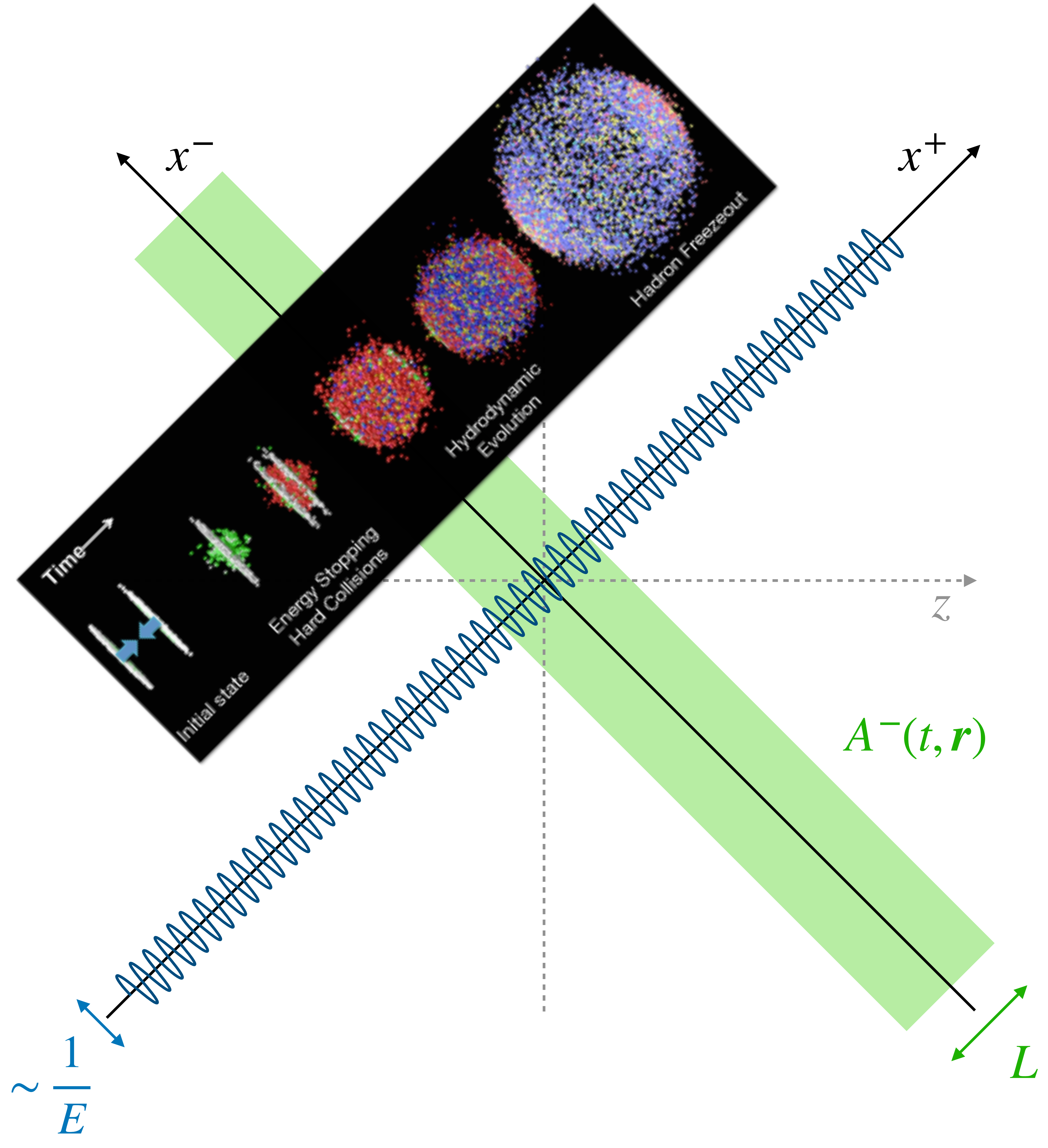


The single parton wave function satisfies

Coupling to matter background

$$\left[i\partial_t + \underbrace{\frac{\partial_\perp^2}{2E}}_{\text{Light front kinetic energy}} + \overline{gA(\boldsymbol{r}, t)} \right] \psi(\boldsymbol{r}, t) = 0$$

Light front kinetic energy

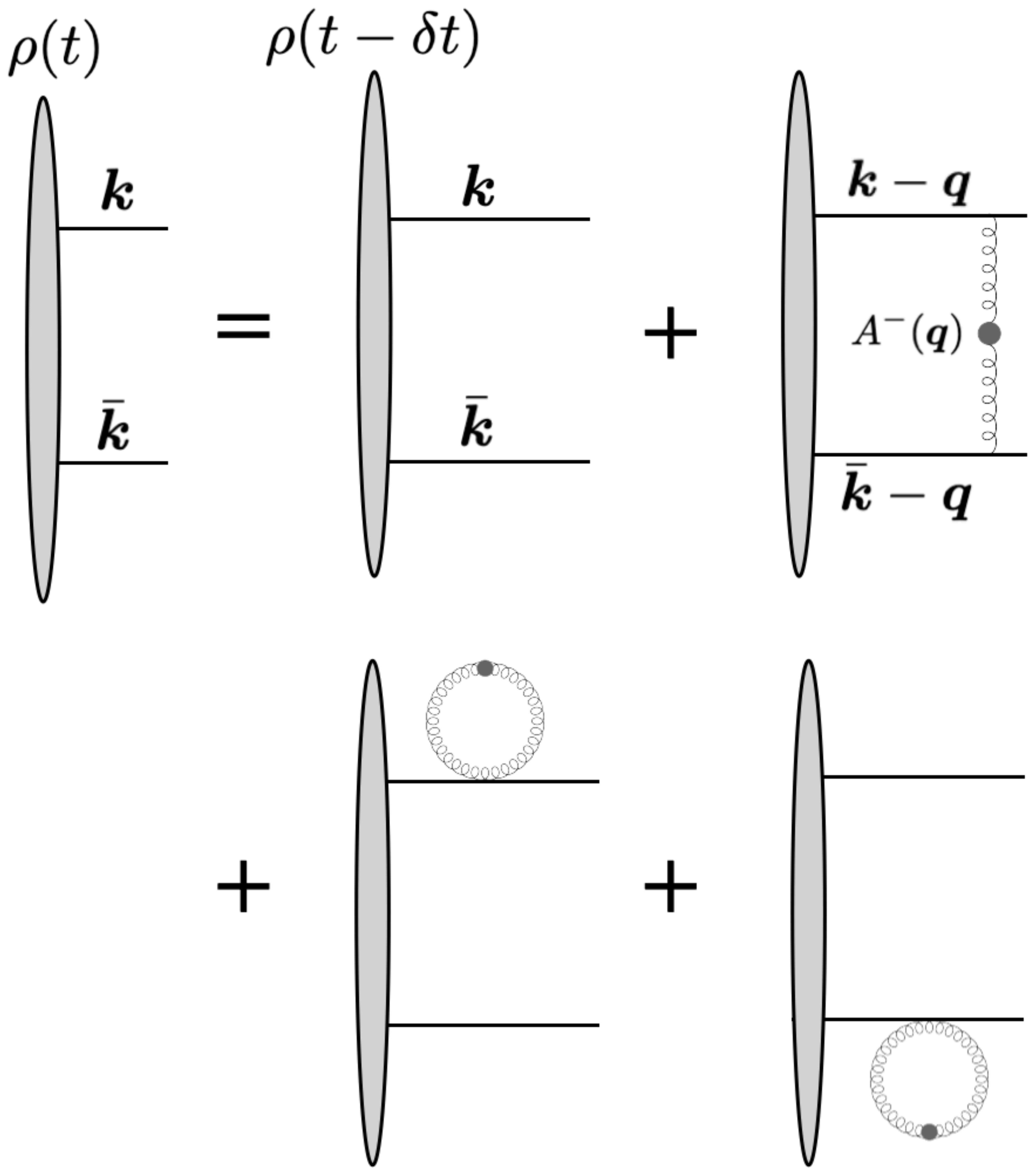
The reduced density matrix can be defined as

$$\rho \equiv \mathrm{tr}_A\left(\rho[A]\right) = \Big\langle |\psi_A(t)\rangle\langle\psi_A(t)| \Big\rangle_A$$

We use Gaussian approximation for background field

$$g^2 \Big\langle A^a(\boldsymbol{q}, t) A^{\dagger b}(\boldsymbol{q}', t') \Big\rangle_A = \delta^{ab}\delta(t - t')(2\pi)^2\delta^{(2)}(\boldsymbol{q} - \boldsymbol{q}')\,\gamma(\boldsymbol{q})$$

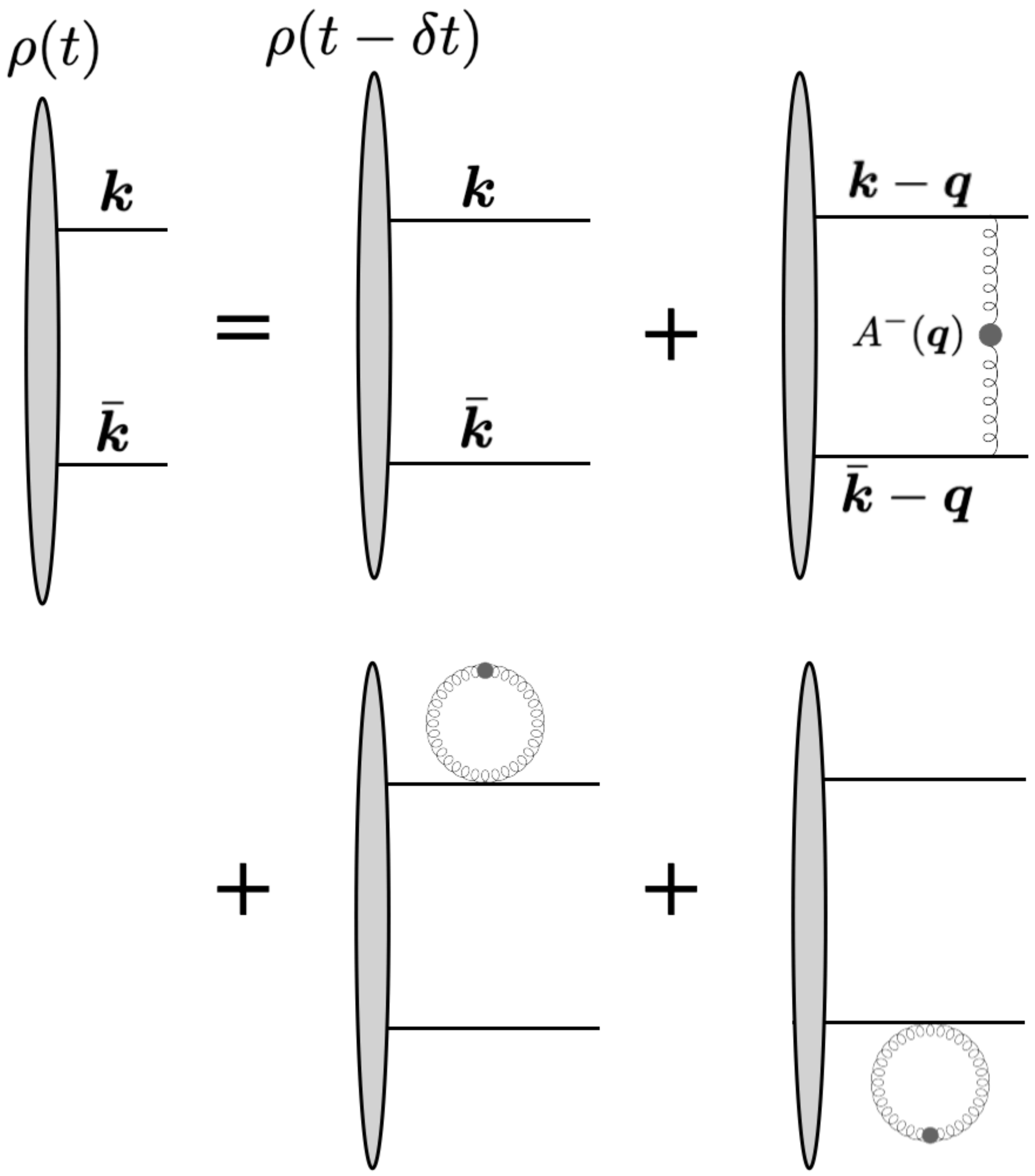
Constructing the evolution equations



$$\boldsymbol{b} \equiv \frac{\boldsymbol{r}+\bar{\boldsymbol{r}}}{2}\,, \qquad \boldsymbol{x} \equiv \boldsymbol{r}-\bar{\boldsymbol{r}}$$

$$\boldsymbol{K} = \frac{\boldsymbol{k}+\bar{\boldsymbol{k}}}{2}\,, \qquad \boldsymbol{\ell} = \boldsymbol{k}-\bar{\boldsymbol{k}}$$

Constructing the evolution equations



$$3 \otimes \bar{3} = 1 \oplus 8$$

$$\rho(t) \equiv \rho_{\rm s} + t^a \rho_{\rm o}^a = \frac{1}{N_c} \mathrm{Tr}_c(\rho) + 2\, t^a \mathrm{Tr}_c(t^a \rho)$$

For color singlet:

$$\langle \boldsymbol{k} | \rho_{\rm s}(t) | \bar{\boldsymbol{k}} \rangle = C_F \int_{\boldsymbol{q}} \int_0^t dt'\, e^{i \frac{(\boldsymbol{k}^2 - \bar{\boldsymbol{k}}^2)}{2E}(t-t')} \times \gamma(\boldsymbol{q}) \left[\langle \boldsymbol{k} - \boldsymbol{q} | \rho_{\rm s}(t') | \bar{\boldsymbol{k}} - \boldsymbol{q} \rangle - \langle \boldsymbol{k} | \rho_{\rm s}(t') | \bar{\boldsymbol{k}} \rangle \right]$$

For color octet:

$$\langle \boldsymbol{k} | \rho_{\rm o}(t) | \bar{\boldsymbol{k}} \rangle = C_F \int_{\boldsymbol{q}} \int_0^t dt'\, e^{i \frac{(\boldsymbol{k}^2 - \bar{\boldsymbol{k}}^2)}{2E}(t-t')} \times \gamma(\boldsymbol{q}) \left[\langle \boldsymbol{k} - \boldsymbol{q} | \rho_{\rm o}(t') | \bar{\boldsymbol{k}} - \boldsymbol{q} \rangle + \frac{1}{2 N_c C_F} \langle \boldsymbol{k} | \rho_{\rm o}(t') | \bar{\boldsymbol{k}} \rangle \right]$$

Constructing the evolution equations

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The matrix elements of the singlet and octet components satisfy Boltzmann transport

$$\gamma(\boldsymbol{q}) \approx g^4 n/\boldsymbol{q}^4$$

$$\partial_t \rho_{\mathrm{s,o}}(\boldsymbol{\ell}, \boldsymbol{x}, t) = -\left[\frac{\boldsymbol{\ell}\cdot\partial_{\boldsymbol{x}}}{E} + \Gamma_{\mathrm{s,o}}(\boldsymbol{x})\right]\rho_{\mathrm{s,o}}(\boldsymbol{\ell}, \boldsymbol{x}, t)$$

$$\Gamma_{\mathrm{s}}(\boldsymbol{x}) = C_F \int_{\boldsymbol{q}} \left(1 - e^{i\boldsymbol{q}\cdot\boldsymbol{x}}\right)\gamma(\boldsymbol{q})\,,$$

$$\Gamma_{\mathrm{o}}(\boldsymbol{x}) = \int_{\boldsymbol{q}} \left(C_F + \frac{1}{2N_c} e^{i\boldsymbol{q}\cdot\boldsymbol{x}}\right)\gamma(\boldsymbol{q})$$

This form allows to settle the evolution in color space

$$\rho_{\mathrm{s,o}}(\boldsymbol{b}, \boldsymbol{x}, t) = \rho^{(0)}_{\mathrm{s,o}}(\boldsymbol{b}, \boldsymbol{x})\, \mathrm{e}^{-t\,\Gamma_{\mathrm{s,o}}(\boldsymbol{x})} \qquad E \to \infty$$

$$\Gamma_{\mathrm{s}}(\boldsymbol{x}) \approx 4\pi\alpha_s^2 C_F n \log\left(\frac{Q^2}{m_D^2}\right)\frac{\boldsymbol{x}^2}{4} \equiv \frac{\hat{q}}{4}\boldsymbol{x}^2\,,$$

$$\Gamma_{\mathrm{o}}(\boldsymbol{x}) \approx \frac{4\pi\alpha_s^2 C_A n}{m_D^2}$$

$\xrightarrow{\boldsymbol{x} \to 0}$

Singlet $\longrightarrow$ Neutral to matter

Octet $\longrightarrow$ Damping

Blaizot, Iancu, Braaten, Pisarski, ...

Only true in the absence of coherent background fields

One can also show that singlet subspaces become equally probable Zakharov, Blaizot, Escobedo, ...

Singlet evolution

We consider a simple initial condition for the reduced singlet density matrix

$$\rho_W(\boldsymbol{b}, \boldsymbol{K}, 0) = 4\mathrm{e}^{-\mu^2 \boldsymbol{b}^2} \mathrm{e}^{-\frac{\boldsymbol{K}^2}{\underline{\mu^2}}}$$

Characteristic momentum
scale of initial wave packet

akin to a coherent state in QM and **positive definite**

$$a(t) = \mu^2 + \hat{q}t \equiv \mu^2 \left(1 + \frac{t}{t_1}\right),$$

$$b(t) = -2\mu^2 t - \hat{q}t^2 \equiv -2\mu^2 \left(1 + \frac{t}{2t_1}\right) t,$$

$$c(t) = \frac{E^2}{\mu^2} + \mu^2 t^2 + \frac{\hat{q}t^3}{3} \equiv \frac{E^2}{\mu^2} \left[1 + \frac{t^2}{t_0^2} + \frac{1}{3}\frac{t^3}{t_2^3}\right]$$

Using the harmonic approximation, the matrix elements reduce to

$$\rho_W(\boldsymbol{b}, \boldsymbol{K}) = \frac{4}{D} \exp\left\{-\frac{1}{D}\left(a\,\boldsymbol{b}^2 + \frac{b}{E}\,\boldsymbol{b}\cdot\boldsymbol{K} + \frac{c}{E^2}\boldsymbol{K}^2\right)\right\}$$

$$a = \langle \boldsymbol{K}^2 \rangle_t, \quad \frac{c}{E^2} = \langle \boldsymbol{b}^2 \rangle_t, \quad \frac{b}{E} = -2\langle \boldsymbol{b}\cdot\boldsymbol{K}\rangle_t$$

$$D = \langle \boldsymbol{b}^2 \rangle \langle \boldsymbol{K}^2 \rangle - \langle \boldsymbol{K}\cdot\boldsymbol{b} \rangle = \frac{ac}{E^2} - \frac{b}{2E}$$

Singlet evolution

In the absence if interactions with the background, one obtains free streaming

$$\rho_W(\boldsymbol{b} - (\boldsymbol{K}/E)t, \boldsymbol{K}, 0)$$

which is dominated by the natural spreading of the wave packet

$$\rho(\boldsymbol{b}, t) = \frac{1}{\pi \langle \boldsymbol{b}^2 \rangle_t^{(0)}} \mathrm{e}^{-\frac{\boldsymbol{b}^2}{\langle \boldsymbol{b}^2 \rangle_t^{(0)}}}, \quad \langle \boldsymbol{b}^2 \rangle_t^{(0)} \equiv \frac{1}{\mu^2} \left(1 + \frac{t^2}{t_0^2} \right) \qquad t_0 = \frac{E}{\mu^2}$$

Singlet evolution

When including interactions, more scales emerge. Consider first **diagonal elements**

Momentum space:

$$\rho(\boldsymbol{\ell}, \boldsymbol{K}) = \frac{4\pi}{a} \exp\left\{ -\frac{1}{4a}\boldsymbol{K}^2 - \frac{1}{4E^2}\left(c - \frac{b^2}{4a}\right)\boldsymbol{\ell}^2 - i\frac{b}{4Ea}\boldsymbol{\ell}\cdot\boldsymbol{K} \right\}$$

For the sectors $\ell = 0$ we recover classical broadening distribution but with

$$\langle \boldsymbol{k}^2 \rangle_t = \mu^2 + \hat{q}t = \mu^2\left(1 + \frac{t}{t_1}\right) \qquad t_1 = \frac{\mu^2}{\hat{q}}$$

Singlet evolution

When including interactions, more scales emerge. Consider first **diagonal elements**

Position space:

The diagonal terms evolve as

$$\rho(\boldsymbol{b}, t) = \frac{1}{\pi \langle \boldsymbol{b}^2 \rangle_t} \, \mathrm{e}^{-\frac{\boldsymbol{b}^2}{\langle \boldsymbol{b}^2 \rangle_t}}$$

where

$$\langle \boldsymbol{b}^2 \rangle_t = \frac{c(t)}{E^2} = \frac{1}{\mu^2} \left[1 + \frac{t^2}{t_0^2} + \frac{1}{3} \frac{t^3}{t_2^3} \right]$$

$$t_2^3 = \frac{E^2}{\hat{q} \mu^2}$$

$$t_2^3 = t_1 t_0^2$$

Asymptotically, the distribution spreads rapidly as

$$\rho(\boldsymbol{b}, t) \approx \frac{3E^2}{\pi \hat{q} t^3} \exp\left\{ -\frac{3E^2}{\hat{q} t^3} \boldsymbol{b}^2 \right\} \qquad (t \gg t_2)$$

Singlet evolution

When including interactions, more scales emerge. Now let us look at the **off-diagonal elements**

Momentum space:

$$\rho(\boldsymbol{\ell}, \boldsymbol{K}=0, t) = \frac{4\pi}{\mu^2(1+(t/t_1))} \exp\left\{-\frac{\boldsymbol{\ell}^2}{4\mu^2} d(t)\right\}$$

$$d(t) = 1 + \frac{1}{12}\left(\frac{t}{t_2}\right)^3 \frac{t+4t_1}{t+t_1}$$

At late times, the initial condition is lost and off-diagonal terms vanish rapidly

$$\rho(\boldsymbol{\ell}, \boldsymbol{K}=0, t) \approx \frac{4\pi}{\mu^2 + \hat{q}t} \exp\left\{-\frac{\boldsymbol{\ell}^2\, \hat{q}t^3}{48E^2}\right\}$$

Position space:

At late times, the off-diagonal terms also vanish

$$\rho(\boldsymbol{b}=0, \boldsymbol{x}, t) \approx \frac{1}{\pi\langle \boldsymbol{b}^2\rangle_t} \exp\left\{-\frac{\langle \boldsymbol{k}^2\rangle_t\, \boldsymbol{x}^2}{4}\right\}$$

Singlet evolution: numerical example

$$t_0 > t_2 > t_1$$

Medium-parton interactions dominate evolution

vs

$$t_0 < t_2 < t_1$$

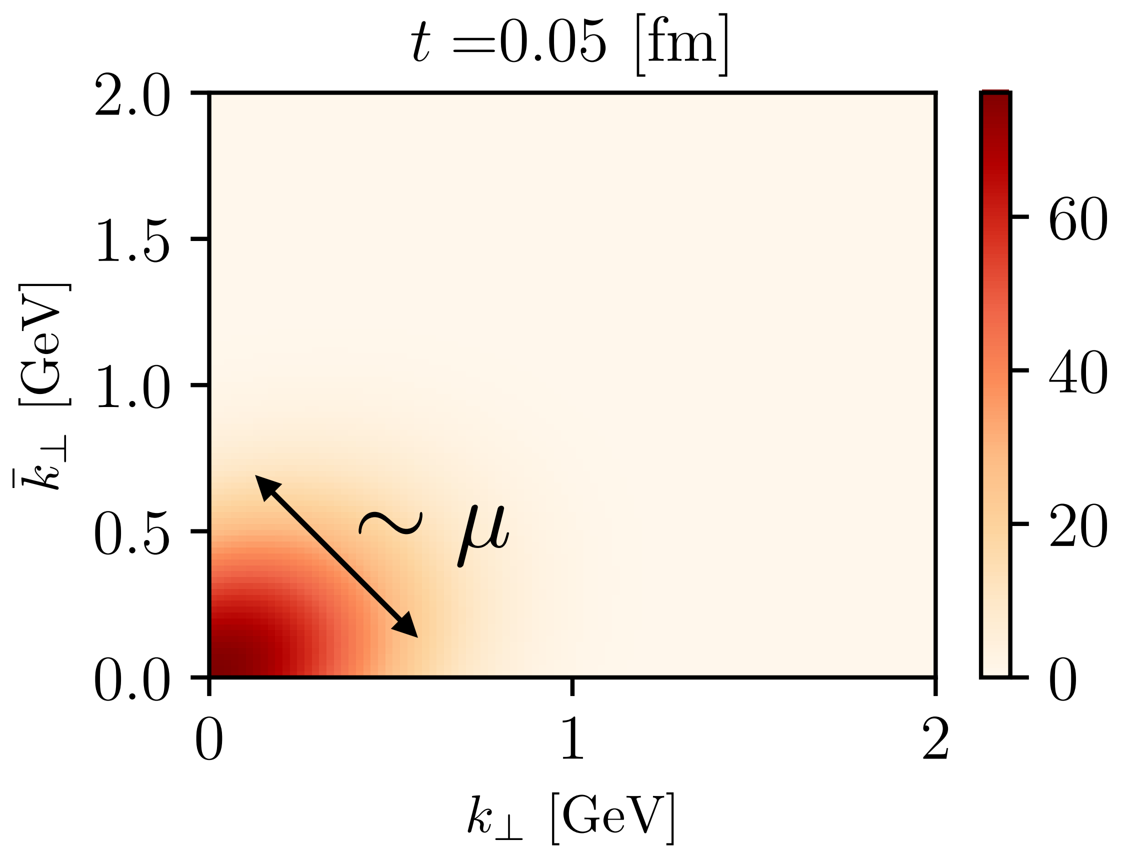
Natural wave packet spreading determines evolution

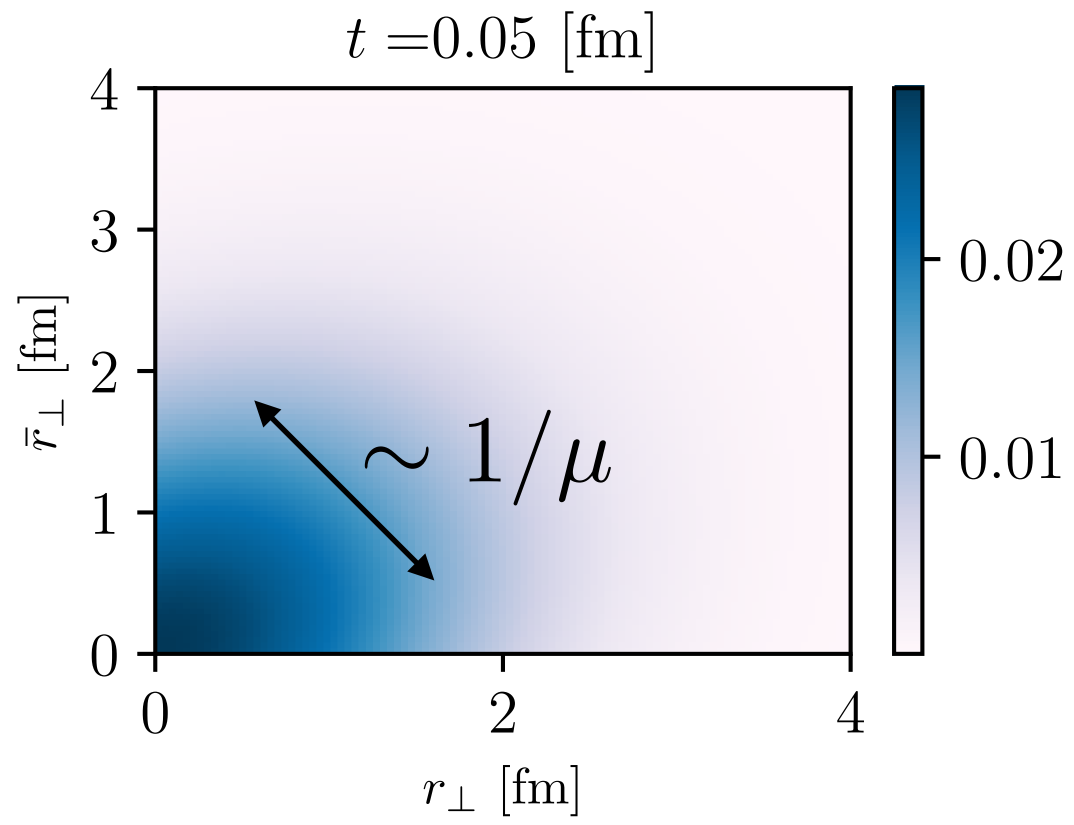
$\hat{q} = 0.3\,\mathrm{GeV}^3$, $\mu = 0.3$ GeV, and $E = 200\,\mathrm{GeV}$

$t_1 \simeq 0.06$ fm $\quad t_2 \simeq 22.80$ fm, $\quad t_0 \simeq 444.44$ fm,

Singlet evolution: numerical example

$$t_1 \simeq 0.06 \text{ fm} \qquad t_2 \simeq 22.80 \text{ fm} \qquad t_0 \simeq 444.44 \text{ fm}$$





Singlet evolution: numerical example

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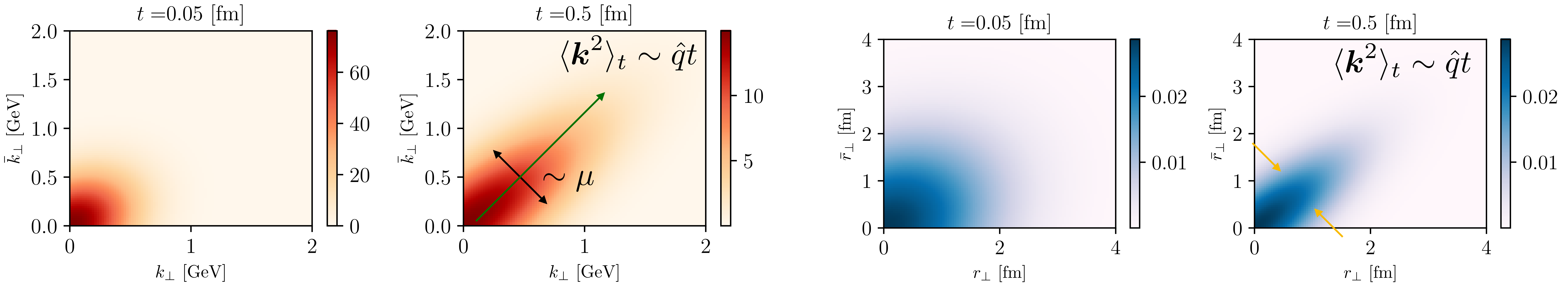

Singlet evolution: numerical example

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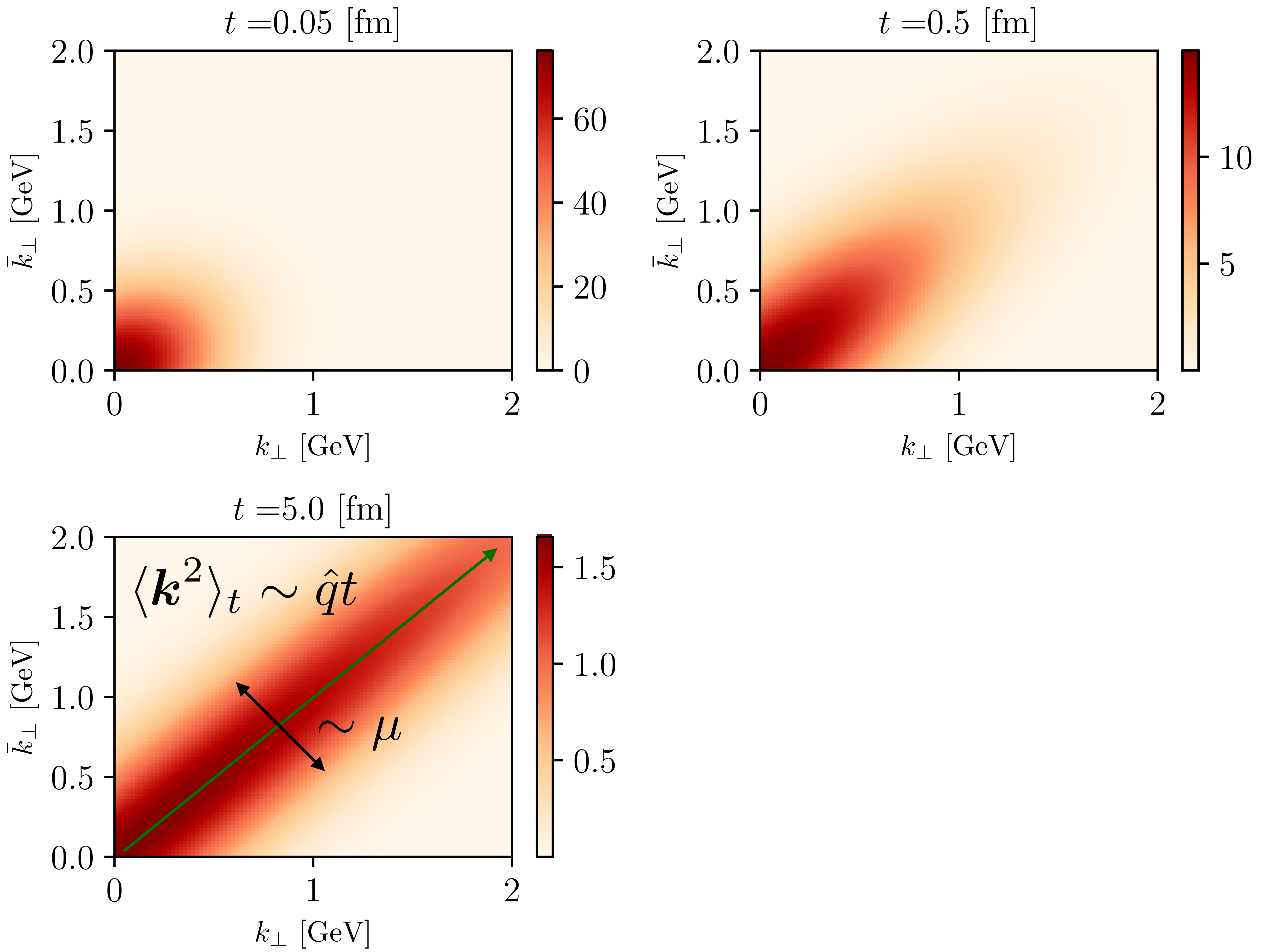

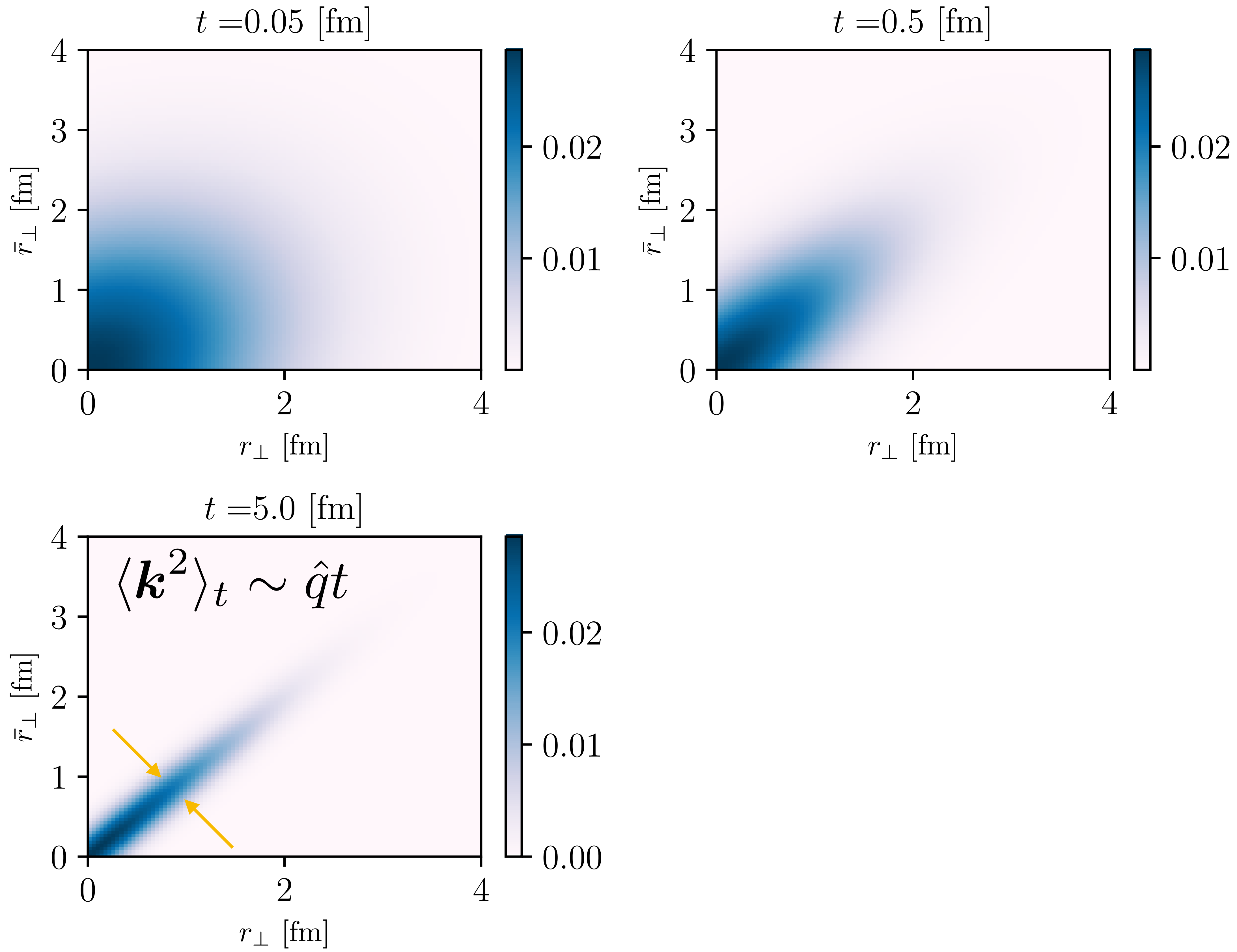

Singlet evolution: numerical example

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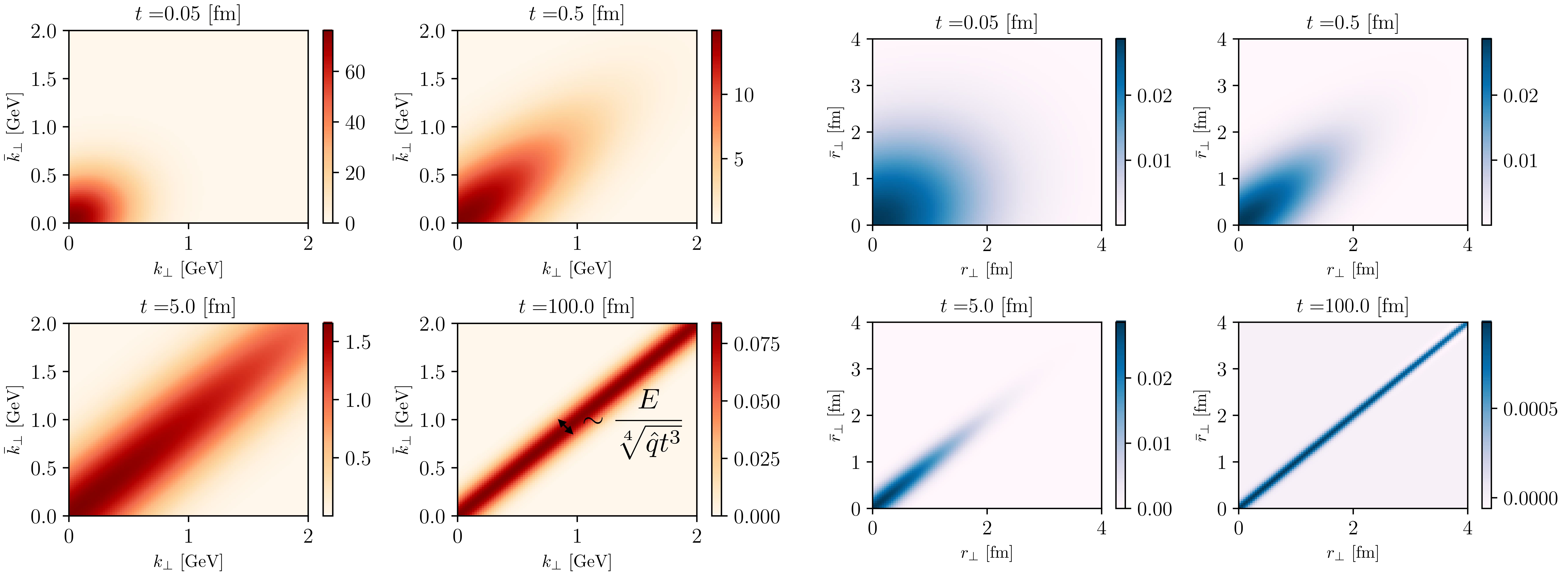
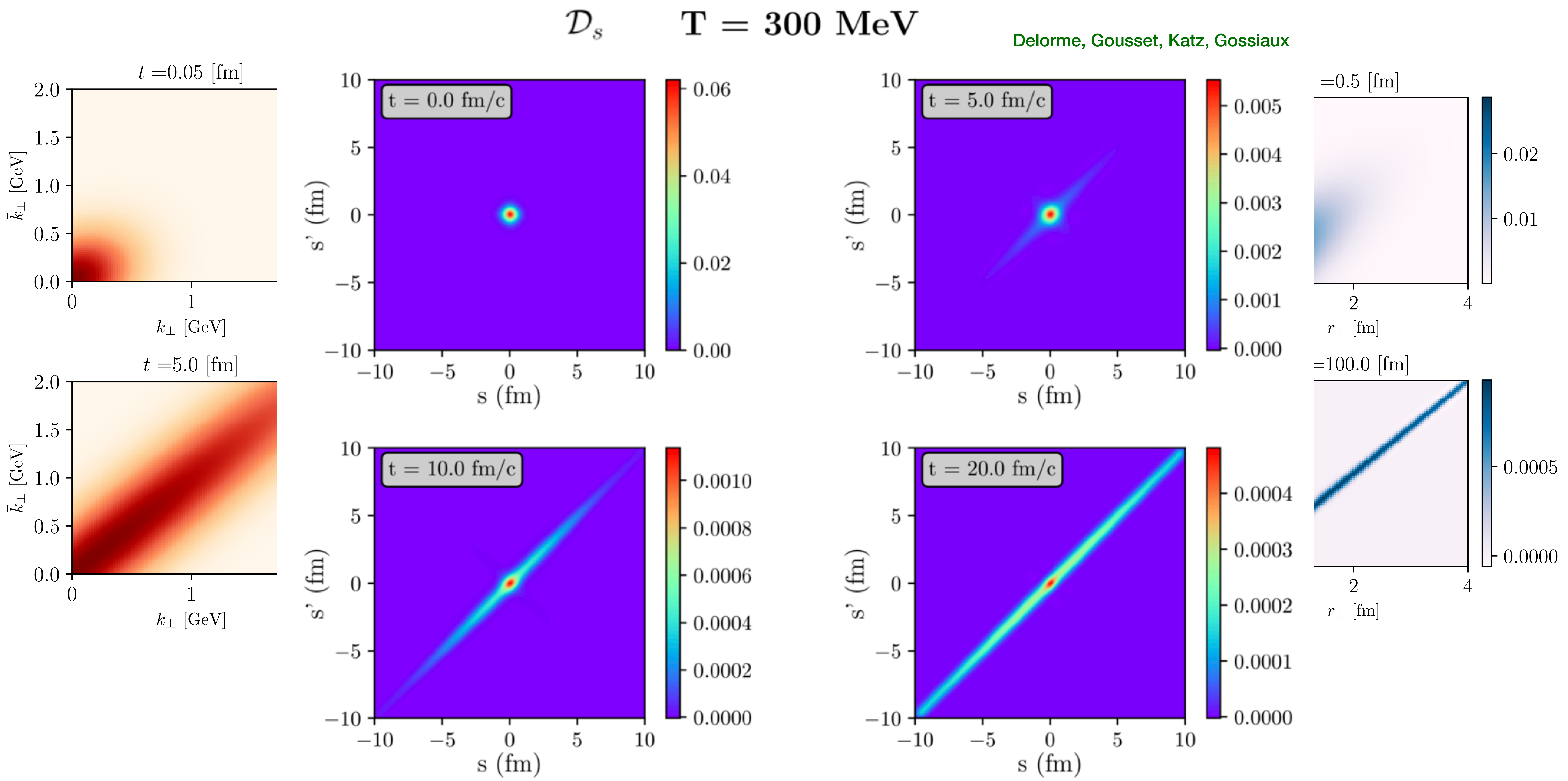


Singlet evolution: numerical example



Relation to QCD LPM effect

Production of gluon radiation in QCD backgrounds, at high energies, can be split into 3 regimes

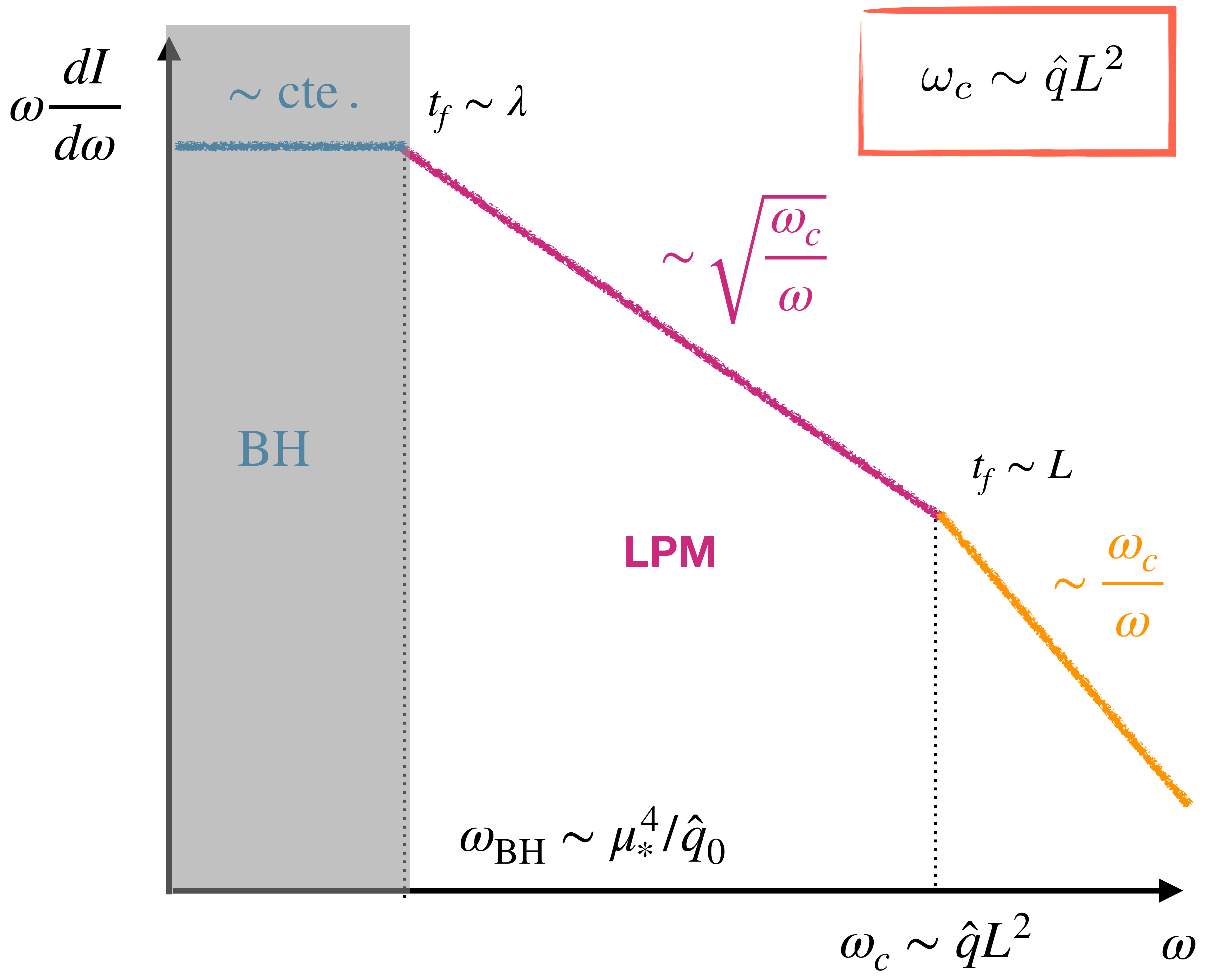
$$\omega \frac{dI}{d\omega} \sim \frac{L}{t_{coh,f}}$$

$$t_f \sim \frac{\omega}{\boldsymbol{k}^2} \qquad \langle \boldsymbol{k}^2 \rangle_t \sim \hat{q} t \qquad t_{coh,f} \sim \sqrt{\frac{\omega}{\hat{q}}}$$

LPM regime: for coherence times larger than m.f.p.

$$\omega \frac{dI}{d\omega} \sim \alpha_s \frac{L}{t_f} \sim \alpha_s \sqrt{\frac{\boxed{\omega_c}}{\omega}}$$

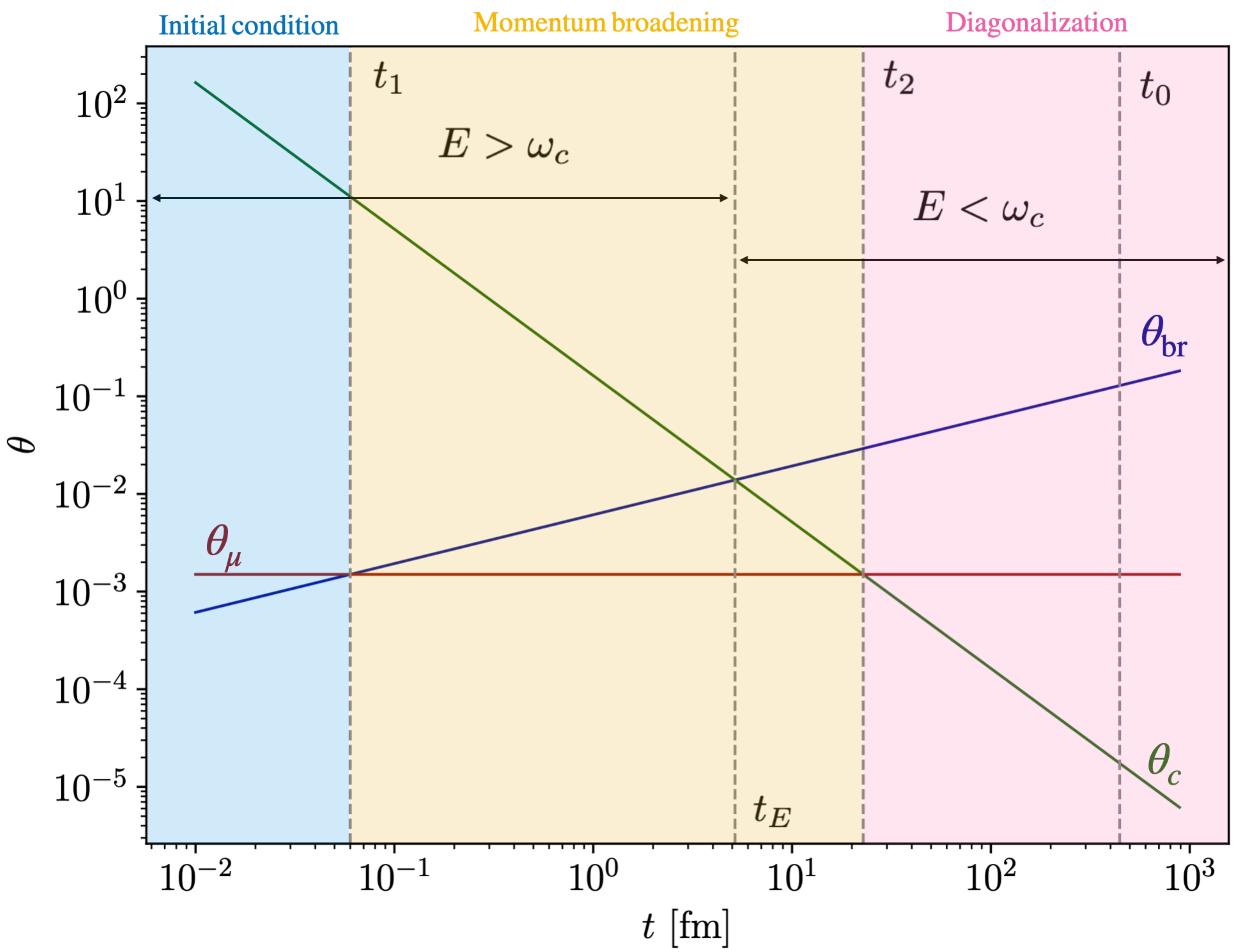
Can also be given angular interpretation in terms of the scale $\boxed{\theta_c \sim \frac{1}{\hat{q} L^3}}$



Relation to QCD LPM effect

$$\theta_\mu^2 = \frac{\mu^2}{E^2}\,, \quad \theta_c^2(t) = \frac{1}{\hat{q}t^3}\,, \quad \theta_{\rm br}^2(t) = \frac{\hat{q}t}{E^2}$$

$$t_E \equiv \sqrt{\frac{E}{\hat{q}}} = \sqrt{t_0 t_1}, \qquad t_1 < t_E < t_2$$



Entropy as measure of quantum classical transition

Having access to the reduced density matrix we compute the associated **von-Neumann entropy**,

$$S_{\mathrm{vN}}[\rho] = -\mathrm{Tr}\rho \ln \rho \qquad S_{\mathrm{vN}} = \log\left(\frac{1-p}{4p}\right) + \frac{1}{\sqrt{p}} \ln \frac{1+p+2\sqrt{p}}{(1-p)}$$

the **purity**,

$$p \equiv \mathrm{Tr}\rho^2 \qquad \frac{1}{p} = \left(1+\frac{t}{t_1}\right)\left(1+\frac{t^3}{12t_2^3}\frac{t+4t_1}{t+t_1}\right)$$

and the **Wigner entropy**

$$S_{\mathrm{W}} \equiv -\int_{\boldsymbol{K},\boldsymbol{b}} \rho_{\mathrm{W}}(\boldsymbol{b},\boldsymbol{K}) \log \rho_{\mathrm{W}}(\boldsymbol{b},\boldsymbol{K}) \qquad S_{\mathrm{W}} = \ln\frac{1}{p} + 2 - \ln 4$$

Entropy as measure of quantum classical transition

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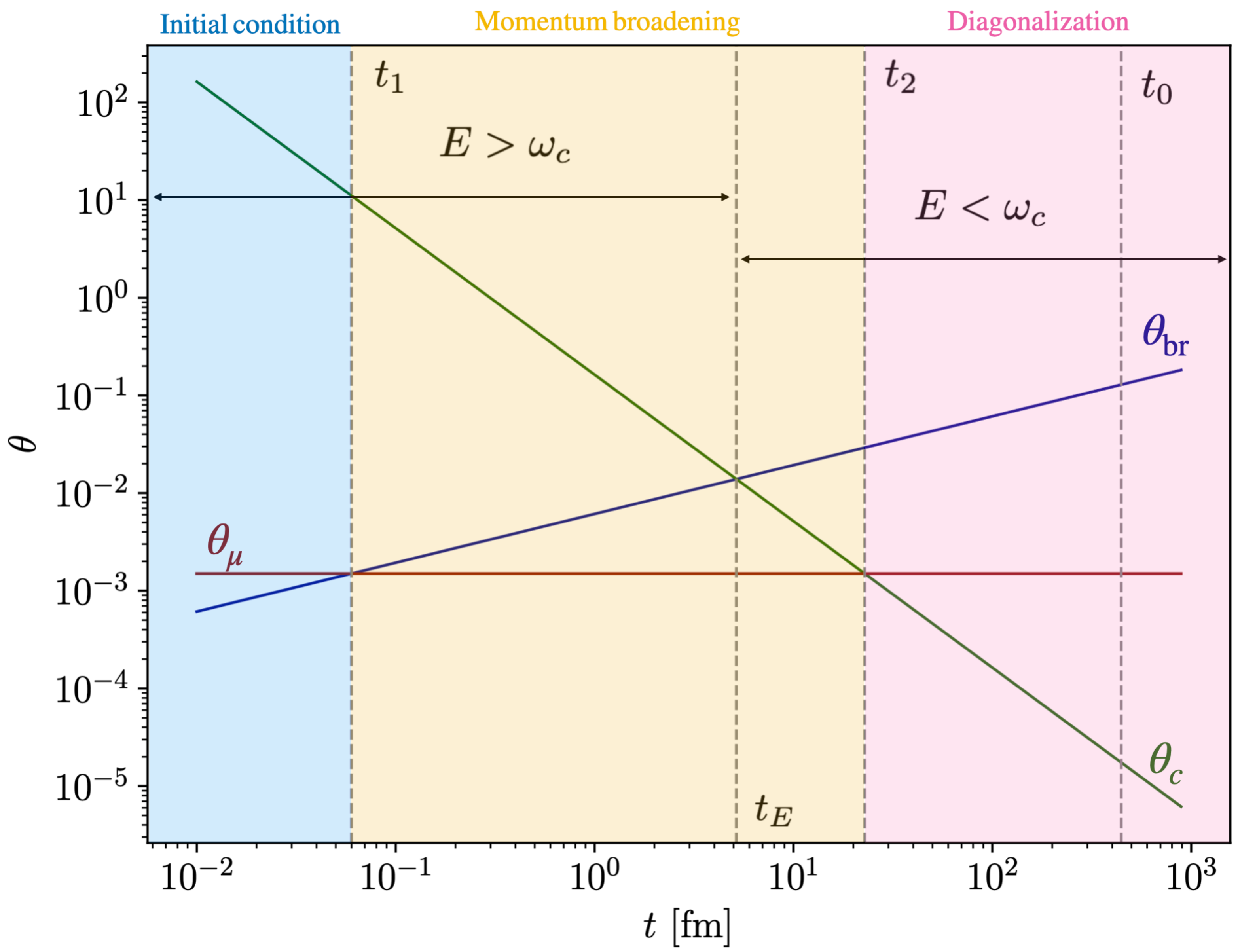
In each of the previous regions we find



Initial stage: $t \ll t_1, \quad p \simeq 1 \quad S_{\rm vN} \to 0$

Spatial decoherence: $t_1 \ll t \ll t_2$

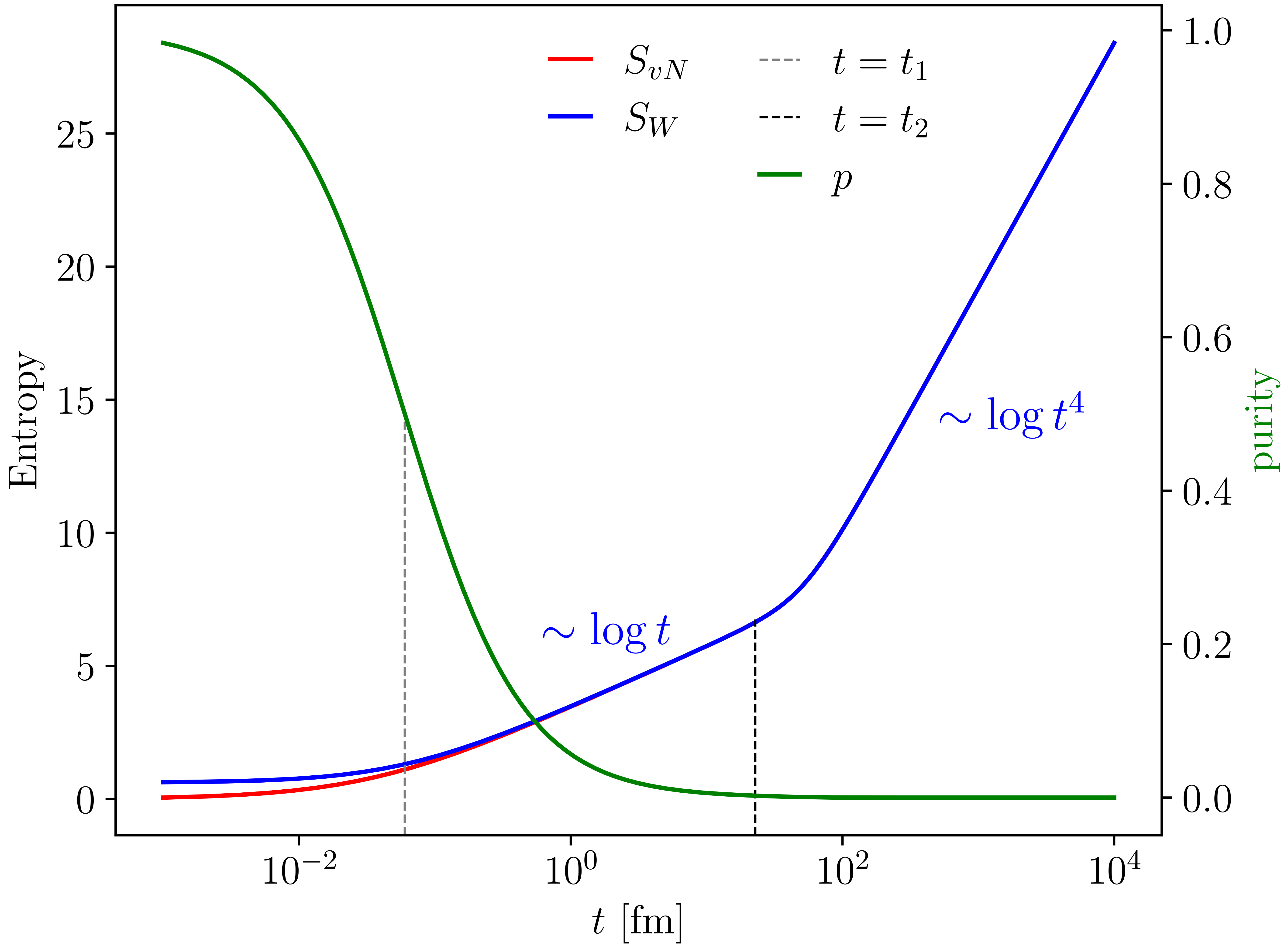
$$p \simeq \frac{1}{\left(\frac{t}{t_1}\right)\left(1+\frac{t^3}{12t_2^3}\right)} \simeq \frac{t_1}{t} \ll 1 \qquad S_{\rm vN} \sim \ln\frac{t}{t_1} = \ln\frac{\langle \boldsymbol{K}^2\rangle_t}{\mu^2}$$

Memory loss: $t \gg t_2$

$$p \simeq \frac{12t_1t_2^3}{t^4} \ll 1 \qquad S_{\rm vN} \simeq \ln\frac{1}{p} \simeq \ln\frac{\hat{q}^2t^4}{E^2} \sim \ln\langle \boldsymbol{k}^2\rangle_t\langle \boldsymbol{b}^2\rangle_t$$

Entropy as measure of quantum classical transition

$t_1 \simeq 0.06$ fm $\quad t_2 \simeq 22.80$ fm $\quad t_0 \simeq 444.44$ fm $\quad t_{\rm rel} \simeq 66.7$ fm



Asymptotically, one has that

$$\frac{S_{\rm W} - S_{\rm vN}}{S_{\rm W}} \approx \frac{\sqrt{p}}{\ln(1/p)}$$

thus, the **entropy content of the density matrix coincides with that of a classical distribution**

In reality, the entropy growth is bounded

$$\left[\frac{\partial}{\partial t} - \frac{\partial}{\partial \boldsymbol{K}}\left(\frac{\hat{q}}{4}\frac{\partial}{\partial \boldsymbol{K}} + \gamma_f \frac{\boldsymbol{K}}{E}\right)\right]\mathcal{P}(\boldsymbol{K}, t) = 0$$

This occurs roughly after a time $\gamma_f = \hat{q}/4T \sim T^2$

$$t_{\rm rel} \equiv ET/\hat{q}$$

Is single parton evolution essentially classical ?

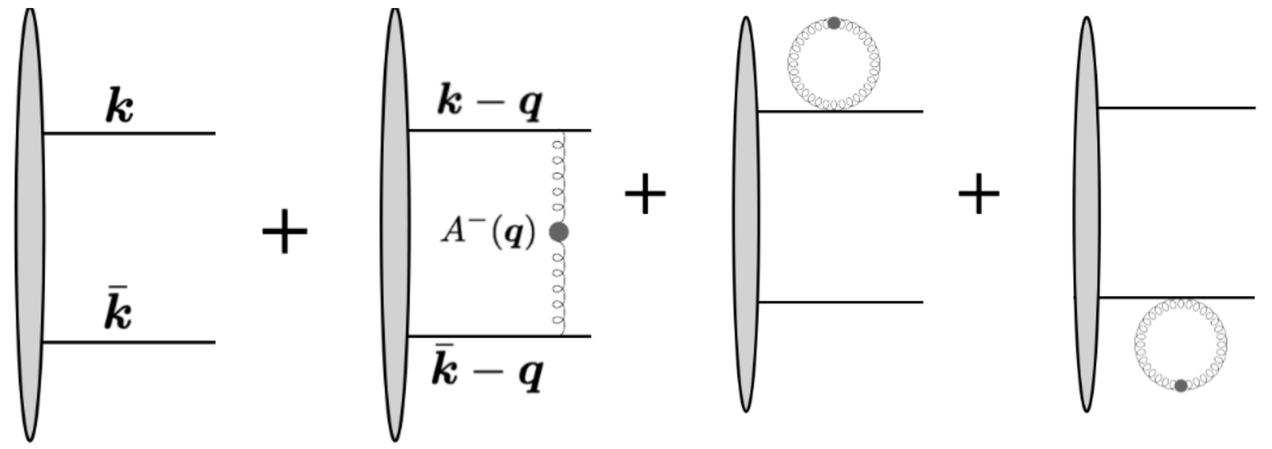
The previous calculation seemed to indicate that, at high energies, the real time dynamics of quark in a large QCD background are essentially classical. **Is this generally true?** **No**

Example: One can construct the same evolution equation but for non-isotropic matter 2210.06519, J.B., A. Sadofyev, X.-N. Wang

$$\partial_L W(\boldsymbol{k},\bar{\boldsymbol{k}}) = -i\,\frac{\boldsymbol{k}^2-\bar{\boldsymbol{k}}^2}{2E} W(\boldsymbol{k},\bar{\boldsymbol{k}}) - \int_{\boldsymbol{q},\bar{\boldsymbol{q}},\boldsymbol{l},\bar{\boldsymbol{l}}} \mathcal{K}(\boldsymbol{q},\bar{\boldsymbol{q}};\boldsymbol{l},\bar{\boldsymbol{l}}) W(\boldsymbol{l},\bar{\boldsymbol{l}})$$

$$\mathcal{K}(\boldsymbol{q},\bar{\boldsymbol{q}};\boldsymbol{l},\bar{\boldsymbol{l}}) = -(2\pi)^4\, C\, v(\boldsymbol{q})v(\bar{\boldsymbol{q}}) \times \left\{ \rho(\boldsymbol{q}-\bar{\boldsymbol{q}})\,\delta^{(2)}(\boldsymbol{k}-\boldsymbol{q}-\boldsymbol{l})\delta^{(2)}(\bar{\boldsymbol{k}}-\bar{\boldsymbol{q}}-\bar{\boldsymbol{l}}) - \frac{1}{2}\rho(\boldsymbol{q}+\bar{\boldsymbol{q}})\,\delta^{(2)}(\boldsymbol{k}-\boldsymbol{l})\delta^{(2)}(\bar{\boldsymbol{k}}-\boldsymbol{q}-\bar{\boldsymbol{q}}-\bar{\boldsymbol{l}}) - \frac{1}{2}\underline{\rho^\dagger(\boldsymbol{q}+\bar{\boldsymbol{q}})}\,\delta^{(2)}(\boldsymbol{k}-\boldsymbol{q}-\bar{\boldsymbol{q}}-\boldsymbol{l})\delta^{(2)}(\bar{\boldsymbol{k}}-\bar{\boldsymbol{l}}) \right\}$$

Color charge distribution in matter



Is single parton evolution essentially classical ?

If one expands this relation in gradient powers of the density, it leads to

0th: Boltzmann + diffusion ✅

1st: Boltzmann + diffusion + $\hat{q}(\boldsymbol{Y})$ ✅

2nd: Boltzmann + diffusion + $\hat{q}(\boldsymbol{Y})$ + quantum corrections

$$\kappa = 2\pi^2 C \int_{\boldsymbol{q}} v^2$$

$$\left(\partial_L + \frac{\boldsymbol{p}\cdot\boldsymbol{\nabla}_{\boldsymbol{Y}}}{E} - \frac{\hat{q}(\boldsymbol{Y})}{4}\partial^2_{\boldsymbol{p}}\right) W(\boldsymbol{Y},\boldsymbol{p}) = \boxed{\nabla_i\nabla_j\rho \times \int_{\boldsymbol{q}} \left[\kappa\frac{\partial^2}{\partial p_i \partial p_j}\delta^{(2)}(\boldsymbol{q}) - V_{ij}(\boldsymbol{q})\right] W(\boldsymbol{Y},\boldsymbol{p}-\boldsymbol{q})}$$

$$V_{ij}(\boldsymbol{q}) = \frac{C}{2}\left(\left\{2q_iq_j\left[vv'' - v'v'\right] + vv'\delta_{ij}\right\} - (2\pi)^2\delta^{(2)}(\boldsymbol{q})\int_{\boldsymbol{l}}\left\{2l_il_j\left[vv''-v'v'\right]+vv'\delta_{ij}\right\}\right)$$

New collisional terms imply non-local interactions which goes beyond Boltzmann transport

Is single parton evolution essentially classical ?

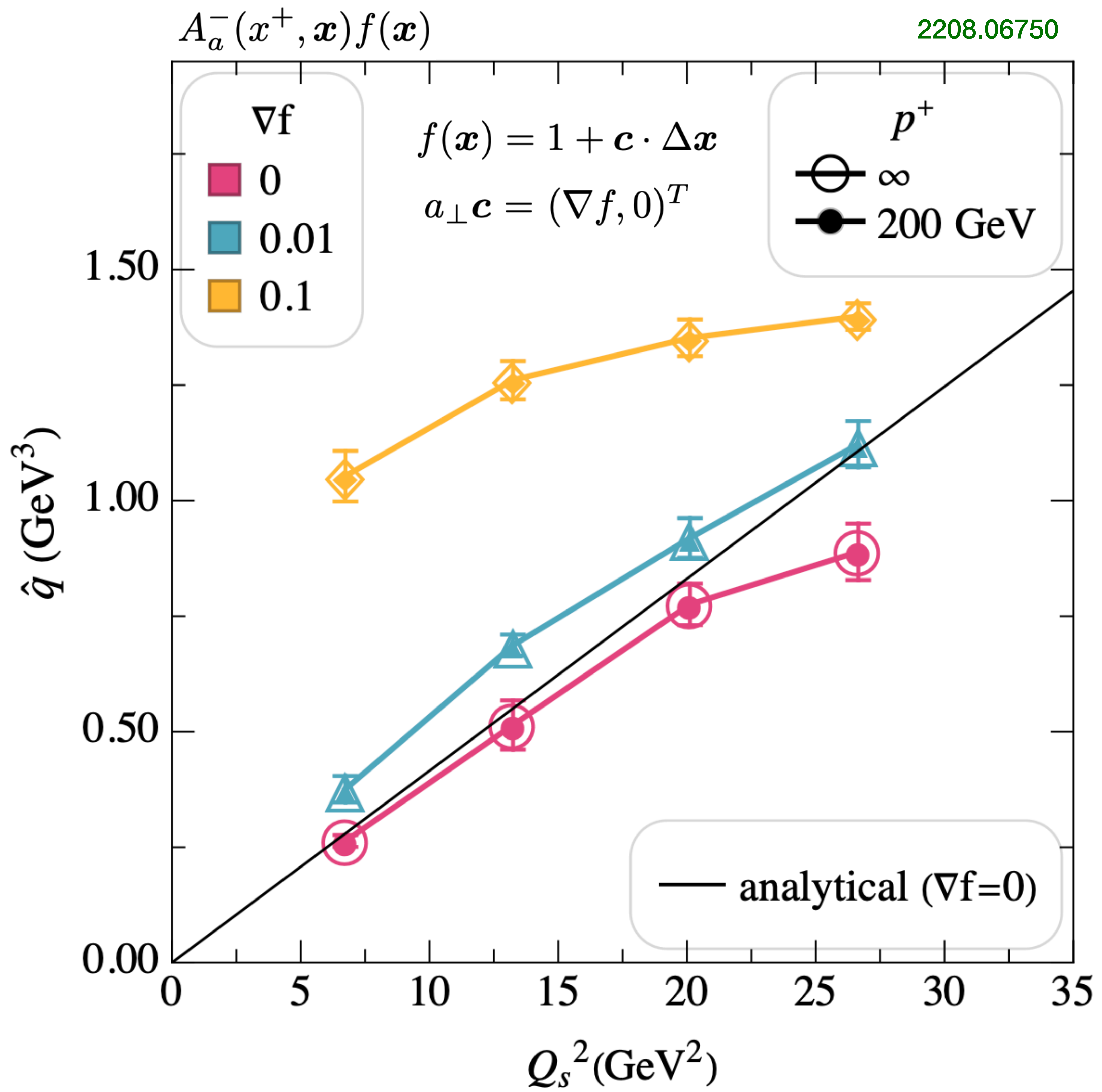
Coef. homogenous case

$$\hat{q}_r = \hat{q} + \boldsymbol{\nabla}^2 \hat{q} \left(\frac{\hat{q} L^3}{12 E^2} + \eta \right)$$

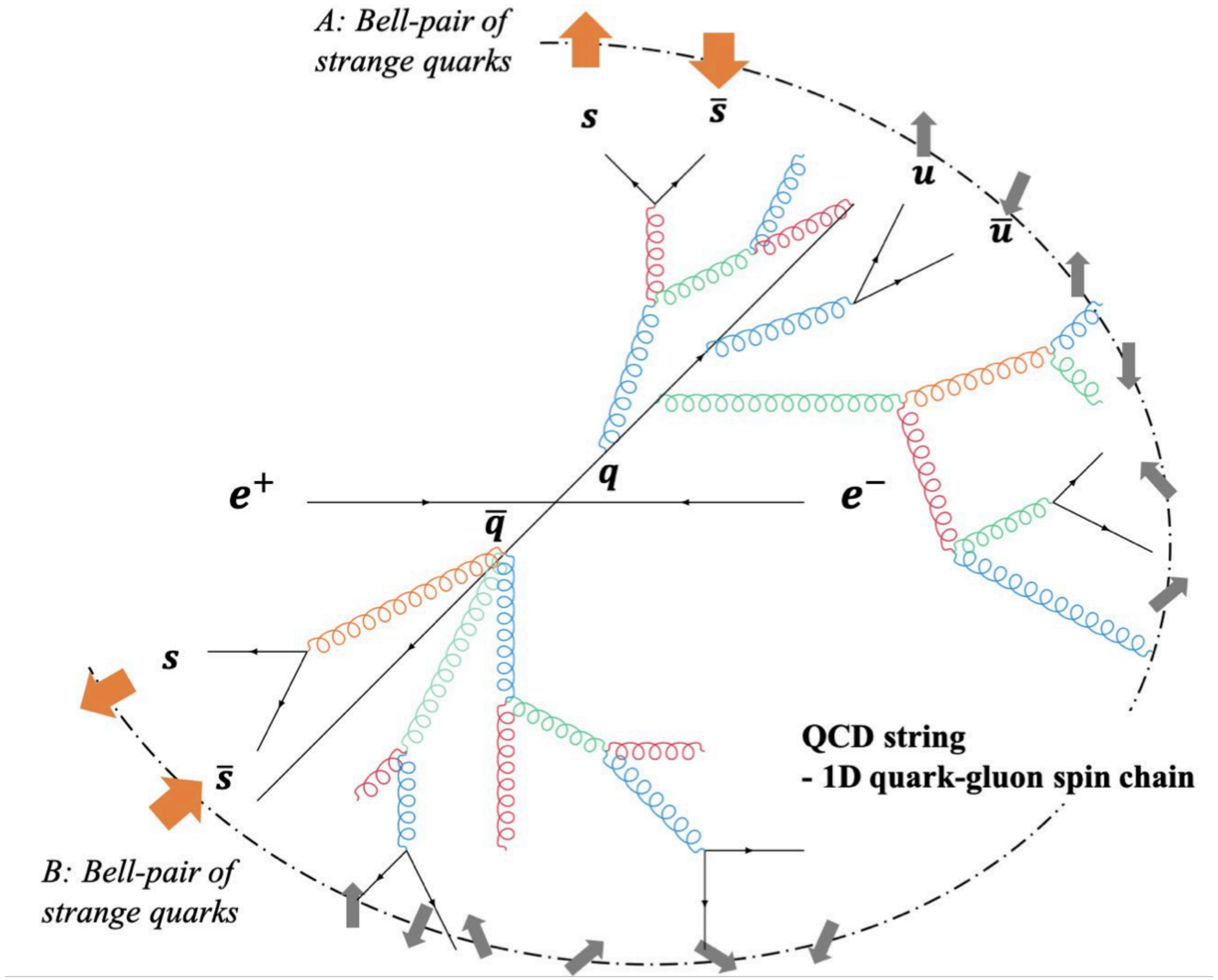
Full coeff.

Coef. due to anisotropy effects

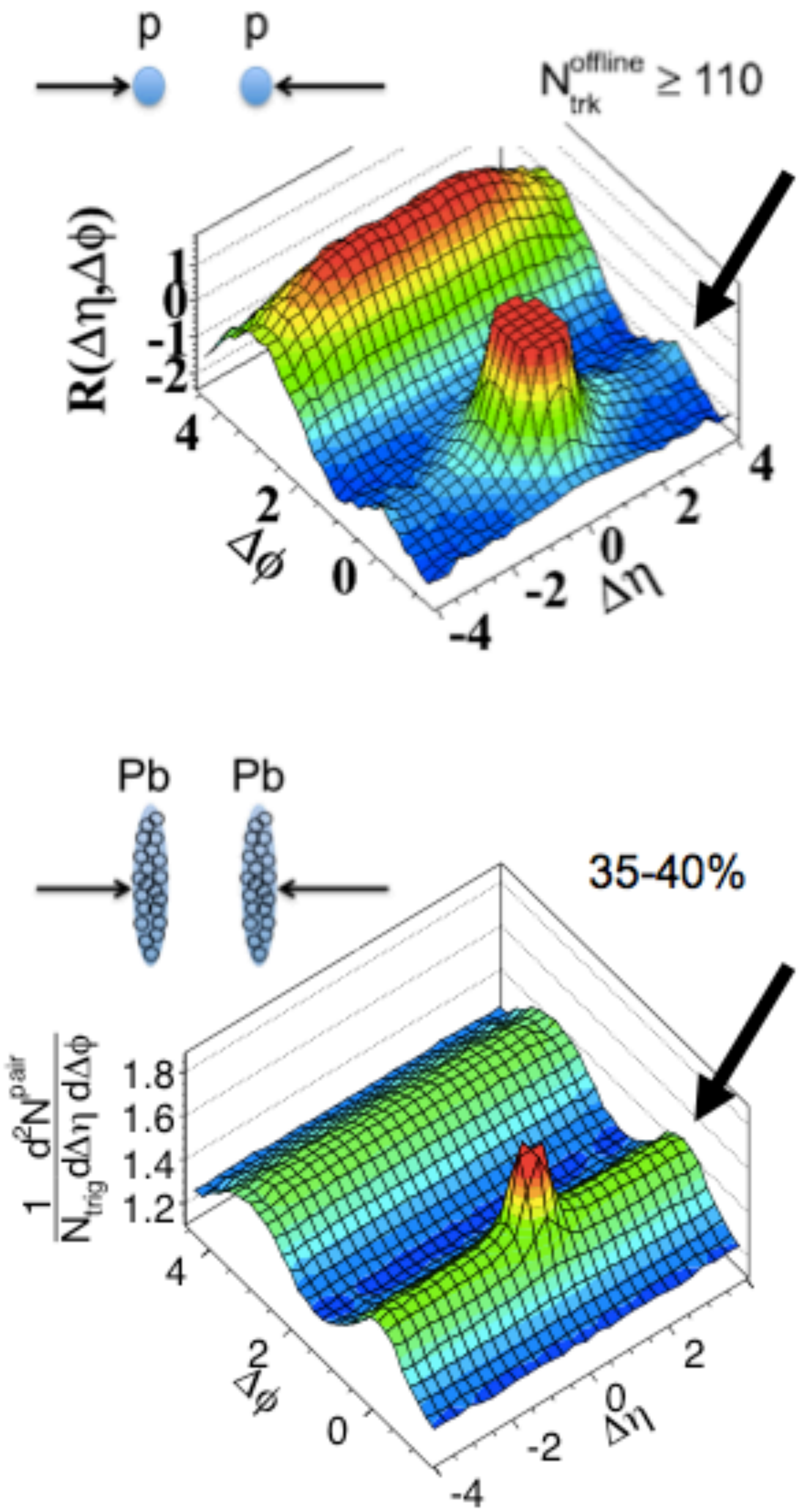
$$\eta = \rho\,\kappa/(2\pi^2 \hat{q}) + \frac{C\rho}{2\hat{q}} \int_{\boldsymbol{q}} \boldsymbol{q}^2 v^2 [\boldsymbol{q}^2 v'/v]'$$

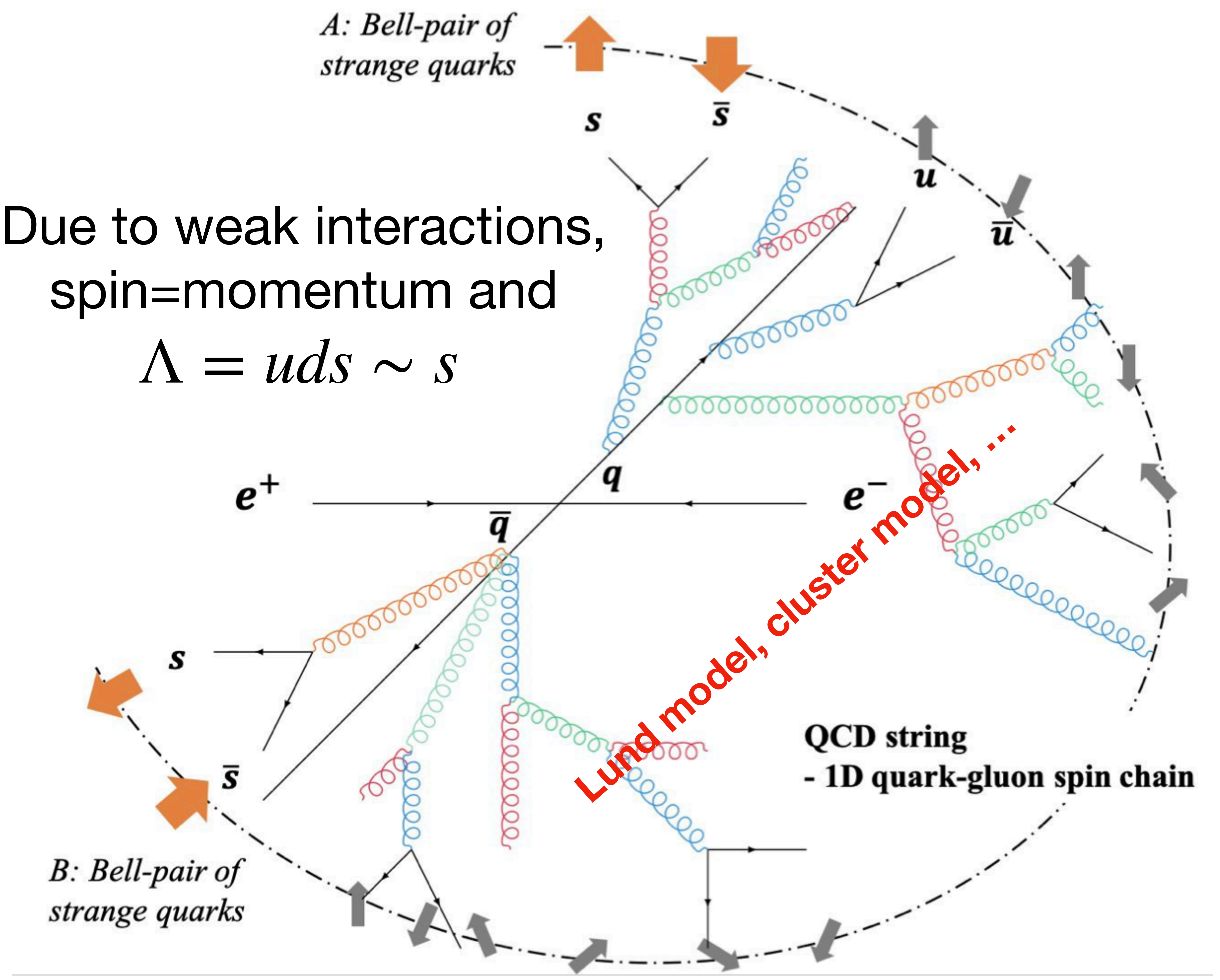


Spin correlations and entanglement in 1+1d models



Spin correlations in a QCD string





Due to weak interactions, spin=momentum and

$\Lambda = uds \sim s$

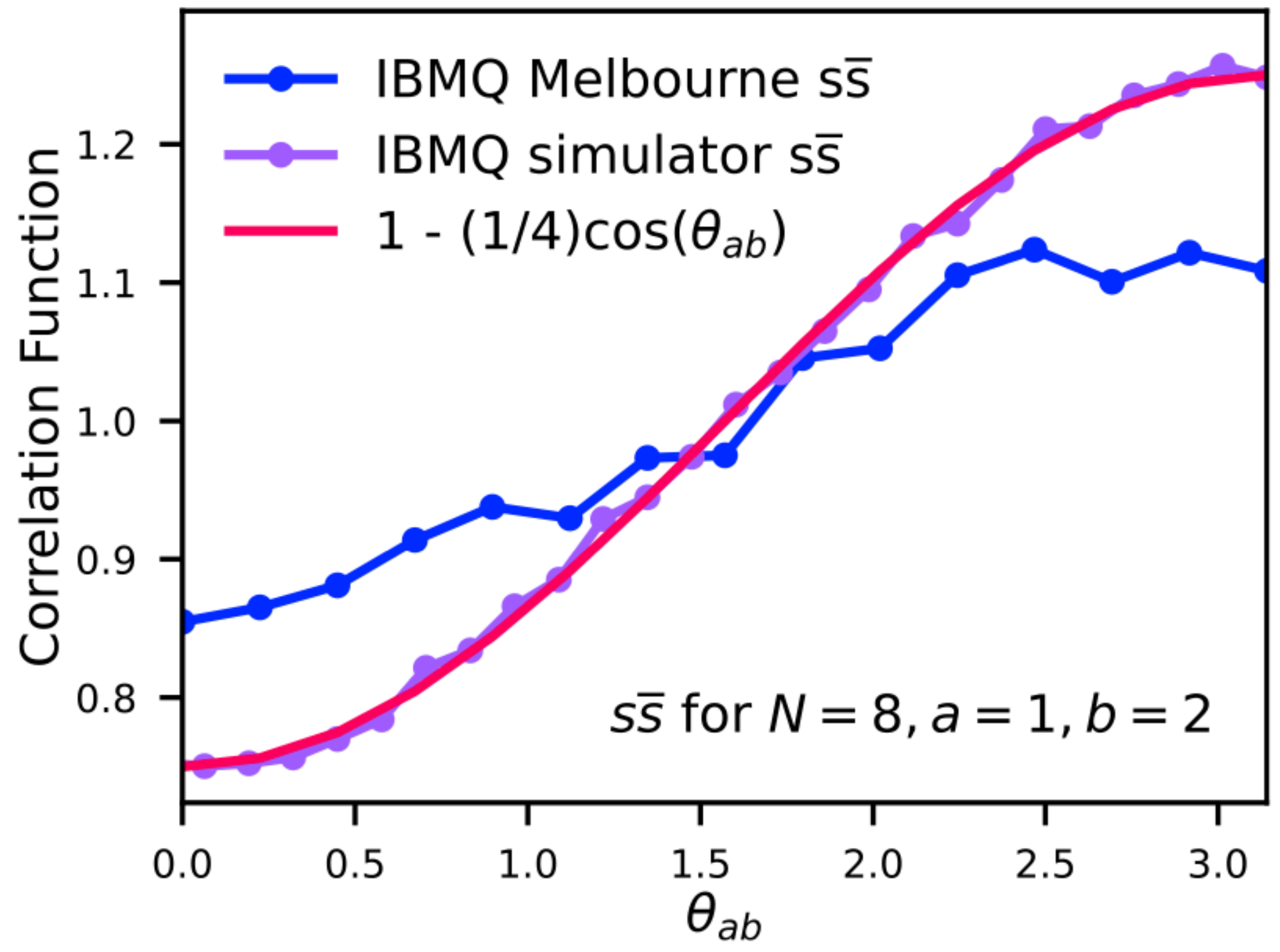
Spin correlations in a QCD string

Previous study showed that spin correlations between heavy quarks **after fragmentation** can be used as a measure of entanglement within the QCD string.

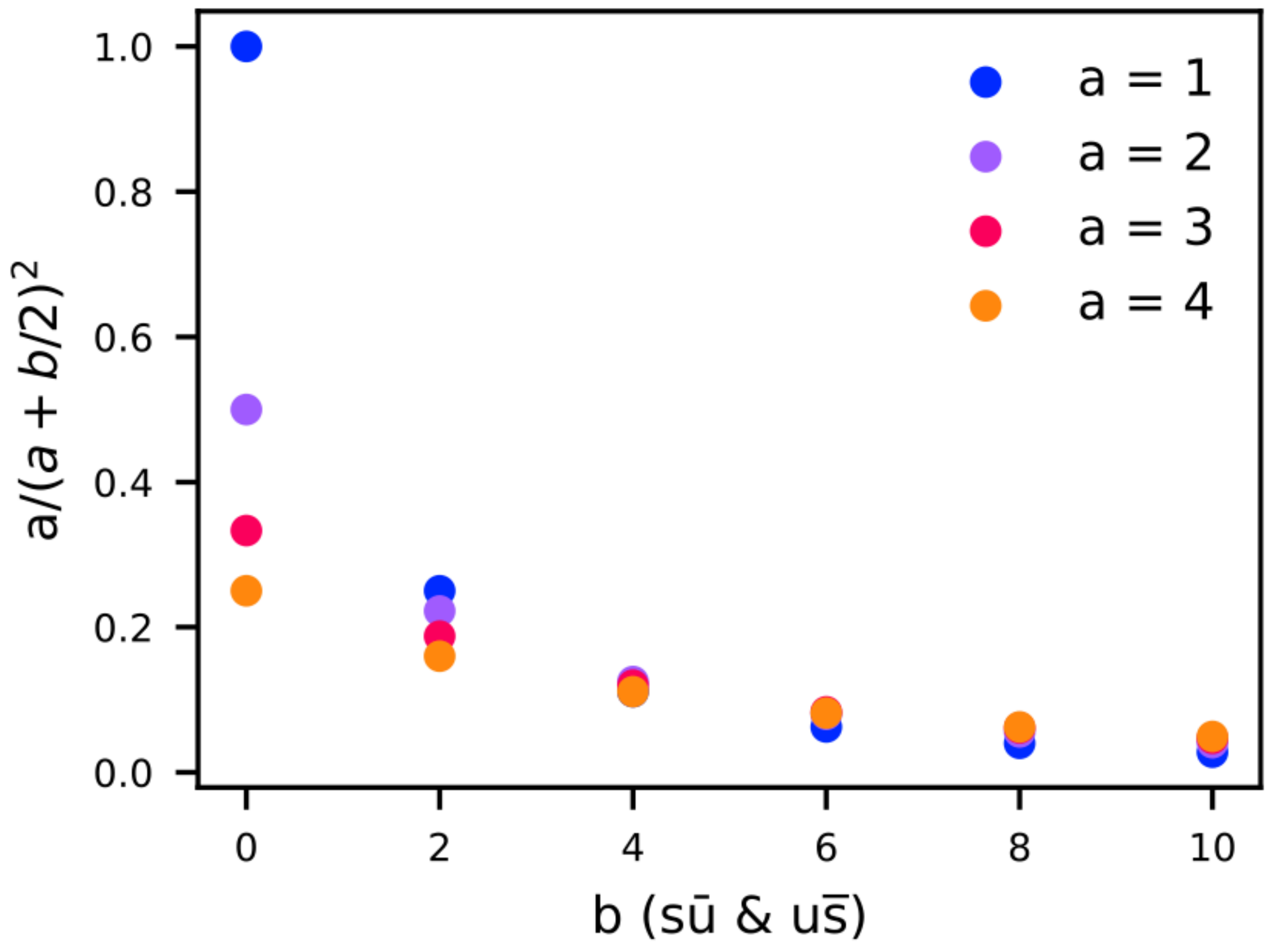
2107.13007, Gong, Parida, Tu, Venugopalan

$$\frac{P(|\hat{n}_1\rangle\,,|\hat{n}_2\rangle)}{P(|\hat{n}_1\rangle)P(|\hat{n}_2\rangle)} = 1 - \frac{a}{(a+b/2)^2}\cos(\theta_2-\theta_1)$$

a= # strange pairs b= # light quarks



Corollary 2. *If the magnitude of the coefficient of* $\cos(\theta_{ab})$ *in a symmetric rotationally invariant correlation function is* $> \frac{1}{2}$, *then the measured state* ρ_{ab} *is entangled.*



Toy model in 1+1d

We would like to understand the real time evolution instead of fixed string configurations

We consider a simple toy model in 1+1d, based on 4 flavor Schwinger model

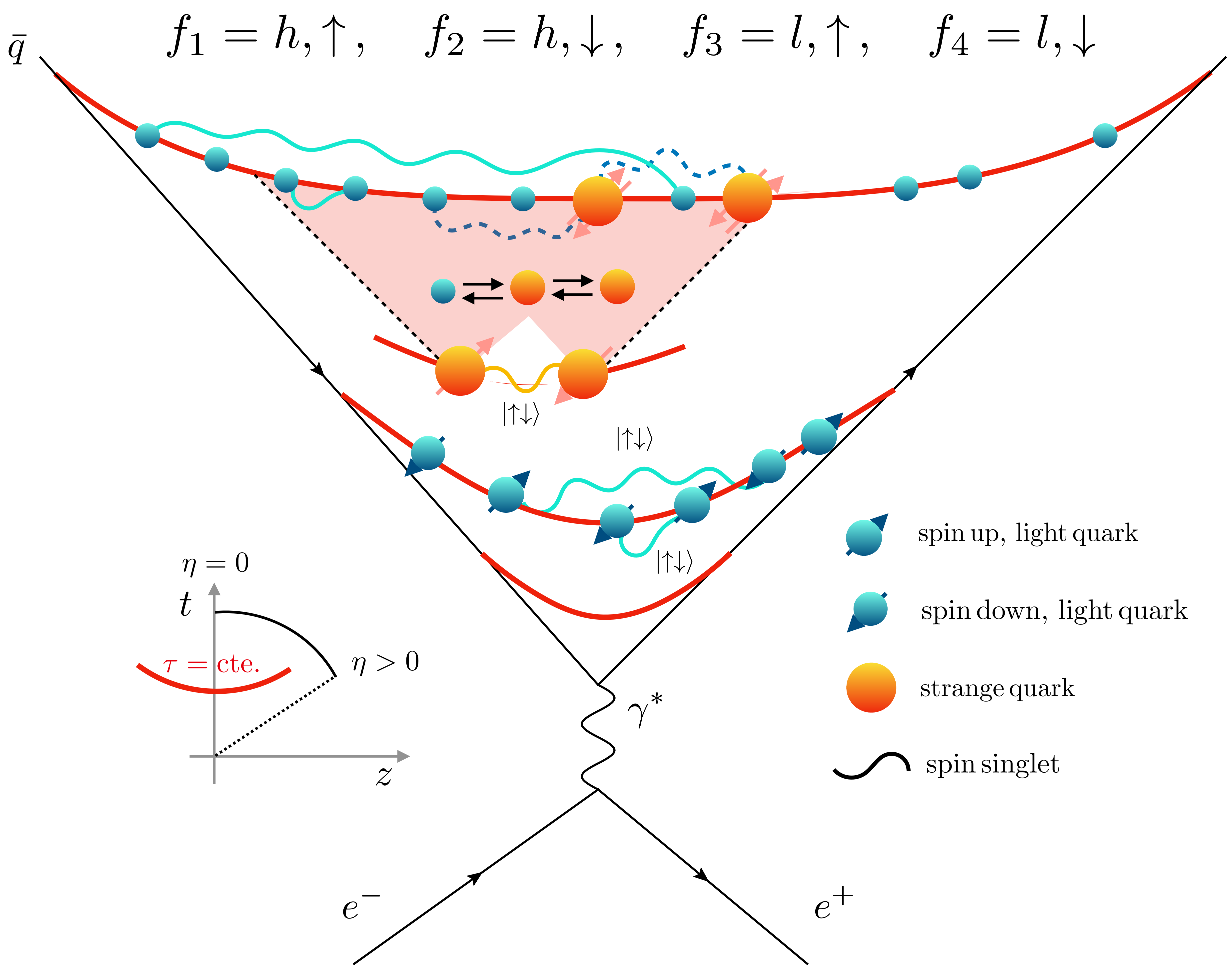
$$H = H_{\text{Schwinger}} + H_{\text{spin}}$$

$$H_{\text{Schwinger}} = \int dx \, \frac{1}{2} E^2(x) + \sum_{f=1}^{N_f} \bar{\psi}_f(x)(-i\gamma^1\partial_1 + g\gamma^1 A_1(x) + m_f)\psi_f(x)$$

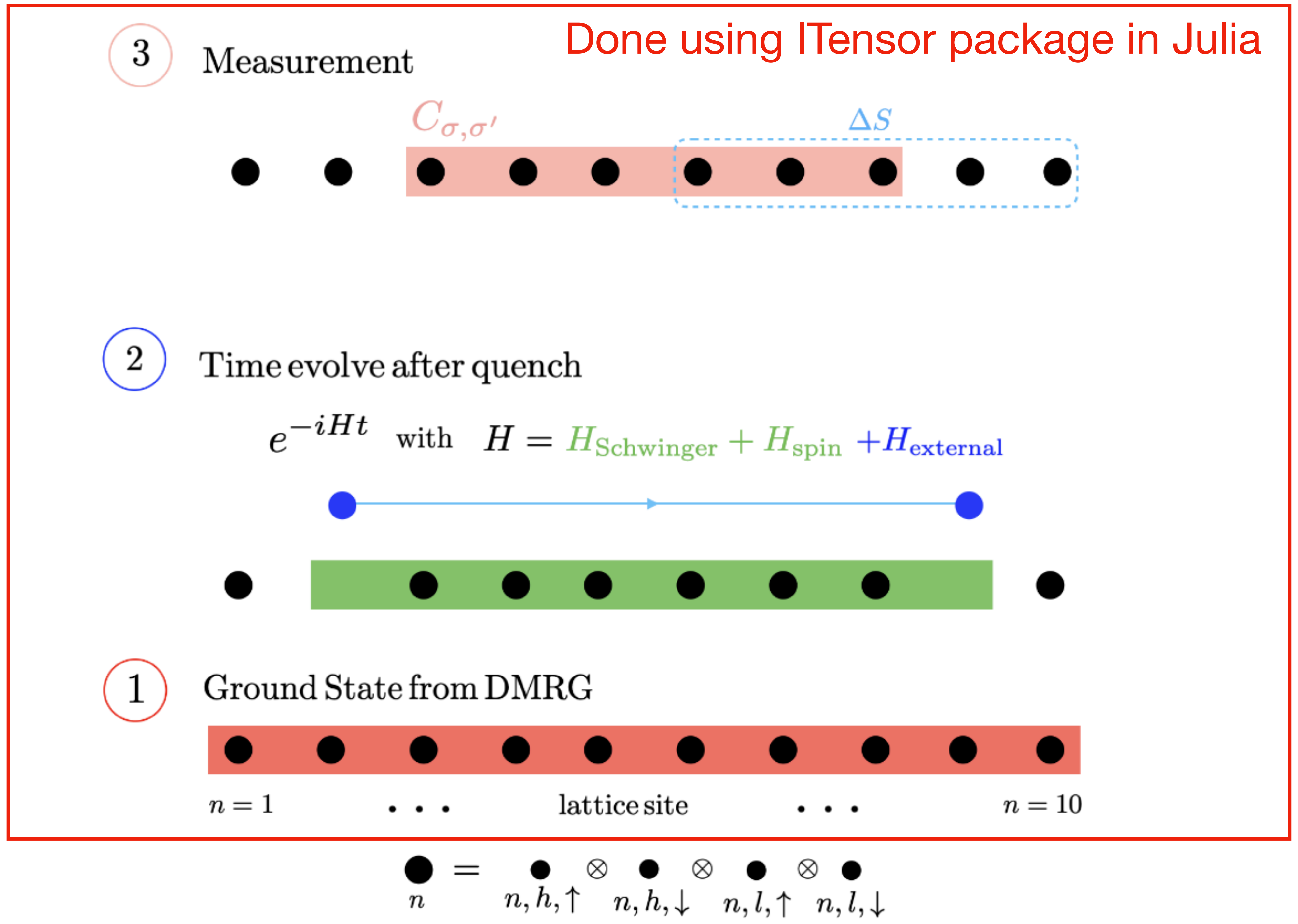
Spin flips between different species are induced by the term

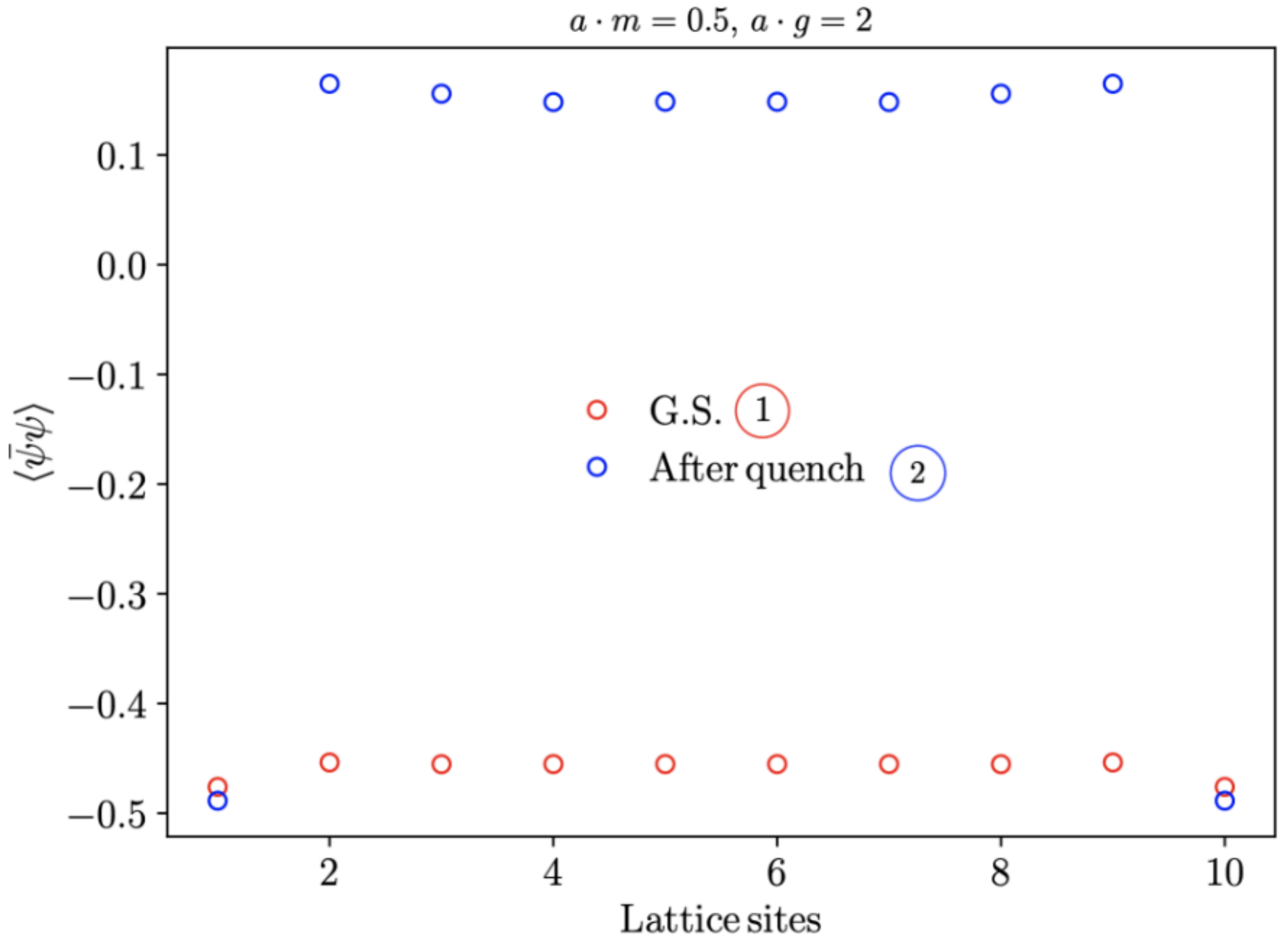
$$\begin{aligned} H_{\text{spin}} = & \, g^0_{ll}(\bar{\psi}_{l,\uparrow}\gamma^0\psi_{l,\downarrow} + h.c.) \\ & + g^1_{ll}(\bar{\psi}_{l,\uparrow}\psi_{l,\downarrow} + \text{h.c.}) \\ & + g^0_{l,h}(\bar{\psi}_{h,\uparrow}\gamma^0\psi_{l,\downarrow} + \bar{\psi}_{h,\downarrow}\gamma^0\psi_{l,\uparrow} + \text{h.c.}) \\ & + g^1_{l,h}(\bar{\psi}_{h,\uparrow}\psi_{l,\downarrow} + \bar{\psi}_{h,\downarrow}\psi_{l,\uparrow} + h.c.) \end{aligned}$$

and the gauge field



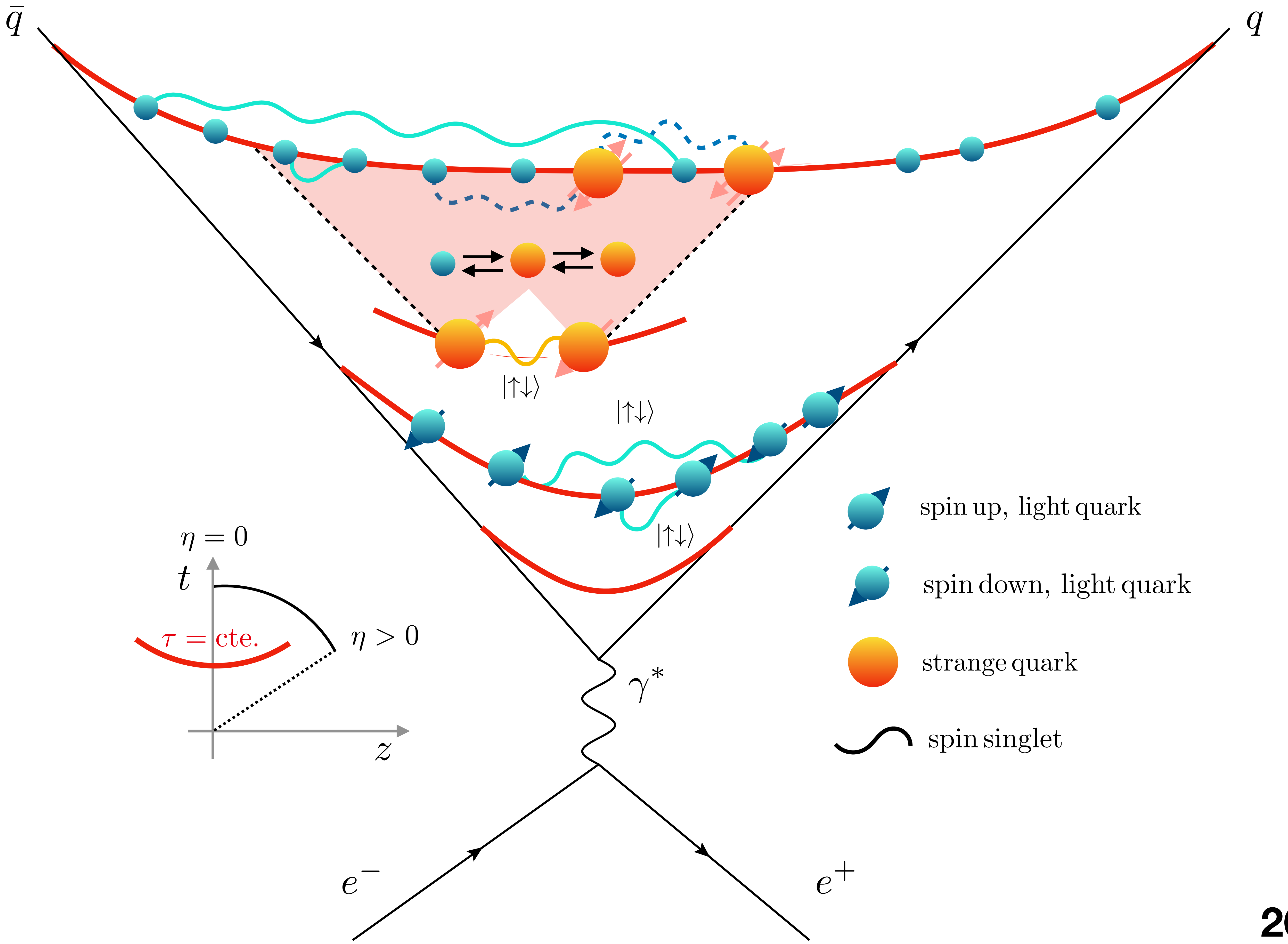
Toy model in 1+1d



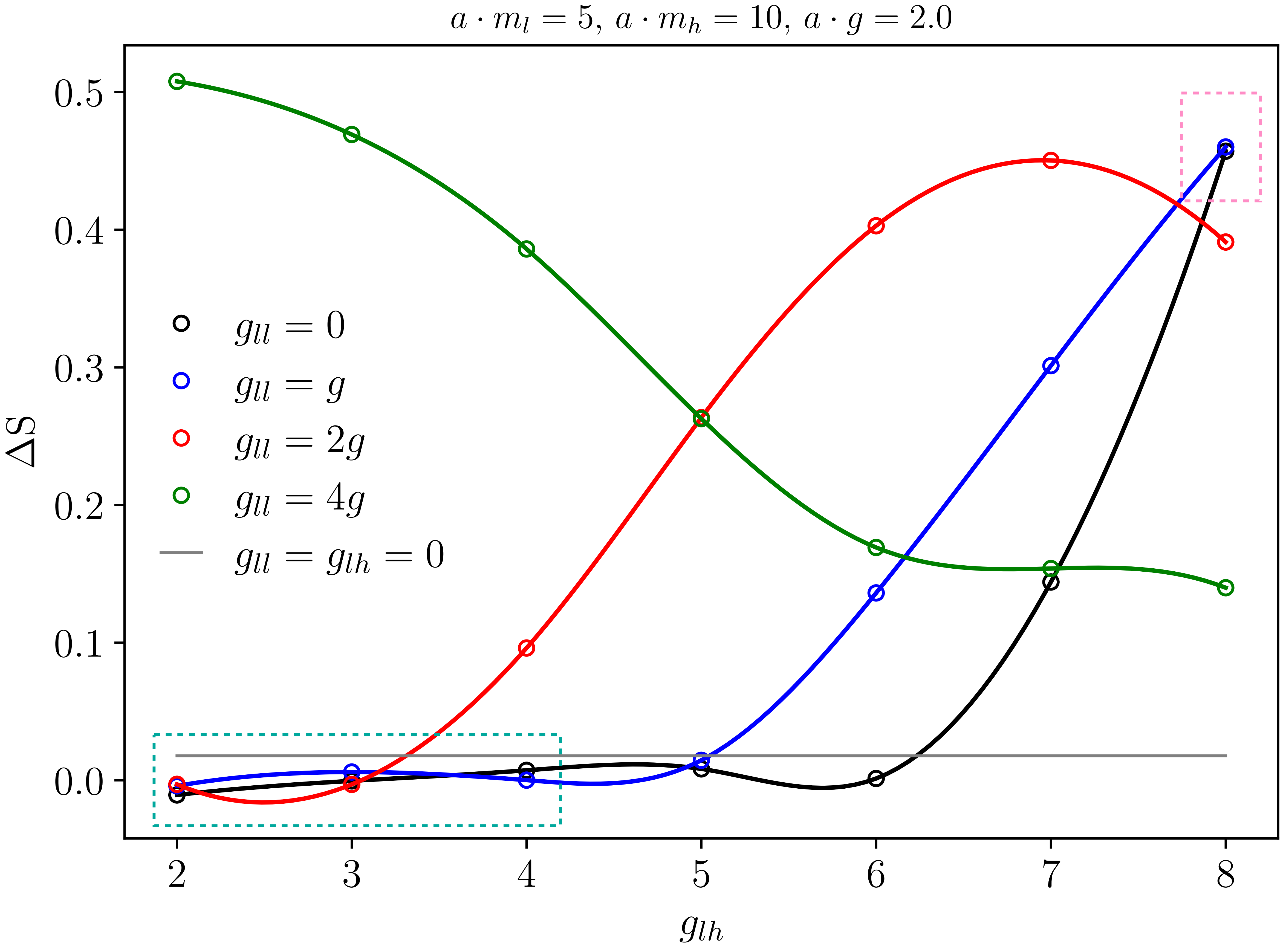


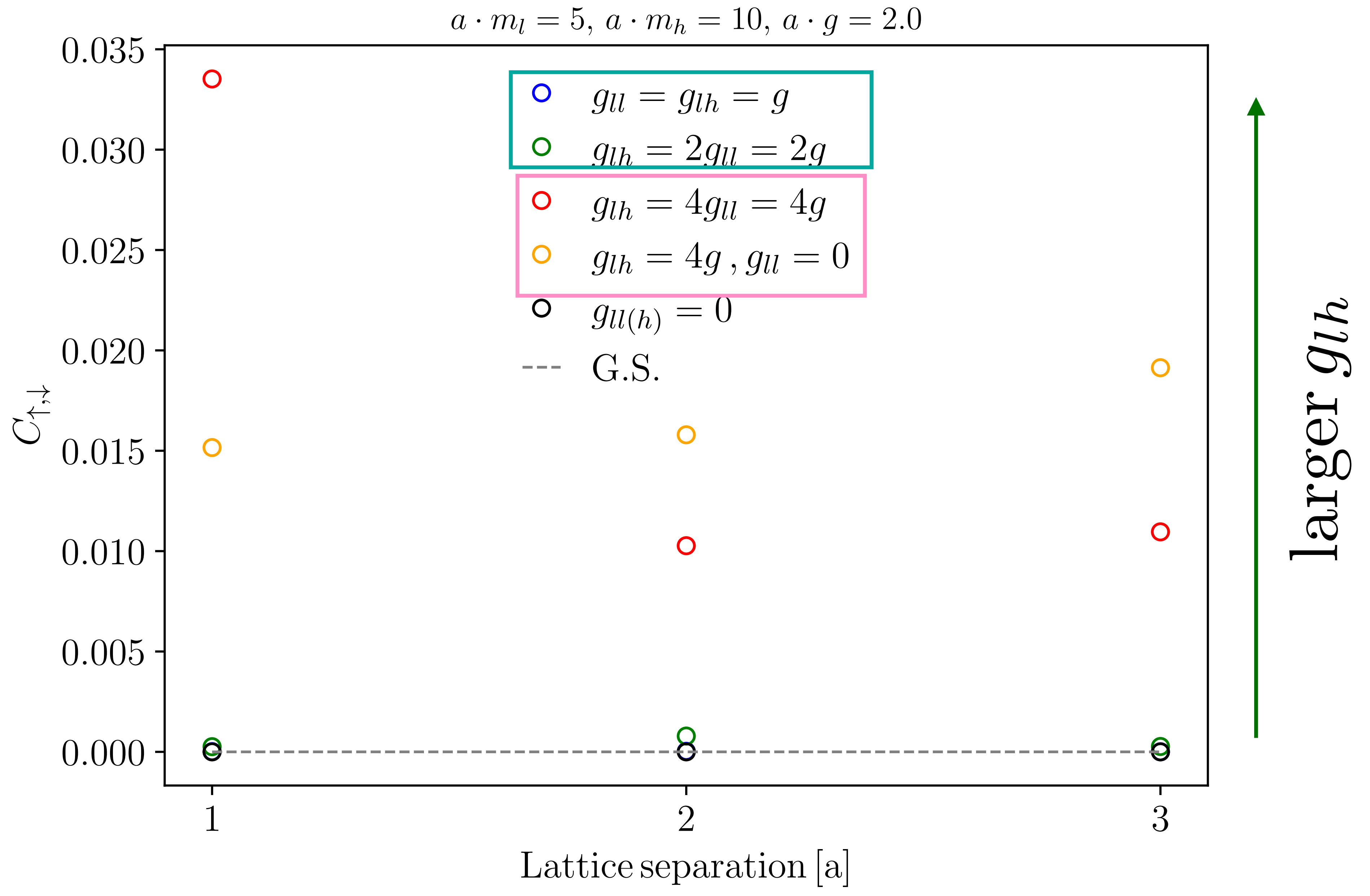
$$C_{\sigma,\sigma'}(\Delta) \equiv \frac{\sum_{n,m} \langle \hat{c}_\sigma(n)\hat{c}_{\sigma'}(m)\rangle \delta(\Delta - n + m)}{\sum_{n,m} \delta(\Delta - n + m)}$$

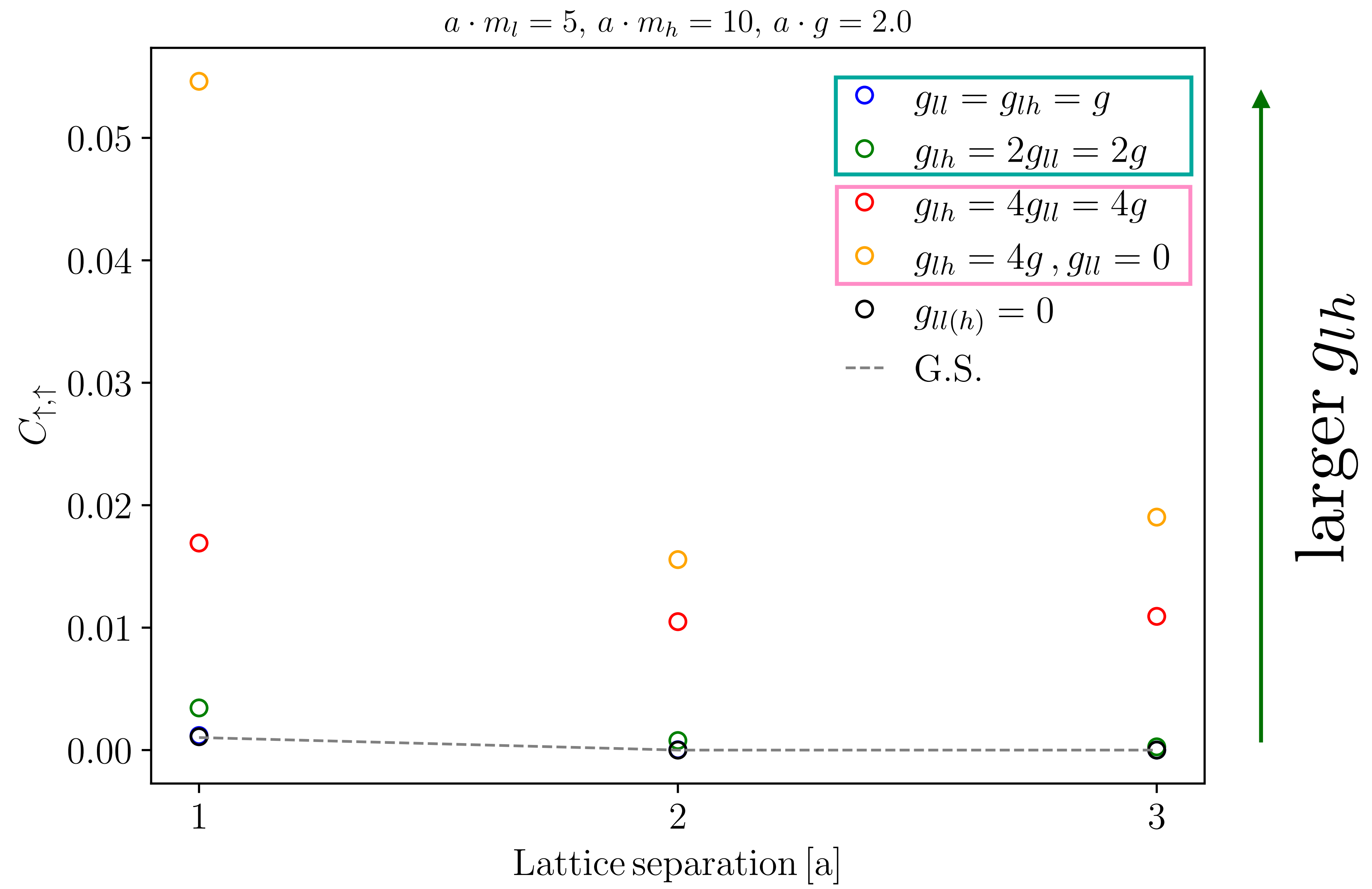
$$\hat{c}_\sigma(n) = \bar{\psi}(n)\psi(n) - \langle \bar{\psi}(n)\psi(n)\rangle_{\text{G.S.}}$$



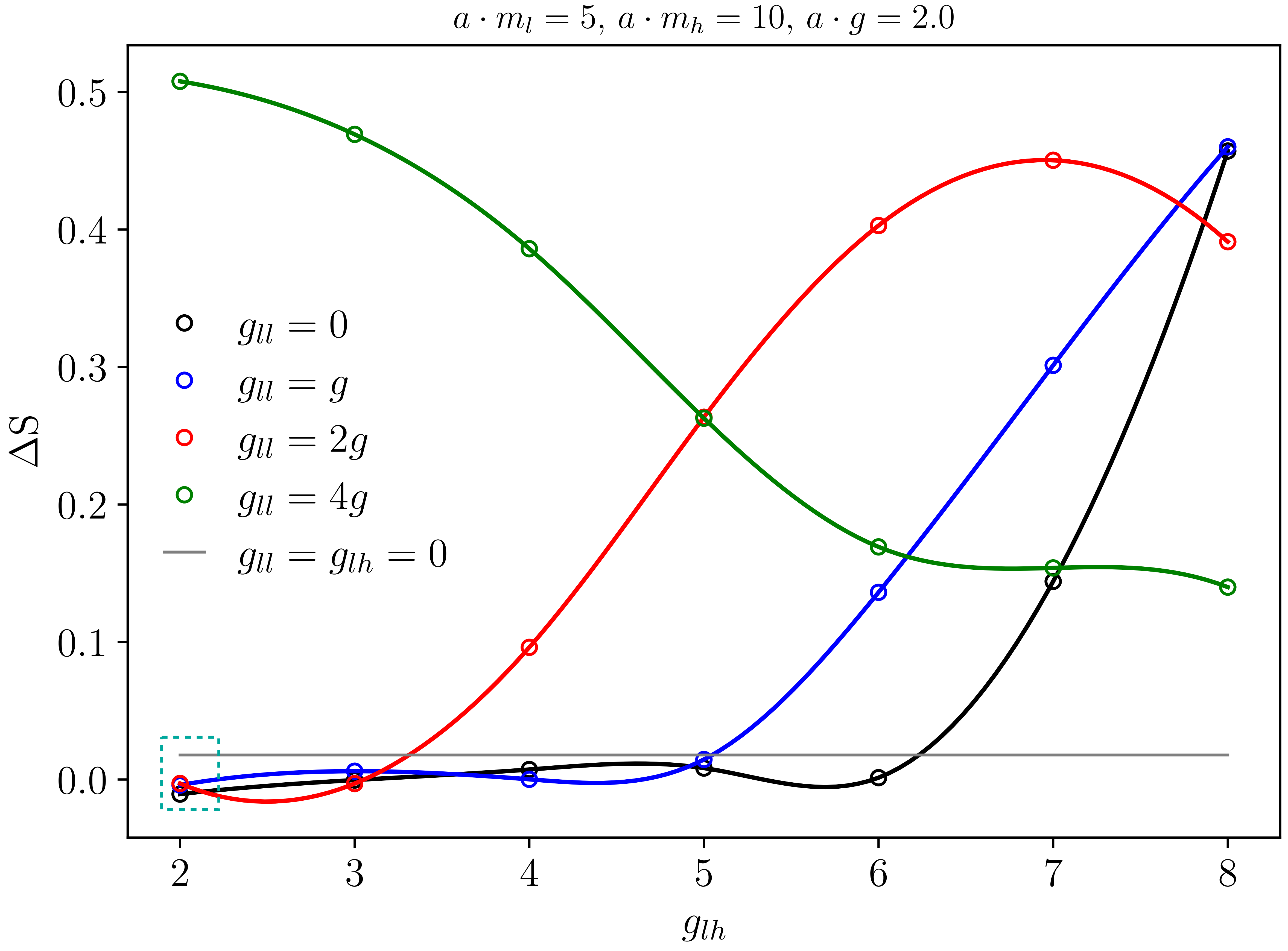
Some preliminary results

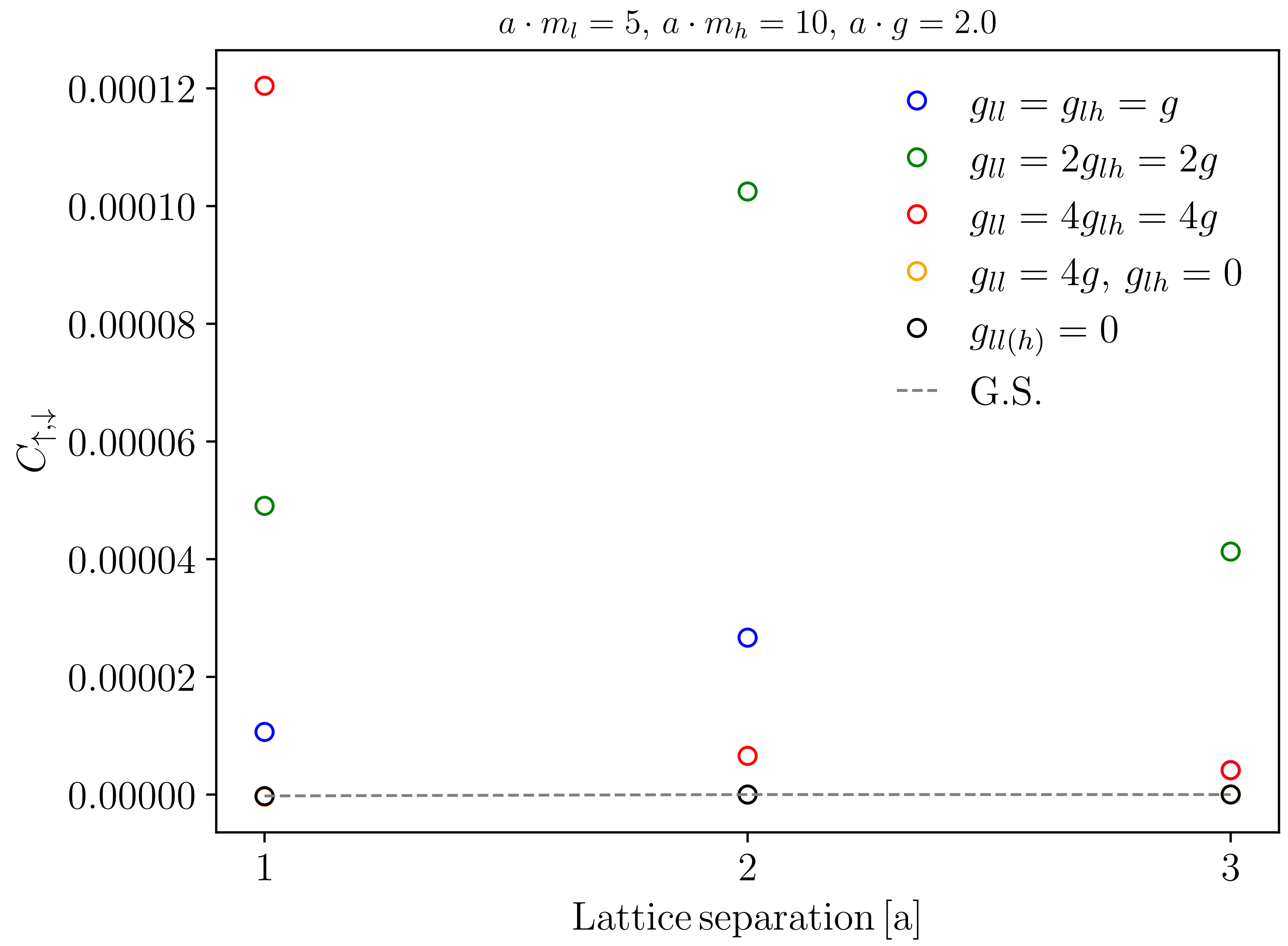


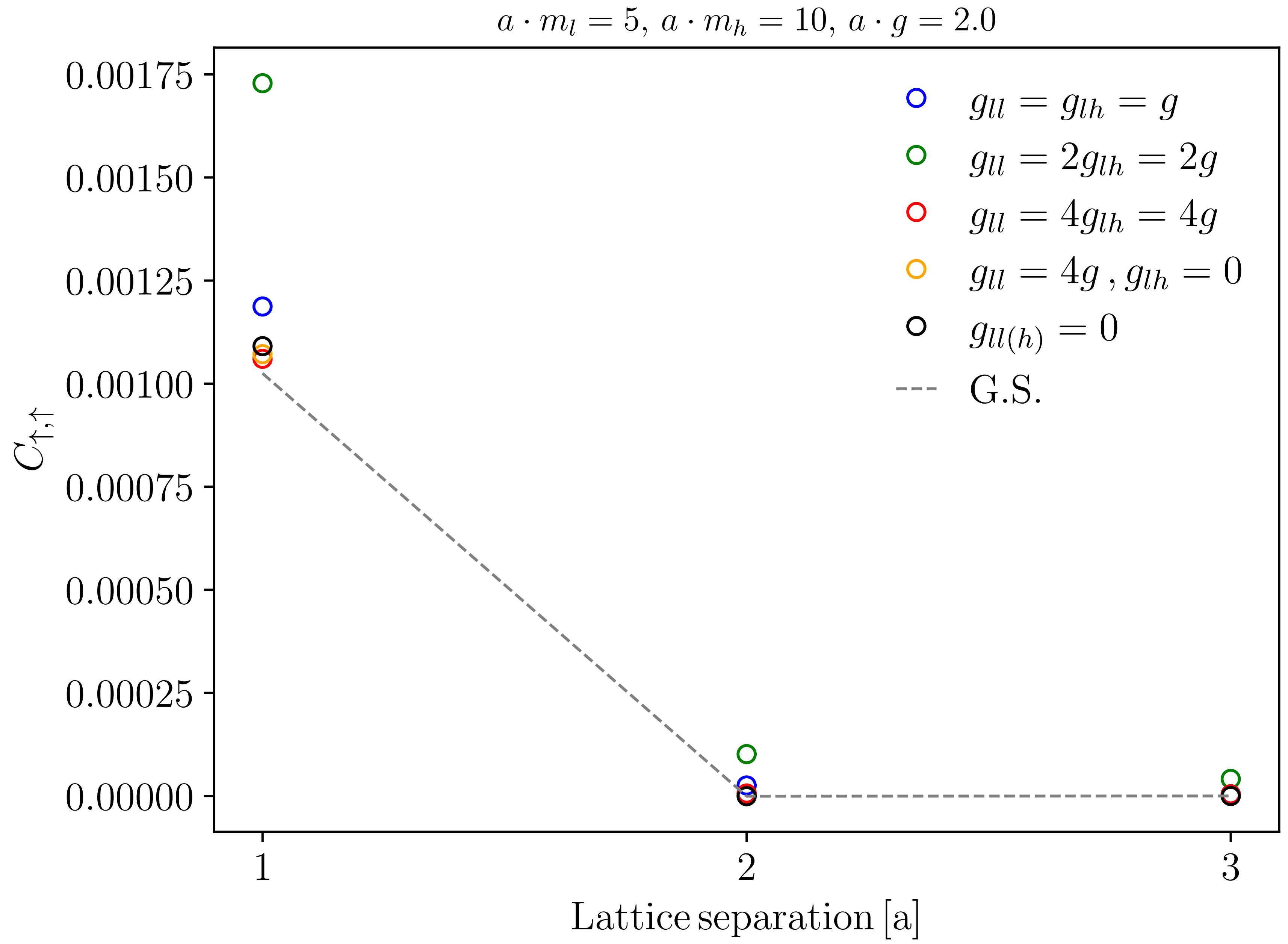




Some preliminary results







Conclusion and Outlook

- Quark density matrix decoheres once medium interactions become significant

- Accessing quantum aspects requires more detailed treatment of both matter and evolution operator

- Spin correlations can be used to probe the entanglement in a QCD string

- For small and at finite lattice spacing simulations, we observe that the increase in entanglement entropy is related to larger spin-spin correlations