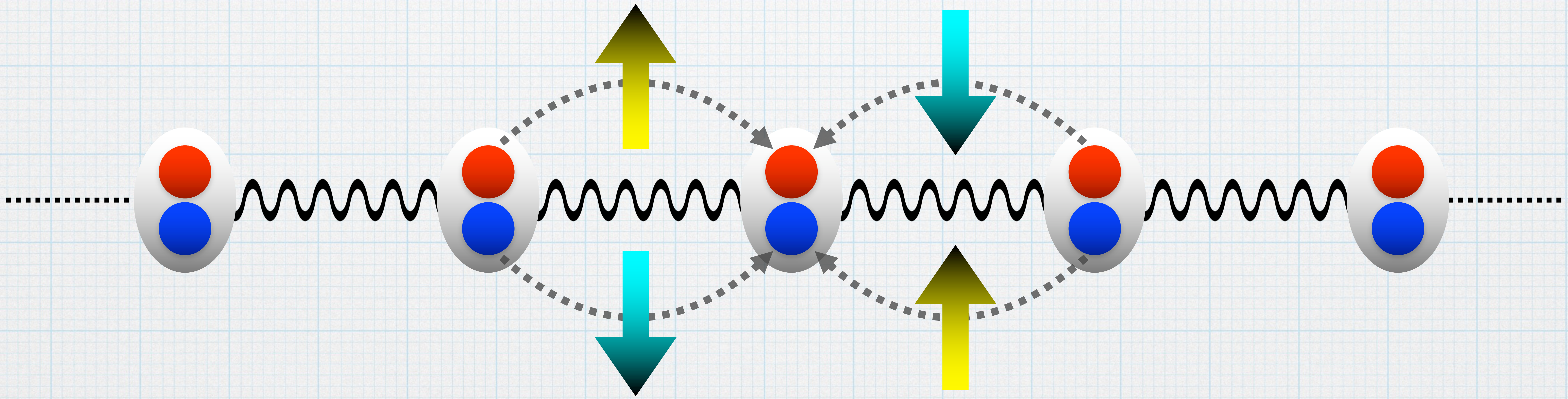
Entanglement entropy in critical Abelian Higgs model (M.A.Nowak)

Towards quantum simulation of
abelian Higgs model in 1+1D on a lattice



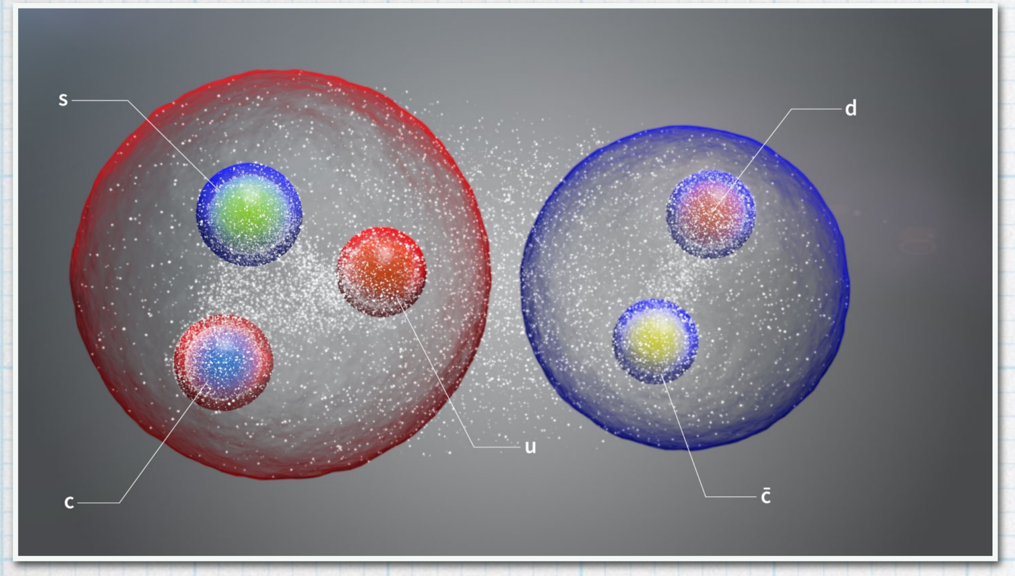
Jakub Zakrzewski
Jagiellonian University

Outline …

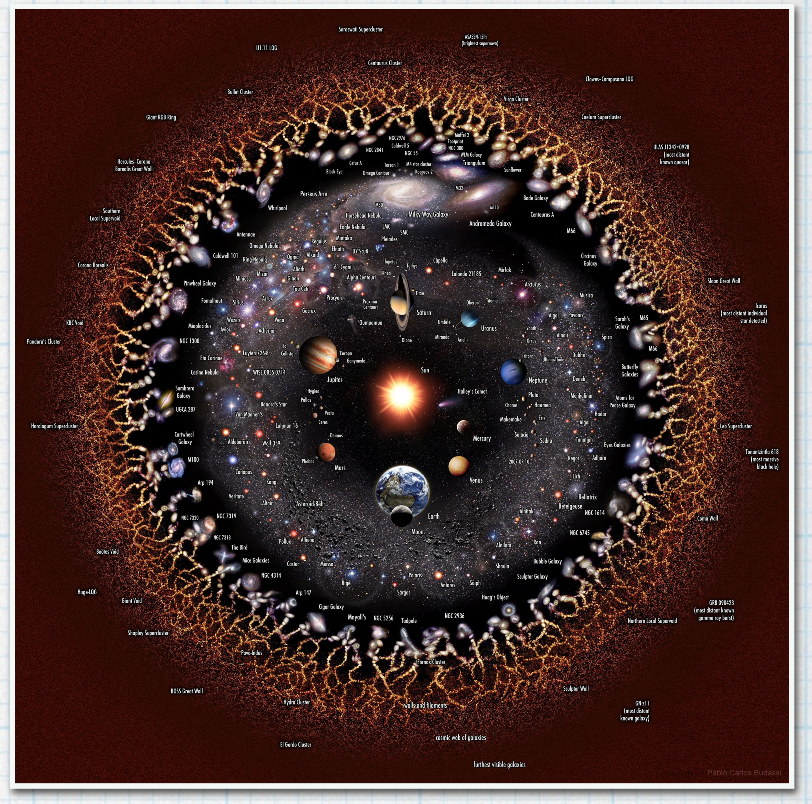
1. Motivation
   (i) Quantum Simulations
   (ii) Tensor Network Algorithms

2. Simulating Lattice Gauge Theories in and out of equilibrium
   (i) Bosonic Schwinger Model Out-Of-Equilibrium
   (ii) Criticality and Higgs Mechanism in 1+1D Abelian-Higgs Model

Motivation



The subject of Physics encompasses a massive range of phenomena and concepts from sub-atomic particles to the observable universe

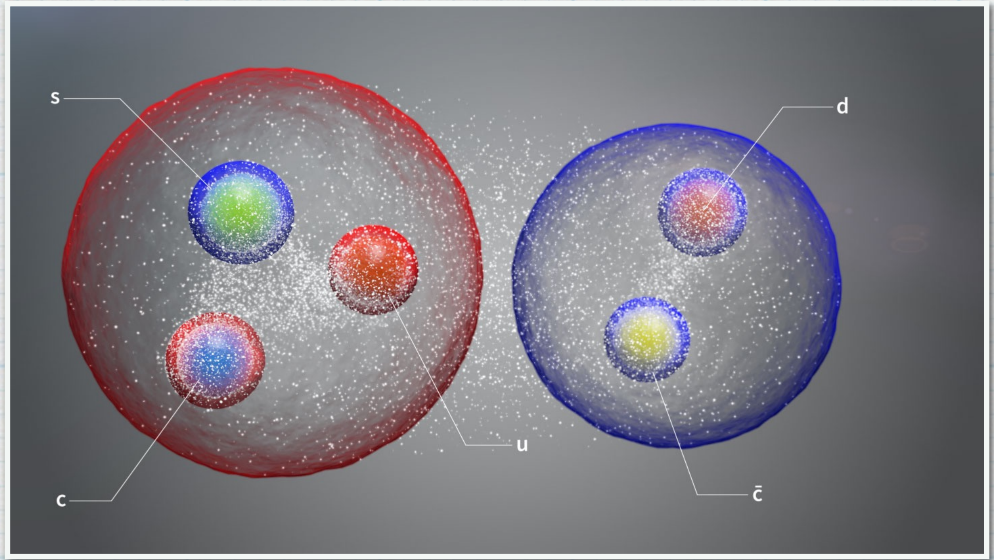


diverse in length-scales…
diverse in time-scales…
diverse in energy-scales…

} Different fields to explore

e.g., energy-scales (in eV) for a few prominent fields of physics…

Motivation



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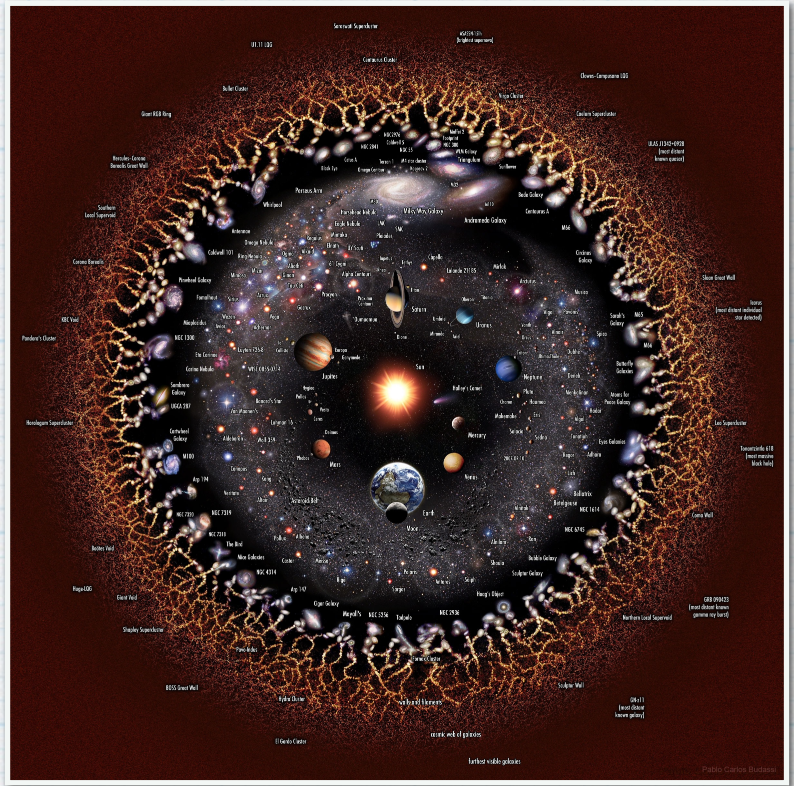

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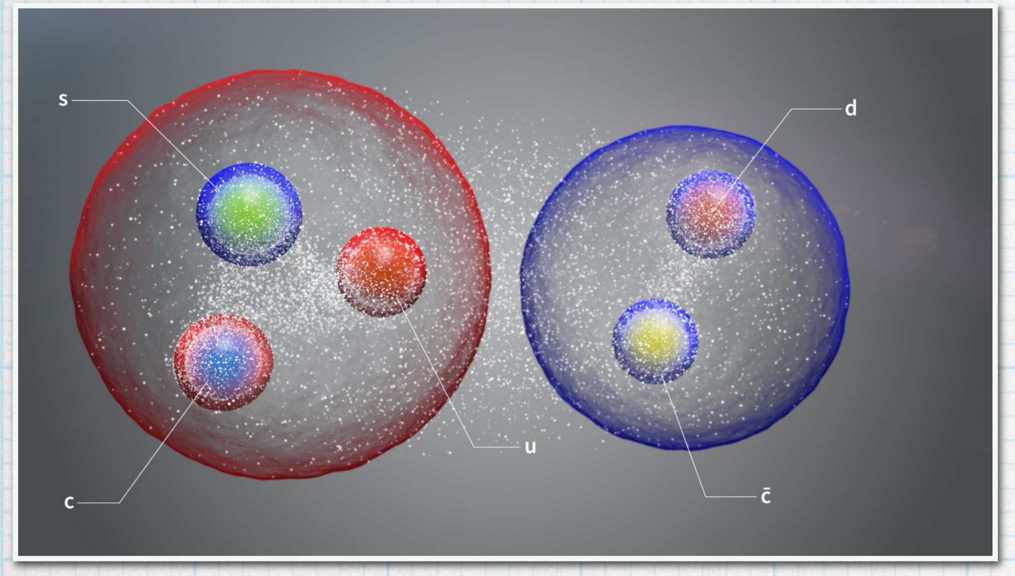

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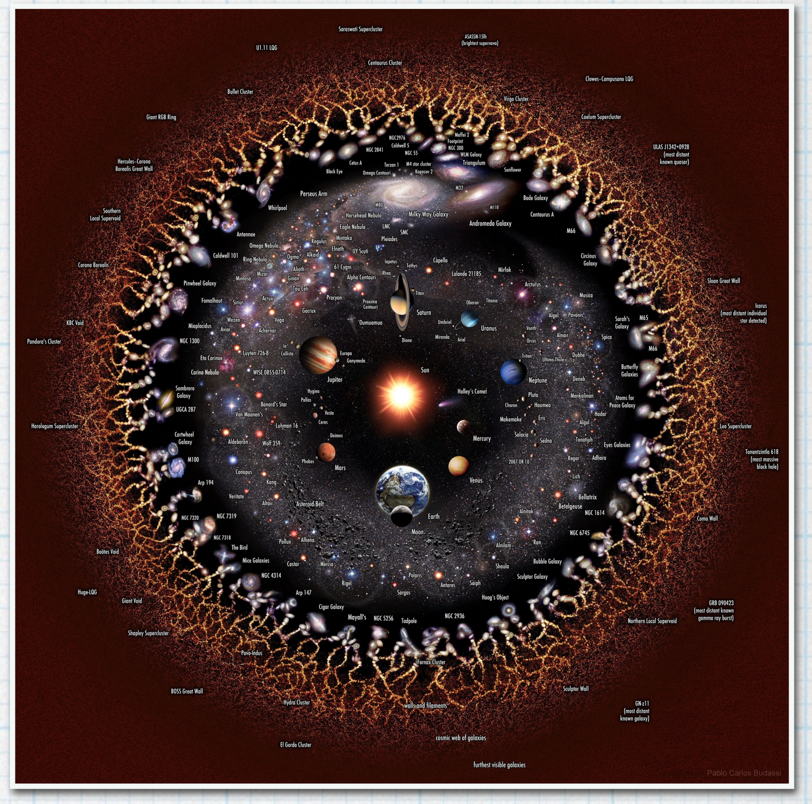

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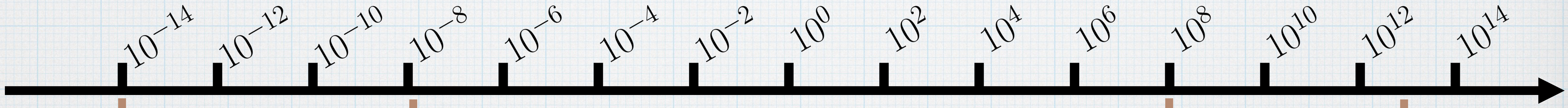


Ultra-cold atomic physics
(~ nK — mK)

Quantum many-body physics
(upto ~ 50K)

Condensed matter physics

Theoretical quantum information and computation (0 K)

Particle Physics
Standard model and beyond

Grand Unified Theory
$10^{23} - 10^{25}$ eV (??)

M Theory
10^{28} eV (??)

Can there exist common interfaces between these topics?

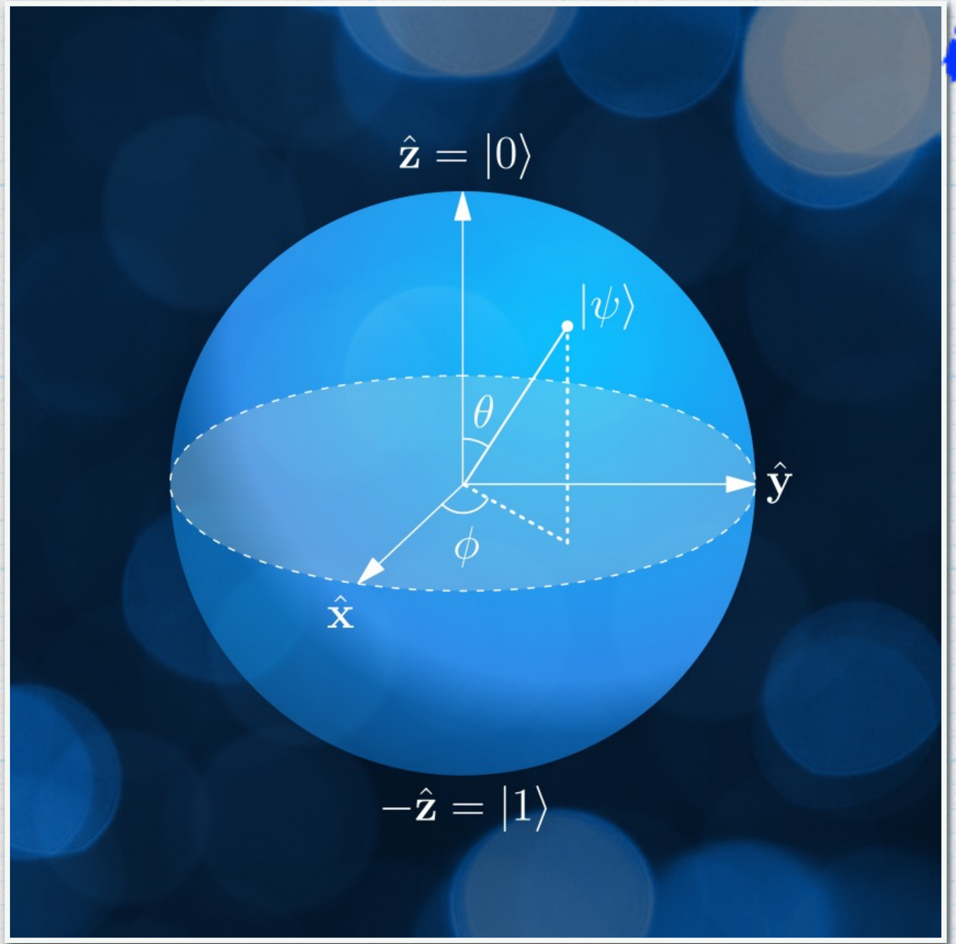
Motivation

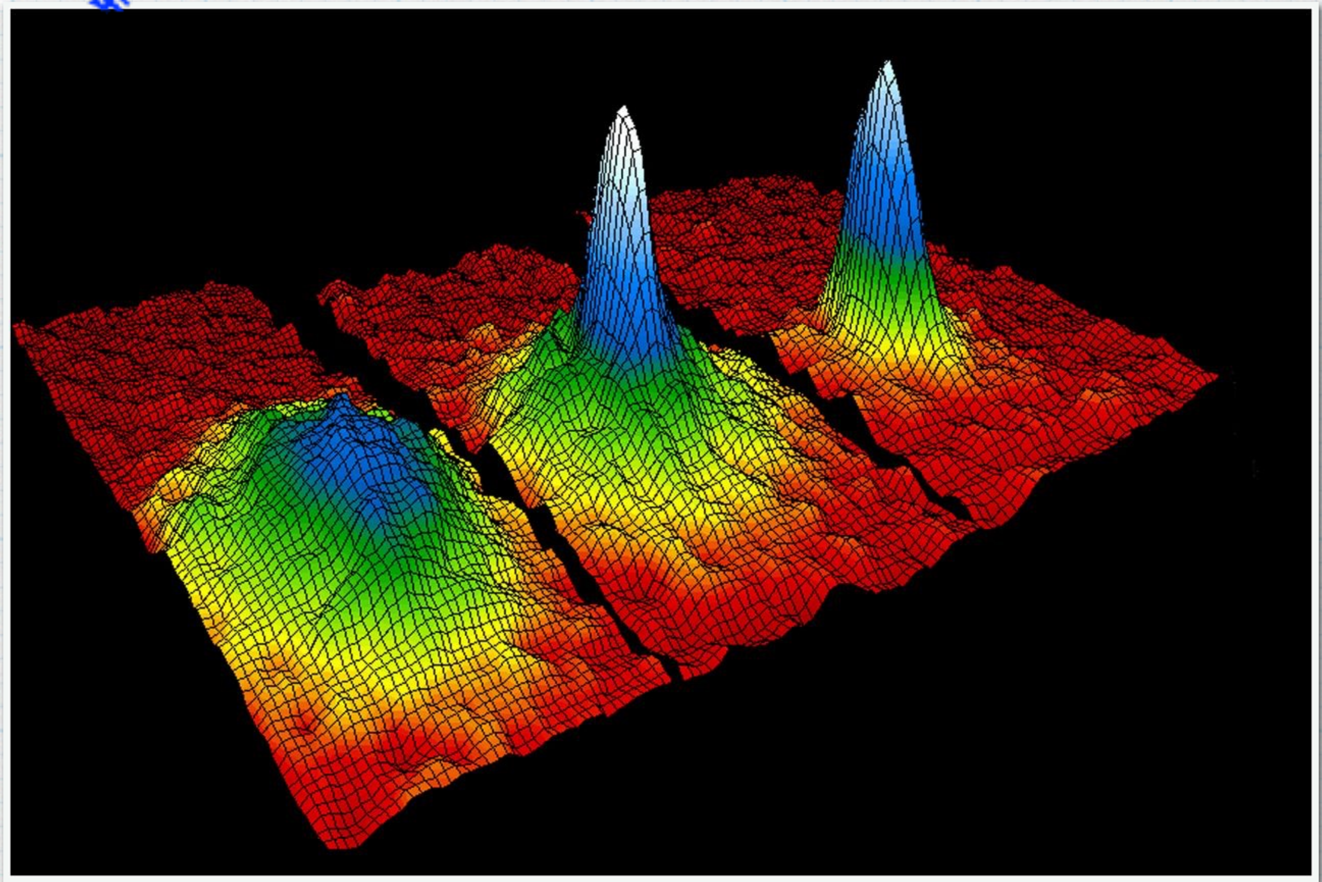
Interface between Quantum Information and Computation Science and Quantum Many-Body Physics

Experimental realization of QIC tasks needs QMB

e.g.,

1. ultra-cold neutral atoms (Bloch, Dalibard, Zwerger, RMP '08)
2. trapped ions (Leibfried, Blatt, Monroe, Wineland, RMP '03; Simon, Kim, Bryan, RMP '12)
3. superconducting qubits (Girvin, Schoelkopf, RMP '11; Devitt, Munro, Nemoto, ROPP '13)





Motivation

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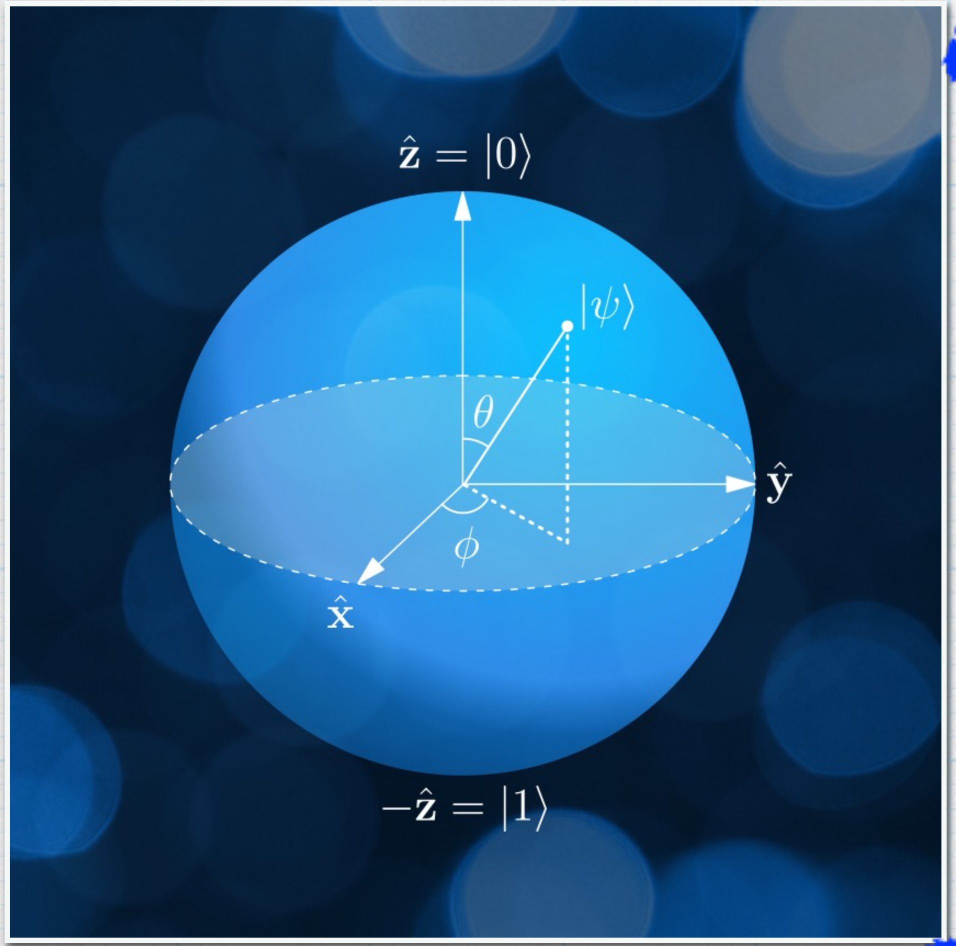
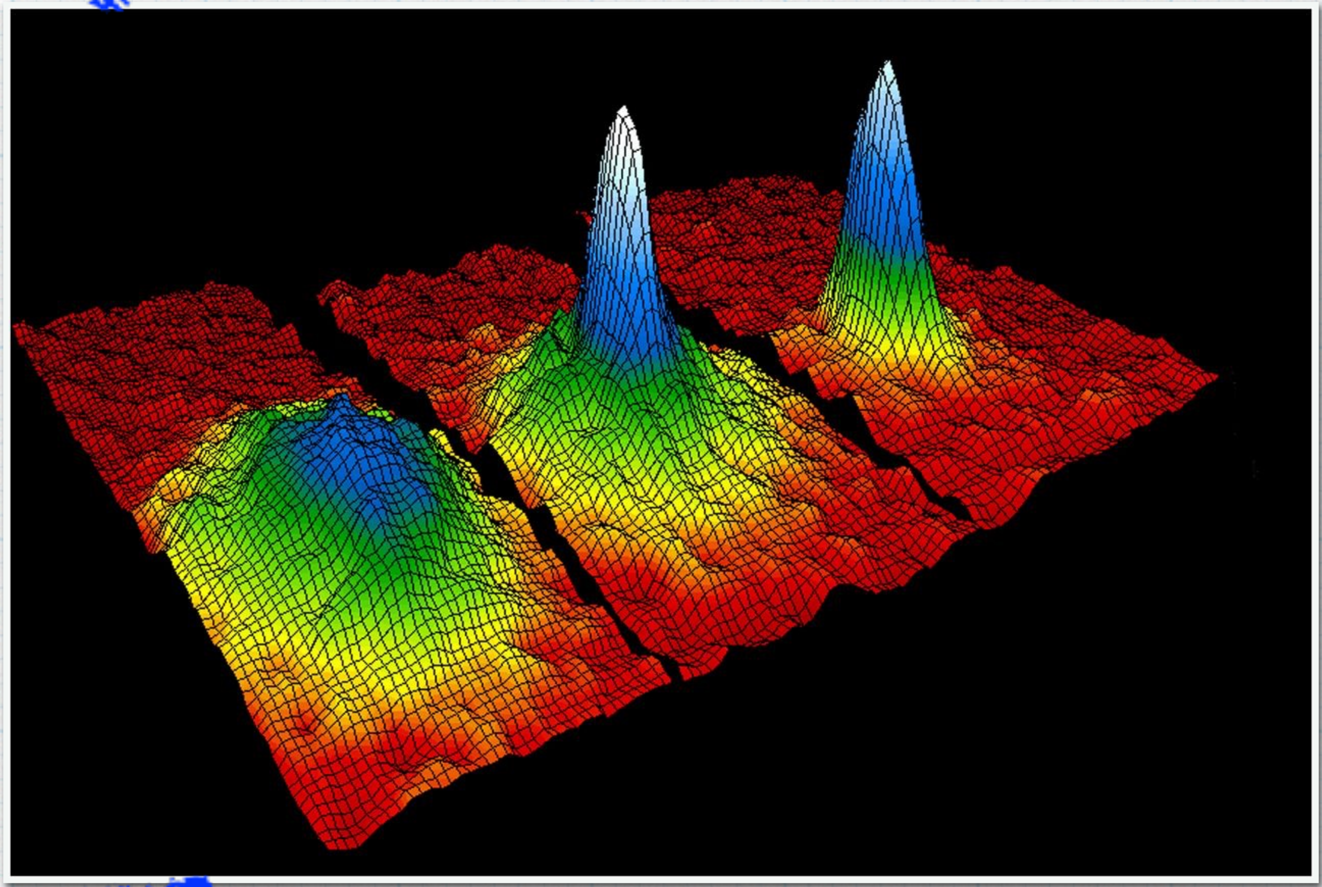


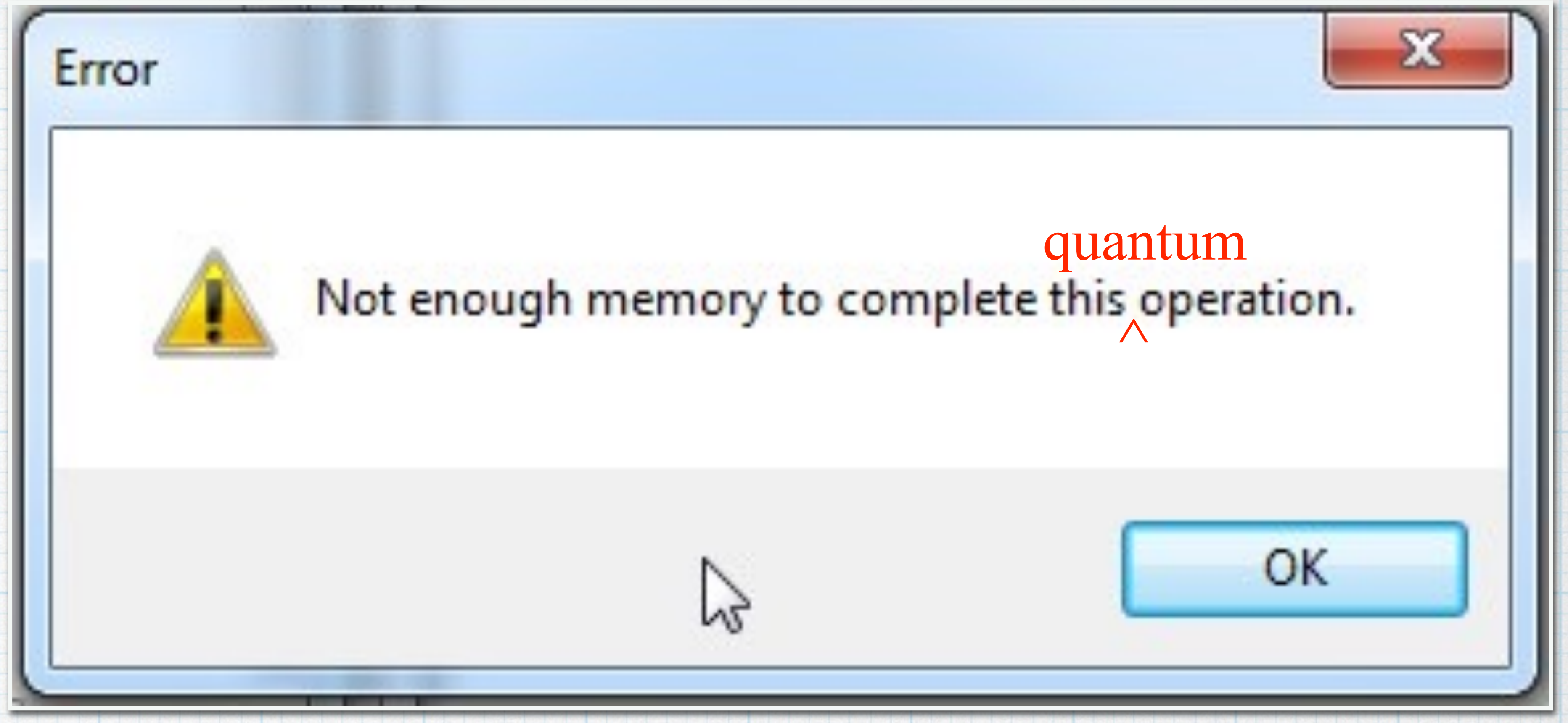


We can use concepts/results from QIC to analyze QMB systems

e.g.,

1. Quantum Simulation of QMB systems
2. Tensor Network methods to tackle QMB problems

Quantum Simulations



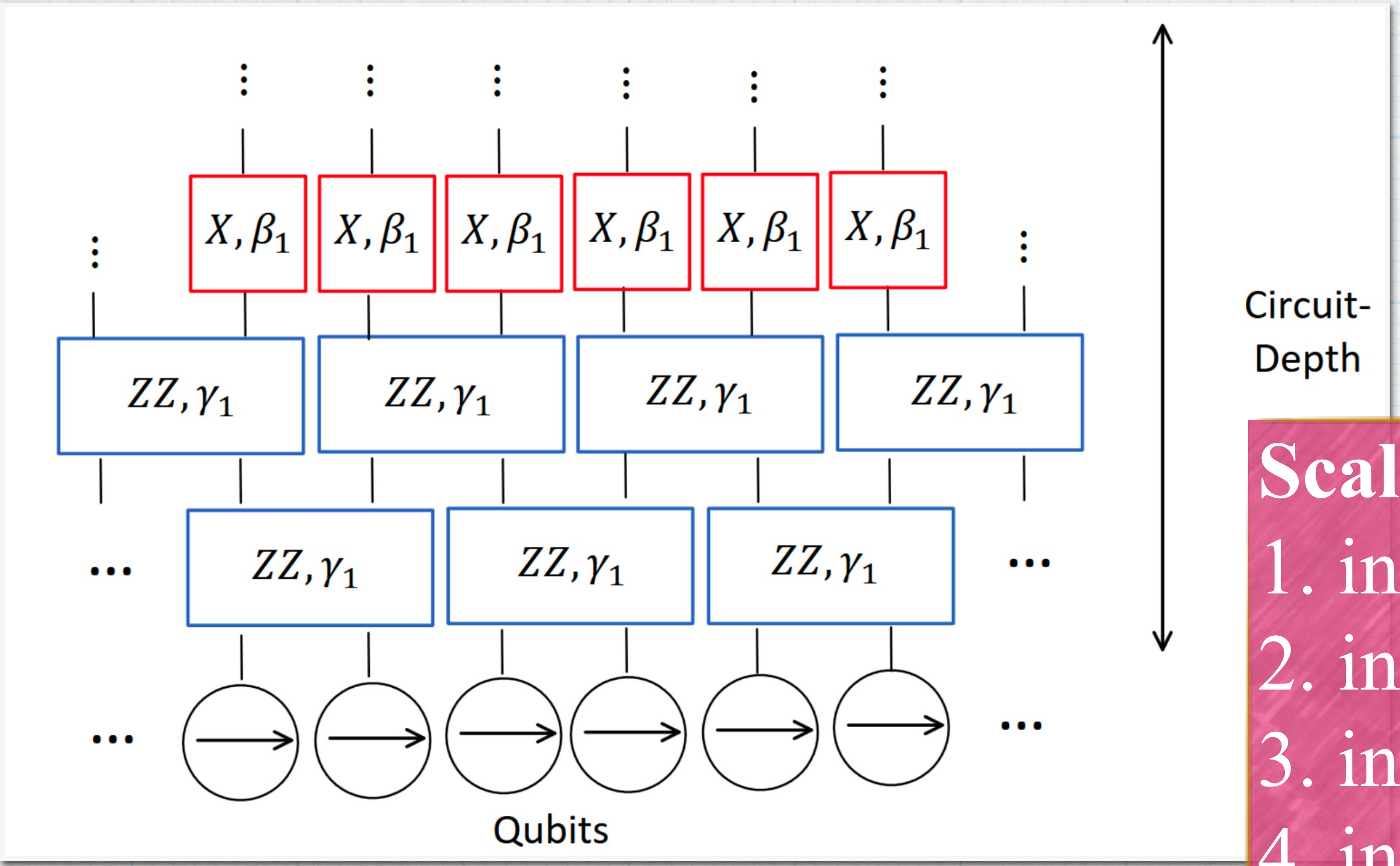
Proposed by Yuri Manin and Richard Feynman around ~ 1980s

Two Kinds

Digital simulations

Unitary (or any other) operators are simulated using quantum gates in a quantum circuit

$$e^{-\tau H} \approx e^{-\tau H_1} e^{-\tau H_2} e^{-\tau H_3} \ldots$$



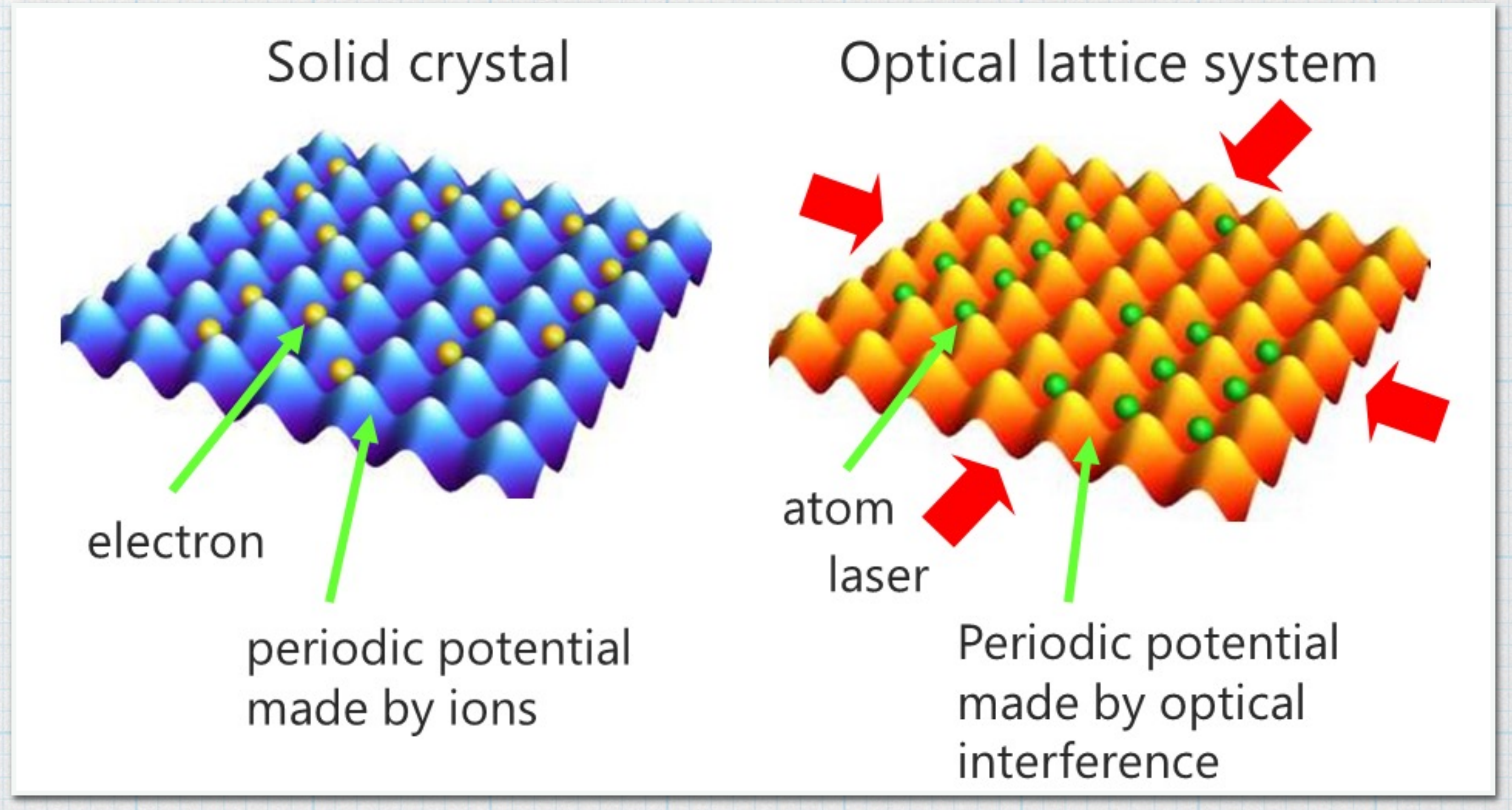
Scalability problems:
1. in physical dimensions
2. in range of interactions
3. in system-size
4. in time

Analog simulations

"True quantum simulations"

An 'analogous' synthetic system are tuned to mimic the physics of a 'target system'

Very successful in simulating solid state physics



Quantum Simulations

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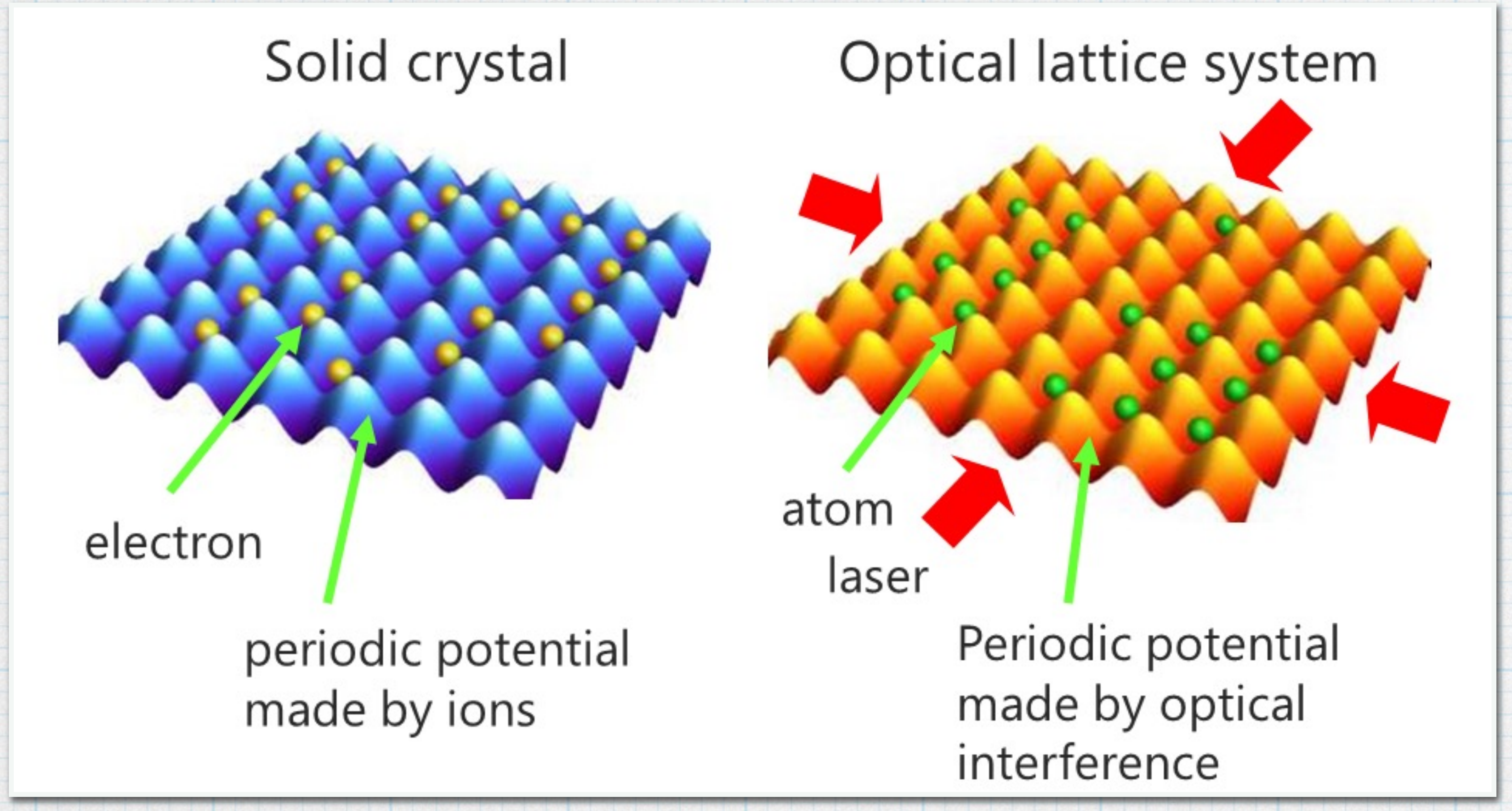

Successful in quantum simulating theoretical models, like

1. Bose- and Fermi-Hubbard models
2. Isotropic Heisenberg model
3. Ising model (thanks to tilted optical lattice and then to Rydberg systems)
4. And very recently, anisotropic XXZ model

SPECIAL ISSUE REVIEW

Quantum simulations with ultracold atoms in optical lattices

CHRISTIAN GROSS AND IMMANUEL BLOCH

SCIENCE • 8 Sep 2017 • Vol 357, Issue 6355 • pp. 995-1001 • DOI: 10.1126/science.aal3837

Published: 02 April 2012

Quantum simulations with ultracold quantum gases

Immanuel Bloch, Jean Dalibard & Sylvain Nascimbène

Nature Physics 8, 267–276 (2012) | Cite this article

26k Accesses | 1329 Citations | 15 Altmetric | Metrics

Advances in Physics ›
Volume 56, 2007 - Issue 2
Journal homepage

Original Articles

Ultracold atomic gases in optical lattices: mimicking condensed matter physics and beyond

Maciej Lewenstein, Anna Sanpera, Veronica Ahufinger, Bogdan Damski, Aditi Sen(De) & Ujjwal Sen

Pages 243-379 | Received 31 May 2006, Accepted 11 Jan 2007, Published online: 04 May 2007
Download citation https://doi.org/10.1080/00018730701223200

Technical Review | Published: 01

Tools for quantum simulation with ultracold atoms in optical lattices

Florian Schäfer, Takeshi Fukuhara, Seiji Sugawa, Yosuke Takasu & Yoshiro Takahashi

Nature Reviews Physics 2, 411–425 (2020) | Cite this article

2833 Accesses | 81 Citations | 15 Altmetric | Metrics

Quantum Simulations

As in 'analogous'

Analogue/Analog simulations

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SPECIAL ISSUE REVIEW

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SCIENCE • 8 Sep 2017 • Vol 357, Issue 6355 • pp. 995-1001 • DOI: 10.1126/science.aal3837

The Coming Decades of Quantum Simulation

Joana Fraxanet, Tymoteusz Salamon, Maciej Lewenstein

Contemporary quantum technologies face major difficulties in fault tolerant quantum computing with error correction, and focus instead on various shades of quantum simulation (Noisy Intermediate Scale Quantum, NISQ) devices, analogue and digital quantum simulators and quantum annealers. There is a clear need and quest for such systems that, without necessarily simulating quantum dynamics of some physical systems, can generate massive, controllable, robust, entangled, and superposition states. This will, in particular, allow the control of decoherence, enabling the use of these states for quantum communications (e.g. to achieve efficient transfer of information in a safer and quicker way), quantum metrology, sensing and diagnostics (e.g. to precisely measure phase shifts of light fields, or to diagnose quantum materials). In this Chapter we present a vision of the golden future of quantum simulators in the decades to come.

Subjects: **Quantum Physics (quant-ph)**; Quantum Gases (cond-mat.quant-gas)
Cite as: arXiv:2204.08905 **[quant-ph]**
(or arXiv:2204.08905v1 **[quant-ph]** for this version)
https://doi.org/10.48550/arXiv.2204.08905

hors, DOI, OF

yond

S

simulating theoretical models, like

1. Bose- and Fermi-Hubbard models
2. Isotropic Heisenberg model
3. Ising model (thanks to tilted optical lattice and then to Rydberg systems)
4. And very recently, anisotropic XXZ model

Tools for optical la

Florian Schäfer

Nature Reviews P

We can even simulate synthetic phases/transitions of matter that does not have any counterpart in nature, or the natural counterpart hasn't been discovered yet!!

e.g., (spoiler alert!!) SUSY

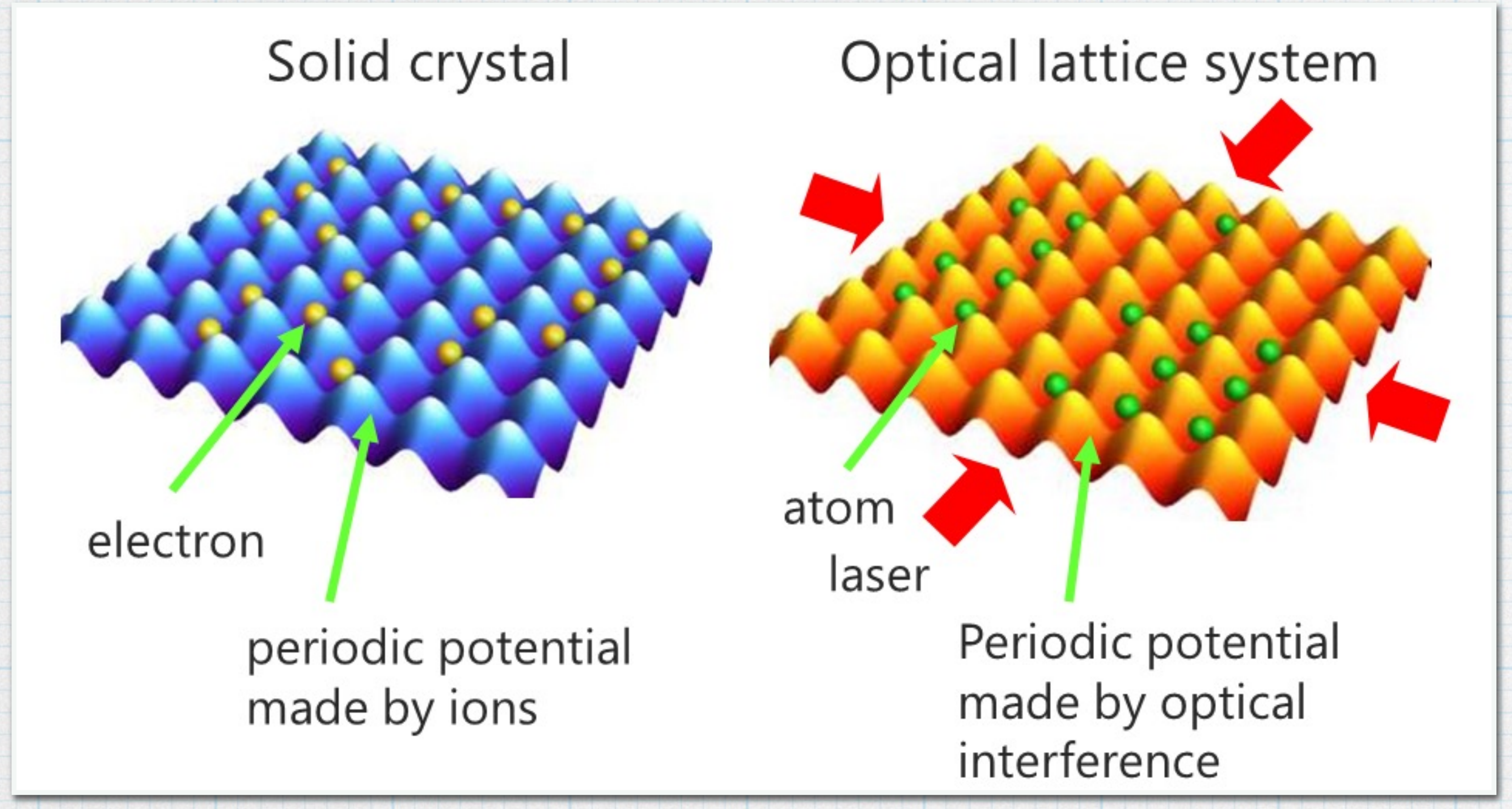
Quantum Simulations

As in 'analogous'

Analogue/Analog simulations

"True quantum simulations"

An 'analogous' synthetic system are tuned to mimic the physics of a 'target system'



Successful in quantum simulating theoretical models, like

1. Bose- and Fermi-Hubbard models
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3. Ising model (thanks to tilted optical lattice and then to Rydberg systems)
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Theoreticians' perspective…

1. Theoretical propositions of experimental setups

2. Theoretical analysis of strongly-correlated many-body systems that are within the reach of present day experiments

(in turn, we peak the interests of our experimental colleagues to quantum simulate the respective systems)

Needs algorithms for classical simulations…

1. Exact diagonalization…
2. Mean field theories, including DMFT
3. Several types of Monte-Carlo: Classical, Quantum…
4. Density functional theory…
5. **Tensor Network Algorithms…**

where we come in along with the ideas from Quantum Information

Tensor Networks

Goal: Efficient representation of quantum many-body states

N body quantum state → Hilbert space dimension $= d^N$, exponential in system size

A generic quantum state… $|\psi\rangle = \sum_{i_1,i_2,i_3..,i_N} C_{i_1 i_2 i_3 \ldots i_N} |i_1 i_2 i_3 \ldots i_N\rangle$

→ d^N terms, **inefficient!!**

Tensor Networks

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Tensor Network: Efficient representation of quantum many-body states with *poly*(N) terms

ELSEVIER
Annals of Physics
Volume 349, October 2014, Pages 117-158

A practical introduction to tensor networks: Matrix product states and projected entangled pair states

Román Orús

ELSEVIER
Annals of Physics
Volume 326, Issue 1, January 2011, Pages 96-192

The density-matrix renormalization group in the age of matrix product states

Ulrich Schollwöck

ELSEVIER
Annals of Physics
Volume 411, December 2019, 167998

Time-evolution methods for matrix-product states

Sebastian Paeckel [a], Thomas Köhler [a, b], Andreas Swoboda [c], Salvatore R. Manmana [a], Ulrich Schollwöck [c, d], Claudius Hubig [e, d]

SciPost Physics Lecture Notes

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The Tensor Networks Anthology: Simulation techniques for many-body quantum lattice systems

Pietro Silvi, Ferdinand Tschirsich, Matthias Gerster, Johannes Jünemann, Daniel Jaschke, Matteo Rizzi, Simone Montangero

SciPost Phys. Lect. Notes 8 (2019) · published 18 March 2019

Tensor Networks

Tensor Network: Efficient representation of quantum many-body states with $poly(N)$ terms

Providing answers to long-standing open problems

Haldane spin-gap in spin-1 Heisenberg chain

PHYSICAL REVIEW B
covering condensed matter and materials physics

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Numerical renormalization-group study of low-lying eigenstates of the antiferromagnetic S=1 Heisenberg chain

Steven R. White and David A. Huse
Phys. Rev. B **48**, 3844 – Published 1 August 1993

REPORT

Spin-Liquid Ground State of the S = 1/2 Kagome Heisenberg Antiferromagnet

SIMENG YAN, DAVID A. HUSE, AND , STEVEN R. WHITE Authors Info & Affiliations

SCIENCE · 28 Apr 2011 · Vol 332, Issue 6034 · pp. 1173-1176 · DOI

Quantum spin liquids in frustrated antiferromagnets

PHYSICAL REVIEW LETTERS

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Nature of the Spin-Liquid Ground State of the $S = 1/2$ Heisenberg Model on the Kagome Lattice

Stefan Depenbrock, Ian P. McCulloch, and Ulrich Schollwöck
Phys. Rev. Lett. **109**, 067201 – Published 7 August 2012

Conclusive evidence of stripe order in 2D Hubbard model

RESEARCH ARTICLE

Stripe order in the underdoped region of the two-dimensional Hubbard model

BO-XIAO ZHENG, CHIA-MIN CHUNG, PHILIPPE CORBOZ, GEORG EHLERS, MING-PU QIN, REINHARD M. NOACK, HAO SHI, STEVEN R. WHITE, SHIWEI ZHANG, [...] GARNET KIN-LIC CHAN
+1 authors Authors Info & Affiliations

SCIENCE · 1 Dec 2017 · Vol 358, Issue 6367 · pp. 1155-1160 · DOI: 10.1126/science.aam7127

'Hard' problems in strongly correlated systems

PHYSICAL REVIEW X

Highlights Recent Subjects Accepted Collections

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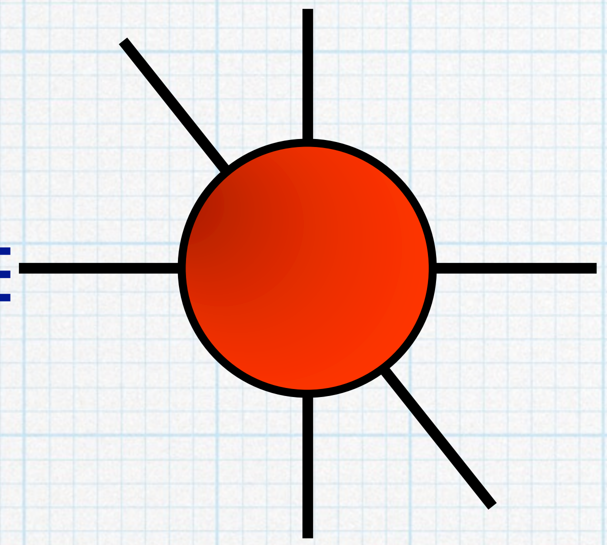
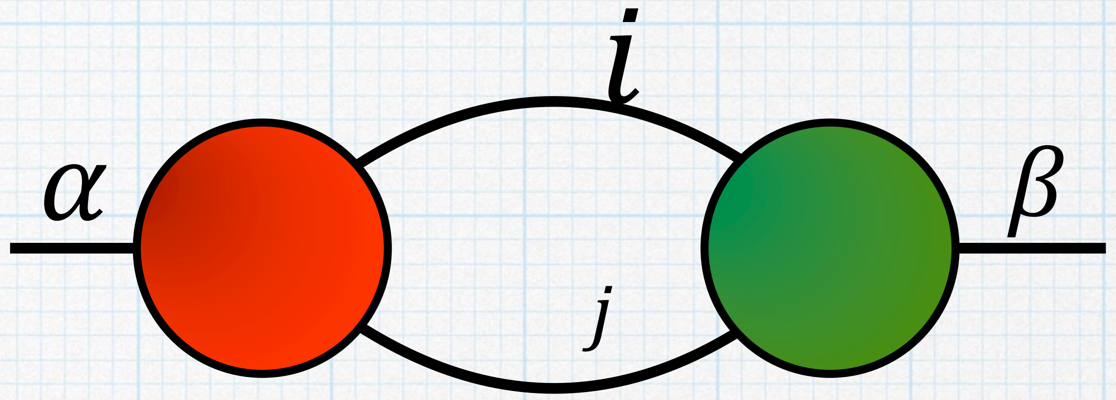
Solutions of the Two-Dimensional Hubbard Model: Benchmarks and Results from a Wide Range of Numerical Algorithms

J. P. F. LeBlanc, Andrey E. Antipov, Federico Becca, Ireneusz W. Bulik, Garnet Kin-Lic Chan, Chia-Min Chung, Youjin Deng, Michel Ferrero, Thomas M. Henderson, Carlos A. Jiménez-Hoyos, E. Kozik, Xuan-Wen Liu, Andrew J. Millis, N. V. Prokof'ev, Mingpu Qin, Gustavo E. Scuseria, Hao Shi, B. V. Svistunov, Luca F. Tocchio, I. S. Tupitsyn, Steven R. White, Shiwei Zhang, Bo-Xiao Zheng, Zhenyue Zhu, and Emanuel Gull (Simons Collaboration on the Many-Electron Problem)
Phys. Rev. X **5**, 041041 – Published 14 December 2015

Tensor Networks

Tensors ≡ Multi-dimensional arrays ≡ 



$$\equiv \sum_{i,j} A_{\alpha i j} B_{j i \beta}$$

Tensor Networks

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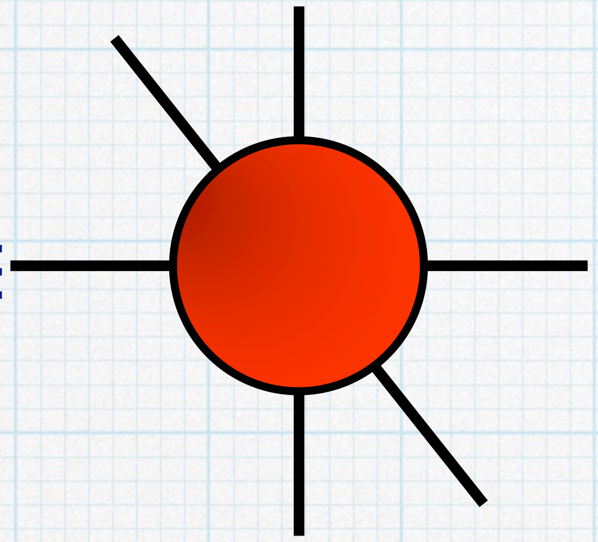

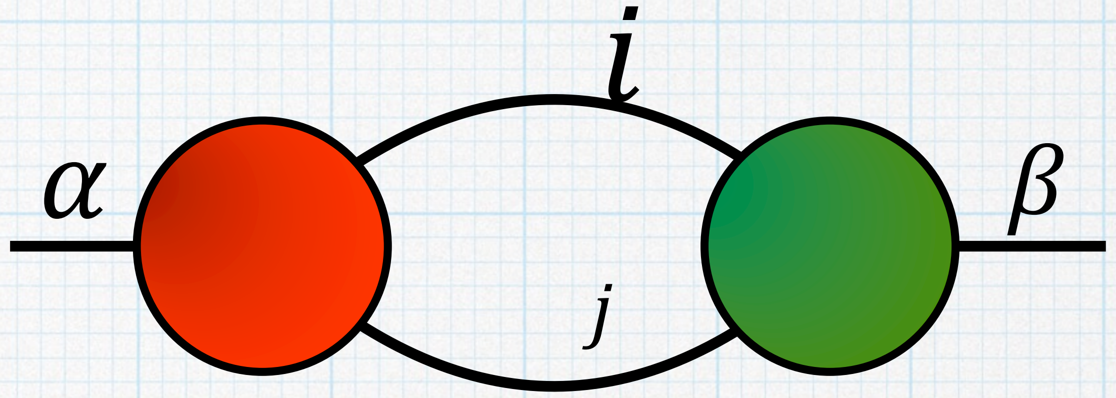

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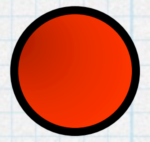
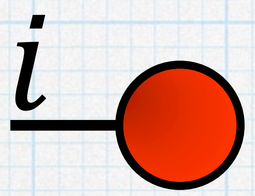
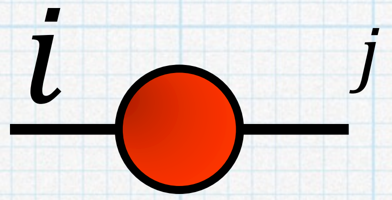


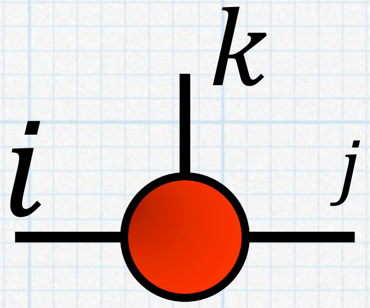
Scalar c
rank 0



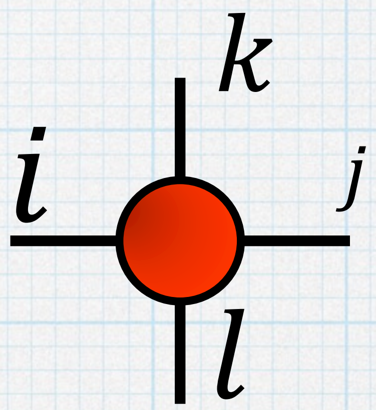
Vector v_i
rank 1



Matrix M_{ij}
rank 2



A_{ijk}
rank 3

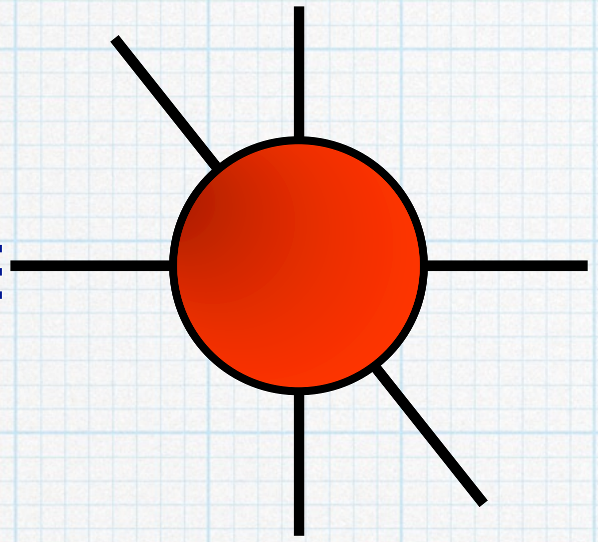


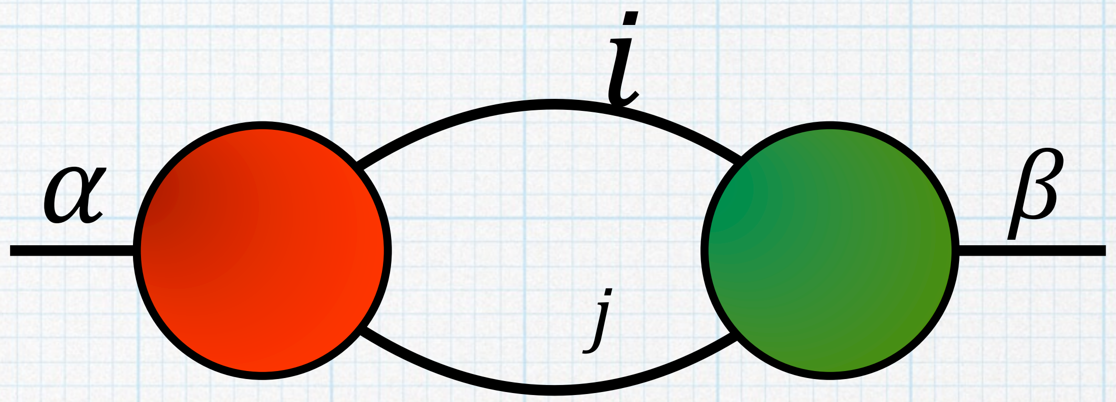
B_{ijkl}
rank 4

Do not need to write down formulas with tensors with many indices

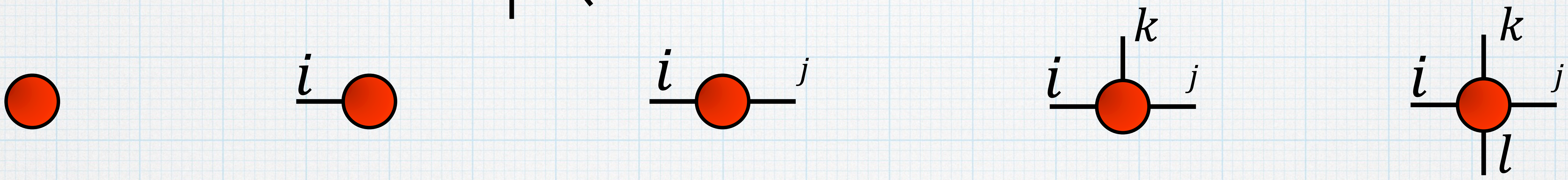
Tensor Networks

Tensors ≡ Multi-dimensional arrays ≡





$\equiv \sum_{i,j} A_{\alpha i j} B_{j i \beta}$



Scalar c
rank 0

Vector v_i
rank 1

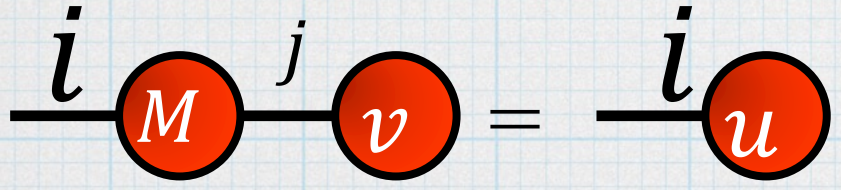
Matrix M_{ij}
rank 2

A_{ijk}
rank 3

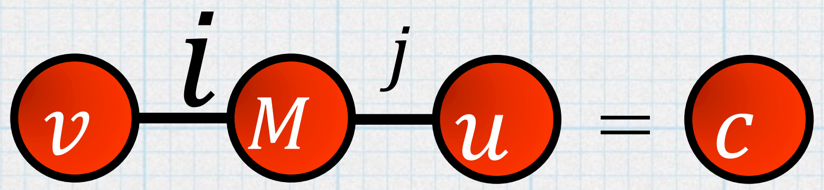
B_{ijkl}
rank 4

Do not need to write down formulas with tensors with many indices

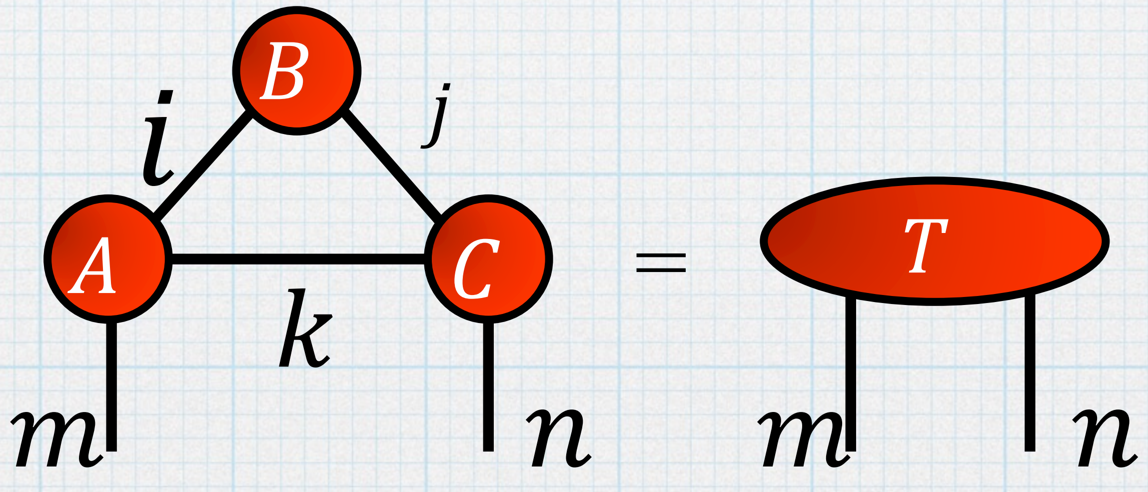
Examples…



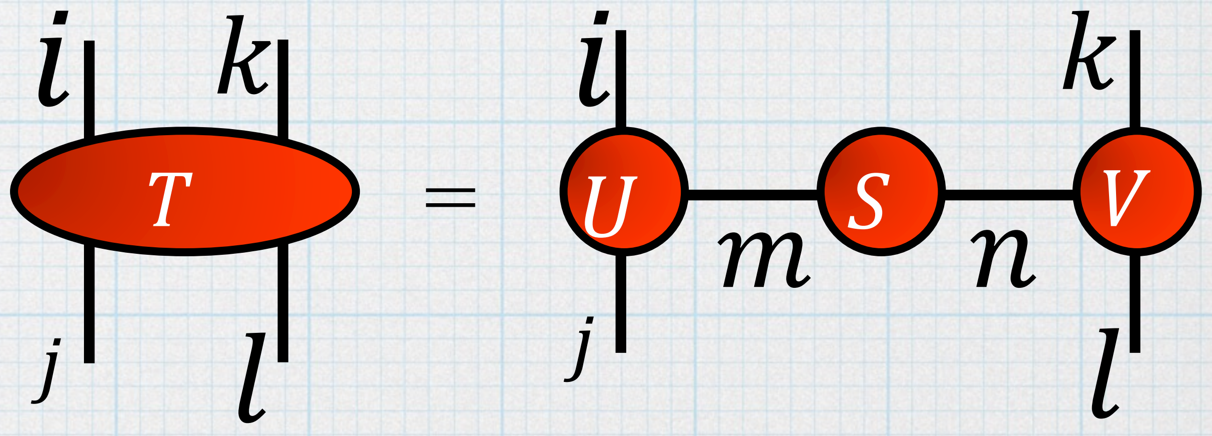
$\sum_j M_{ij} v_j = u_i$



$\sum_{ij} v_i M_{ij} u_j = c$



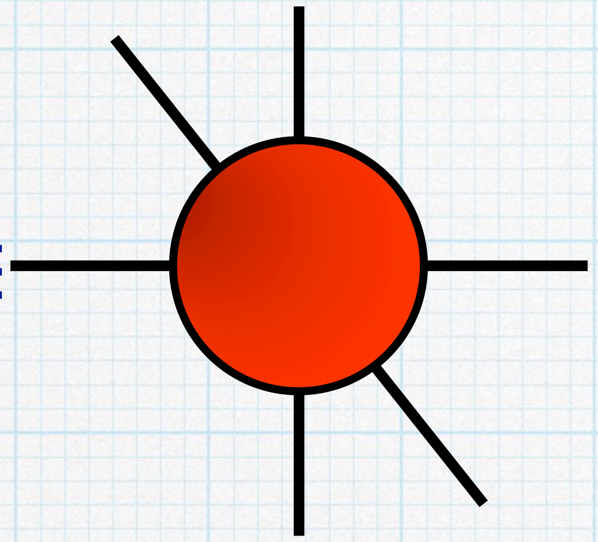
$\sum_{ijk} A_{mik} B_{ij} C_{jkn} = T_{mn}$

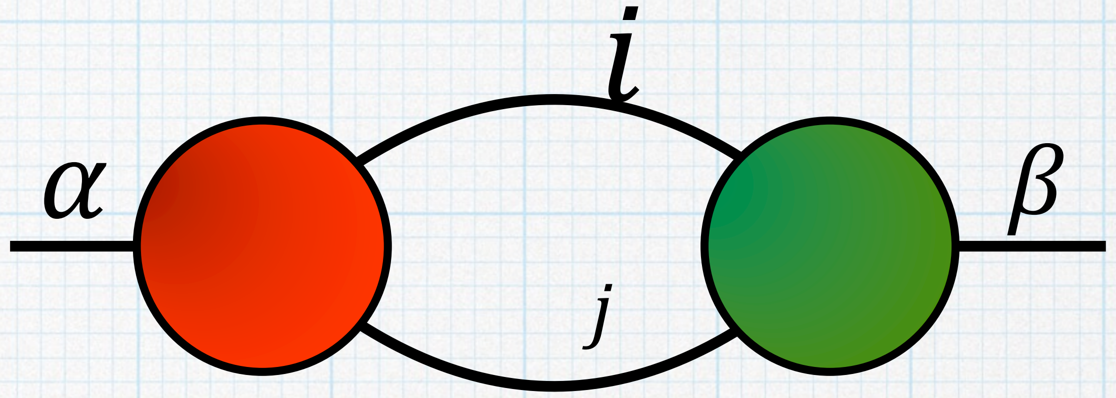


SVD, $T_{ijkl} = \sum_{mn} U_{ijm} S_{mn} V_{nkl}$

Connected lines: sum over corresponding indices

Tensor Networks

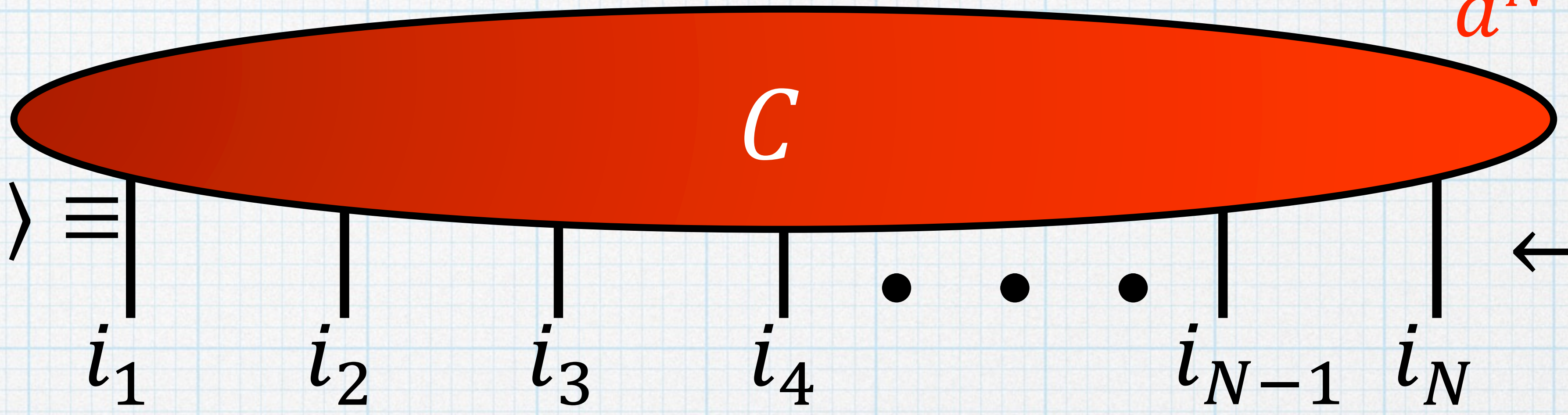
Tensors ≡ Multi-dimensional arrays ≡ 



$\equiv \sum_{i,j} A_{\alpha i j} B_{j i \beta}$

d^N terms…

A generic quantum state… $|\psi\rangle = \sum_{i_1,i_2,i_3..,i_N} C_{i_1 i_2 i_3 \ldots i_N} |i_1 i_2 i_3 \ldots i_N\rangle \equiv$



←Physical indices

Tensor Networks

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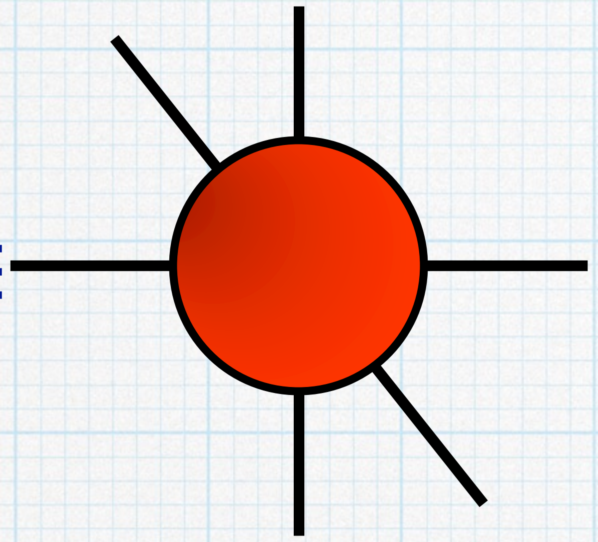

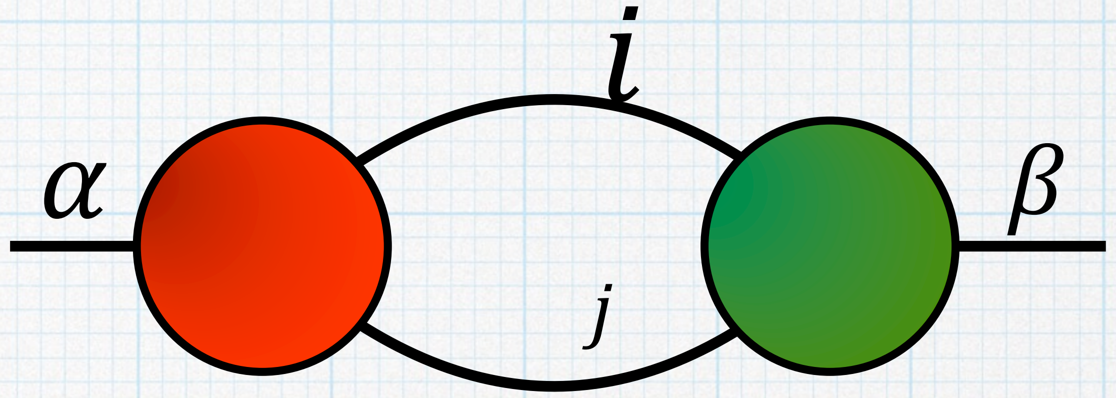

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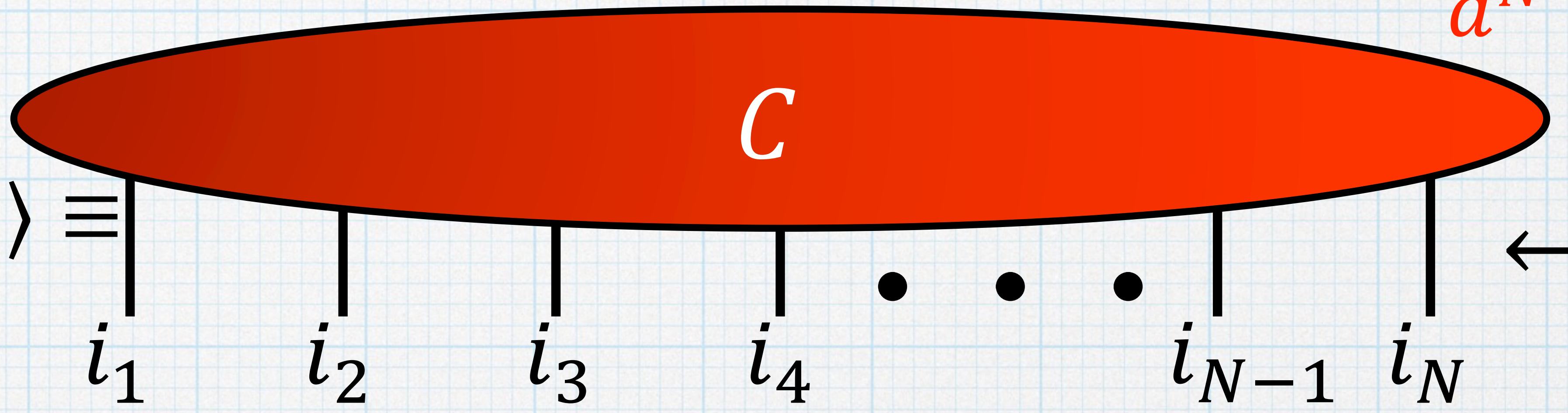
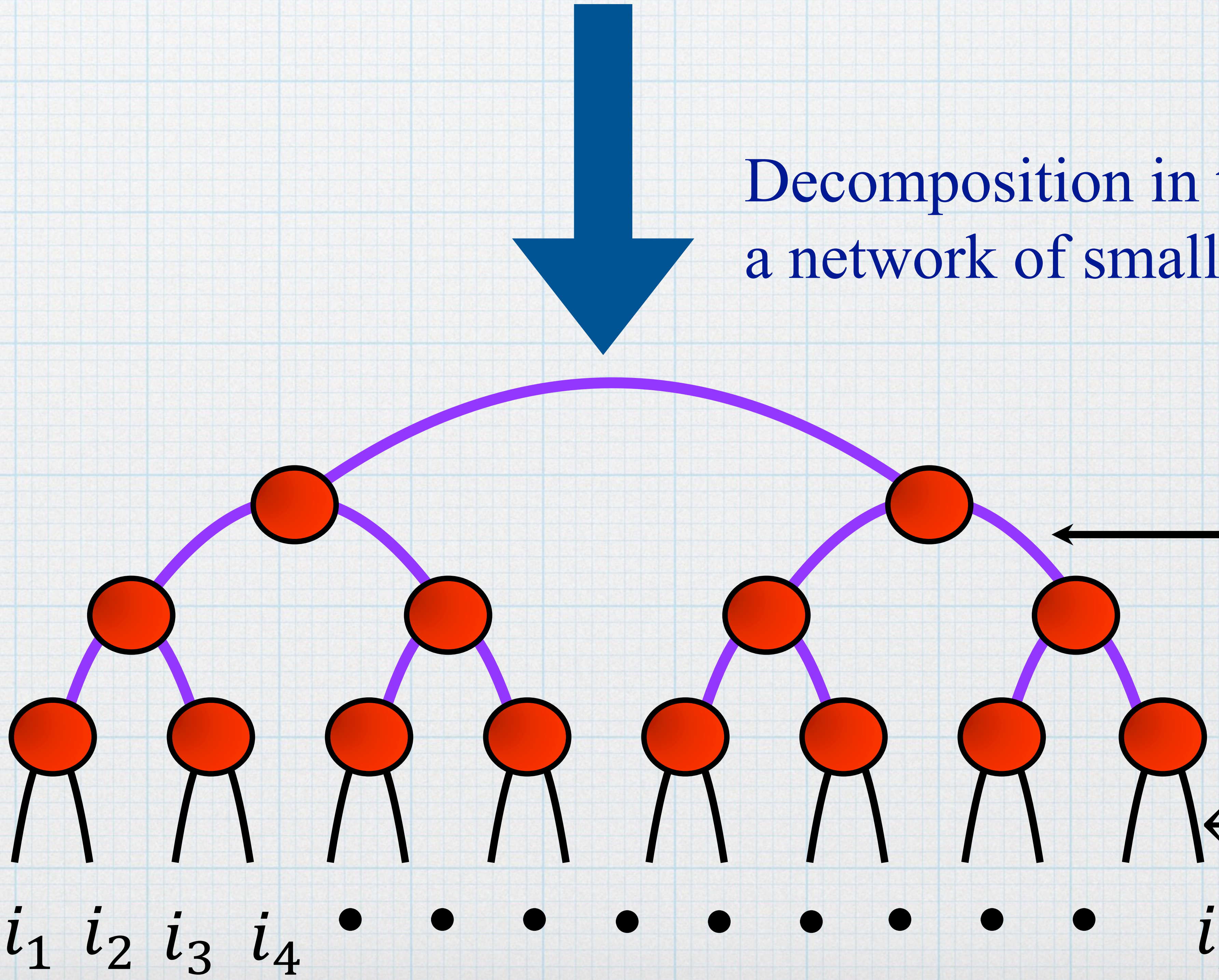


←Physical indices

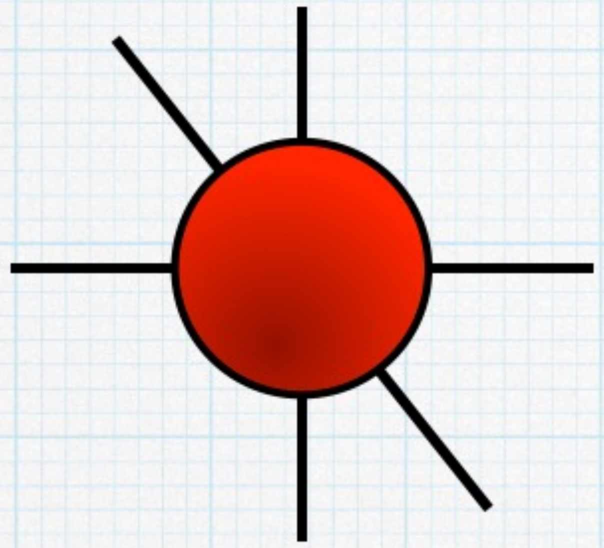
Decomposition in terms of
a network of smaller tensors

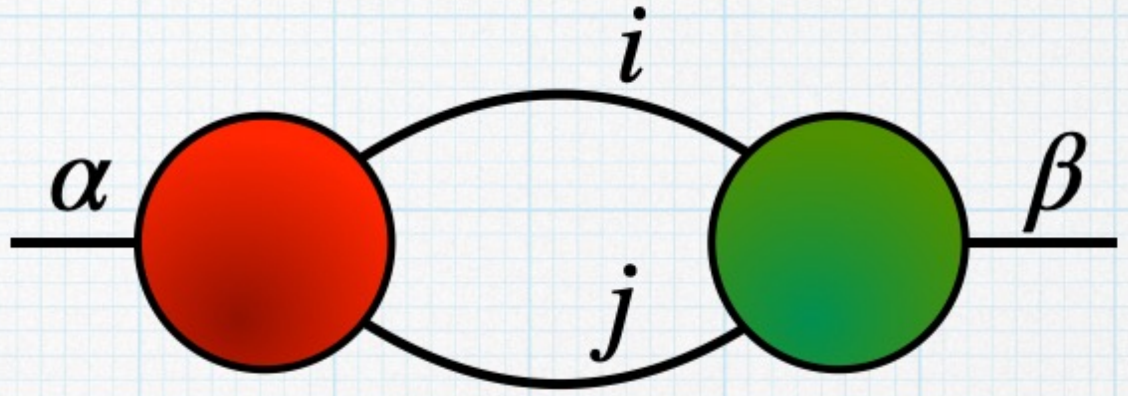


Virtual (bond/link) indices

←Physical indices

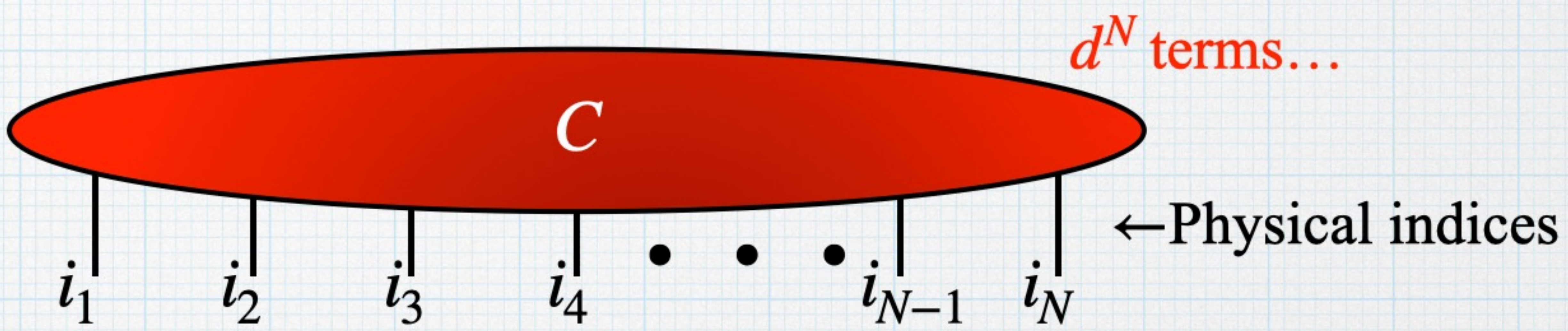
Tensor Networks

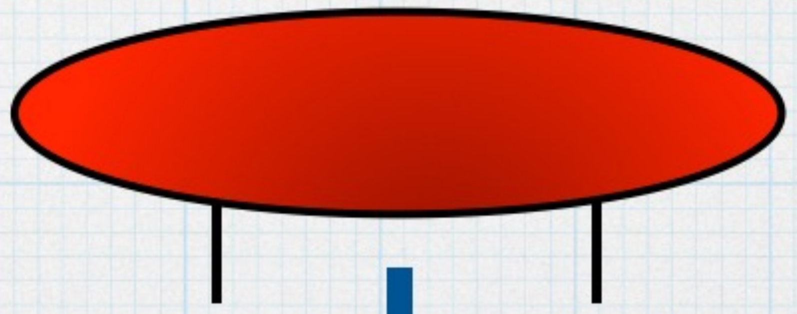
Tensors ≡ Multi-dimensional arrays ≡ 

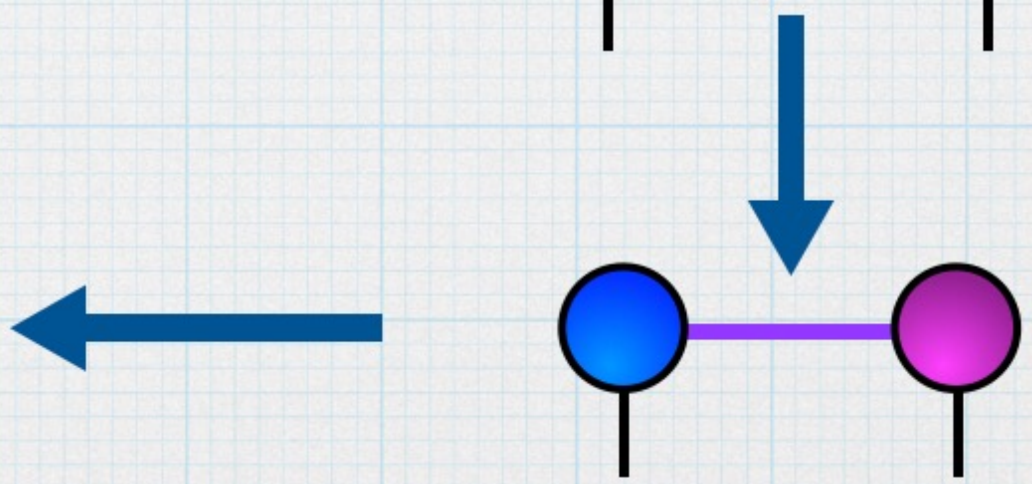


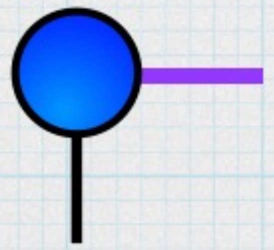
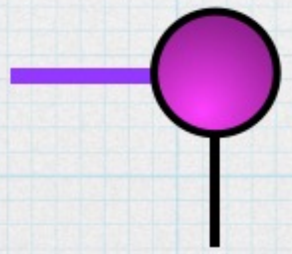
$$\equiv \sum_{i,j} A_{\alpha i j} B_{j i \beta}$$

A generic quantum state… $|\psi\rangle = \sum_{i_1,i_2,i_3..,i_N} C_{i_1 i_2 i_3 \ldots i_N} | i_1 i_2 i_3 \ldots i_N \rangle \equiv$



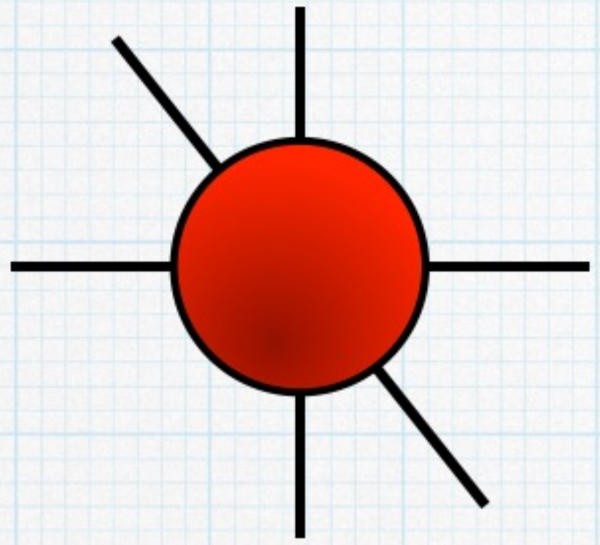
A simple example… generic two qubit state… $|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle \equiv$  $\equiv \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

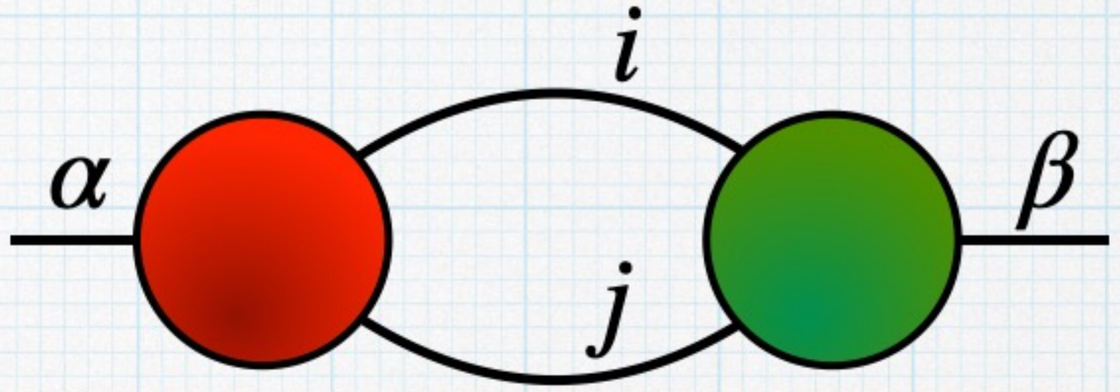


where…  $= \begin{bmatrix} \sqrt{a} & 0 \\ 0 & \sqrt{d} \end{bmatrix}$  $= \begin{bmatrix} \sqrt{a} & b/\sqrt{a} \\ c/\sqrt{d} & \sqrt{d} \end{bmatrix}$

Not a unique decomposition… Gauge freedom of TN…

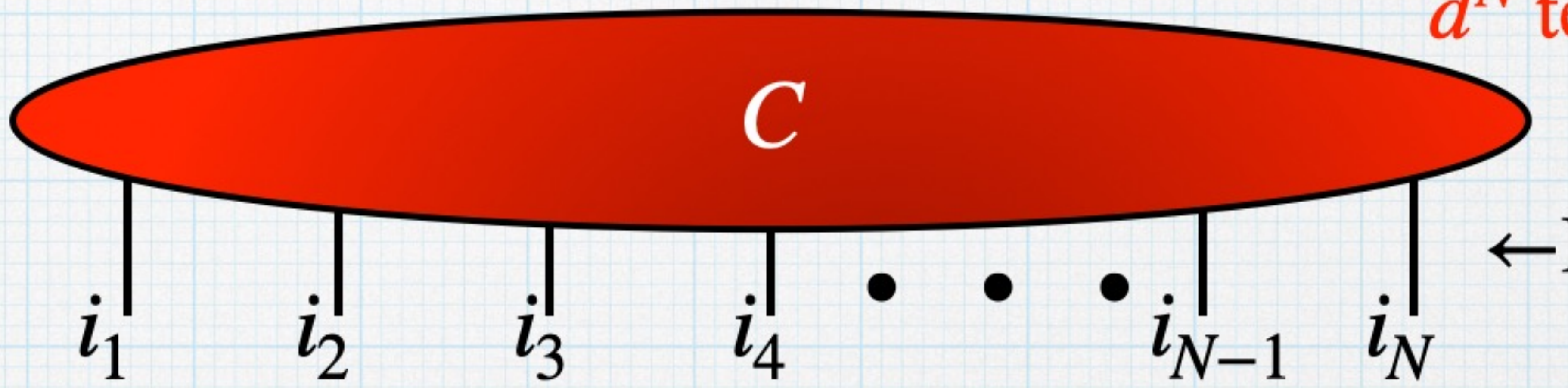
Tensor Networks

Tensors $\equiv$ Multi-dimensional arrays $\equiv$ 

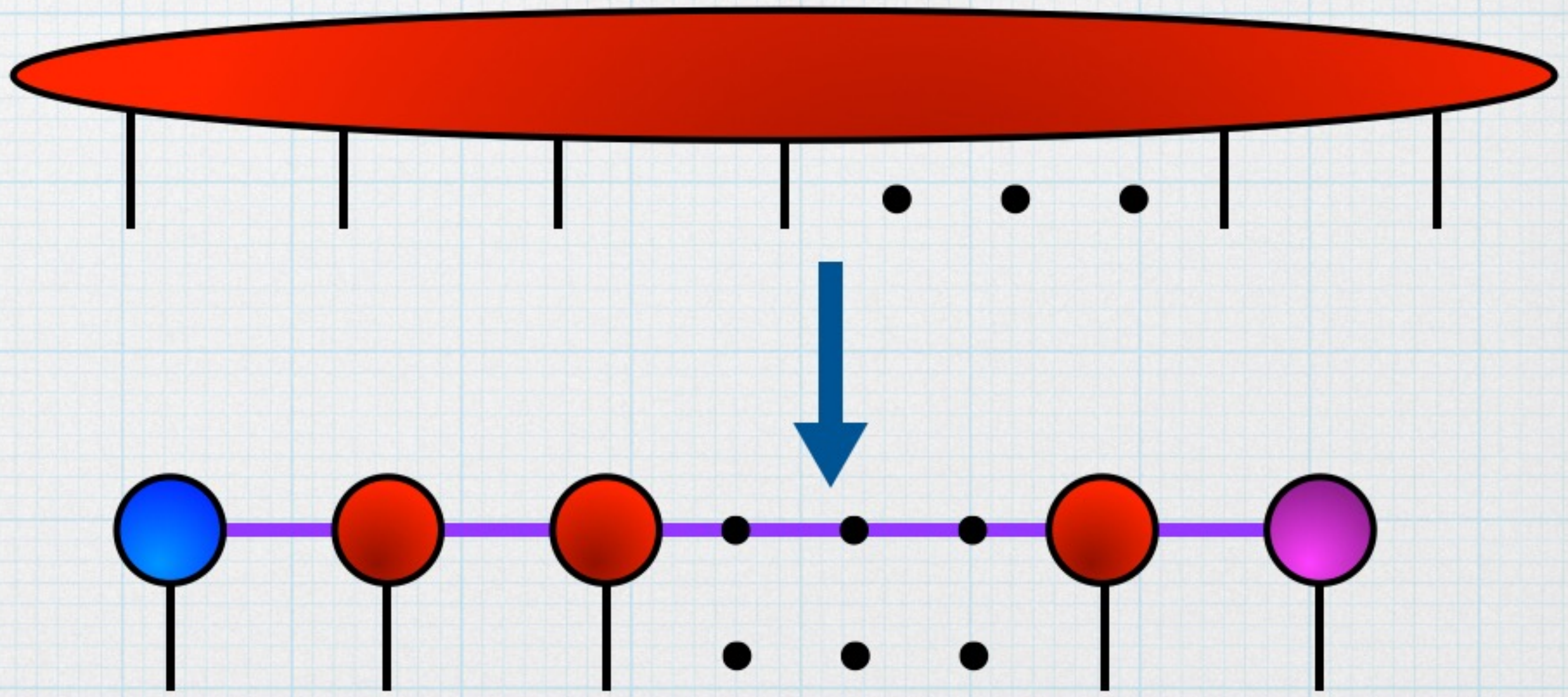


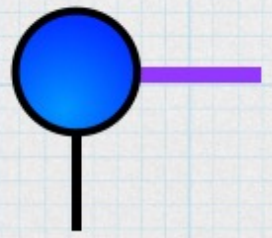
$\equiv \sum_{i,j} A_{\alpha i j} B_{j i \beta}$

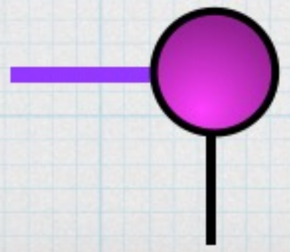
d^N terms…

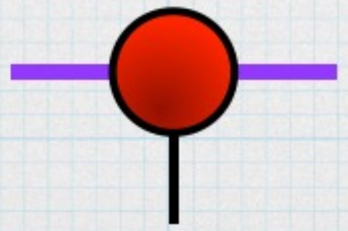
A generic quantum state… $|\psi\rangle = \sum_{i_1,i_2,i_3..,i_N} C_{i_1 i_2 i_3 \ldots i_N} |i_1 i_2 i_3 \ldots i_N\rangle \equiv$ 

←Physical indices

A simple example…. N qubit GHZ state….. $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|00...0\rangle + |11...1\rangle\right) \equiv$ 

where…  $= \begin{bmatrix} \frac{|0\rangle}{\sqrt{2}} & \frac{|1\rangle}{\sqrt{2}} \end{bmatrix}$

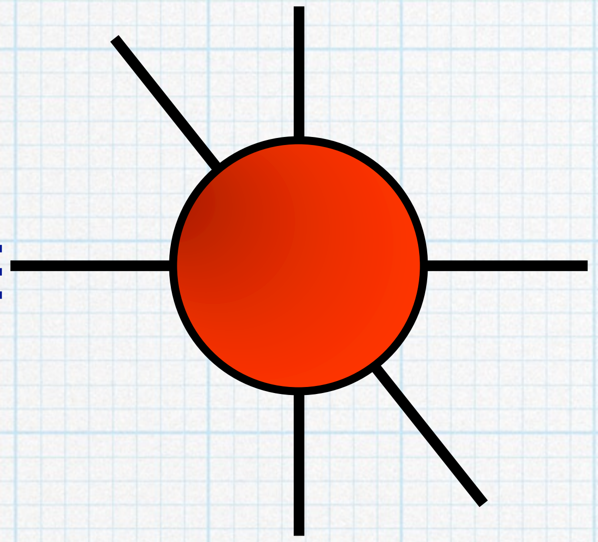
 $= \begin{bmatrix} |0\rangle \\ |1\rangle \end{bmatrix}$

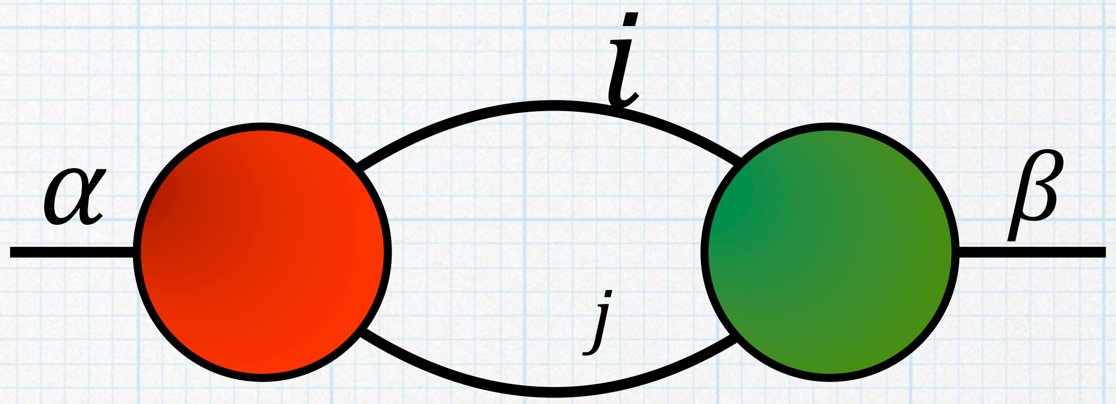
 $= \begin{bmatrix} |0\rangle & 0 \\ 0 & |1\rangle \end{bmatrix}$

This is a trivial example, where the decomposition is exact

Tensor Networks

Tensors ≡ Multi-dimensional arrays ≡





$\equiv \sum_{i,j} A_{\alpha i j} B_{j i \beta}$

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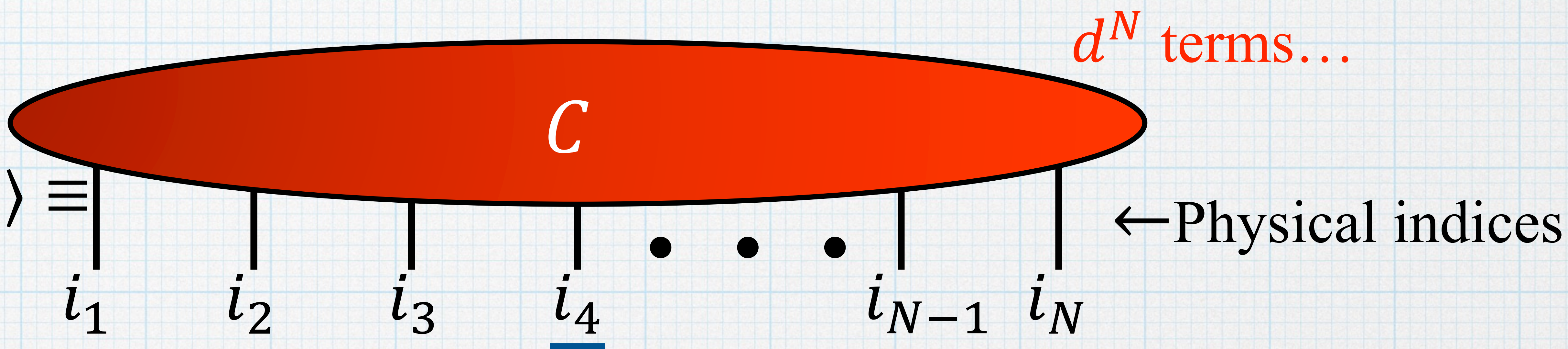
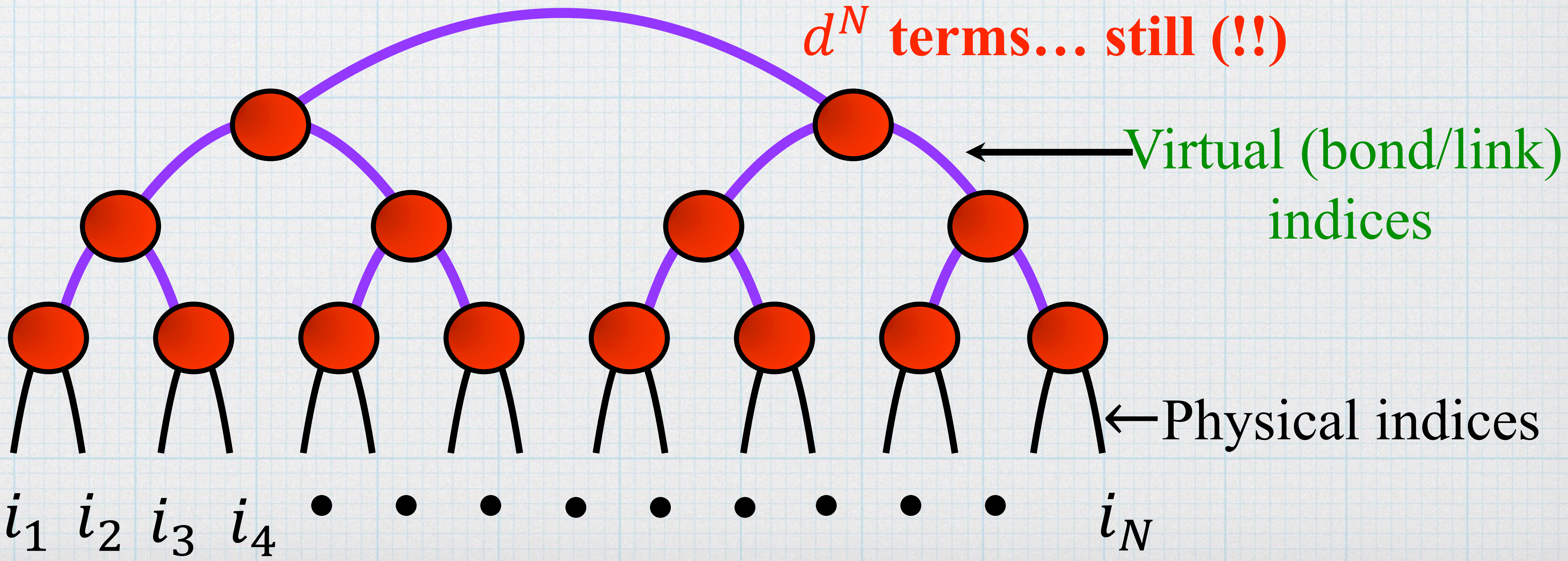


Decomposition in terms of a network of smaller tensors



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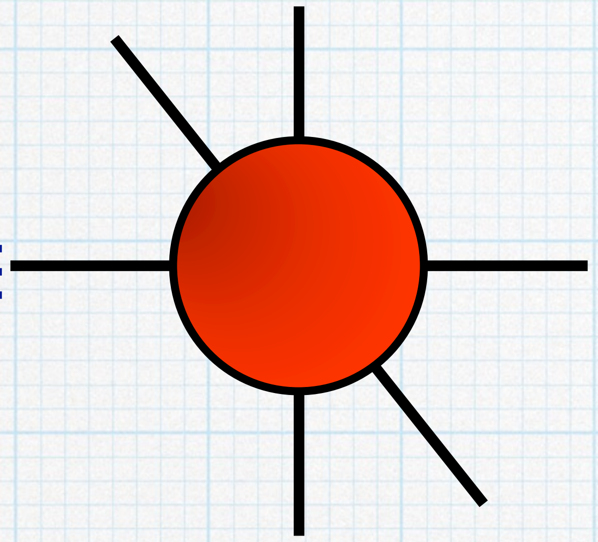

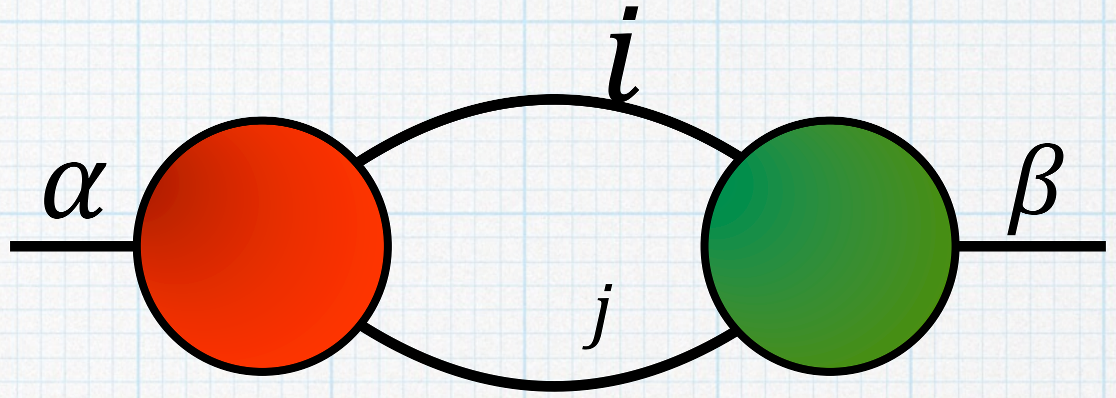

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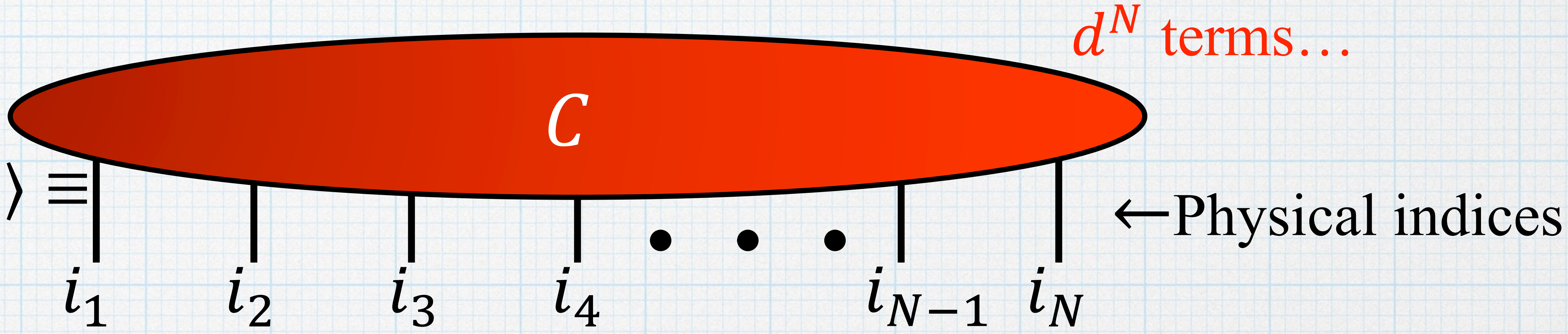

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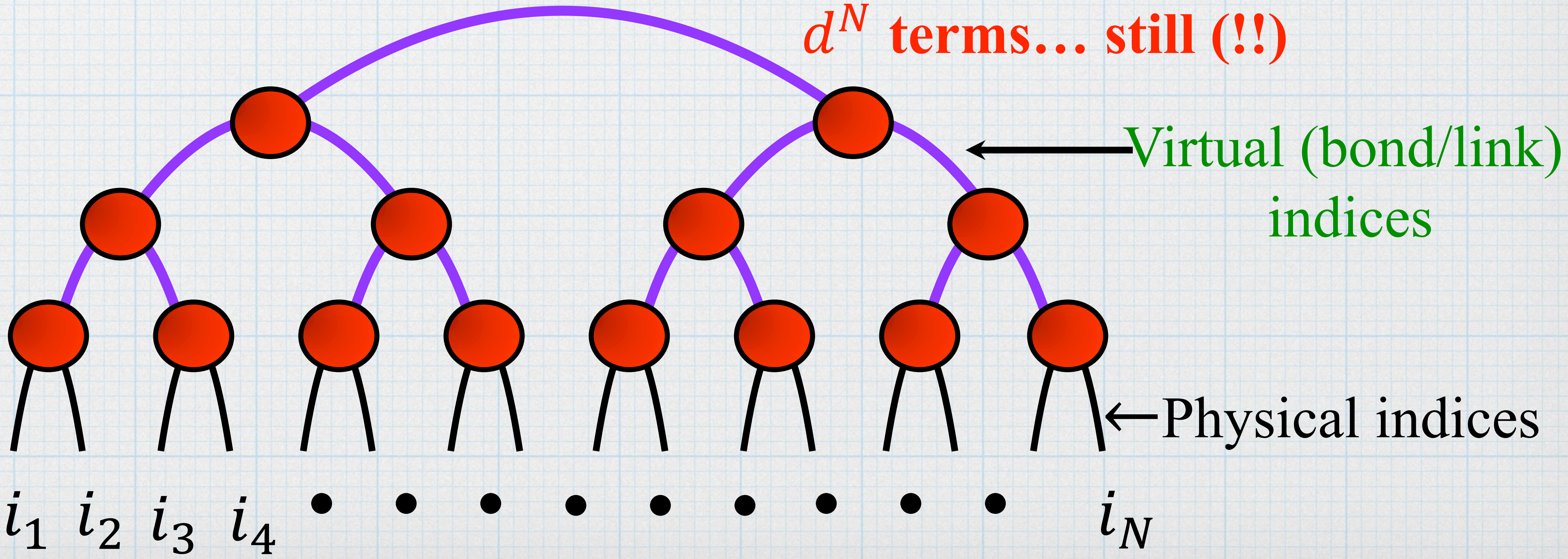

Key Idea:

1. Systematically restricting the virtual dimensions ⇒ No. of terms in TN ~ *poly*(N)
2. A variational ansatz for the many-body wavefunction

Tensor Networks

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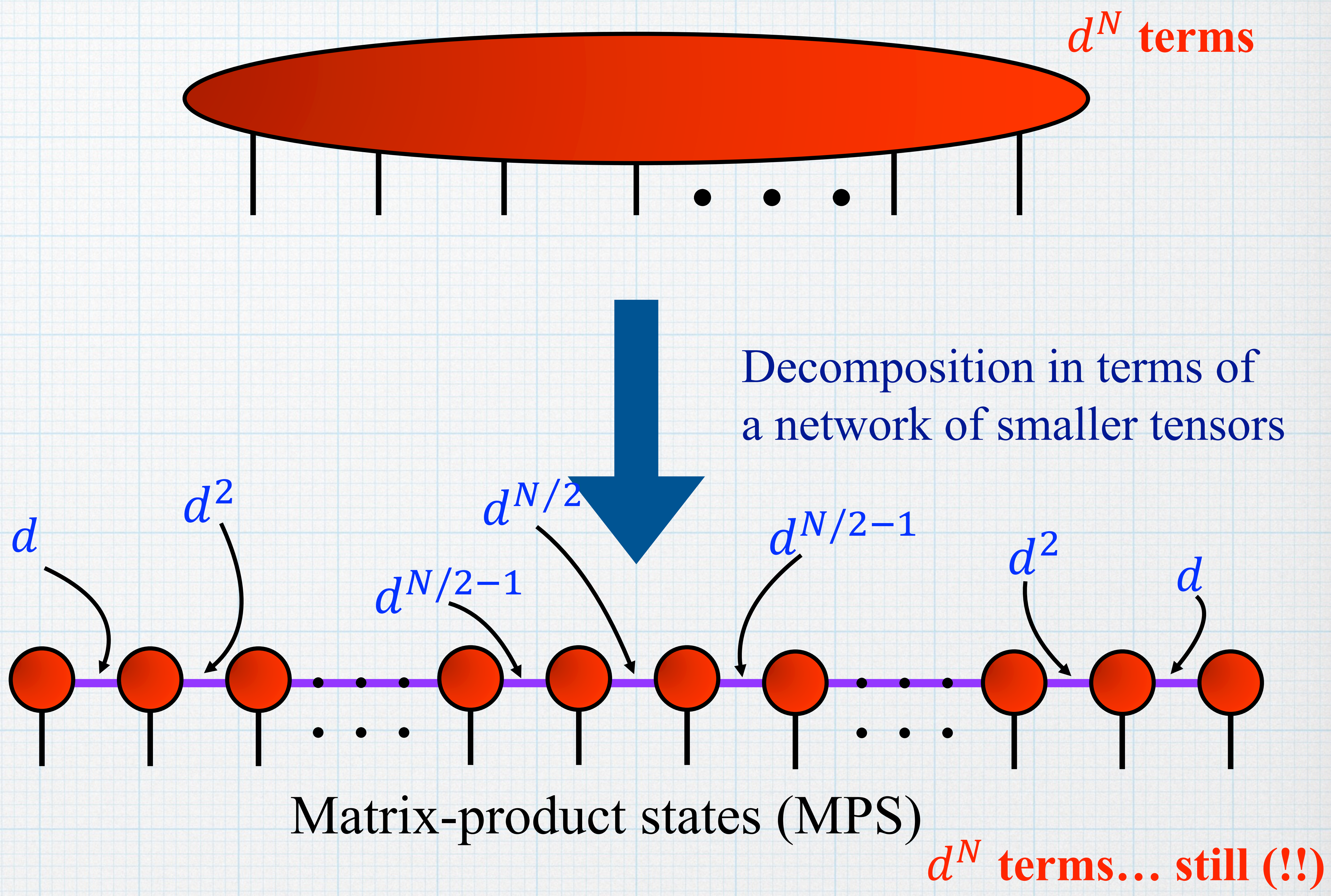
1. Systematically restricting the virtual dimensions $\Rightarrow$ No. of terms in TN $\sim$ $poly(N)$
2. A variational ansatz for the many-body wavefunction



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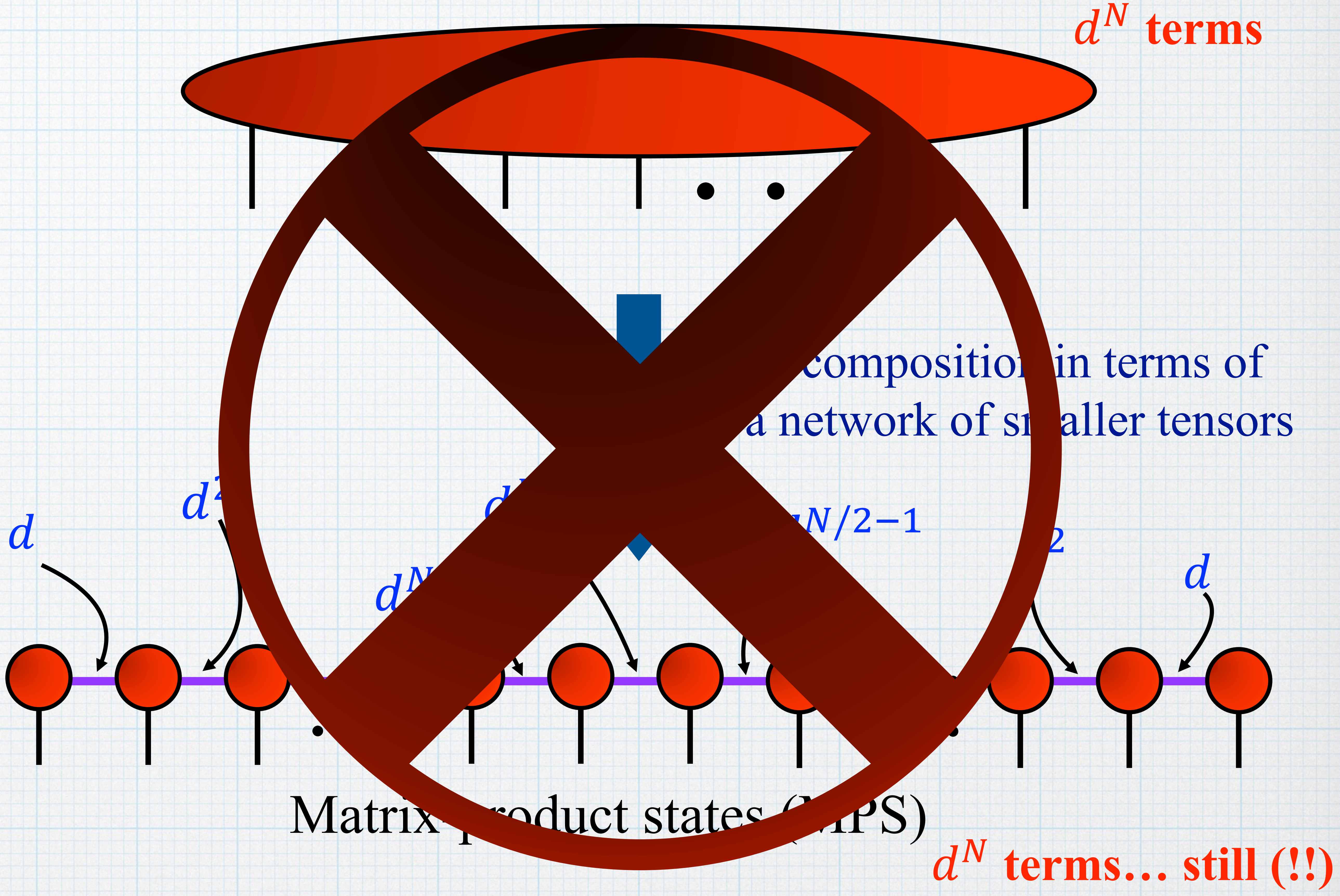
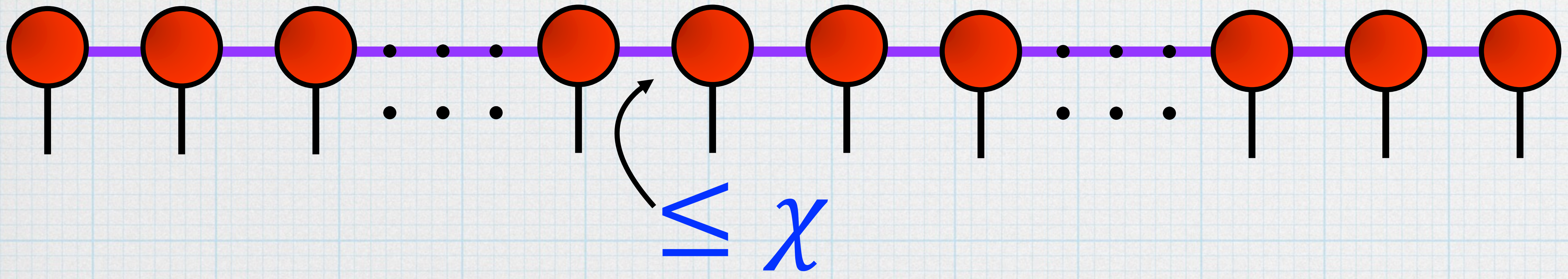


Instead… an MPS with **maximum** bond dimension χ



Number of non-zero elements $\leq Nd\chi^2$

Linear in system-size (!!!)

How do we determine the value of χ ?

That is the 'systematic' part of the restriction.

Tensor Networks

Key Idea:

1. Systematically restricting the virtual dimensions $\Rightarrow$ No. of terms in TN $\sim$ $poly(N)$
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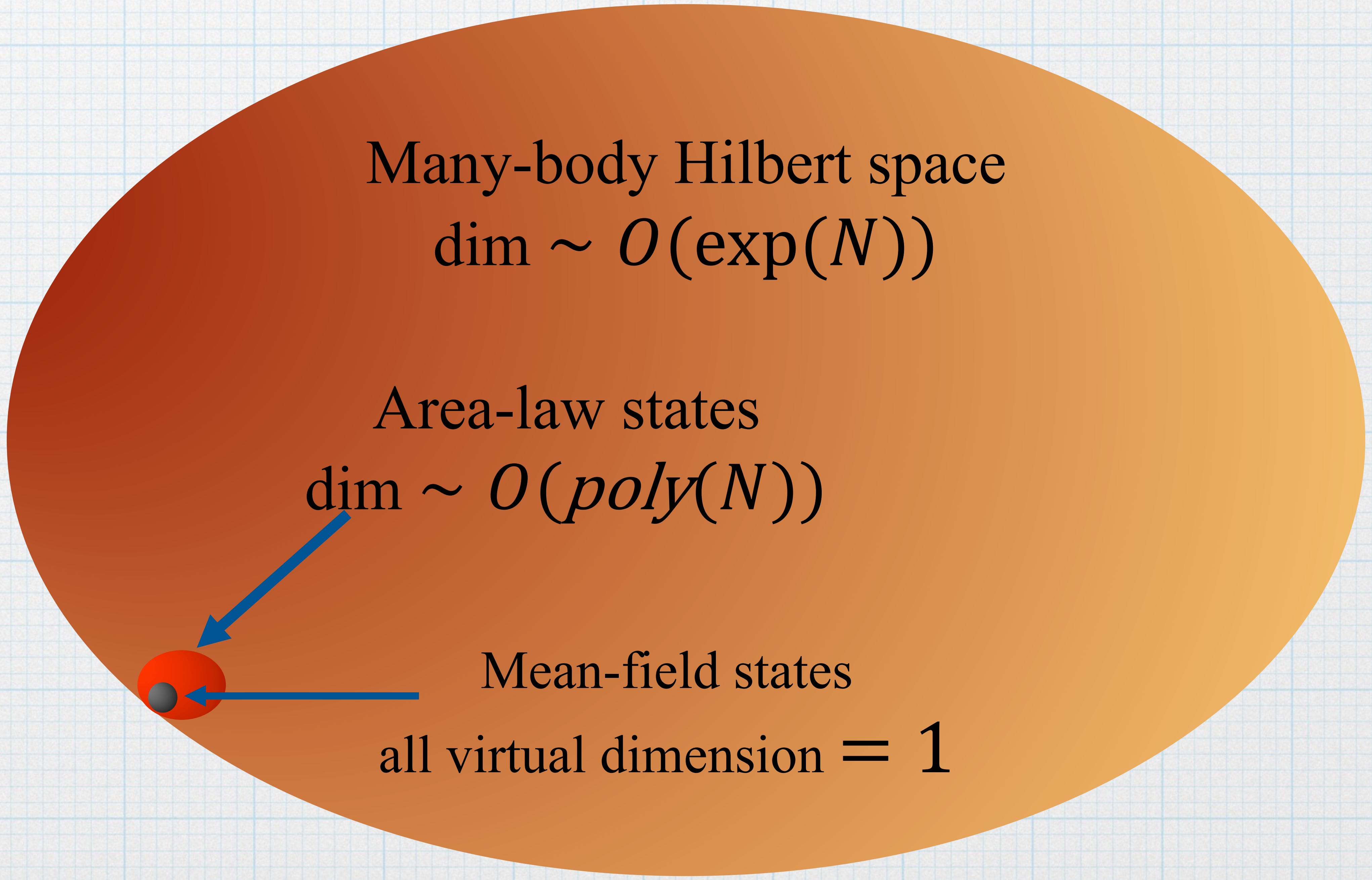
Why it works!!!

The answer comes from quantum information theory

Specifically, from the entanglement structure of low-lying eigenstates of many-body Hamiltonians…

They follow **Area-law of entanglement**
Entanglement grows proportional to the Area of the bipartition, not the volume.



Tensor Networks

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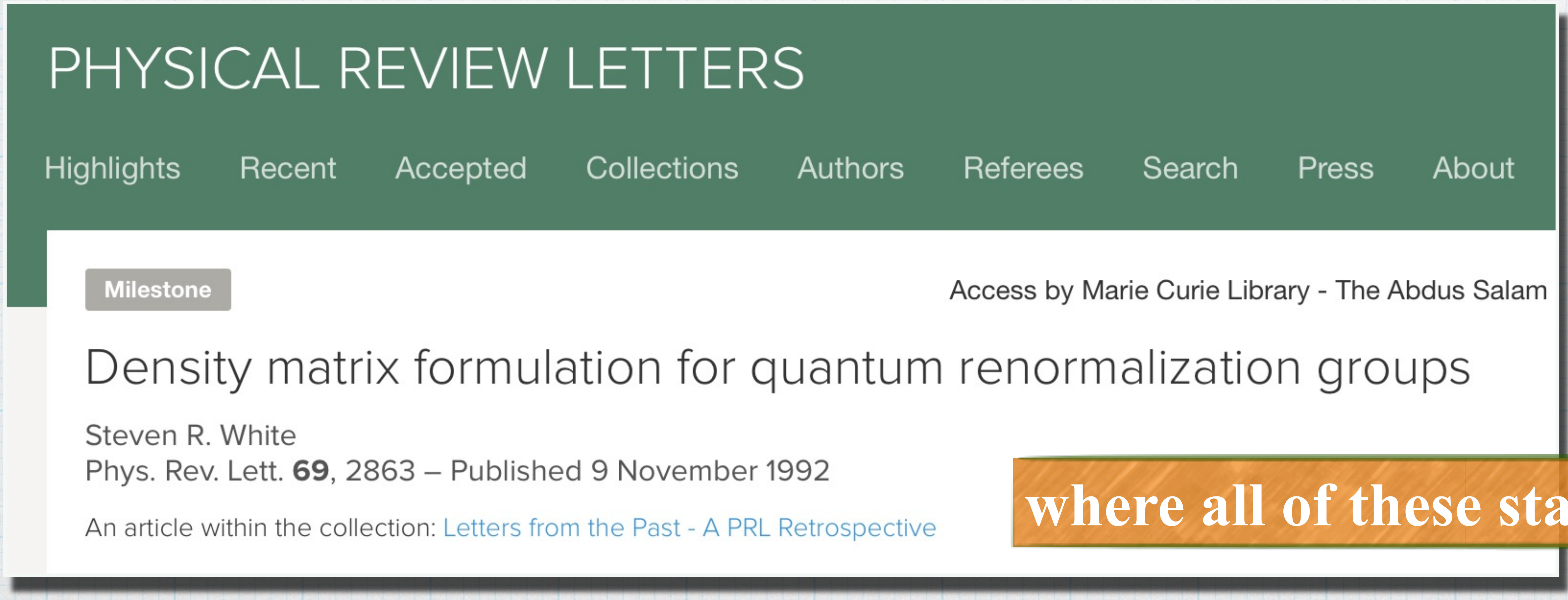
Renormalization of entanglement content or 'entanglement degrees of freedom'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Milestone

Access by Marie Curie Library - The Abdus Salam

Density matrix formulation for quantum renormalization groups

Steven R. White

Phys. Rev. Lett. **69**, 2863 – Published 9 November 1992

An article within the collection: Letters from the Past - A PRL Retrospective

where all of these started…

Why it works!!!

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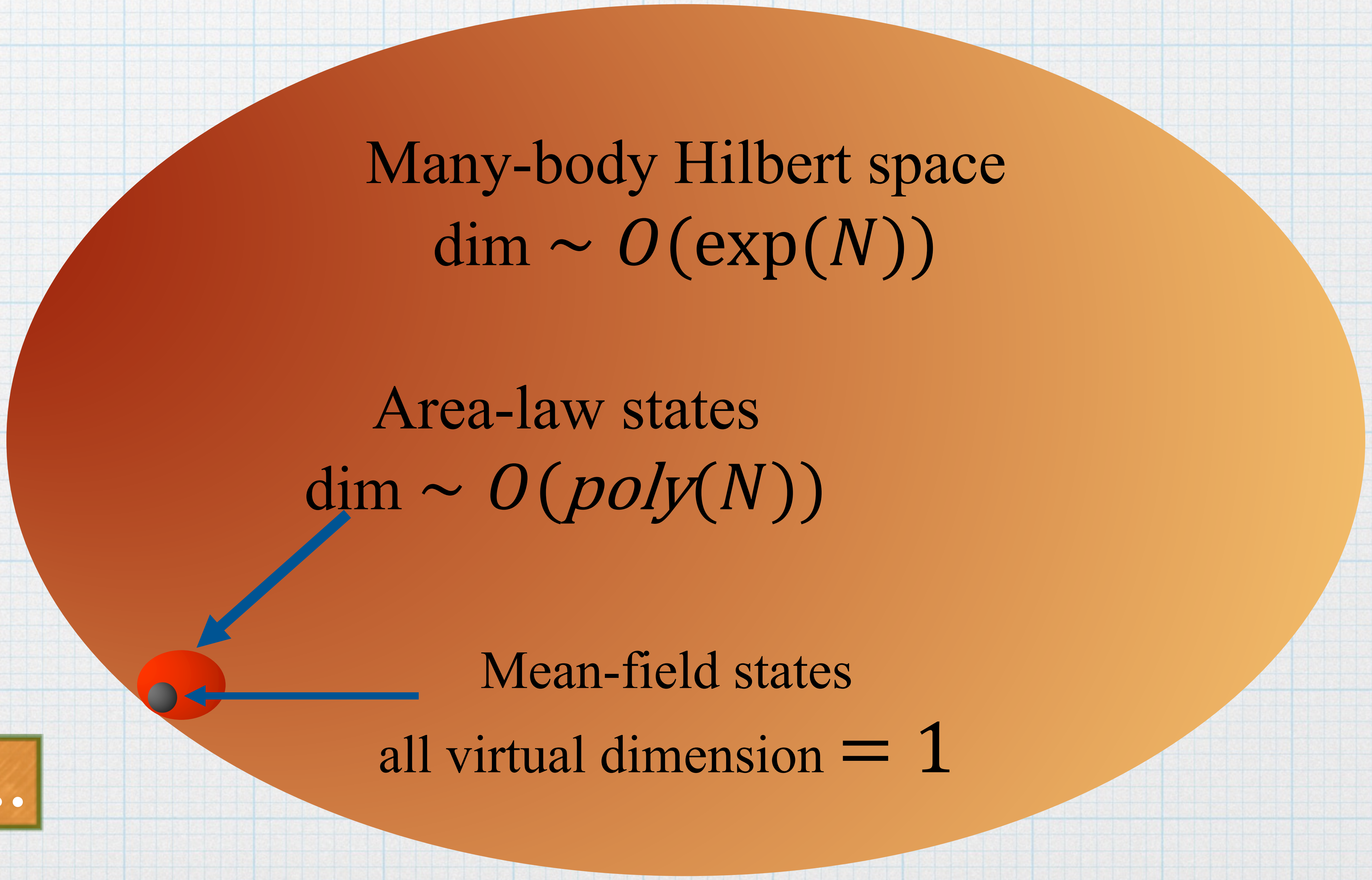
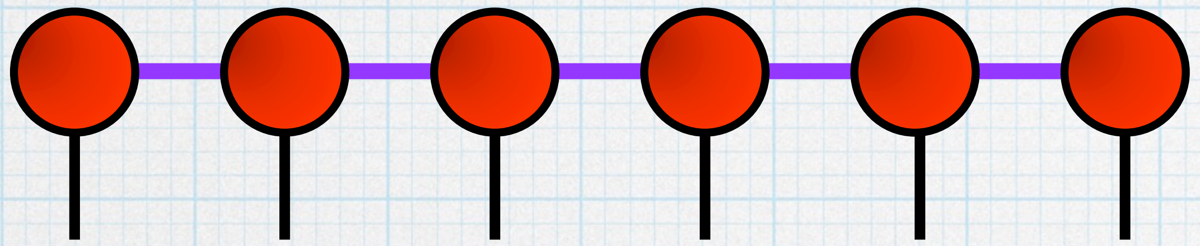


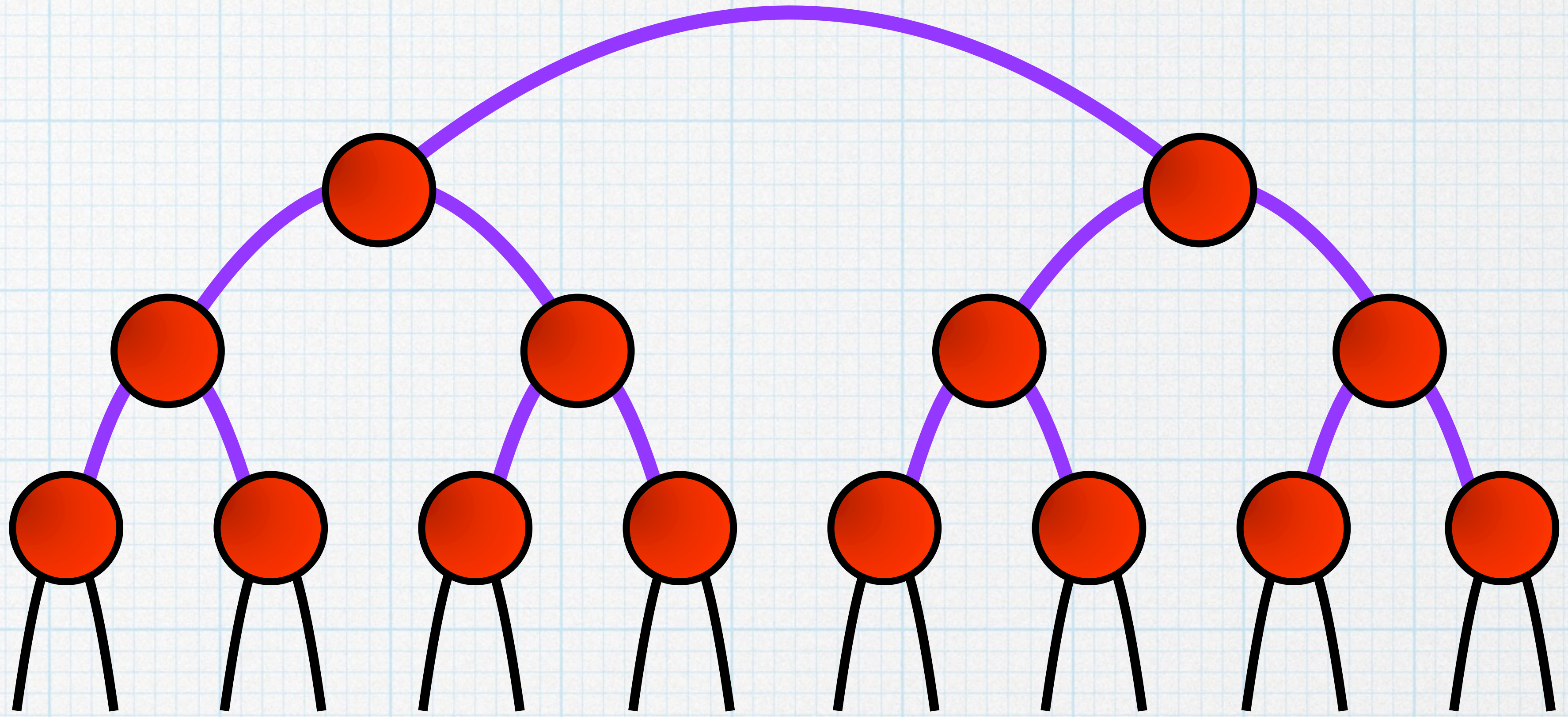
Tensor Networks

Various types… for different systems/geometries

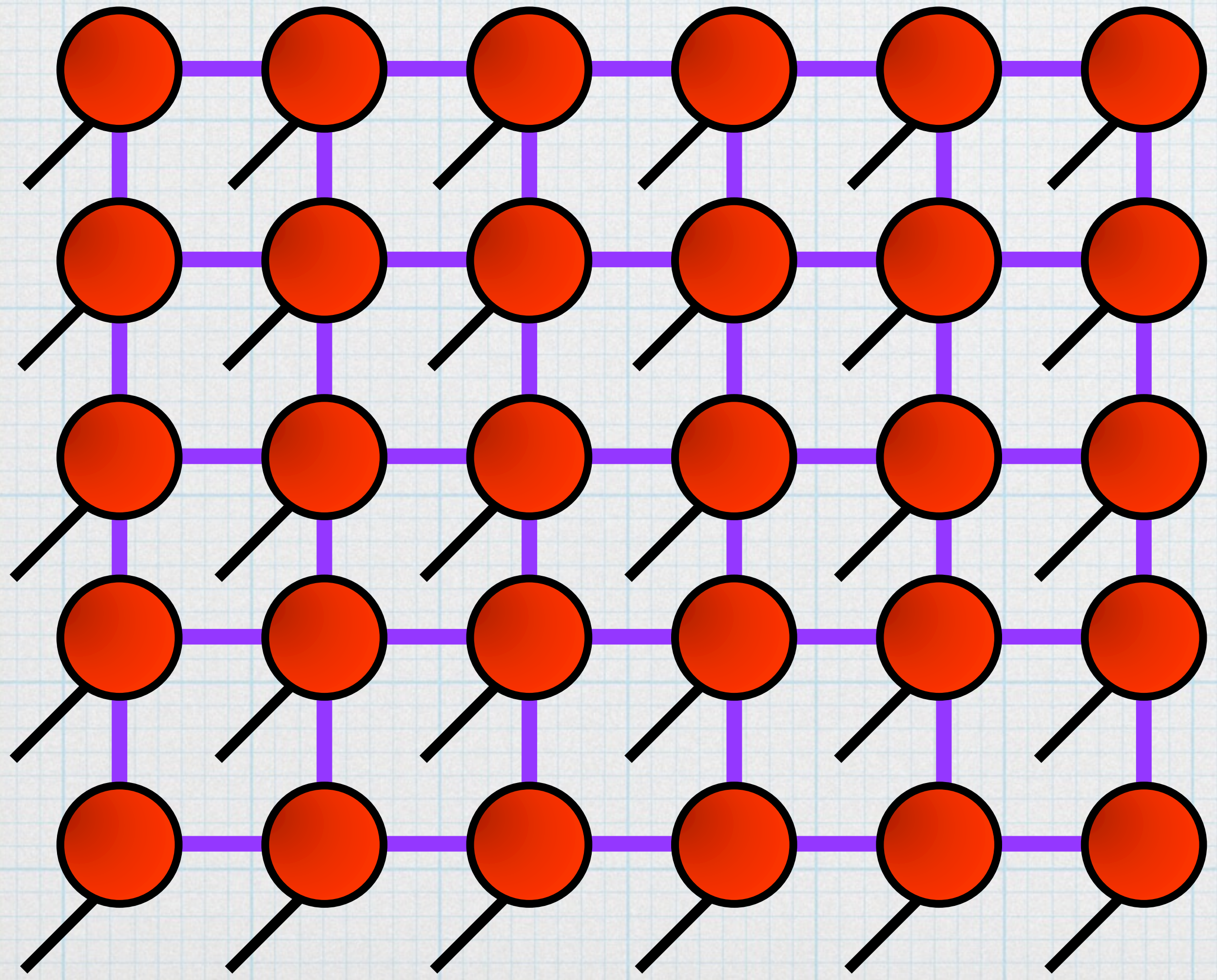
Matrix-product states (MPS)



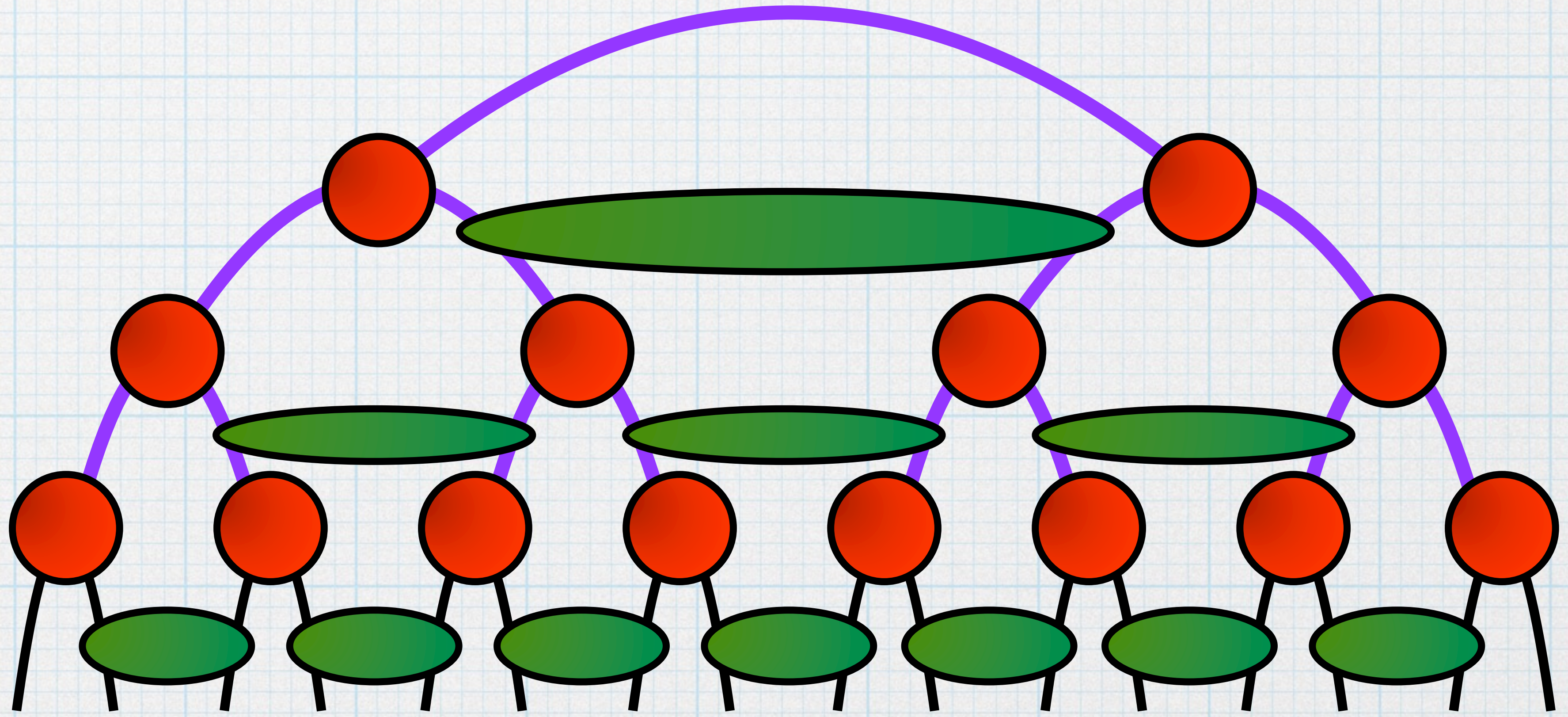
Tree tensor network (TTN)



Projected entangled pair states (PEPS)



Multi-scale entanglement renormalization ansatz (MERA)



1. Annals of Physics **326**, 96 (2011)
2. Annals of Physics **349,** 117 (2014)
3. Annals of Physics **411**, 167998 (2019)
4. SciPost Phys. Lect. Notes **8** (2019)

Tensor Networks

Various types… for different systems/geometries

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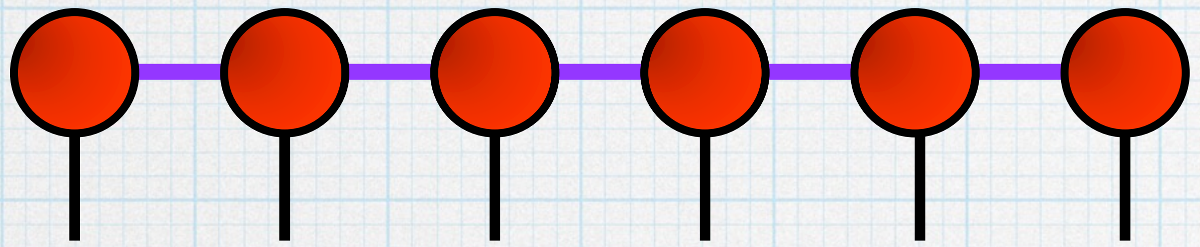
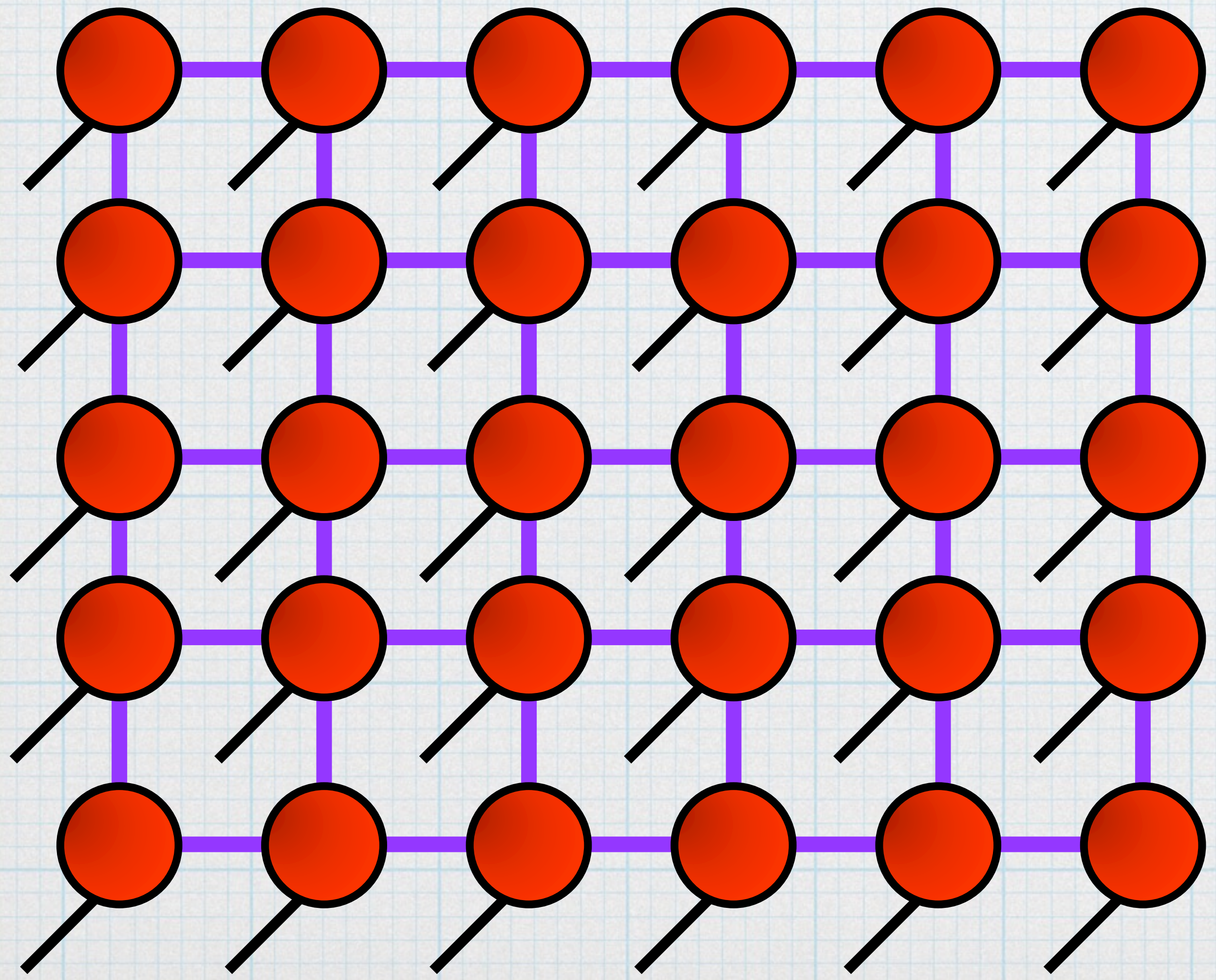
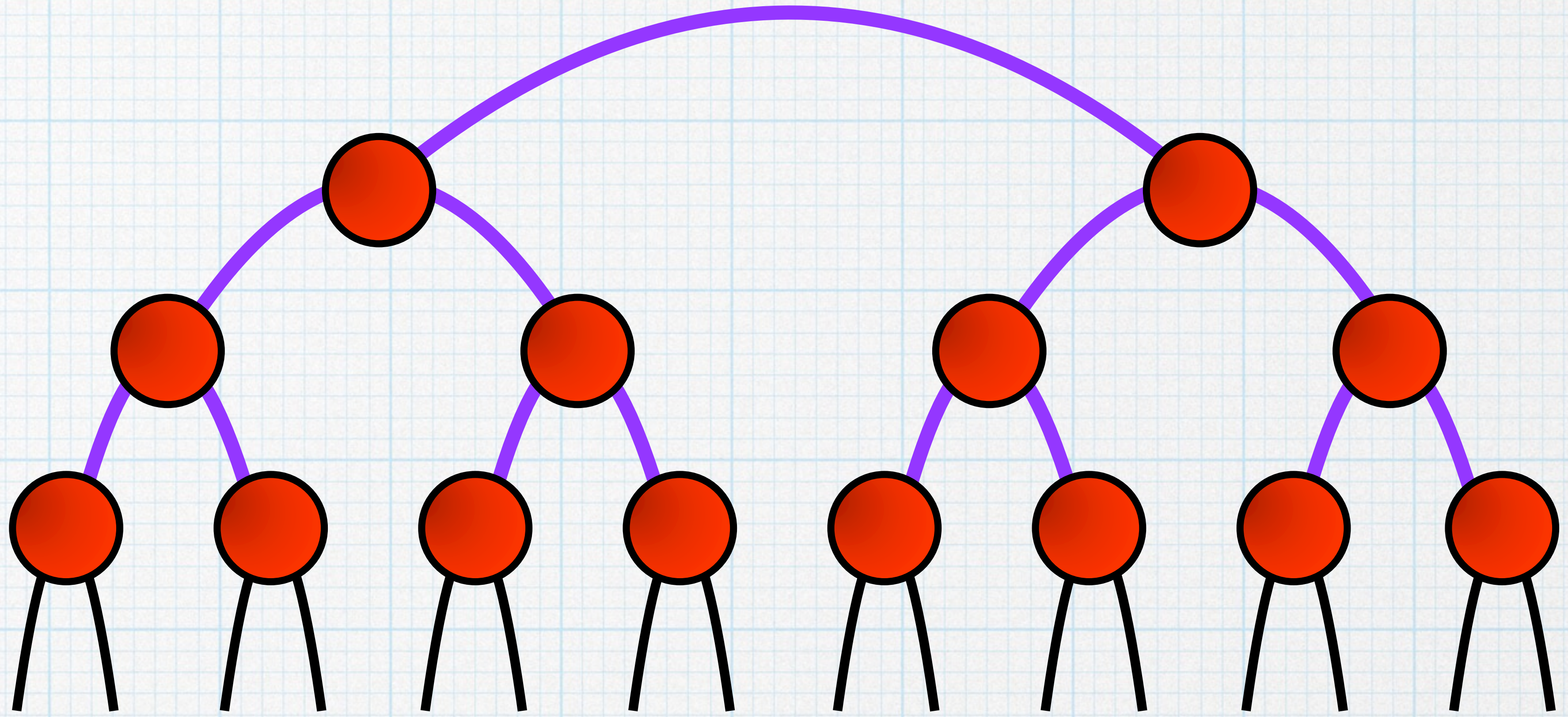


Till now… Most successful and complete
But only suitable for 1D systems

Projected entangled pair states (PEPS)



Tree tensor network (TTN)



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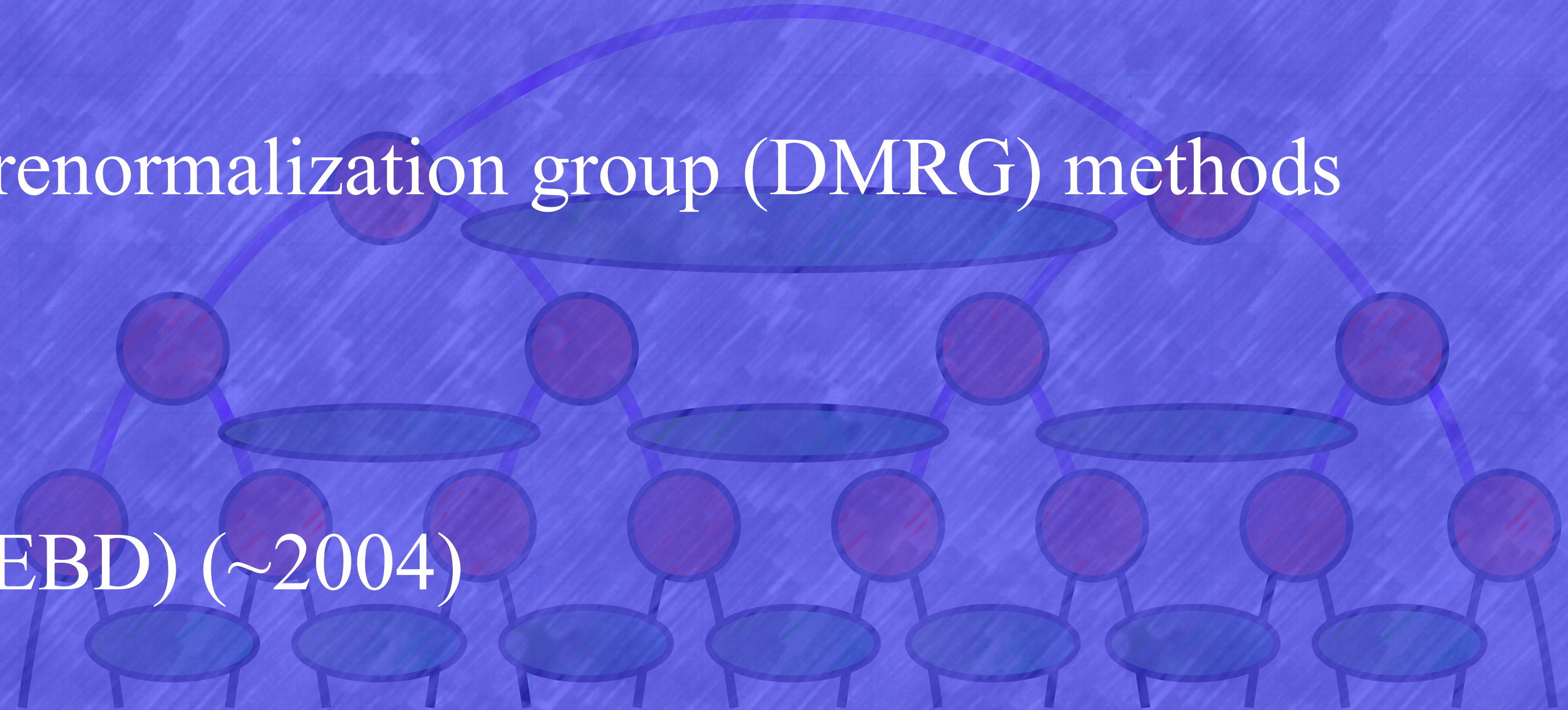

State of the Art algorithms…

For equilibrium physics…
Different variations of density-matrix renormalization group (DMRG) methods

Out-of-equilibrium…
1. Time-evolving block decimation (TEBD) (~2004)

2. Tangent-space method of time-dependent variational principle (TDVP) (2011 - 2016)

1. Annals of Physics **326**, 96 (2011)
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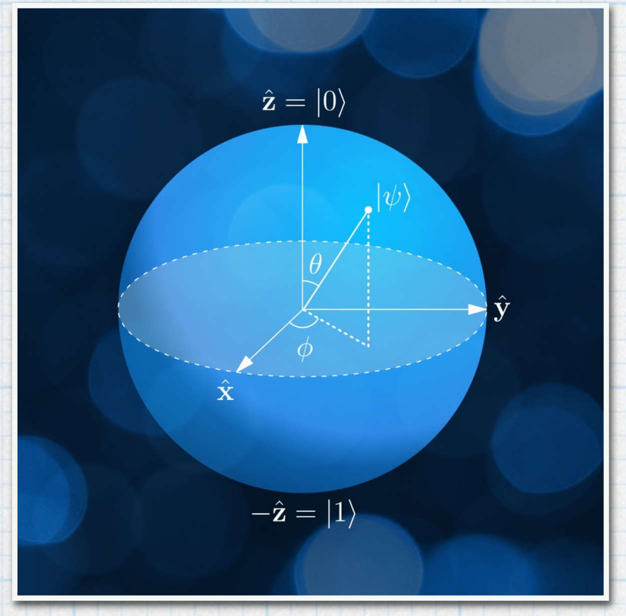
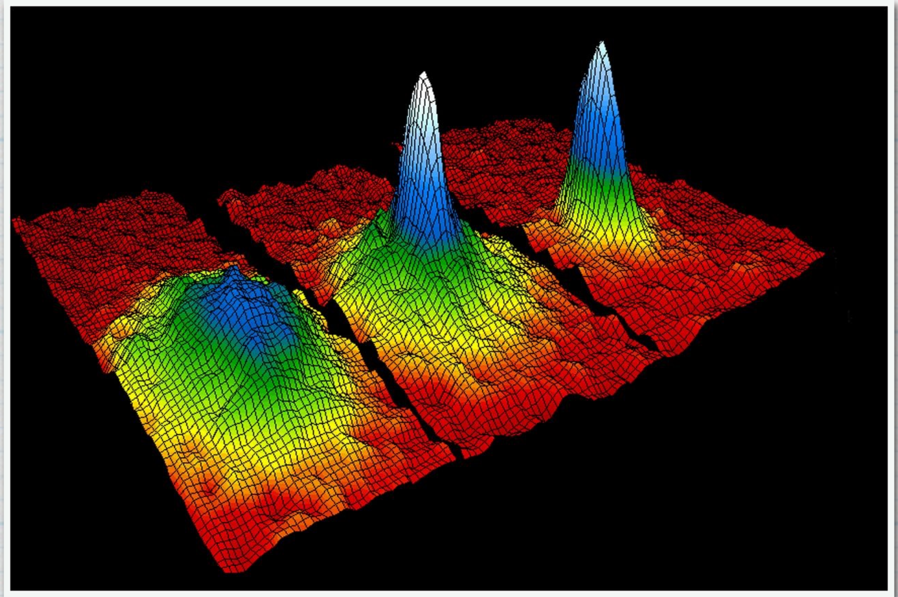
An entirely new domain…

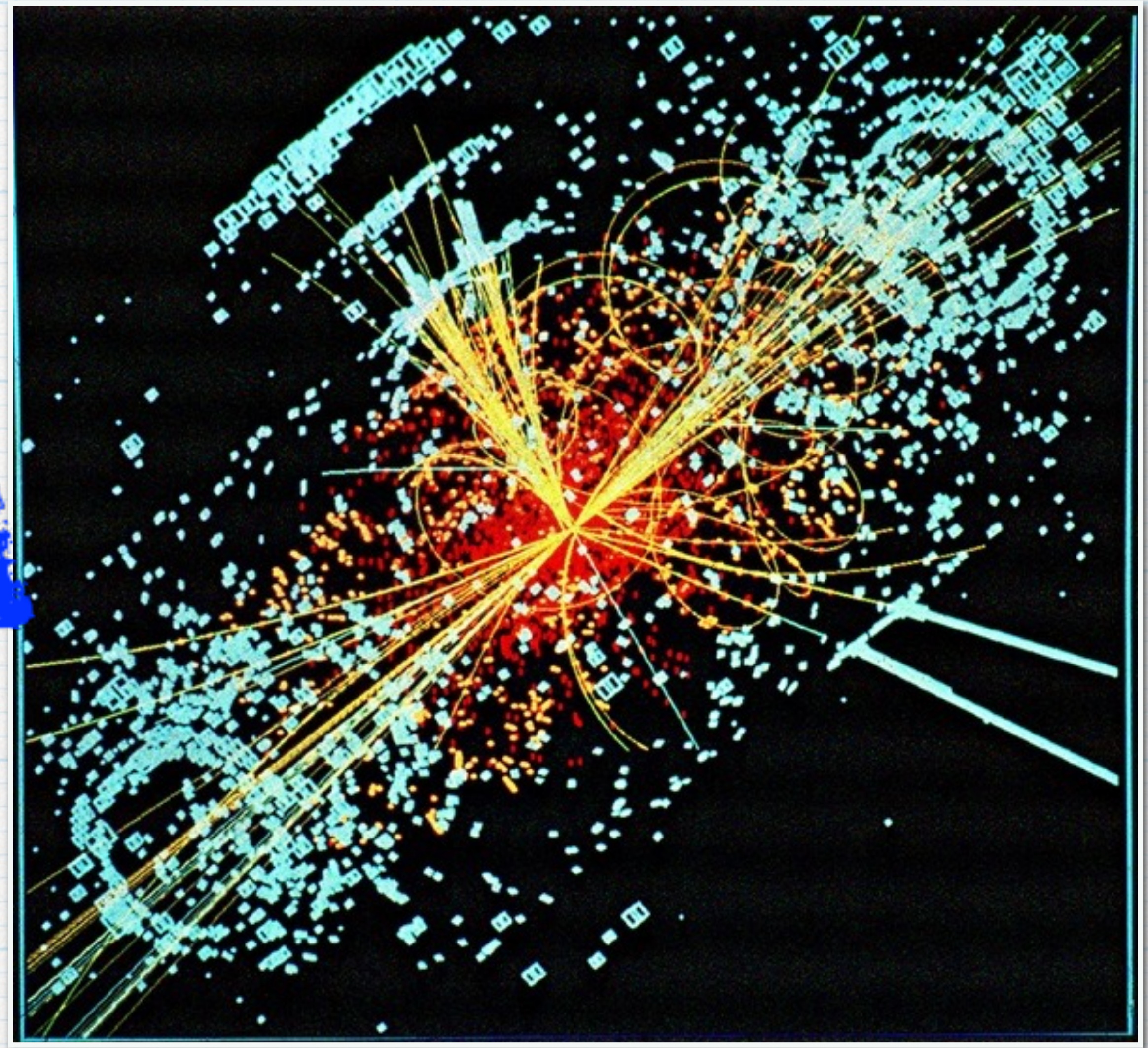
Quantum simulations and tensor networks are successful in strongly-correlated many-body systems… Great… but…





Can we simulate High-Energy Physics described by gauge theories??

1. Interdisciplinary
2. Source of new ideas —
   - (a) conceptual understanding
   - (b) numerical developments
   - (c) experimental advancements



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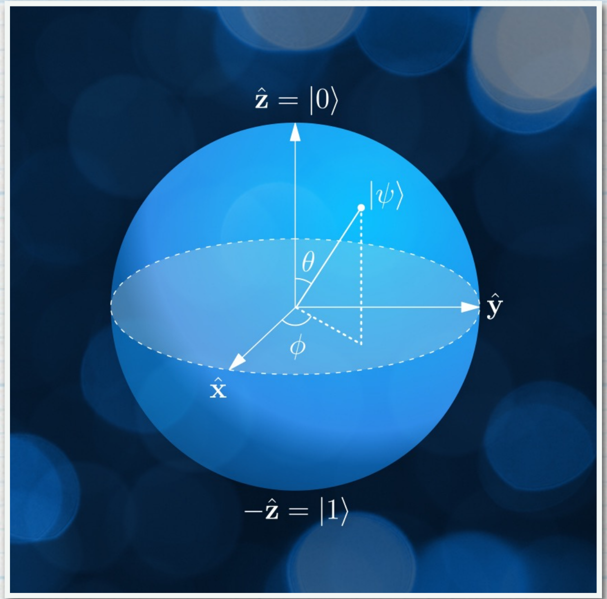

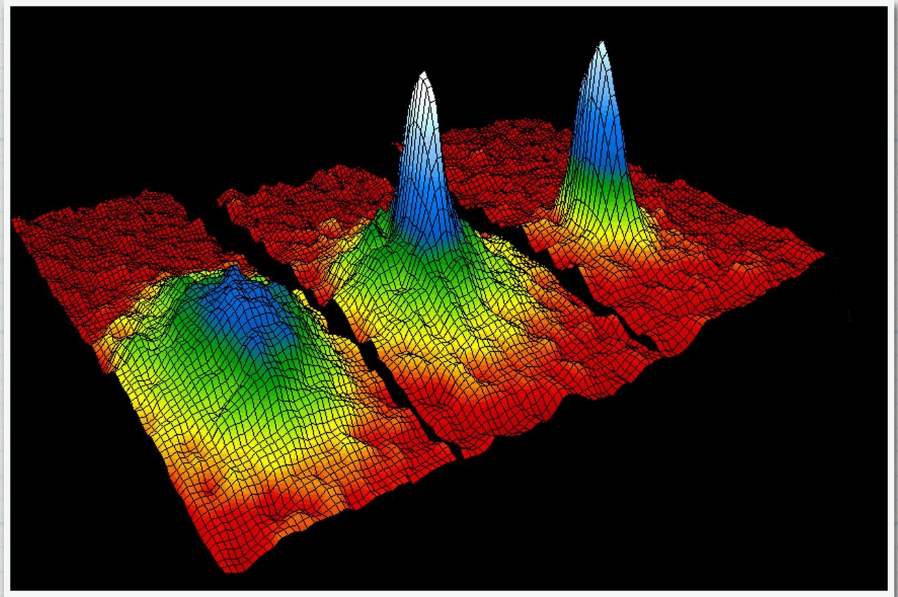

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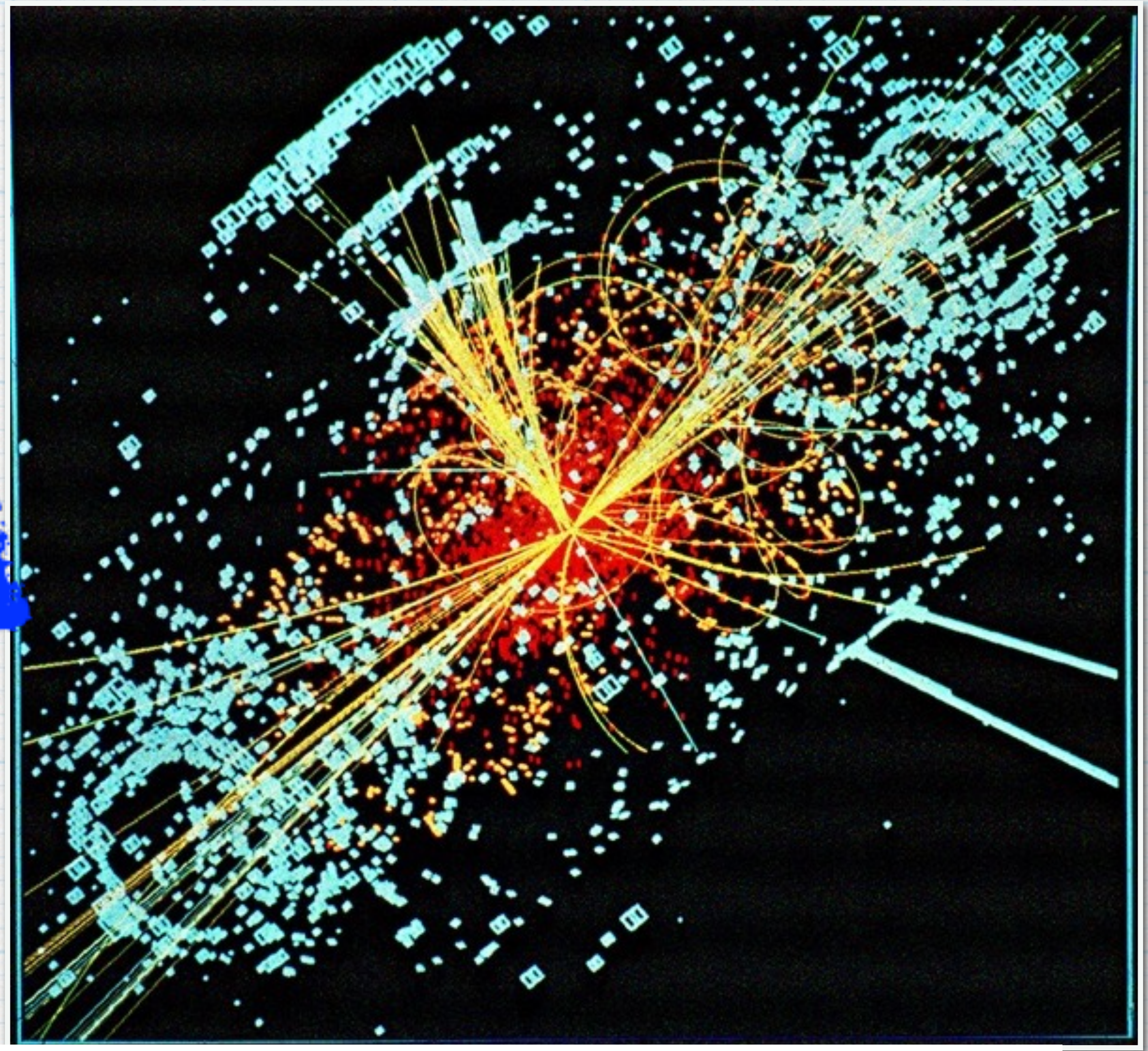

QUANTERA

QTFLAG

Quantum Technologies For LAttice Gauge theories

QUANTERA

DYNAMITE

Next Generation Quantum Simulators: From DYNAMIcal Gauge Fields to Lattice Gauge ThEory

Eur. Phys. J. D (2020) 74: 165
https://doi.org/10.1140/epjd/e2020-100571-8

THE EUROPEAN PHYSICAL JOURNAL D

Colloquium

Simulating lattice gauge theories within quantum technologies

Mari Carmen Bañuls[1,2], Rainer Blatt[3,4], Jacopo Catani[5,6,7], Alessio Celi[3,8], Juan Ignacio Cirac[1,2], Marcello Dalmonte[9,10], Leonardo Fallani[5,6,7], Karl Jansen[11], Maciej Lewenstein[8,12,13], Simone Montangero[14,15,a], Christine A. Muschik[3], Benni Reznik[16], Enrique Rico[17,18], Luca Tagliacozzo[19], Karel Van Acoleyen[20], Frank Verstraete[20,21], Uwe-Jens Wiese[22], Matthew Wingate[23], Jakub Zakrzewski[24,25], and Peter Zoller[3]

PHILOSOPHICAL TRANSACTIONS A

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Review

Check for updates

Cite this article: Aidelsburger M *et al.* 2021 Cold atoms meet lattice gauge theory. *Phil. Trans. R. Soc. A* **380**: 20210064.
https://doi.org/10.1098/rsta.2021.0064

Received: 11 June 2021
Accepted: 23 August 2021

One contribution of 13 to a theme issue 'Quantum technologies in particle physics'.

Cold atoms meet lattice gauge theory

Monika Aidelsburger[1,2], Luca Barbiero[3,4], Alejandro Bermudez[5], Titas Chanda[6,7], Alexandre Dauphin[3], Daniel González-Cuadra[3], Przemysław R. Grzybowski[8], Simon Hands[9,10], Fred Jendrzejewski[11], Johannes Jünemann[12], Gediminas Juzeliūnas[13], Valentin Kasper[3], Angelo Piga[3,14], Shi-Ju Ran[15], Matteo Rizzi[16,17], Germán Sierra[18], Luca Tagliacozzo[19], Emanuele Tirrito[20], Torsten V. Zache[21,22], Jakub Zakrzewski[6], Erez Zohar[23] and Maciej Lewenstein[3,24]

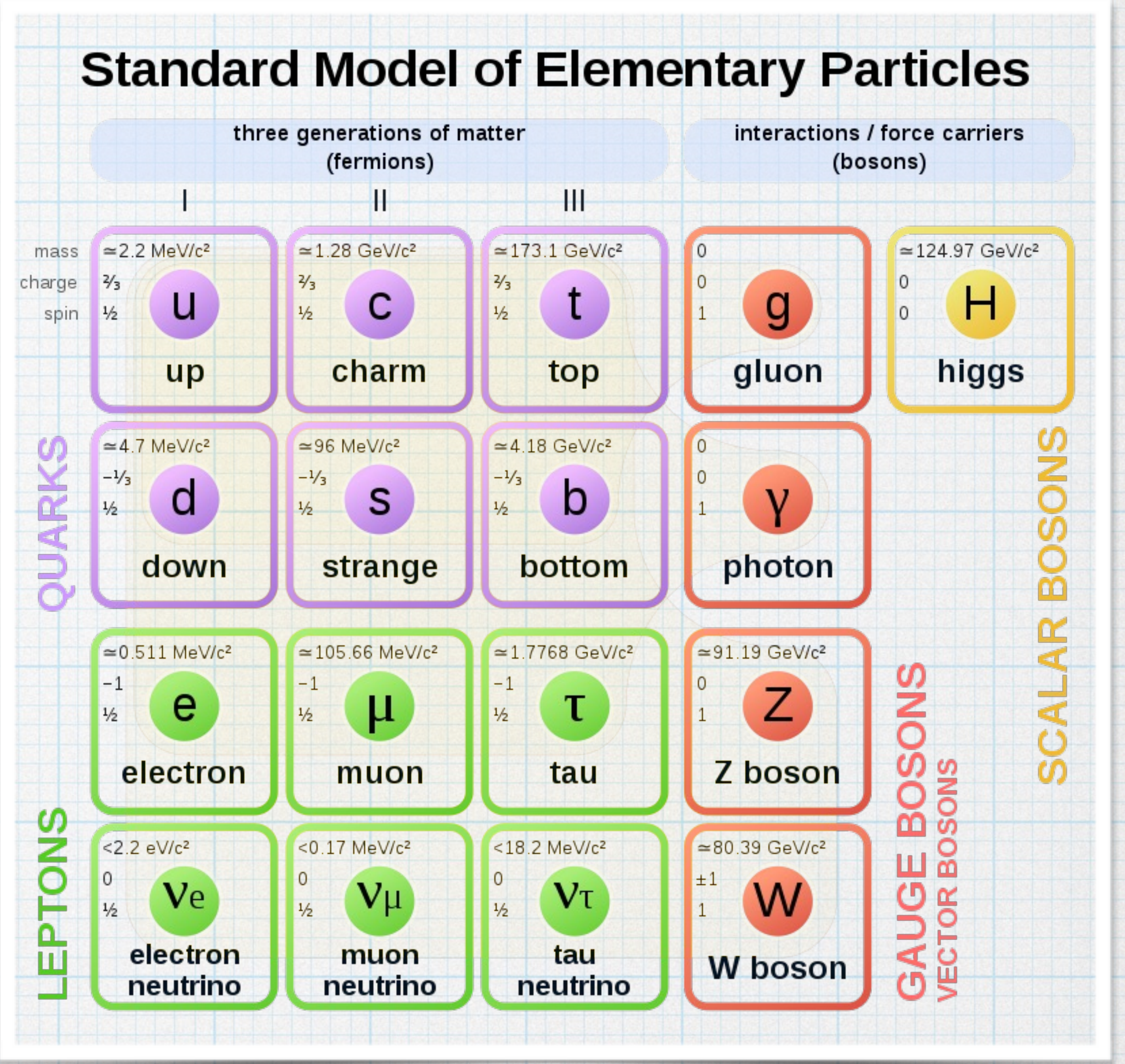
Gauge Theories on Lattice

Gauge Theories —> Theories with **Local** conservation laws (Gauss law)

e.g., classical electrodynamics … $U(1)$ gauge theory

(Quantum) Gauge theories came in the form of quantum electrodynamics, non-Abelian Yang-Mills theories etc.

Standard model of particle physics is a non-Abelian gauge theory with the symmetry group U(1) × SU(2) × SU(3).



Lattice gauge theory (LGT) on Euclidean space-time

PHYSICAL REVIEW D VOLUME 10, NUMBER 8 15 OCTOBER 1974

Confinement of quarks*

Kenneth G. Wilson

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850

(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice) joining quark paths. This structure is reminiscent of relativistic string models of hadrons.

Hamiltonian formulation of LGT

PHYSICAL REVIEW D VOLUME 11, NUMBER 2 15 JANUARY 1975

Hamiltonian formulation of Wilson's lattice gauge theories

John Kogut*

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Leonard Susskind†

Belfer Graduate School of Science, Yeshiva University, New York, New York
and Tel Aviv University, Ramat Aviv, Israel
and Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received 9 July 1974)

Wilson's lattice gauge model is presented as a canonical Hamiltonian theory. The structure of the model is reduced to the interactions of an infinite collection of coupled rigid rotators. The gauge-invariant configuration space consists of a collection of strings with quarks at their ends. The strings are lines of non-Abelian electric flux. In the strong-coupling limit the dynamics is best described in terms of these strings. Quark confinement is a result of the inability to break a string without producing a pair.

Discretized space, but real continuous time

LGT to approach non-perturbative limits…. e.g., by quantum Monte Carlo

Gauge Theories on Lattice

In present days… from quantum mar

Advancements in
(digital

Published: 28 October 2013

Simulation of non-Abelian gauge theories with optical lattices

L. Tagliacozzo, A. Celi, P. Orland, M. W. Mitchell & M. Lewenstein

Nature Communications **4**, Article number: 2615 (2013) | Cite this article

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Non-Abelian SU(2) Lattice Gauge Theories in Superconducting Circuits

A. Mezzacapo, E. Rico, C. Sabín, I. L. Egusquiza, L. Lamata, and E. Solano

Phys. Rev. Lett. **115**, 240502 – Published 9 December 2015

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Atomic Quantum Simulation of $\mathbf{U}(N)$ and $\mathbf{SU}(N)$ Non-Abelian Lattice Gauge Theories

D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, U.-J. Wiese, and P. Zoller

Phys. Rev. Lett. **110**, 125303 – Published 21 March 2013

nature

Letter | Published: 22 June 2016

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Esteban A. Martinez, Christine A. Muschik, Philipp Schindler, Daniel Nigg, Alexander Erhard, Markus Heyl, Philipp Hauke, Marcello Dalmonte, Thomas Monz, Peter Zoller & Rainer Blatt

Nature **534**, 516–519 (23 June 2016) | Download Citation

Article | Published: 16 September 2019

Floquet approach to $\mathbb{Z}_2$ lattice gauge theories with ultracold atoms in optical lattices

Christian Schweizer, Fabian Grusdt, Moritz Berngruber, Luca Barbiero, Eugene Demler, Nathan Goldman, Immanuel Bloch & Monika Aidelsburger

Nature Physics **15**, 1168–1173(2019) | Cite this article

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REPORT

A scalable realization of local U(1) gauge invariance in cold atomic mixtures

Alexander Mil[1,*], Torsten V. Zache[2], Apoorva Hegde[1], Andy Xia[1], Rohit P. Bhatt[1], Markus K. Oberthaler[1], Philipp Hauke[1,2,3], Jürgen Berges[2], Fred Jendrzejewski[1]

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[2]Institut für Theoretische Physik, Heidelberg University, Philosophenweg 16, 69120 Heidelberg, Germany.
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– Hide authors and affiliations

Science 06 Mar 2020:
Vol. 367, Issue 6482, pp. 1128-1130
DOI: 10.1126/science.aaz5312

New experimental results
and propositions are coming very frequently

Long-term goal being the scalable simulation
of non-Abelian theories

Gauge Theories on Lattice

In present days… form quantum many-body perspective…

Advancements in quantum simulation (digital + analog)

Recent developments in tensor network methods

1. Hamiltonian formulation
2. Access to state or wave-function
3. Entanglement entropy becomes almost free
4. **No sign problem**
5. **Real-time dynamics**

nature

First proof of concept

Letter | Published: 22 June 2016

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Esteban A. Martinez, Christine A. Muschik, Philipp Schindler, Daniel Nigg, Alexander Erhard, Markus Heyl, Philipp Hauke, Marcello Dalmonte, Thomas Monz, Peter Zoller & Rainer Blatt

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New experimental results
and propositions are coming very frequently

Long-term goal being the scalable simulation of non-Abelian theories

In 2+1 D…

Some advancement using PEPS, but computationally very hard
e.g., Phys. Rev. D **97**, 034510 (2018)

A better way forward… Tree Tensor Network (TTN)

In 2+1 D…

Phys. Rev. X **10**, 041040 (2020)

In 3+1 D…

Nat. Comm. **12**, 3600 (2021)

Bosonic Schwinger Model

Scalar QED in 1+1D


PHYSICAL REVIEW LETTERS

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Confinement and Lack of Thermalization after Quenches in the Bosonic Schwinger Model

Titas Chanda, Jakub Zakrzewski, Maciej Lewenstein, and Luca Tagliacozzo

Phys. Rev. Lett. **124**, 180602 – Published 6 May 2020

Titas Chanda

Matter particles are also bosonic
→ bosons are easier to cool in cold atomic experiments

Equilibrium characterization of confinement requires calculation of “Wilson loops”

Not possible in experiments

Maciej Lewenstein

Goal:

1. Signatures of confinement out-of-equilibrium, easier to experimentally verify confinement. (Ala Nat. Phys. **13**, 246 (2017))

2. Lack of thermalization and slow dynamics due to confinement.

Luca Tagliacozzo

Lack of thermalization, memory effect, exotic asymptotic states without disorder

Bosonic Schwinger Model

Scalar QED in 1+1D

Lagrangian…. $\mathcal{L} = -[D_\mu\phi]^* D^\mu\phi - m^2|\phi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

$D_\mu = (\partial_\mu + iqA_\mu)$

Metric convention ➞ (-1,1,1,1) or (-1,1)

And then we discretize…

Prescription for discretization: (Kogut-Susskind-1974)

1. Fix temporal gauge $A_t(x,t) = 0$ in 1+1 dimension
2. Canonical quantization, get the Hamiltonian in continuum
3. Discretize the Hamiltonian on a lattice with spacing a
4. Discretization is such that matter fields sit on lattice sites, gauge fields on bonds

Bosonic Schwinger Model

$x = 1/a^2q^2$

$[\hat{\phi}_j, \hat{\Pi}_k] = [\hat{\phi}_j^\dagger, \hat{\Pi}_k^\dagger] = i\delta_{jk}$

Hamiltonian after discretization…

$$\hat{H} = \sum_j \left[\hat{L}_j^2 + 2x\ \hat{\Pi}_j^\dagger \hat{\Pi}_j + (4x + \frac{2m^2}{q^2})\hat{\phi}_j^\dagger \hat{\phi}_j - 2x(\hat{\phi}_{j+1}^\dagger\ \hat{U}_j\ \hat{\phi}_j + \text{h.c.})\right]$$

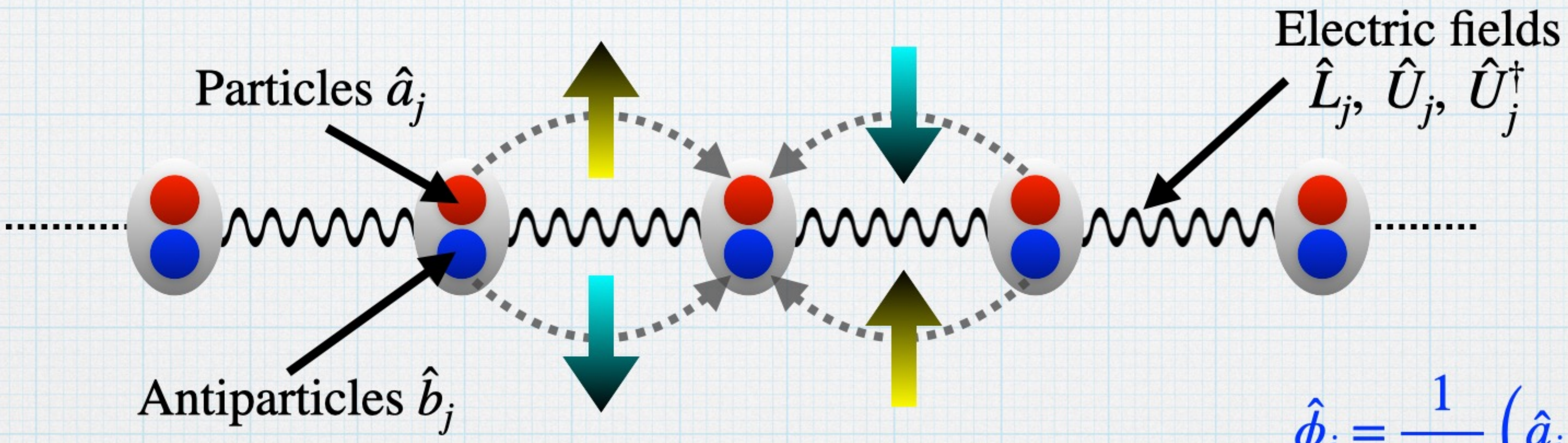
$\hat{L}_j|l_j\rangle = l_j|l_j\rangle,$ with $l_j \in [\ldots,\ -2,\ -1, 0, 1, 2, \ldots]$

$\hat{U}_j|l_j\rangle = |l_j - 1\rangle$

$\hat{U}_j^\dagger|l_j\rangle = |l_j + 1\rangle$

$[\hat{L}_j, \hat{U}_j] = -\hat{U}_j$

$[\hat{L}_j, \hat{U}_j^\dagger] = \hat{U}_j^\dagger$



$$\hat{\phi}_j = \frac{1}{\sqrt{2}}\left(\hat{a}_j + \hat{b}_j^\dagger\right),\ \hat{\Pi}_j = \frac{i}{\sqrt{2}}\left(\hat{a}_j^\dagger - \hat{b}_j\right)$$

$$\hat{\phi}_j^\dagger = \frac{1}{\sqrt{2}}\left(\hat{a}_j^\dagger + \hat{b}_j\right),\ \hat{\Pi}_j^\dagger = \frac{i}{\sqrt{2}}\left(\hat{b}_j^\dagger - \hat{a}_j\right)$$

$$\hat{H} = \sum_j \hat{L}_j^2 + 2\left(x\left(\left(m/q\right)^2 + 2x\right)\right)^{1/2} \sum_j \left(\hat{a}_j^\dagger \hat{a}_j + \hat{b}_j \hat{b}_j^\dagger\right) - \frac{x^{3/2}}{\left(\left(m/q\right)^2 + 2x\right)^{1/2}} \sum_j \left[\left(\hat{a}_{j+1}^\dagger + \hat{b}_{j+1}\right)\hat{U}_j\left(\hat{a}_j + \hat{b}_j^\dagger\right) + \text{h.c.}\right]$$

Bosonic Schwinger Model

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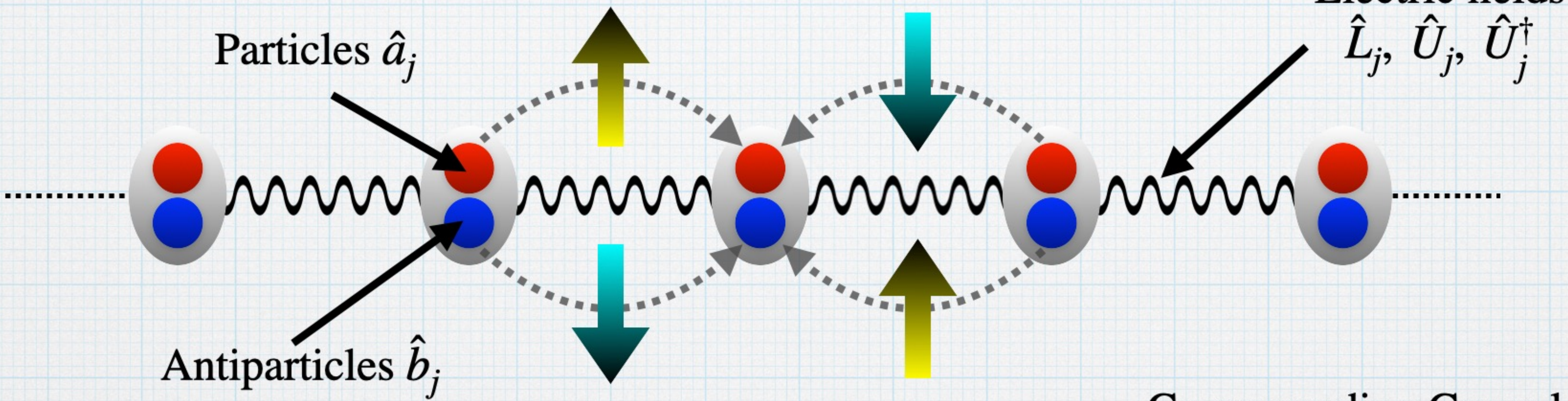

Local $U(1)$ invariance…

$$\hat{\phi}_j \to e^{i\alpha_j}\ \hat{\phi}_j, \quad \hat{a}_j \to e^{i\alpha_j}\ \hat{a}_j$$

$$\hat{\Pi}_j \to e^{-i\alpha_j}\ \hat{\Pi}_j, \quad \hat{b}_j \to e^{-i\alpha_j}\ \hat{b}_j$$

$$\hat{U}_j \to e^{-i\alpha_j}\ \hat{U}_j\ e^{i\alpha_{j+1}}$$

Corresponding Gauss law generators…

$$\hat{G}_j = \hat{L}_j - \hat{L}_{j-1} - \left(\hat{a}_j^\dagger\hat{a}_j - \hat{b}_j^\dagger\hat{b}_j\right)$$

$\hat{Q}_j$

Dynamical charge:
Particle — anti-particle number difference

We restrict ourself to $\hat{G}_j|\psi\rangle = 0$ sector for $\forall j$

Bosonic Schwinger Model

Comment on the ground state…

Dispersion relation without gauge fields (Klein-Gordon theory)…

$$\omega(k) = 2\sqrt{xm^2/q^2 + 2x^2(1 - \cos ka)}$$

$$\lim_{a\to 0} \omega(k) = \sqrt{k^2 + m^2}$$ Gapless in massless scenario

Free theory…
Excitations are free or …in the momentum basis

In the bosonic Schwinger model:

1. Excitations are not free particles, but bound particle-antiparticle pairs (mesons).

Confined theory…
Excitations are…

Bosonic Schwinger Model

Comment on the ground state…

Dispersion relation without gauge fields (Klein-Gordon theory)…

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$$\lim_{a\to 0} \omega(k) = \sqrt{k^2 + m^2}$$ Gapless in massless scenario

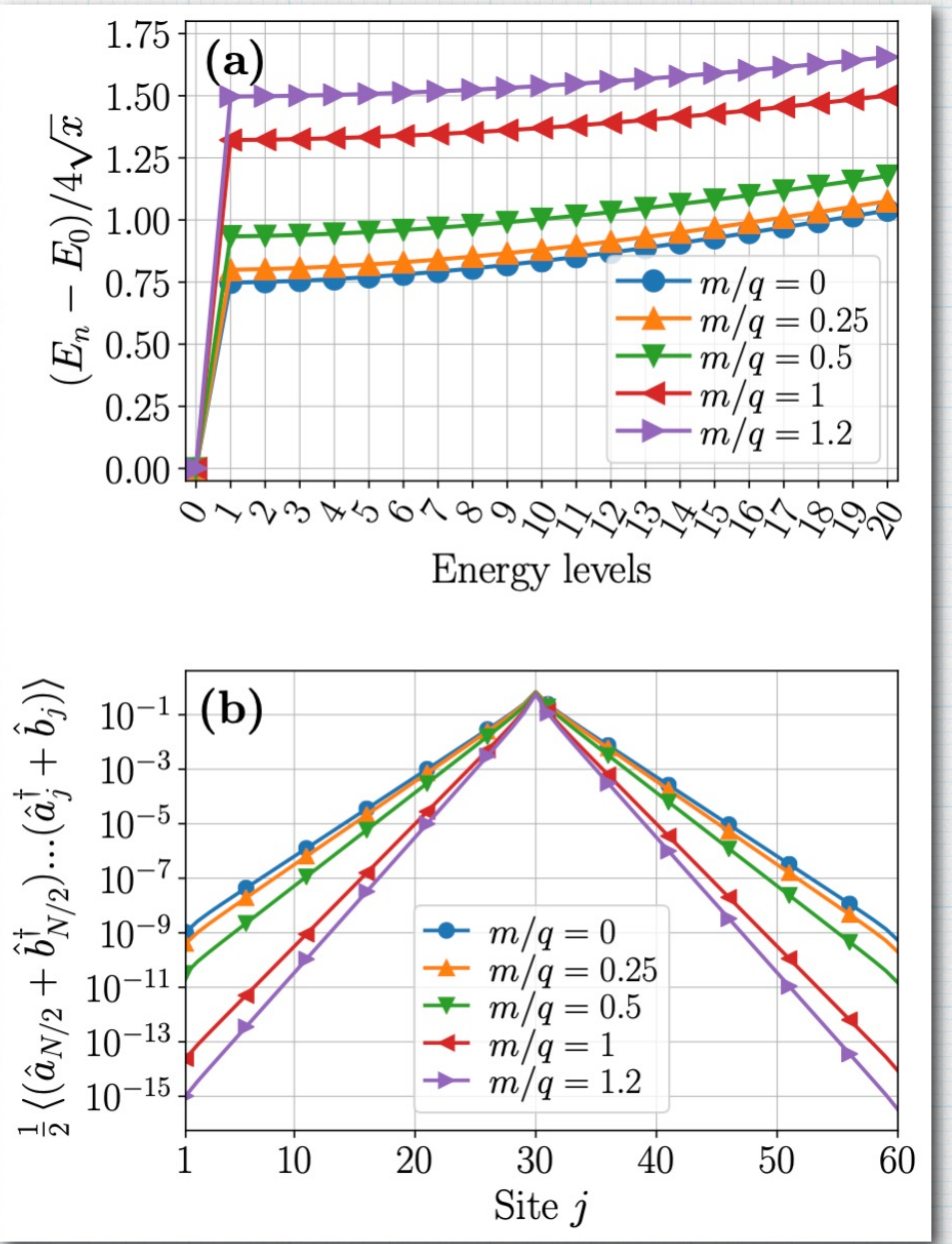
In the bosonic Schwinger model:

1. Excitations are not free particles, but bound particle-antiparticle pairs (mesons).

2. A finite mass-gap is generated due to matter-gauge coupling.
3. Mass-gap, $M/q = (E_1 - E_0)/4\sqrt{x} > m/q$.

4. Extra energy, $E_B/q = M/q - m/q$, arises as binding energy required to tether particle-antiparticle pairs into mesons.

5. Ground state is always gapped with finite correlations.



Bosonic Schwinger Model

Time evolution…

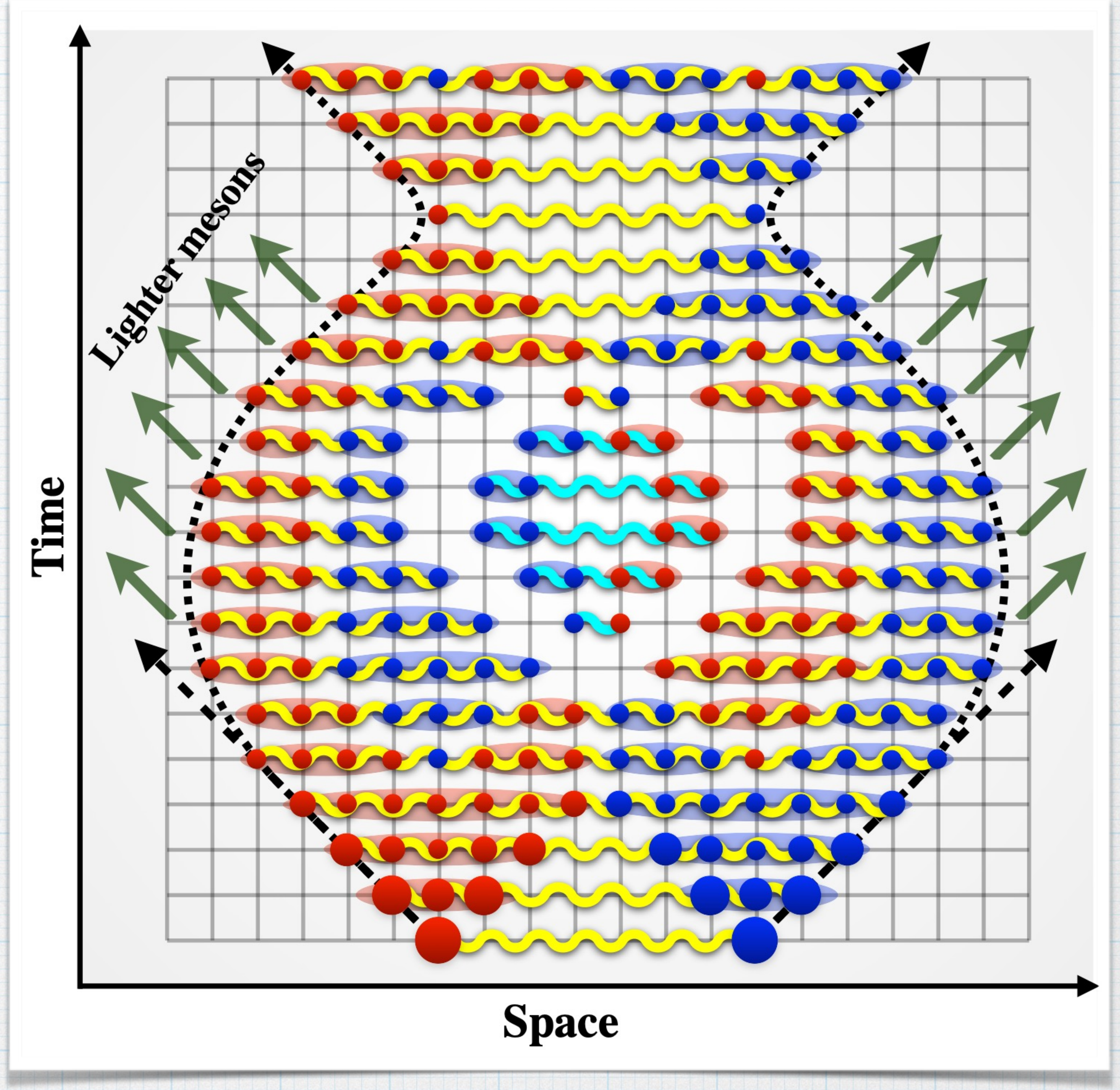
We excite the system out of equilibrium via the non-local operator…

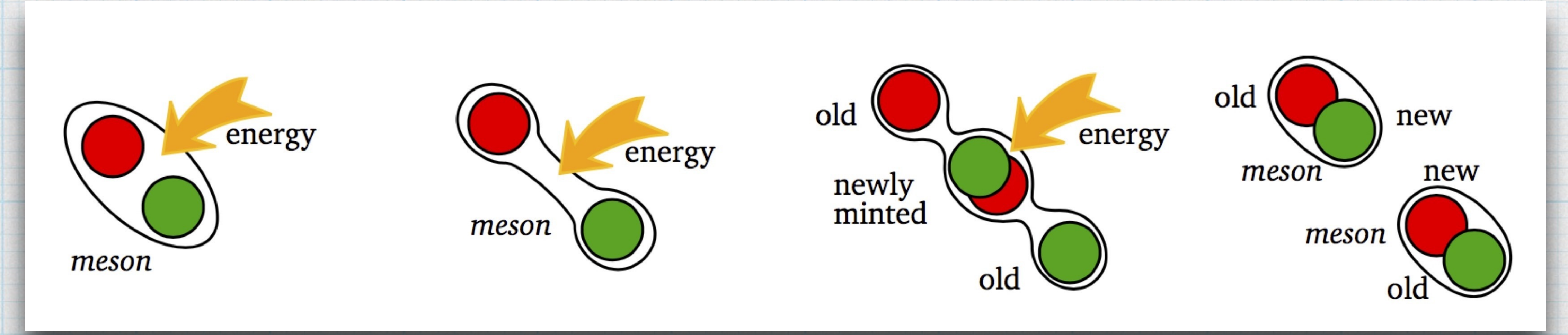
$$\hat{M}_R \equiv \hat{\phi}^{\dagger}_{\frac{N}{2}-R} \left[\prod_{j=\frac{N}{2}-R}^{\frac{N}{2}+R} \hat{U}^{\dagger}_j \right] \hat{\phi}_{\frac{N}{2}+R+1}$$

Creates unit opposite charges separated by a distance of $2R+1$ connected by a string of electric field… **i.e., an extended meson**

Initial state ➞ $|\psi(t=0)\rangle = \mathcal{N} \hat{M}_R |\Omega\rangle$

with extra energy ➞ $\approx (2R+1) + 4(x((m/q)^2 + 2x))^{1/2}$



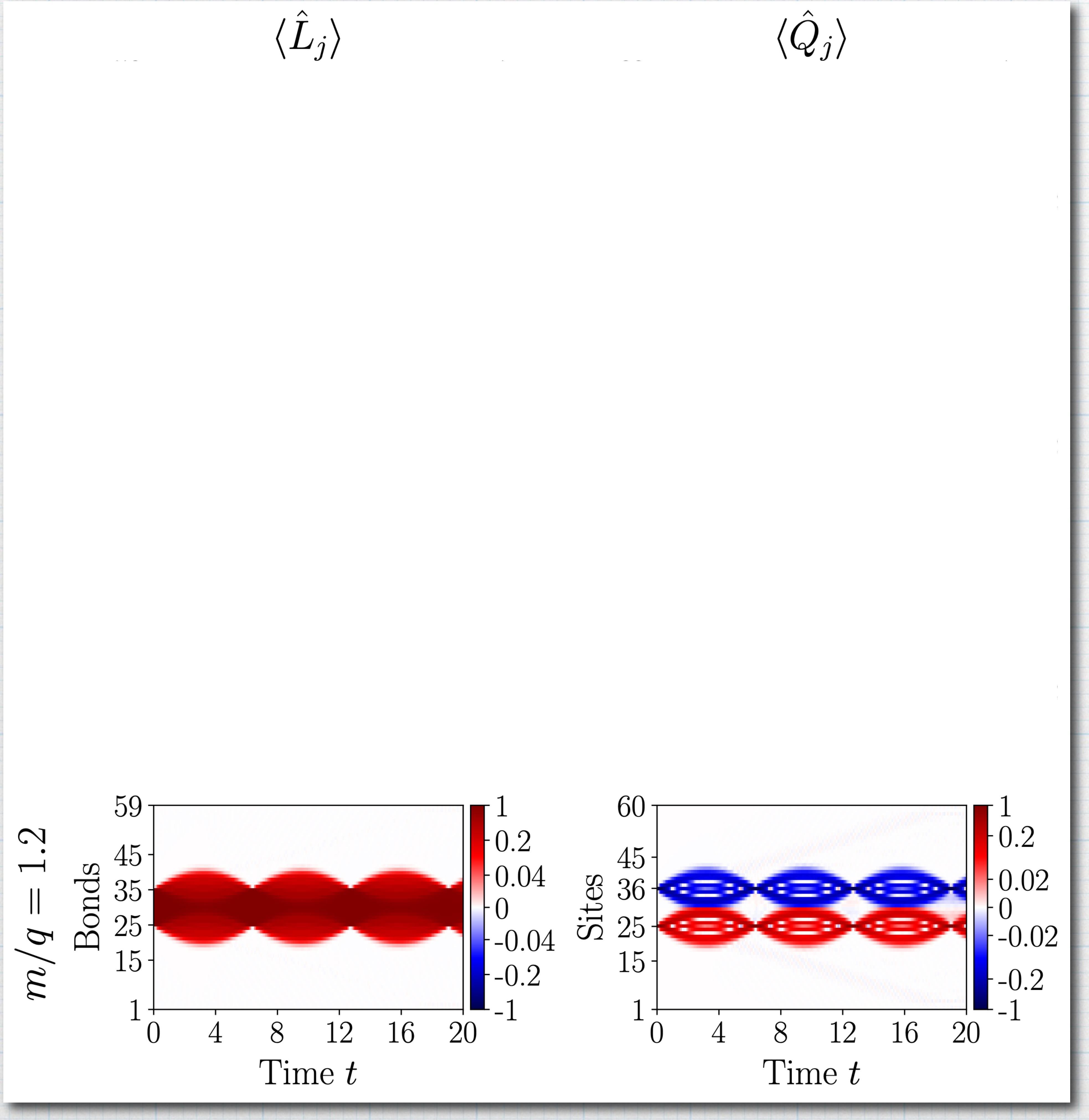


Bosonic Schwinger Model

Time evolution… ***N = 60 sites, N-1 = 59 bonds, R = 5***

Particle and gauge sector

Gauge sector — Matter sector



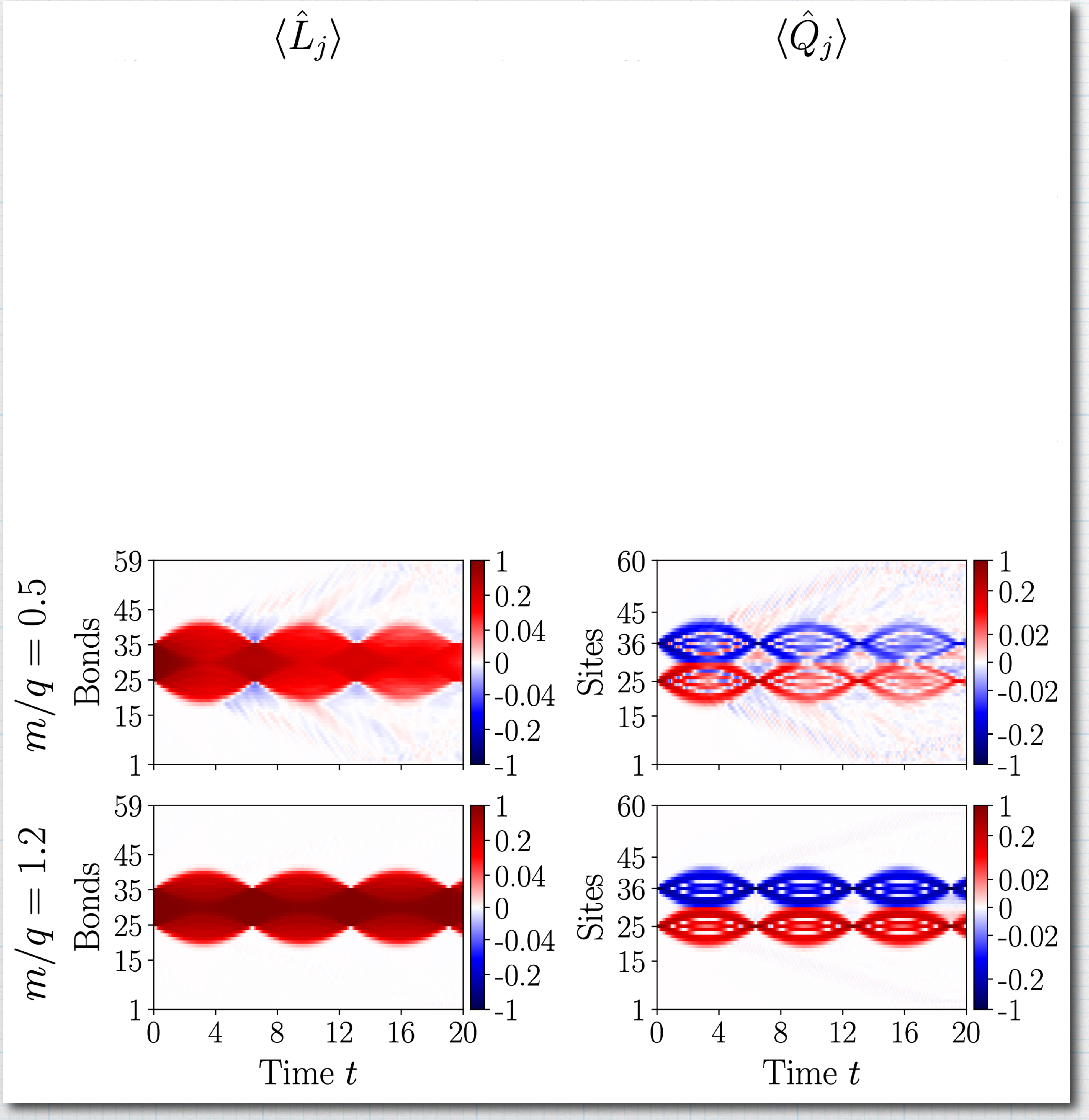
1. No ballistic spreading of the information/excitation
2. **Light-cone bends (signal of confinement)**
3. Periodic and coherent oscillations
4. No thermalization

Bosonic Schwinger Model

Time evolution… ***N = 60 sites, N-1 = 59 bonds, R = 5***

Particle and gauge sector

Gauge sector | Matter sector



1. String breaking from the boundary
2. Radiation of lighter mesons, propagates freely
3. **Two domains — confined core and deconfined outer region**

1. No ballistic spreading of the information/excitation
2. **Light-cone bends (signal of confinement)**
3. Periodic and coherent oscillations
4. No thermalization

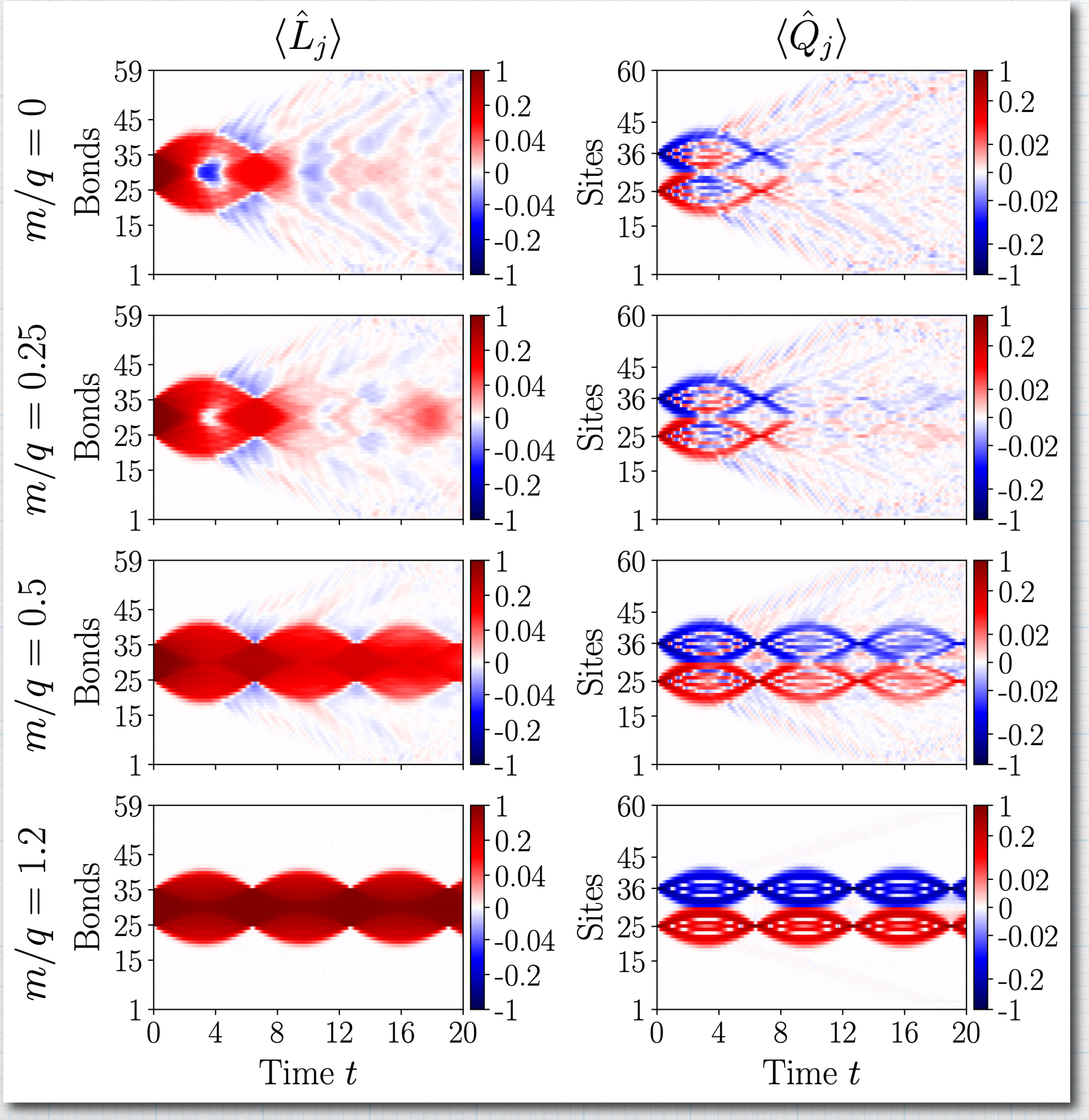
Bosonic Schwinger Model

Time evolution… *$N = 60$ sites, $N-1 = 59$ bonds, $R = 5$*

Particle and gauge sector

Gauge sector

Matter sector



1. String inversion in the bulk
2. Confined core disappears after one oscillation around $t \approx 10$

Concentration of bosons in the core gets depleted after few string-oscillations due to heavy meson radiation

1. String breaking from the boundary
2. Radiation of lighter mesons, propagates freely
3. **Two domains — confined core and deconfined outer region**

1. No ballistic spreading of the information/excitation
2. **Light-cone bends (signal of confinement)**
3. Periodic and coherent oscillations
4. No thermalization

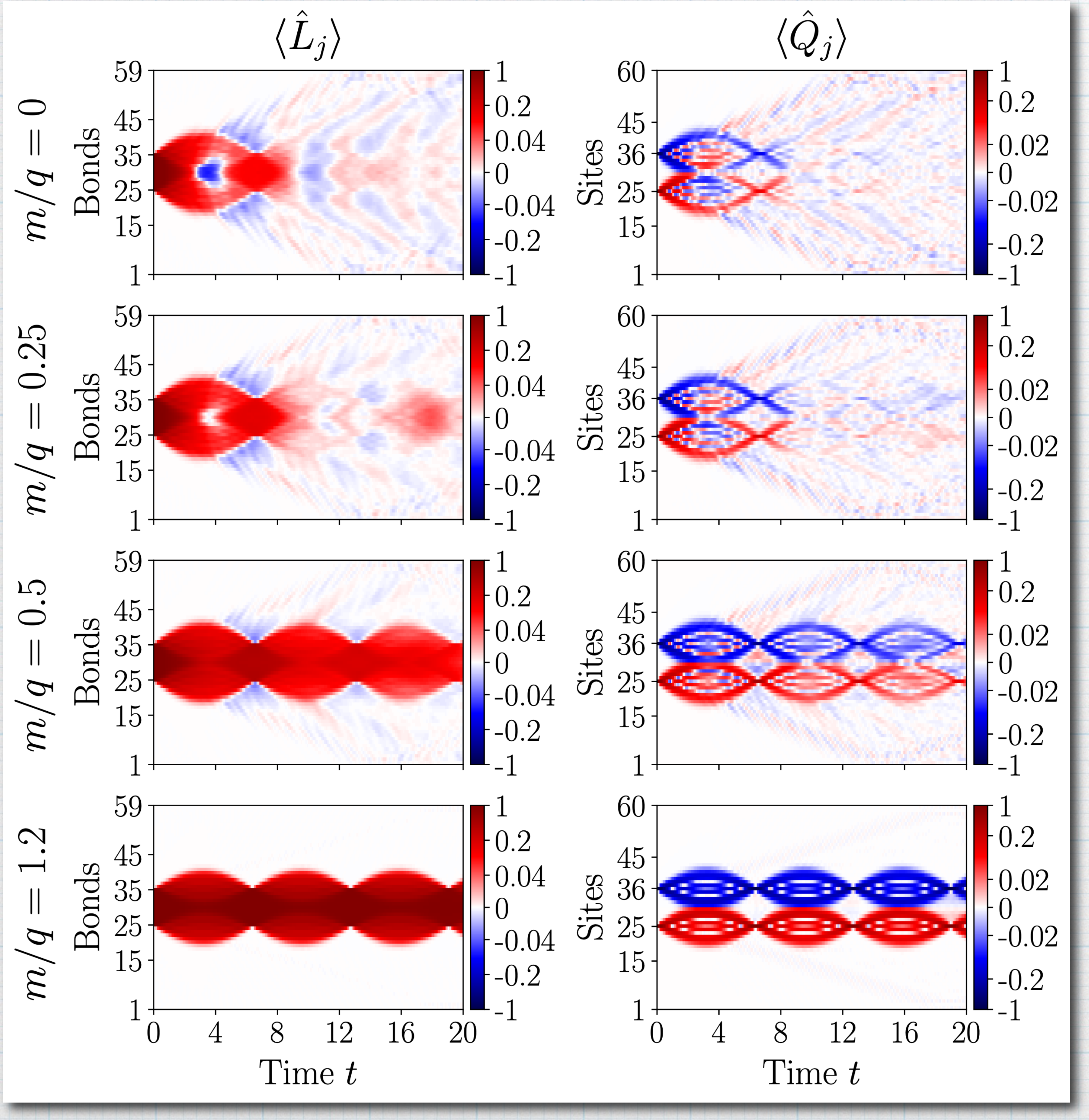
Bosonic Schwinger Model

Time evolution… ***N = 60 sites, N-1 = 59 bonds, R = 5***

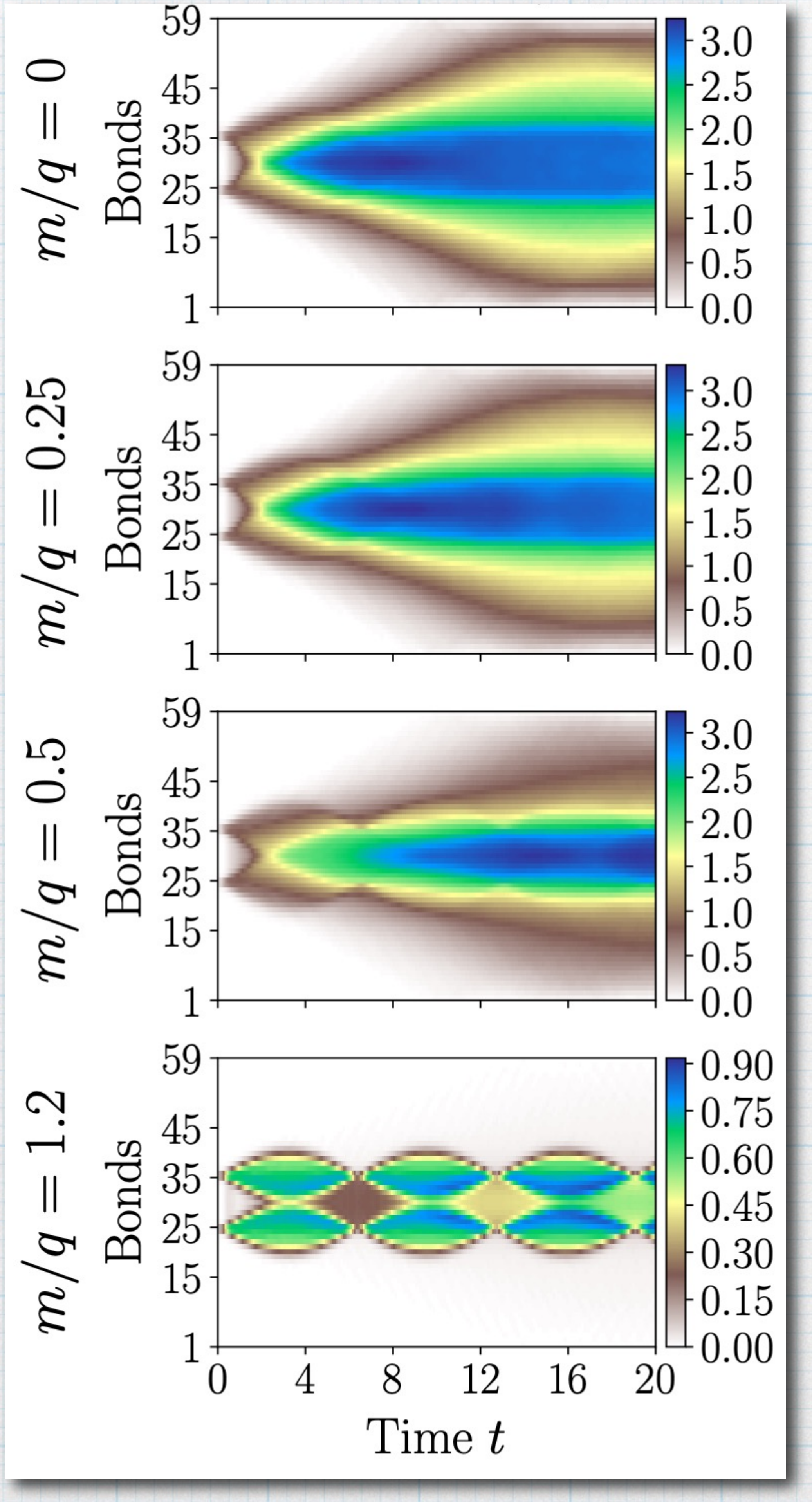
Entanglement entropy at the bond between the sites j and $j+1$…
$$\mathcal{S}_j(t) = -Tr[\rho_j(t)\ln\rho_j(t)]$$
with $\rho_j(t) = Tr_{j+1,j+2,\ldots,N}|\psi(t)\rangle\langle\psi(t)|$

Particle and gauge sector

Gauge sector Matter sector



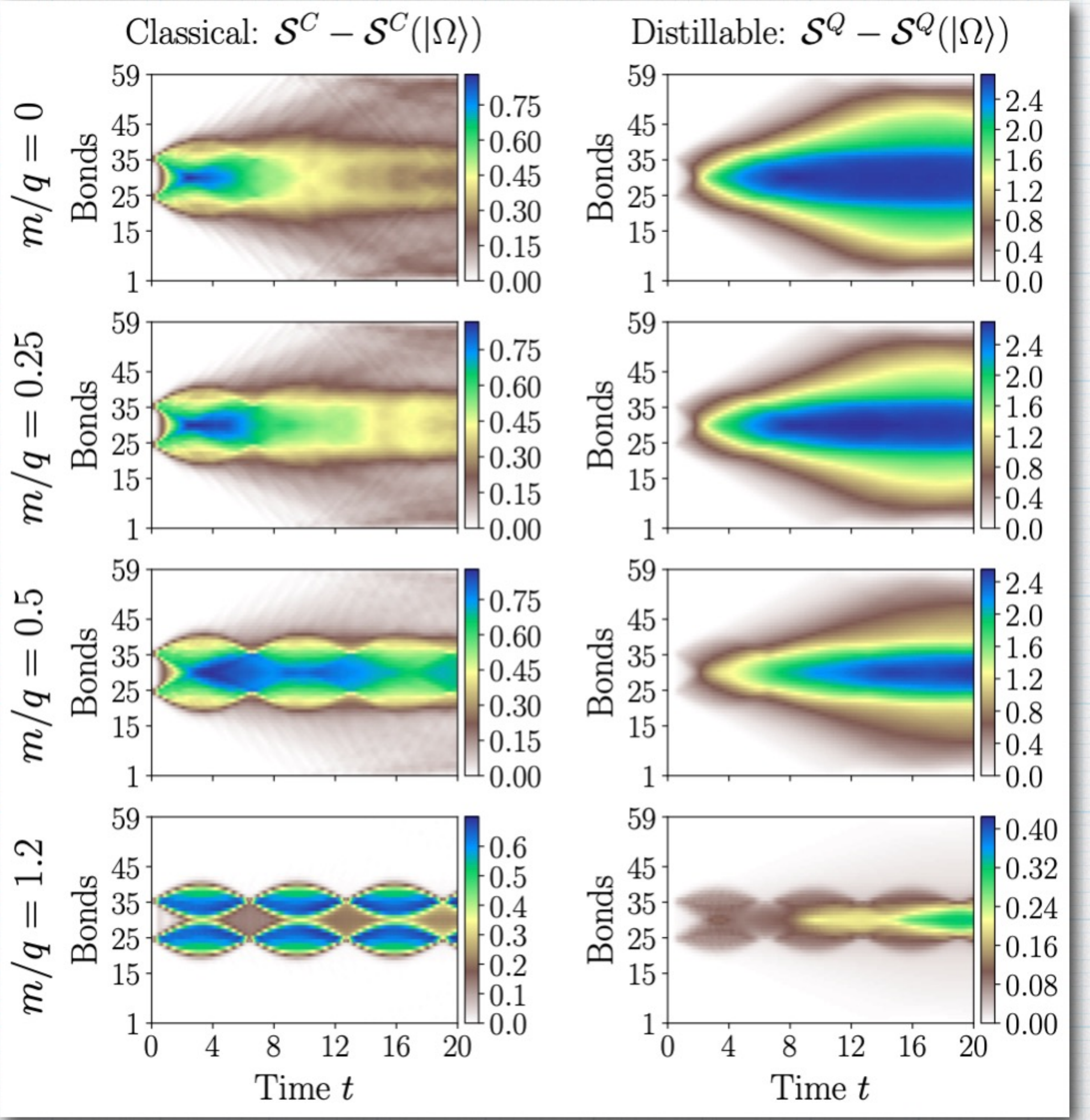
Entanglement entropy at each bond



1. Initial spreading of entanglement slows down.
2. Starts to spread ballistically in correspondence with the radiation of lighter mesons.
3. Entanglement stays concentrated in the confined core, even long after the accumulation of bosons disappears.
4. Strong memory effect.

Bosonic Schwinger Model

Time evolution… *N = 60 sites, N-1 = 59 bonds, R = 5*



Due to global *U(1)* symmetry…

$$\rho = \bigoplus_Q \tilde{\rho}_Q = \bigoplus_Q p_Q \, \rho_Q$$

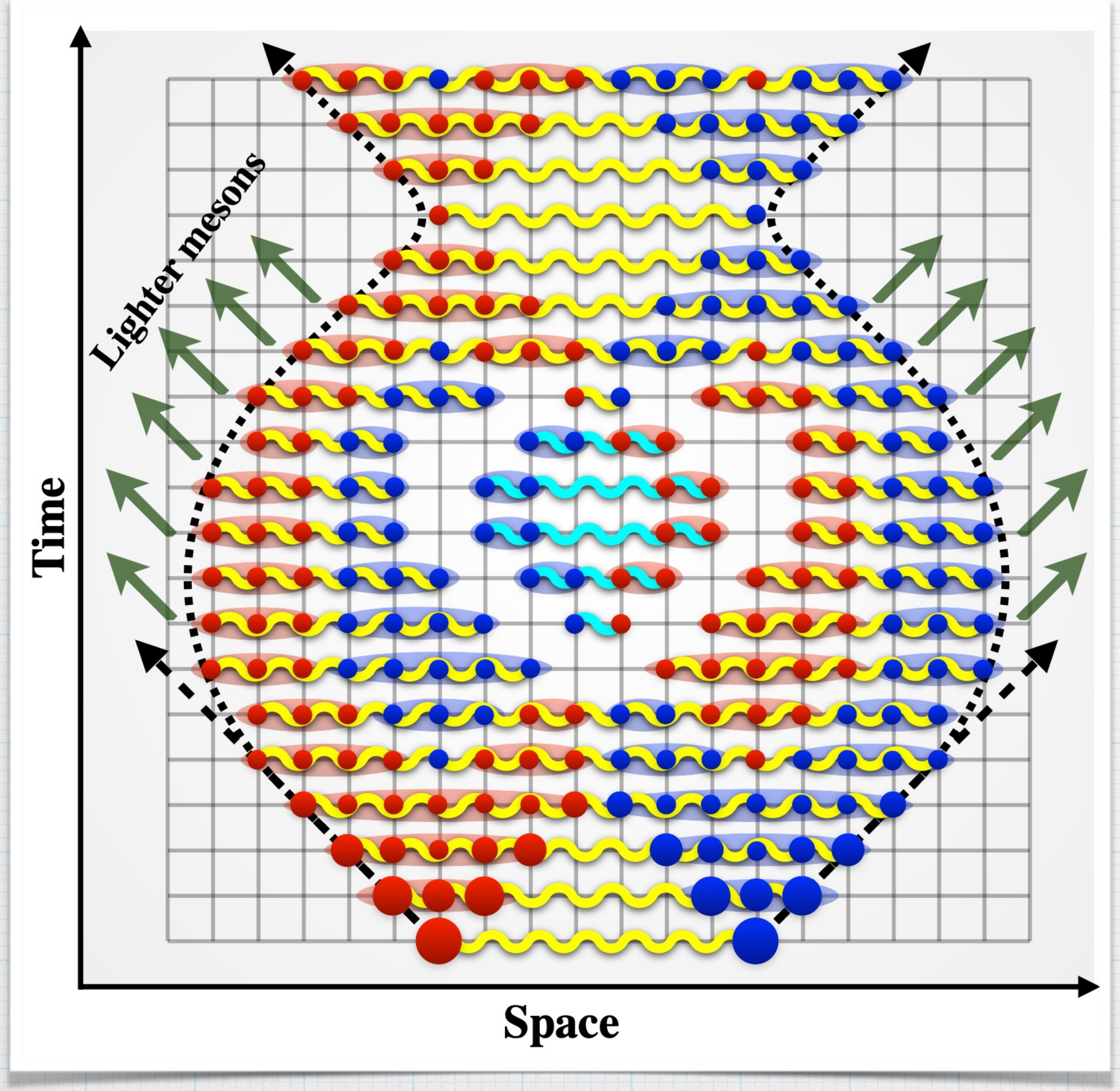
with $p_Q = \mathrm{Tr}\left[\tilde{\rho}_Q\right]$ and $\rho_Q = \tilde{\rho}_Q / p_Q$

$$\mathcal{S}(\rho) = \underbrace{-\sum_Q p_Q \ln p_Q}_{\mathcal{S}^C \text{ (classical)}} + \underbrace{\sum_Q p_Q \mathcal{S}(\rho_Q)}_{\mathcal{S}^Q \text{ (distillable)}}$$

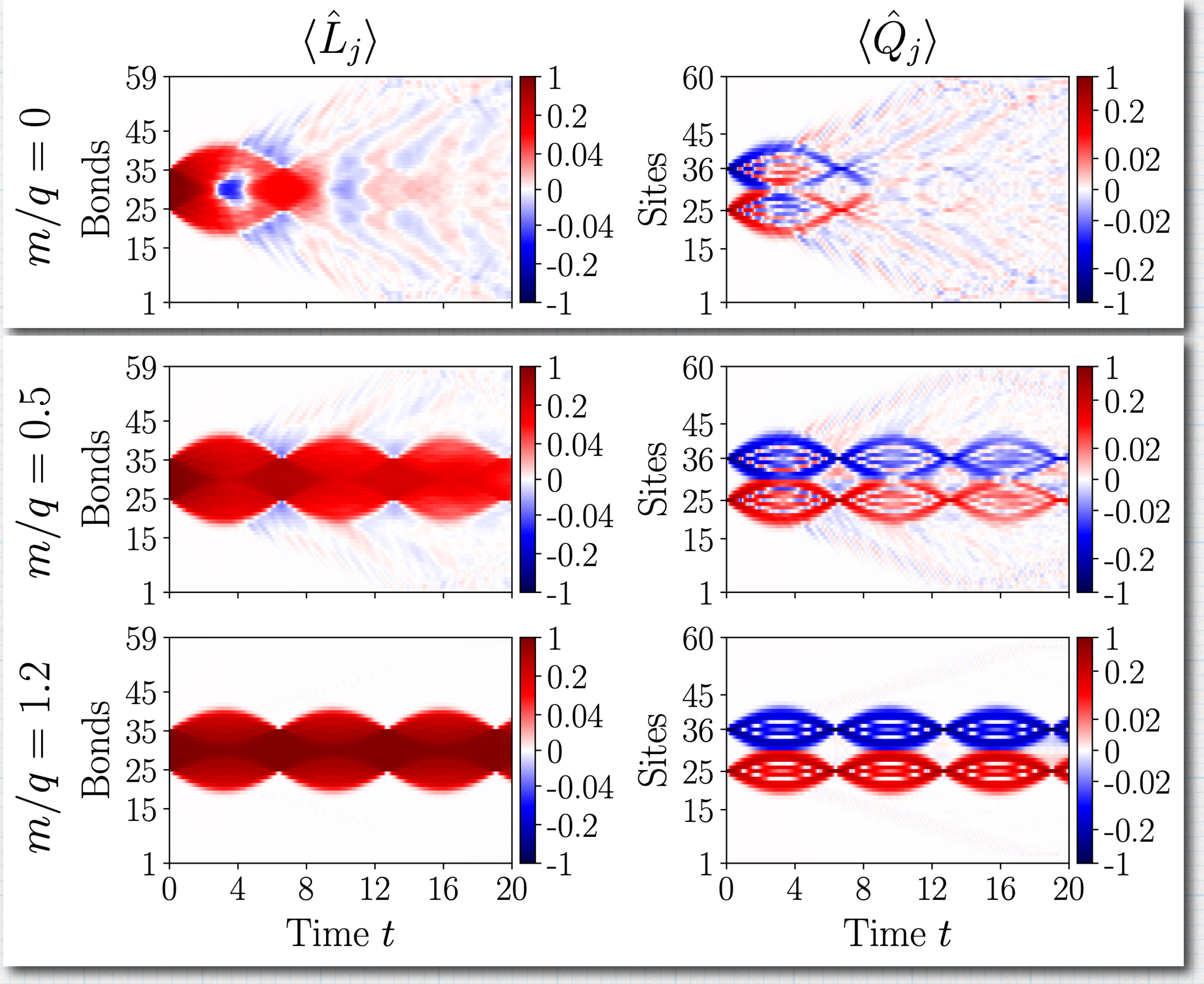
The classical part of the entropy remains *sharply* confined to the confined core, thereby demarcating confined domain from the deconfined one.

Bosonic Schwinger Model

Time evolution…



Particle and gauge sector



1. Light-cone bends.

2. Coherent oscillation of the string.

3. Partial string breaking.

4. String inversion.

5. Radiation of lighter mesons.

6. Two domains — confined core and deconfined outer region.

7. Slow depletion of coherent core.

Bosonic Schwinger Model

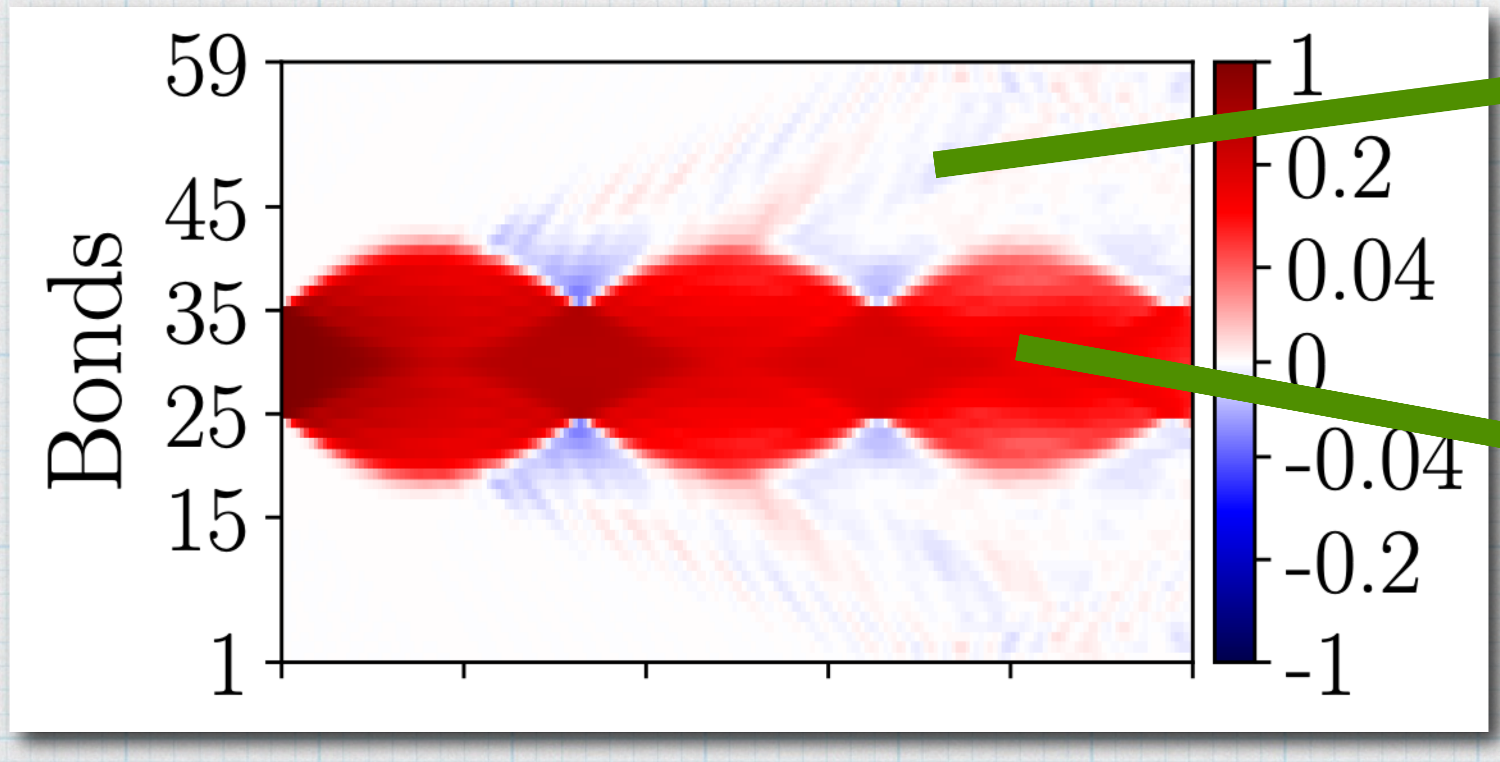
Lack of thermalization…

Thermalization

$\langle \hat{O}(\psi(t))\rangle \rightarrow \overline{O}_{microcann.}$ as $t \rightarrow \infty$… Described by only one parameter (T)… no memory

$\mathcal{S}(t)$ should grow proportional to the bipartition size for sufficiently long t

Expectation…



Deconfined domain.
Populated by freely propagating lighter mesons.
Should 'thermalize'.
Should show volume-law of entropy.

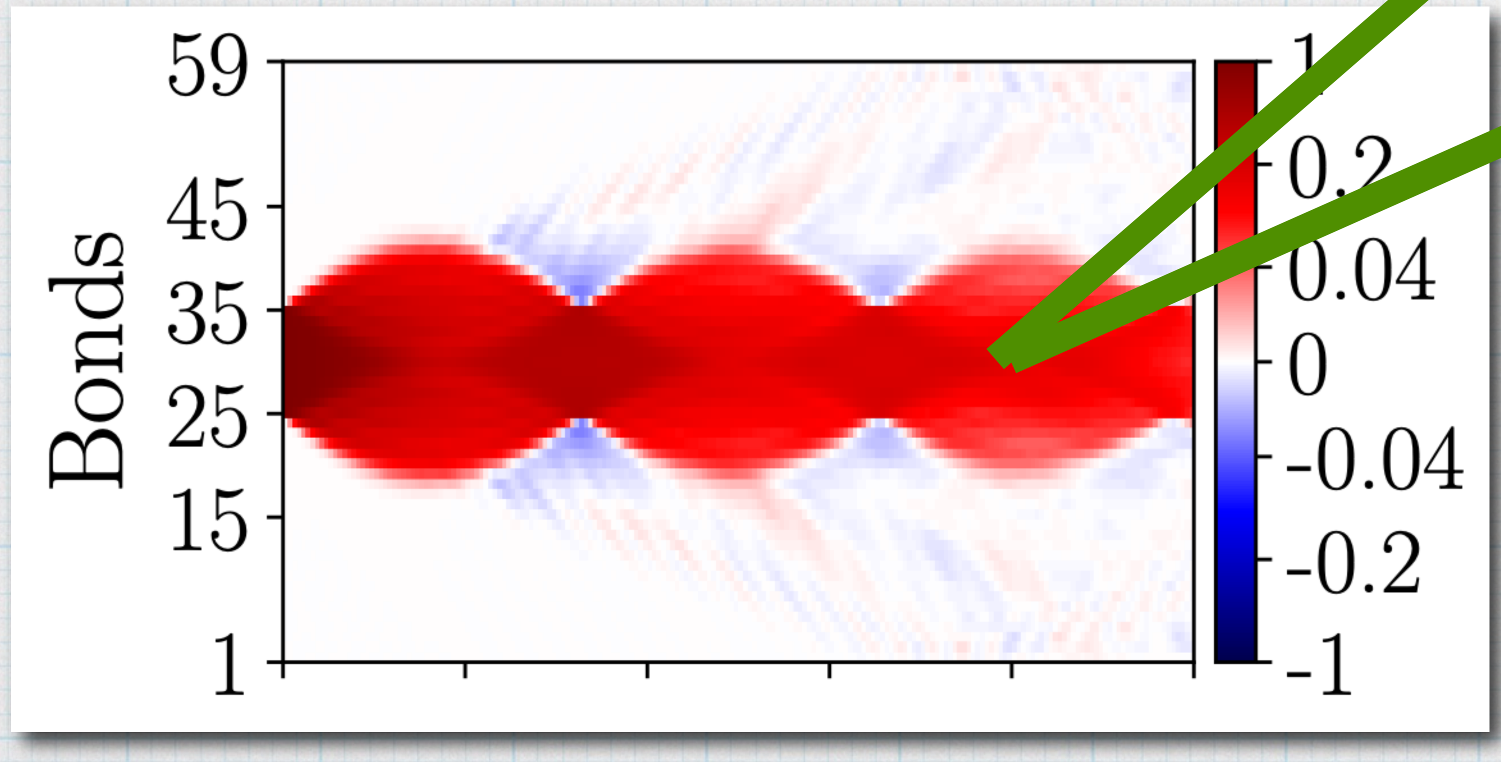
Confined domain.
Coherent oscillations.
Memory effect.
Should remain **non**-thermal.
Entropy should **not** grow proportional to the bipartition size, but slower.

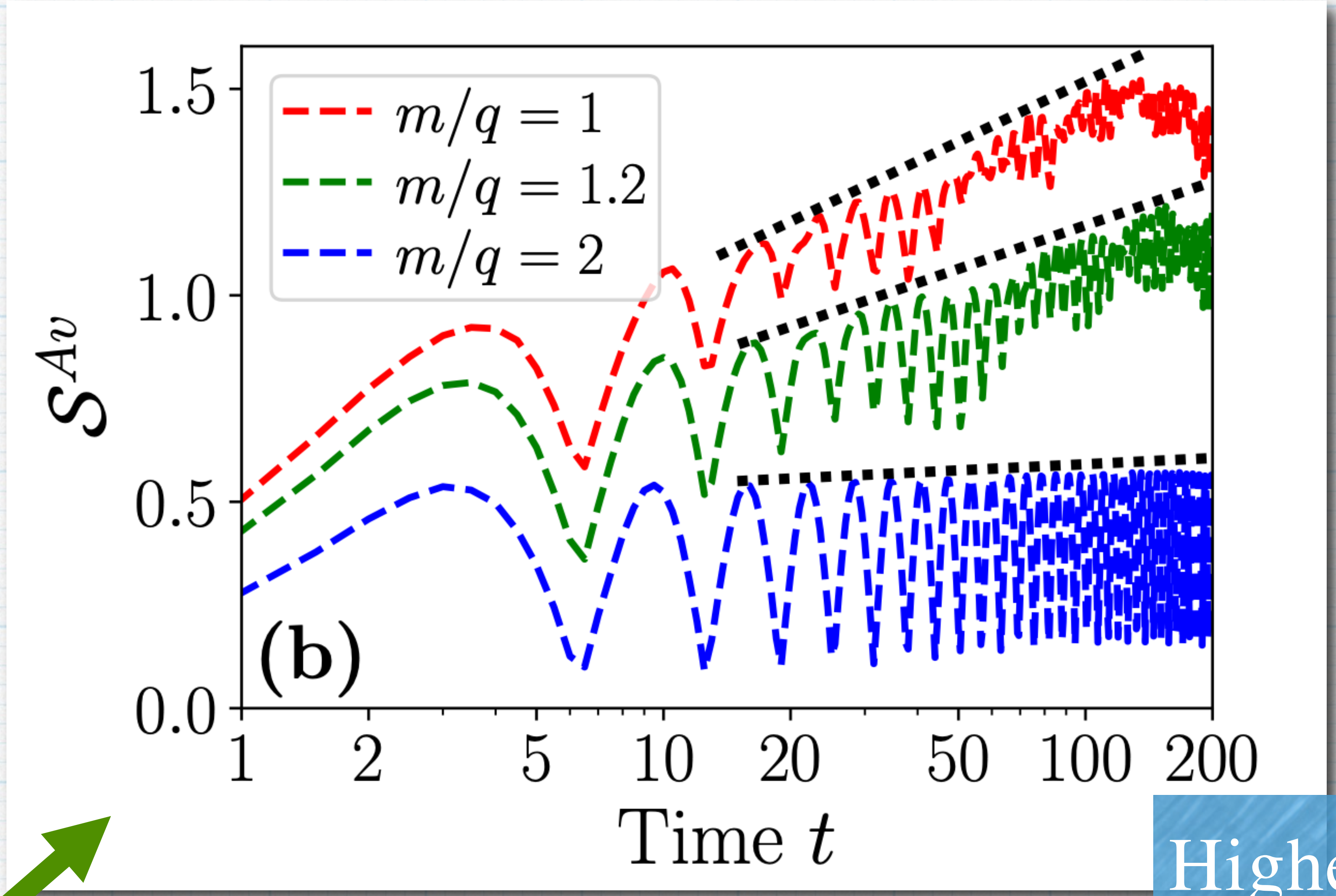
Bosonic Schwinger Model

Lack of thermalization…

N = 60, 80, 100 sites, with R = N/10
Extensive energy in the initial state: required for thermalization

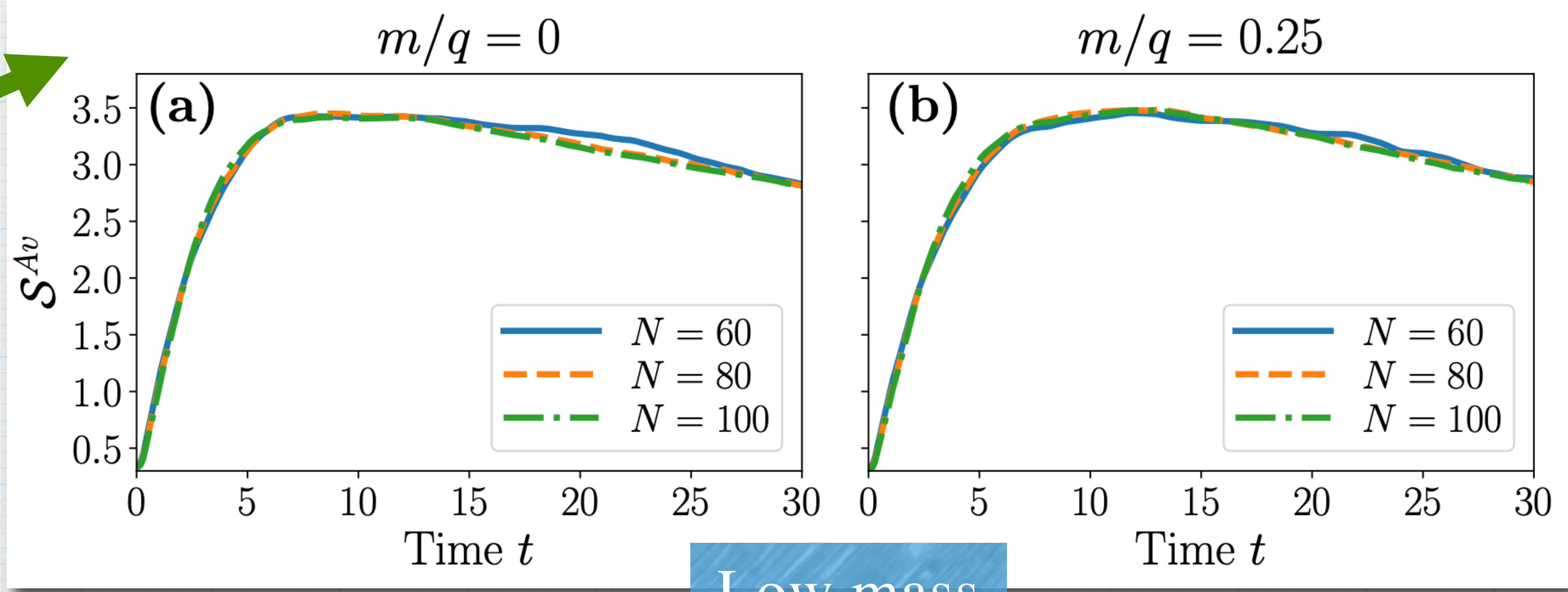
$$\mathcal{S}^{Av} = \frac{1}{2R+1}\sum_{j=N/2-R}^{N/2+R} \mathcal{S}_j$$





Higher mass

Logarithmic growth of entropy
Ala many-body localization
Lack of thermalization…



Low mass

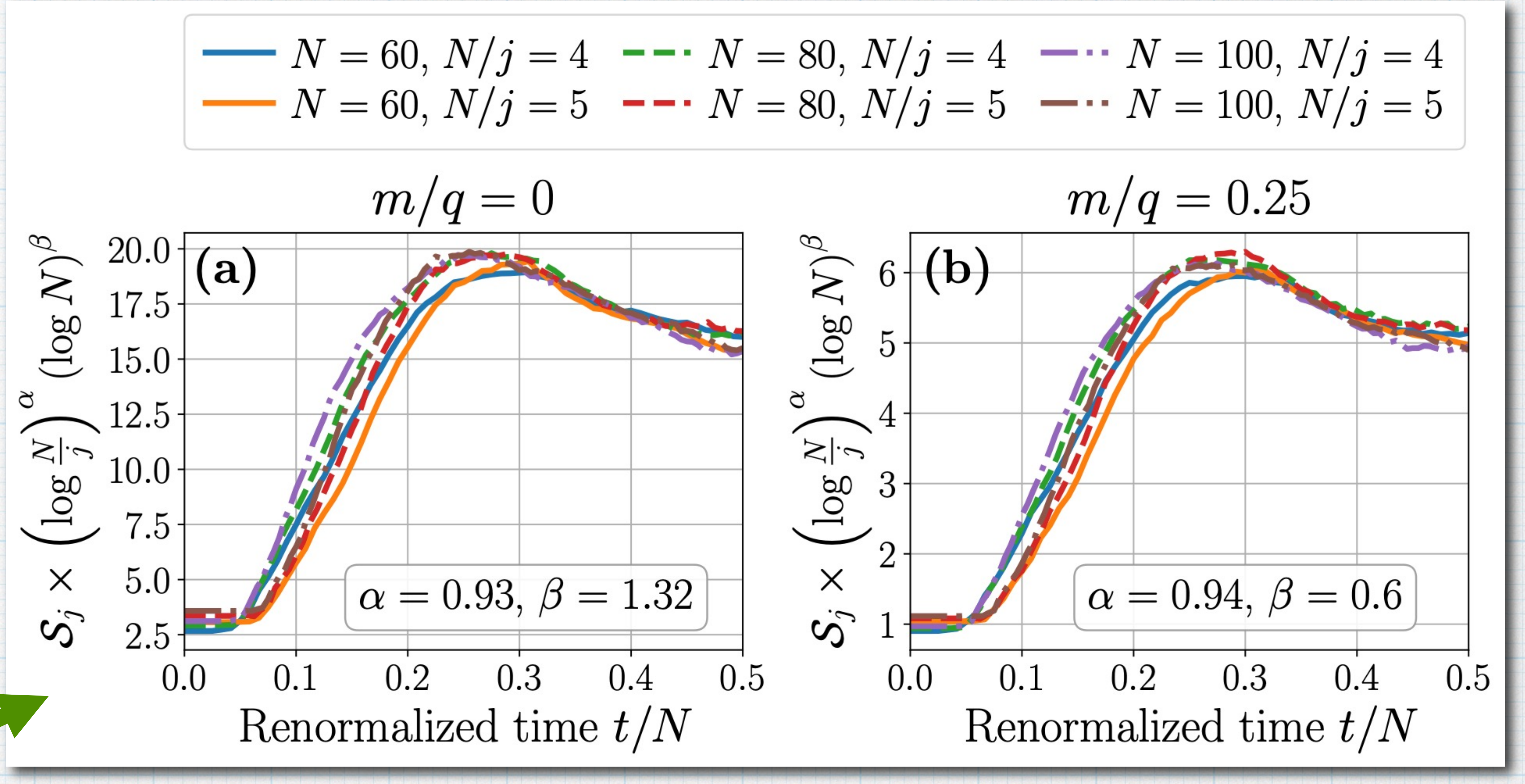
Perfect area-law of entropy
Lack of thermalization…

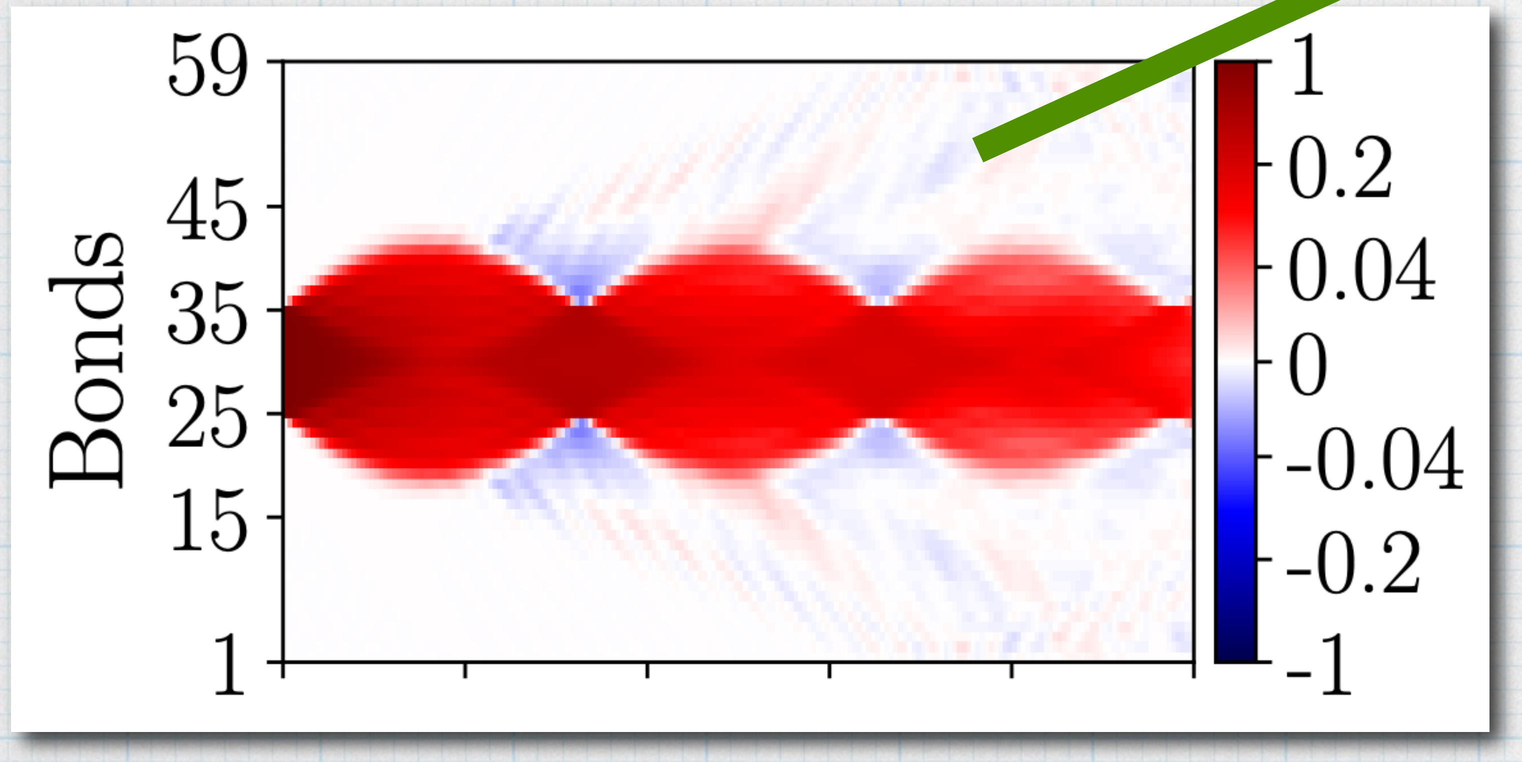
Non-thermal

Bosonic Schwinger Model

Lack of thermalization…

N = 60, 80, 100 sites, with R = N/10
Extensive energy in the initial state: required for thermalization





$$S_j \propto \left(\log \frac{N}{j}\right)^{-\alpha} (\log N)^{-\beta}\ with\alpha \approx 1$$

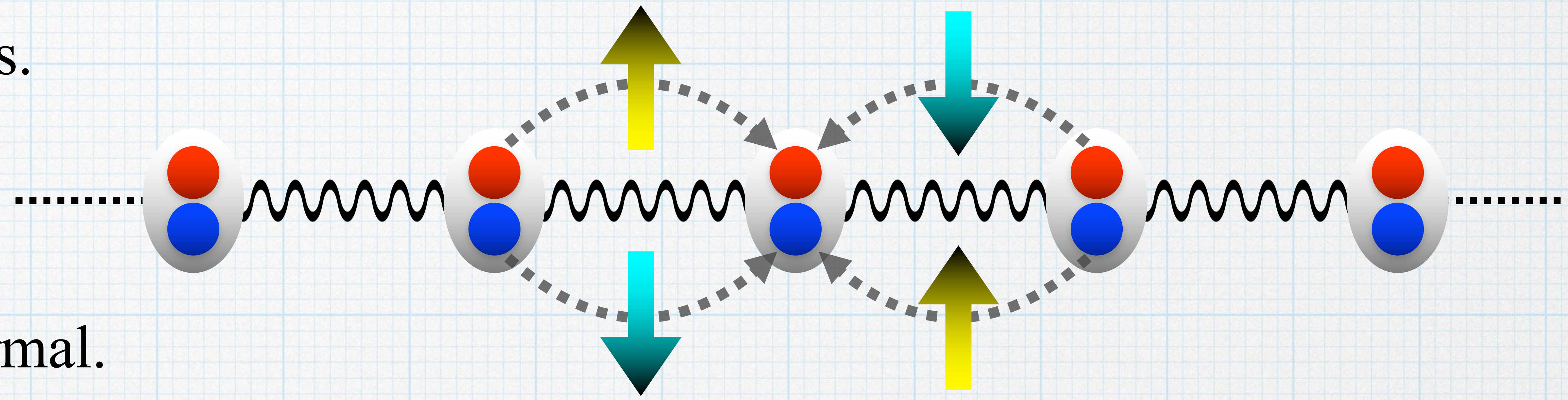
For fixed N:

1. Sub-linear in j for small j.
2. Linear for intermediate j: **volume-law**.
3. Super-linear before saturating into the confined domain.

Deconfined domain behaves like a thermal state

Key Points

Bosonic Schwinger Model



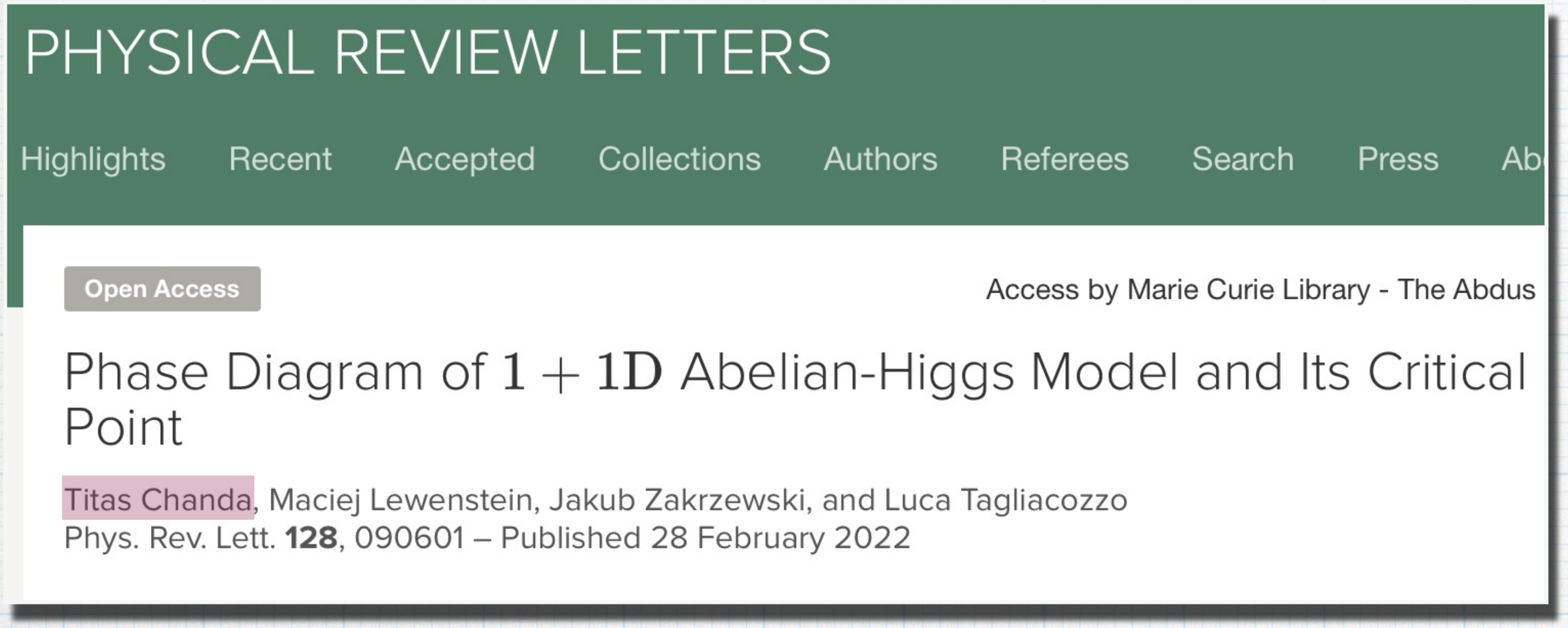
1. Bosonic Schwinger model shows strong confining dynamics.

2. Trajectories of the bosons bends inwards.

3. As a result, asymptotic states are exotic and highly non-thermal.

4. These states are made of —
   i. Strongly correlated confined core that obeys area-law of entropy.
   ii. Almost thermal outer region (for lower masses) or vacuum (higher masses).

Abelian-Higgs Model




Titas Chanda

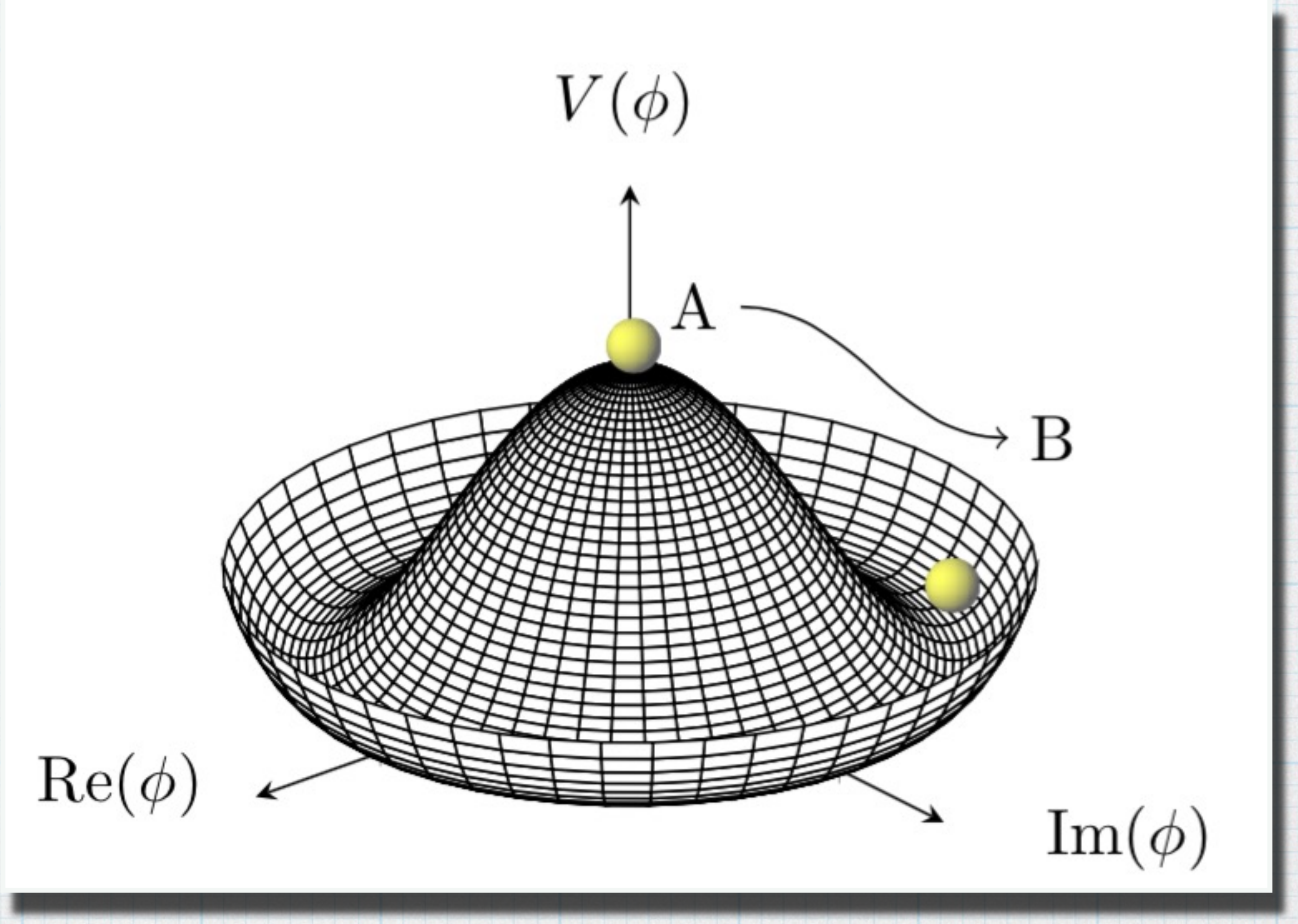

Maciej Lewenstein


Luca Tagliacozzo

Lagrangian…. from… $\mathcal{L} = -[D_\mu \phi]^* D^\mu \phi - m^2|\phi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

Now… $\mathcal{L} = -[D_\mu \phi]^* D^\mu \phi + \mu^2|\phi|^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}|\phi|^4$

the potential term … $V(\phi) = -\mu^2|\phi|^2 + \frac{\lambda}{2}|\phi|^4$



In 3+1 dimensions… Spontaneous symmetry-breaking triggers Higgs mechanism… Gauge fields become massive

What about 1+1 dimensions…?

Abelian-Higgs Model

$$\mathcal{L} = -[D_\mu \phi]^* D^\mu \phi + \mu^2 |\phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{2} |\phi|^4$$

What about 1+1 dimensions…?

No Higgs phase in the continuum theory… only confined phase…

On lattice ??

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covering particles, fields, gravitation, and cosmology

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Phase diagrams of lattice gauge theories with Higgs fields

Eduardo Fradkin and Stephen H. Shenker

Phys. Rev. D **19**, 3682 – Published 15 June 1979

Hamiltonian after discretization…

$$\hat{H} = \sum_j [\hat{L}_j^2 + 2x\hat{\Pi}_j^\dagger \hat{\Pi}_j + (4x - \frac{2\mu^2}{q^2})\hat{\phi}_j^\dagger \hat{\phi}_j + \frac{\lambda}{q^2}(\hat{\phi}_j^\dagger)^2 \hat{\phi}_j^2 - 2x(\hat{\phi}_{j+1}^\dagger \hat{U}_j \hat{\phi}_j + h.c.)]$$

Abelian-Higgs Model

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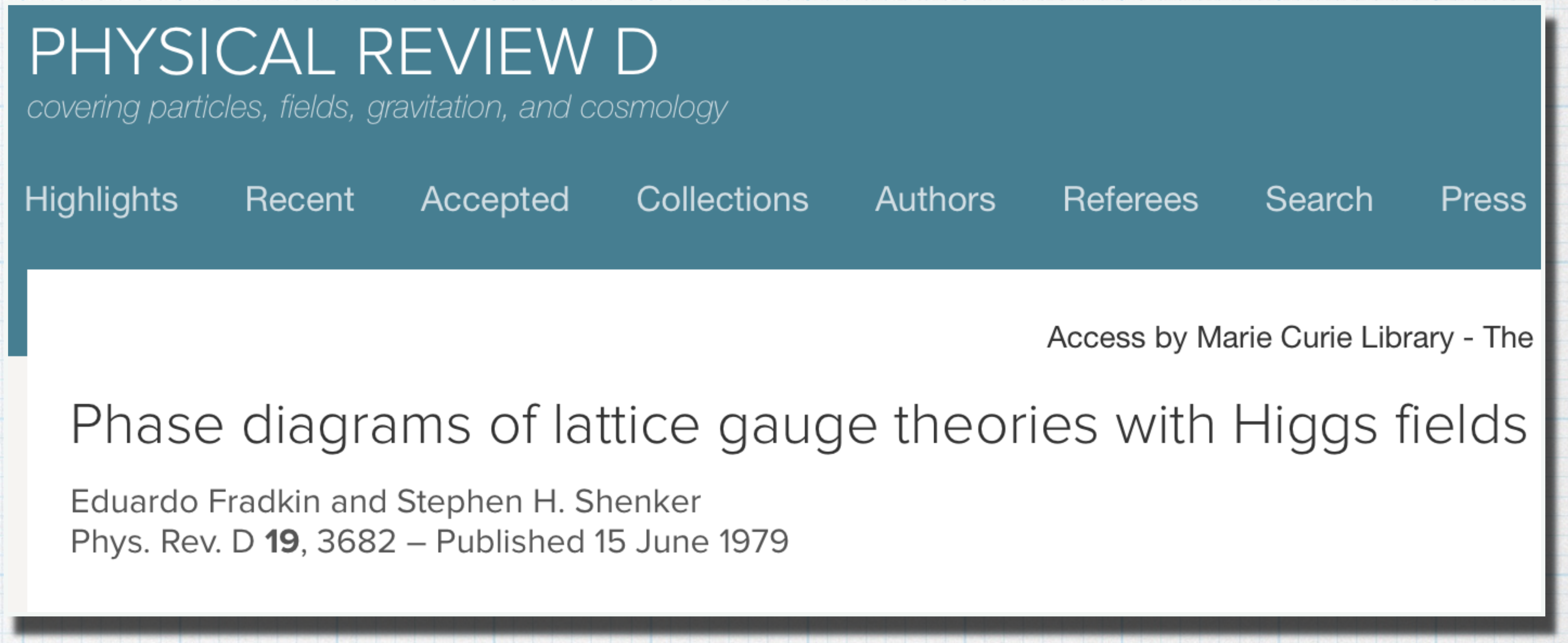
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Phase diagrams of lattice gauge theories with Higgs fields
Eduardo Fradkin and Stephen H. Shenker
Phys. Rev. D **19**, 3682 – Published 15 June 1979

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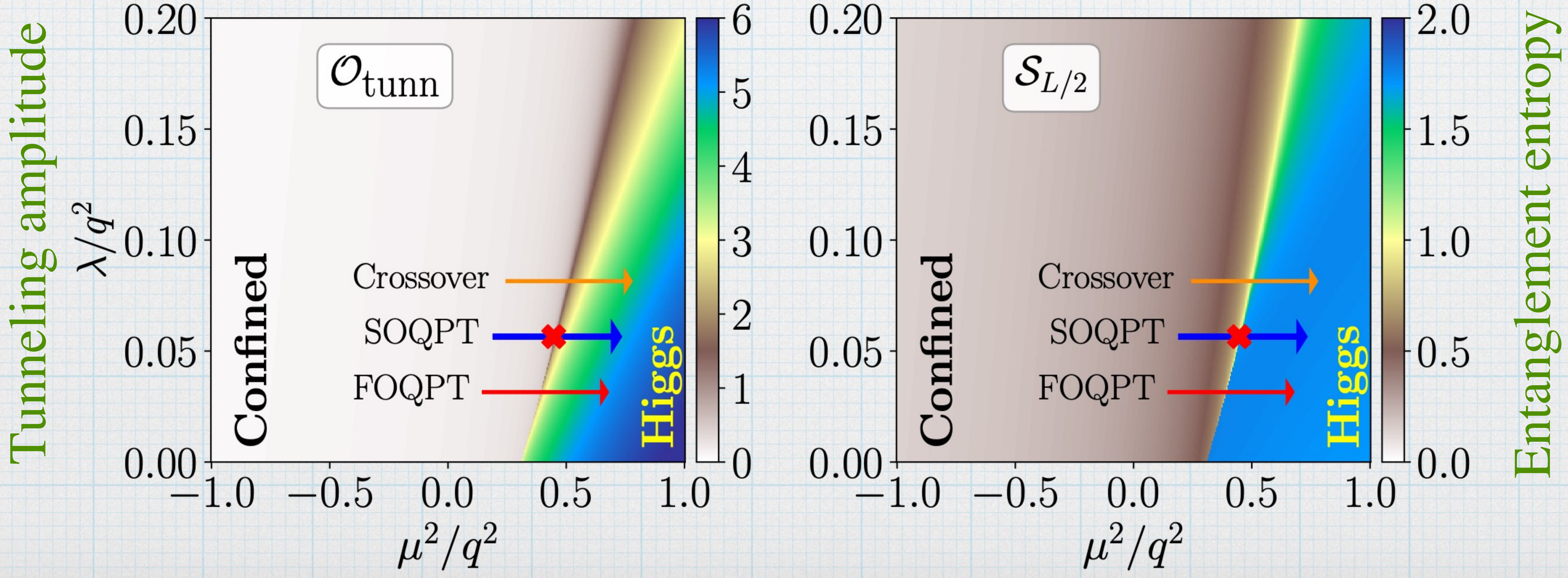
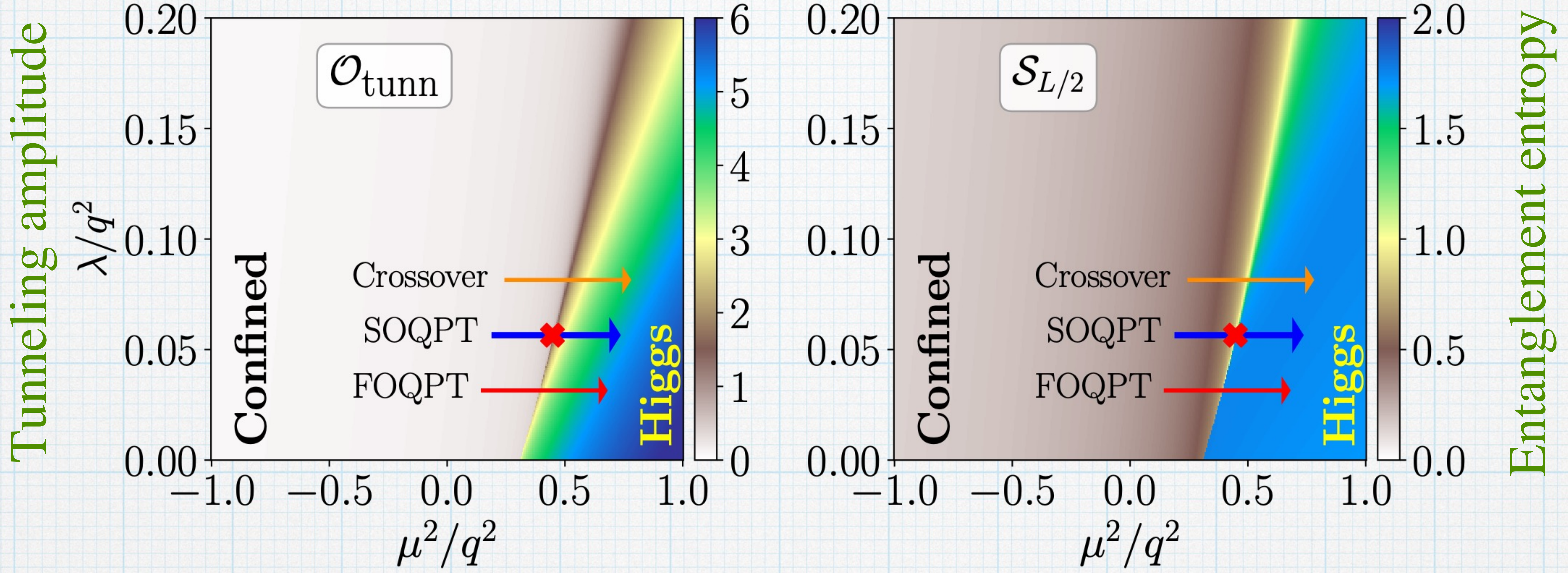


A new phase appears
Higgs phase?

Abelian-Higgs Model



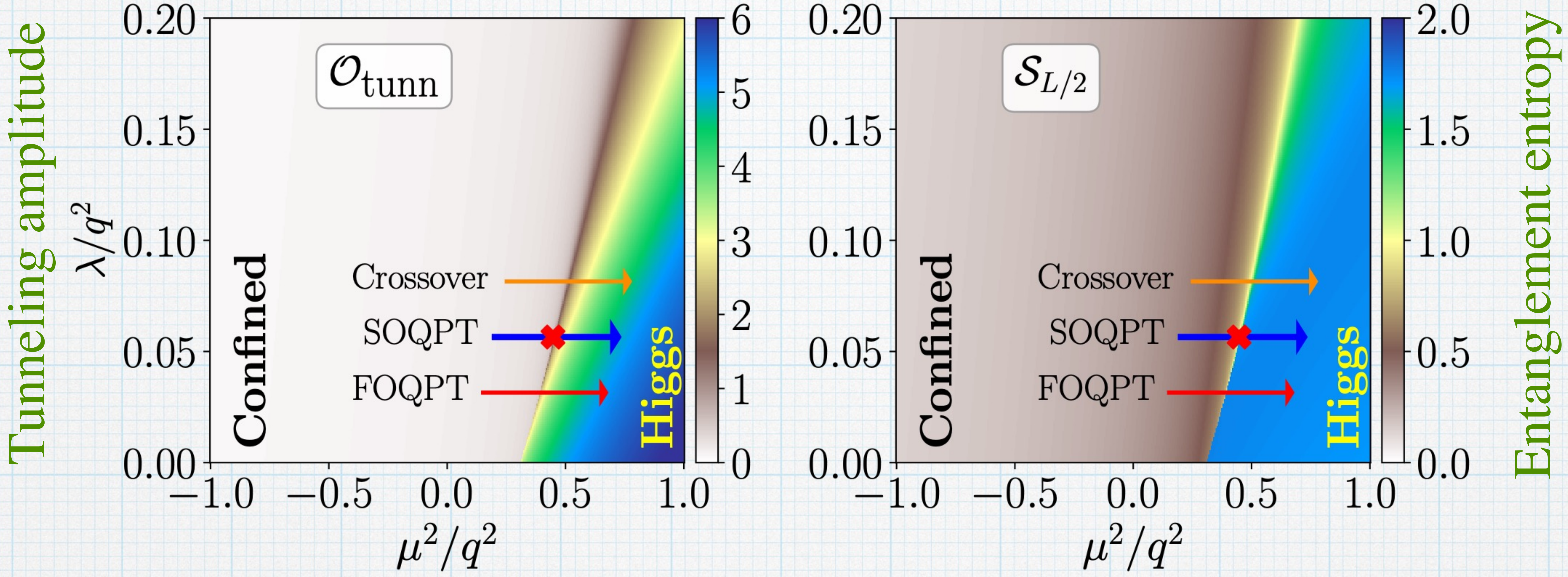
A new phase appears
Higgs phase?

Only on lattice with finite lattice spacing a

For smaller and smaller a, the Higgs phase shifts towards the right

For $a \to 0$, the Higgs phase disappears

Abelian-Higgs Model



A new phase appears
Higgs phase?

Why Higgs phase??

1. In the confined phase, $var(\hat{L}) \approx 0$. In the Higgs phase, $var(\hat{L})$ is large.

2. Tunneling amplitude $\mathcal{O}_{tunn}$ is ≈ 0 in the confined phase, while in the Higgs phase $\mathcal{O}_{tunn}$ is large as confinement disappears.

3. Entanglement entropy is also large in the Higgs phase.

1. For smaller λ/q^2, two phases are separated by first order quantum phase transition (FOQPT)

2. FOQPT line ends at a critical second order quantum phase transition (SOQPT) point

3. Beyond SOQPT point, two phases are smoothly connected by a crossover

Abelian-Higgs Model

Newly discovered critical point is a special one…

We characterize it using the machineries of conformal field theory (CFT)
Scale invariant critical systems in 1+1D are described by CFT…

Scaling of entanglement entropy: $\mathcal{S}(l, L) = \frac{c}{6} W(l, L) + b'$

l →Bipartition size

L →System size

$W(l, L) = \log\left[\frac{2L}{\pi}\sin(\pi l/L)\right]$, the cord length

c →The central charge of the CFT

Journal of Statistical Mechanics: Theory and Experiment

Entanglement entropy and quantum field theory

Pasquale Calabrese[1] and John Cardy[1,2]

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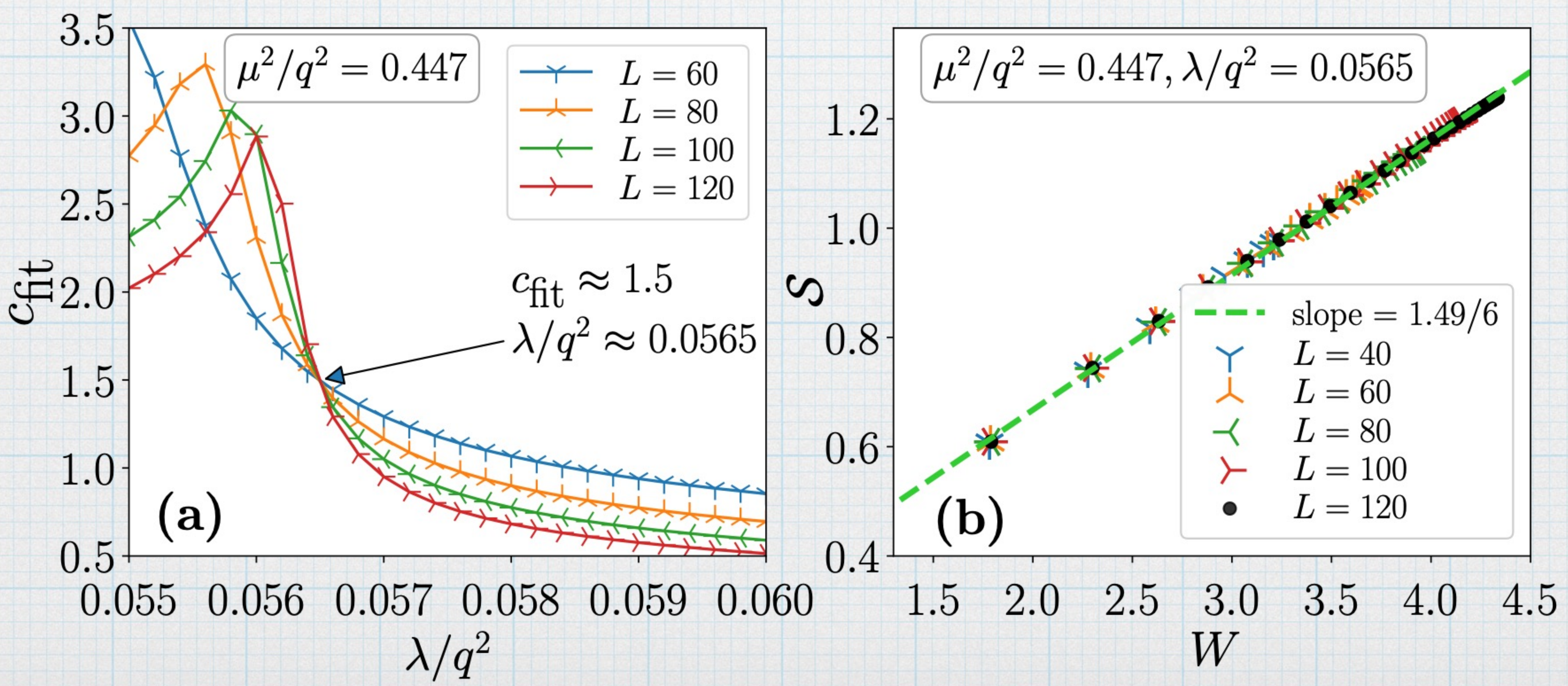
Citation Pasquale Calabrese and John Cardy *J. Stat. Mech.* (2004) P06002



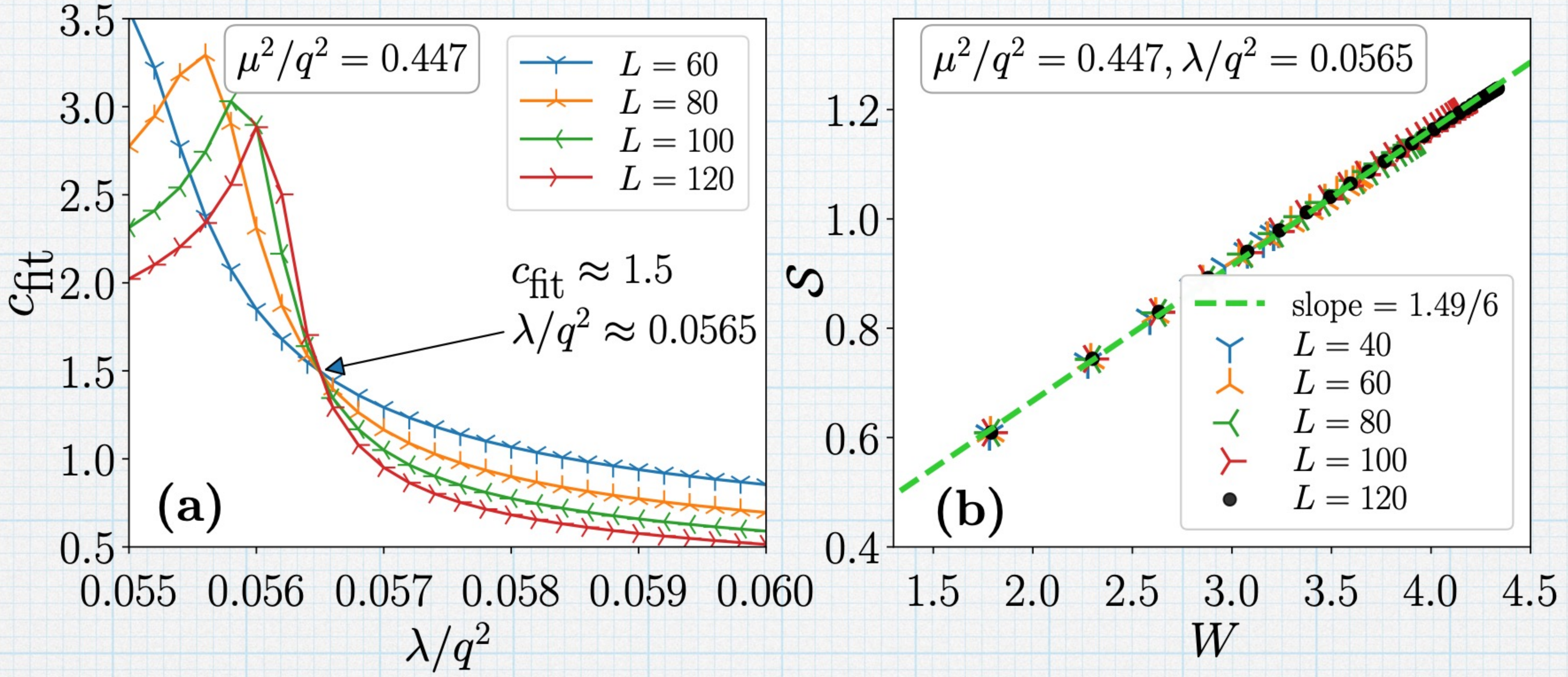
$c = 3/2$ critical point

Abelian-Higgs Model

Newly discovered critical point is a special one…

$c = 3/2$ critical point

Smoking gun for Higgs mechanism



Our picture…

1. $c = 1/2 + 1$. The Ising criticality gives $c_f = 1/2$. $c_b = 1$ comes from free bosons.
2. Due to Higgs mechanism, the complex Higgs field separates into amplitude and phase.
3. The amplitude part → real ϕ^4 theory → Ising transition in 1+1D ($c_f = 1/2$ part).
4. The phase is absorbed by the gauge bosons → massless at the critical point ($c_b = 1$ part).

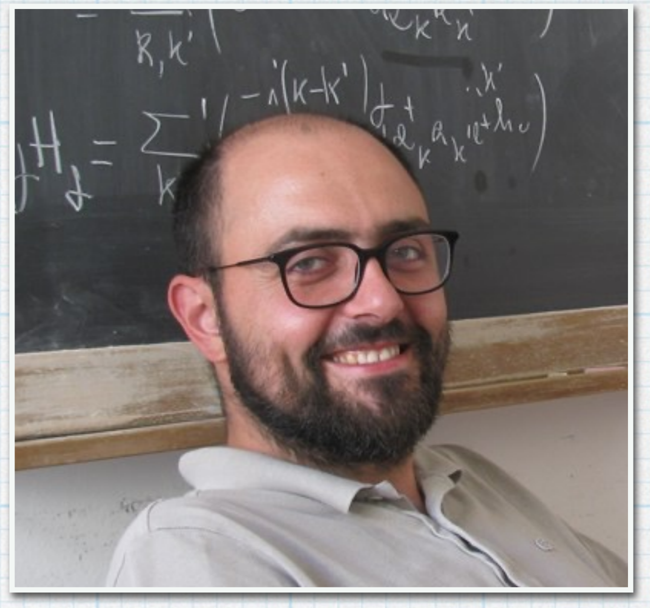
Abelian-Higgs Model

How to verify this and detect these gapless modes?


Titas Chanda


Maciej Lewenstein


Luca Tagliacozzo


Marcello Dalmonte

What is the Luttinger parameter K for the bosonic part?

Local Fluctuations: $\mathcal{F}(l, L) = \langle (\sum_{j\le l} Q_j)^2 \rangle - \langle \sum_{j\le l} Q_j \rangle^2 = \langle \hat{L}_l^2 \rangle - \langle \hat{L}_l \rangle^2$

$K \approx 2$

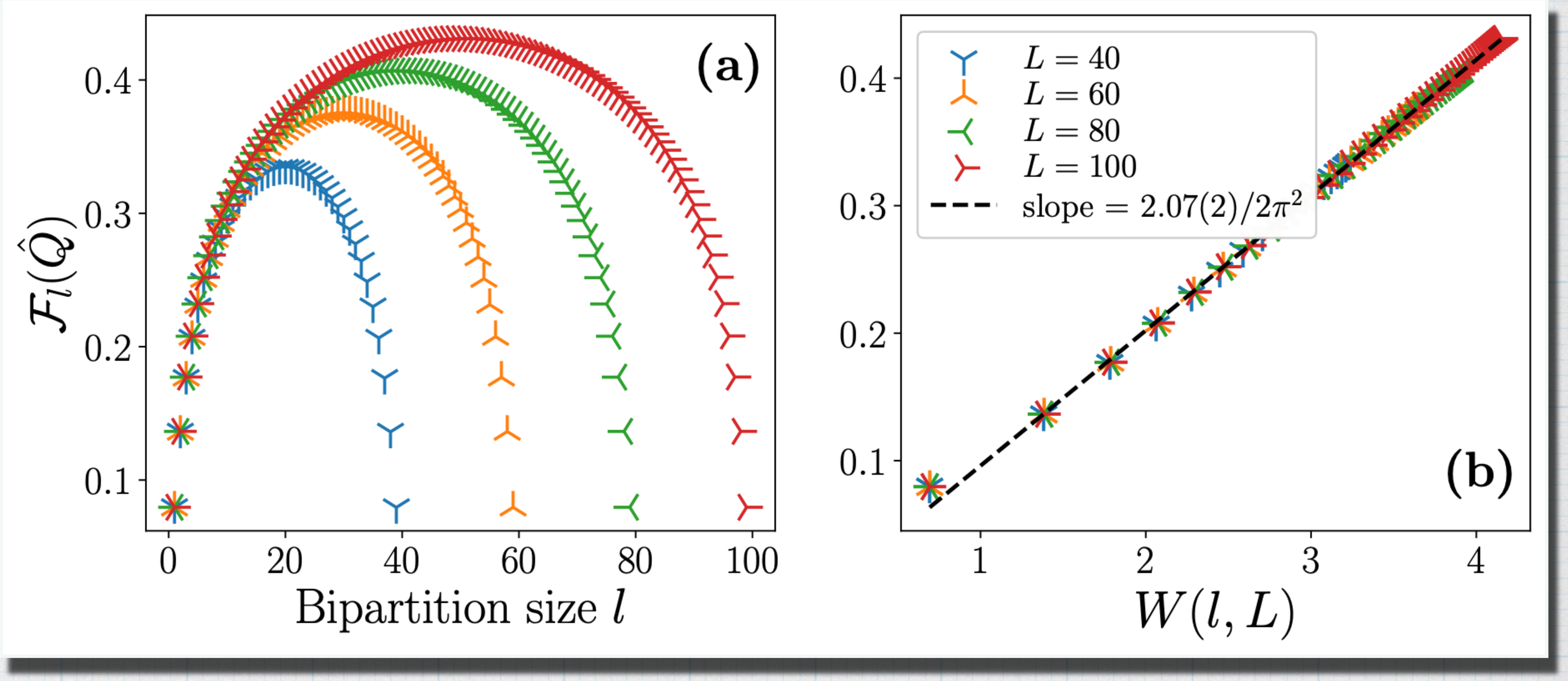
Scaling of **Local Fluctuations**: $\mathcal{F}(l, L) = \frac{K}{2\pi^2} W(l, L) + d'$

l →Bipartition size

L →System size

$W(l, L) = \log\left[\frac{2L}{\pi}\sin(\pi l/L)\right]$, the cord length

K →Luttinger parameter for the free bosonic theory



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Rachel, Laflorencie, Song, Hur, *Phys. Rev. Lett.* ***108****, 116401 (2012)*

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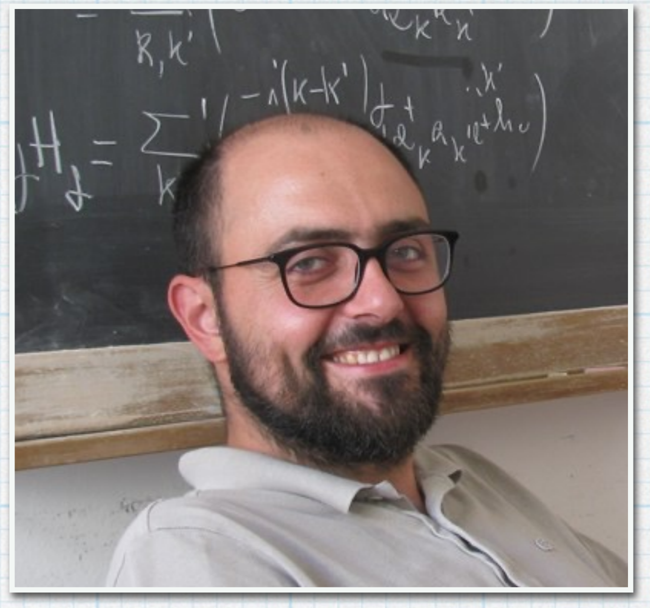

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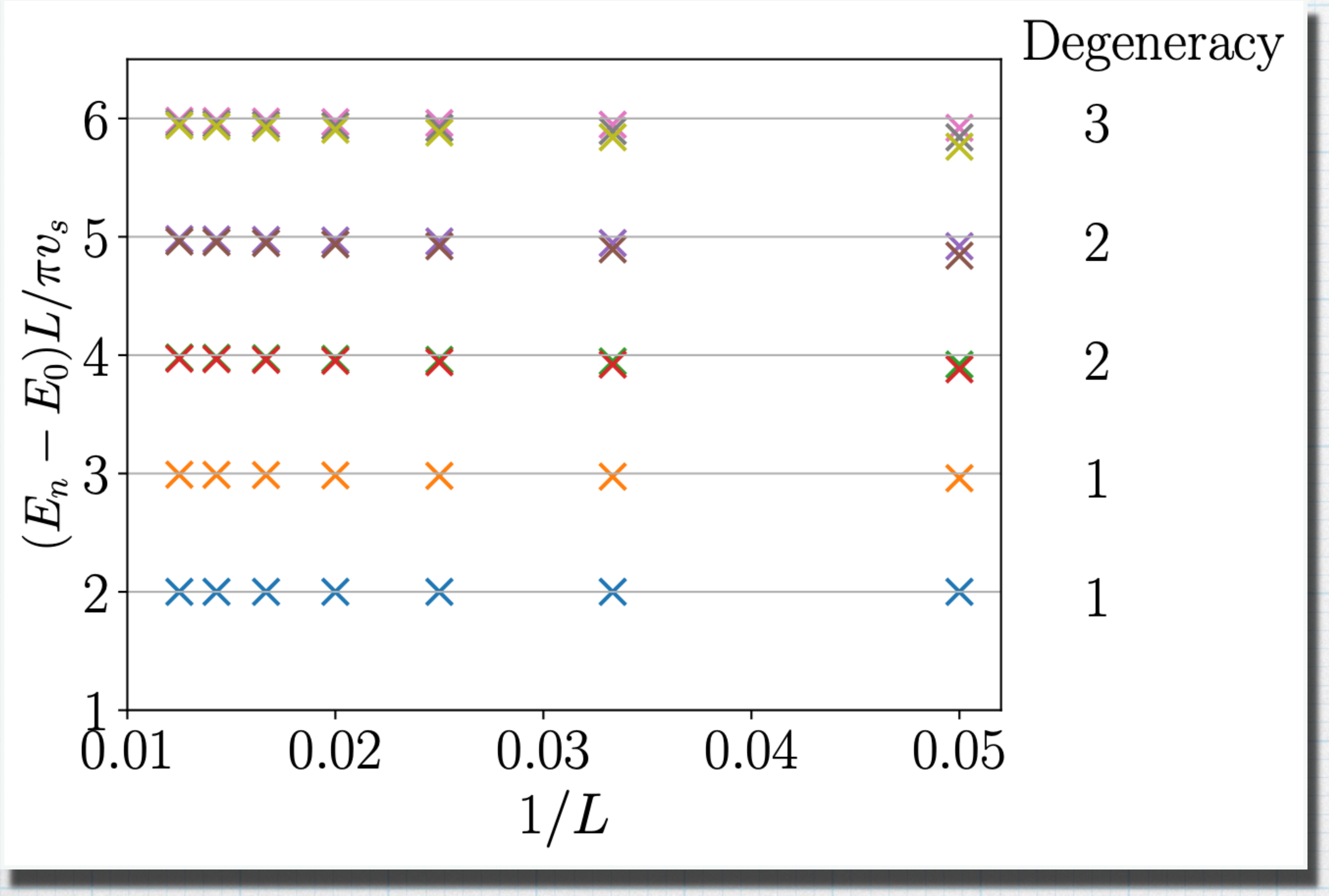
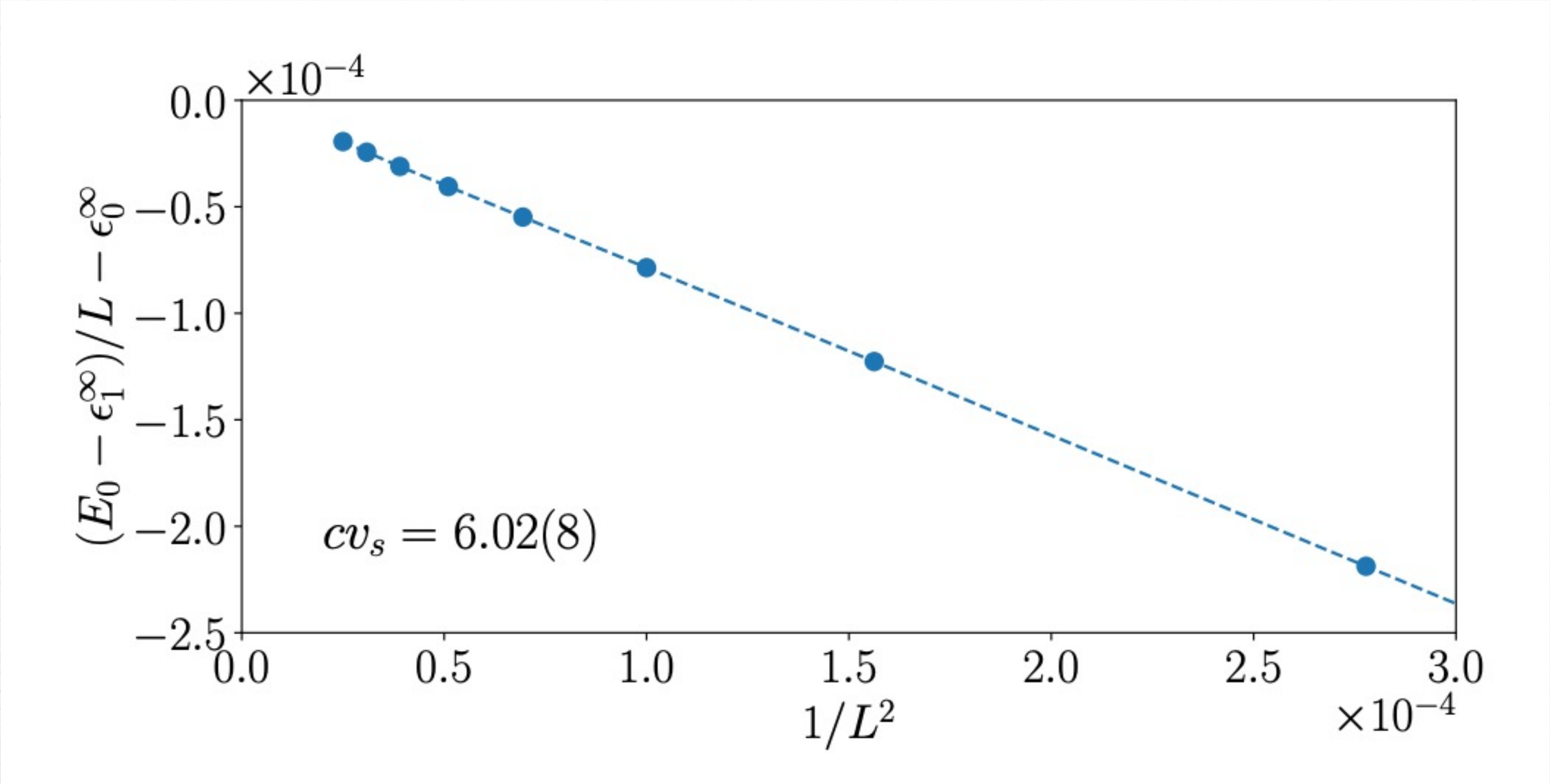
Spectral analysis… equilibrium



$$E_n(L) = E_0(L) + x_n \frac{\pi v_s}{L},$$

$$E_0(L) = \epsilon_0^\infty L + \epsilon_1^\infty - \frac{\pi c v_s}{24L},$$

Conformal towers



Ising Spectrum with fixed boundary condition

bosonic part hidden in different gauge sectors

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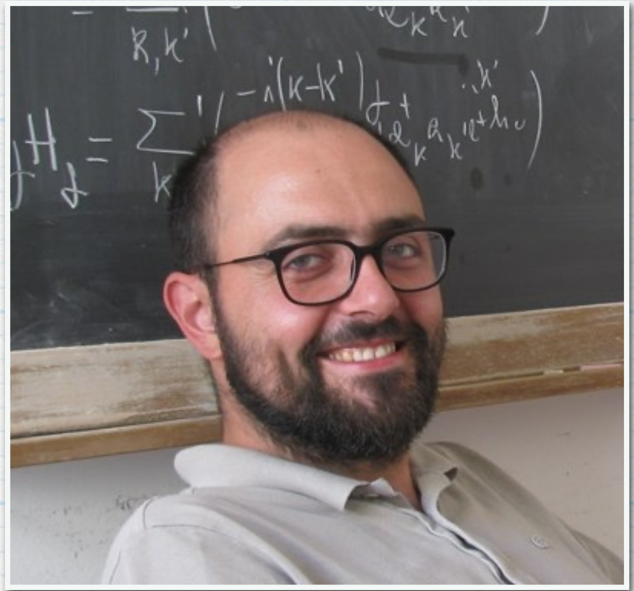
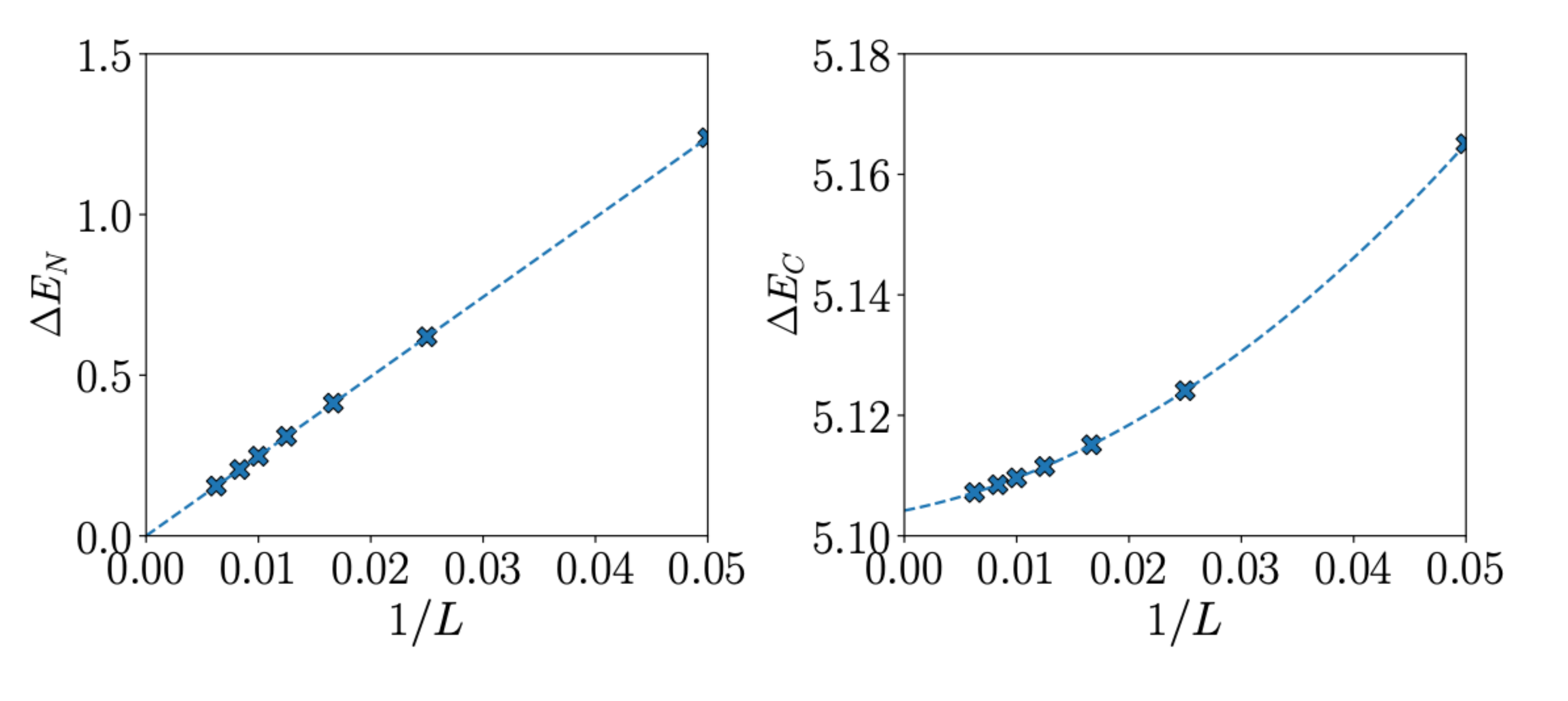


Marcello Dalmonte

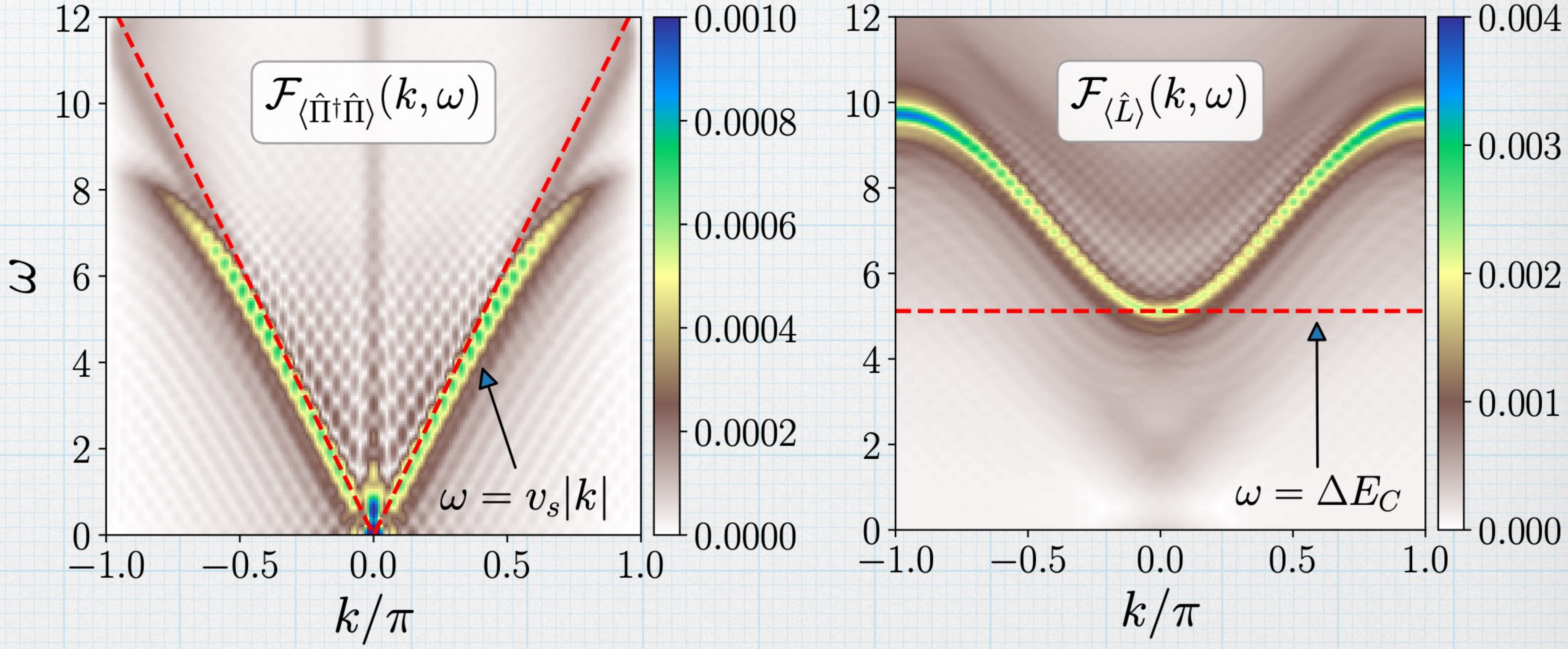
Neutral and charge gaps

$$\Delta E_N = E_1(Q=0) - E_0(Q=0),$$
$$\Delta E_C = E_0(Q=1) + E_0(Q=-1) - 2E_0(Q=0).$$



$$\hat{\mathcal{M}}^Q = \hat{\phi}^\dagger_{L/2}\hat{\phi}_{L/2+1} \qquad |\psi^Q\rangle\,(t=0) = \mathcal{N}\hat{\mathcal{M}}\,|\Omega\rangle$$

$$\mathcal{F}_{\mathcal{O}}(k,\omega) = \frac{2\pi}{LT}\delta t \sum_{j=1}^{L} e^{-ik(j-\frac{L}{2})} \sum_{n=0}^{t_N} e^{-i\omega t_n}\left(\langle \mathcal{O}_j\rangle(t_n) - \langle \mathcal{O}_j\rangle_\Omega\right),$$



Out-of-equilibrium dynamics to find the dispersion relations of this non-integrable model…

Only one gapless signal… **the other one is gapped**

Key Points

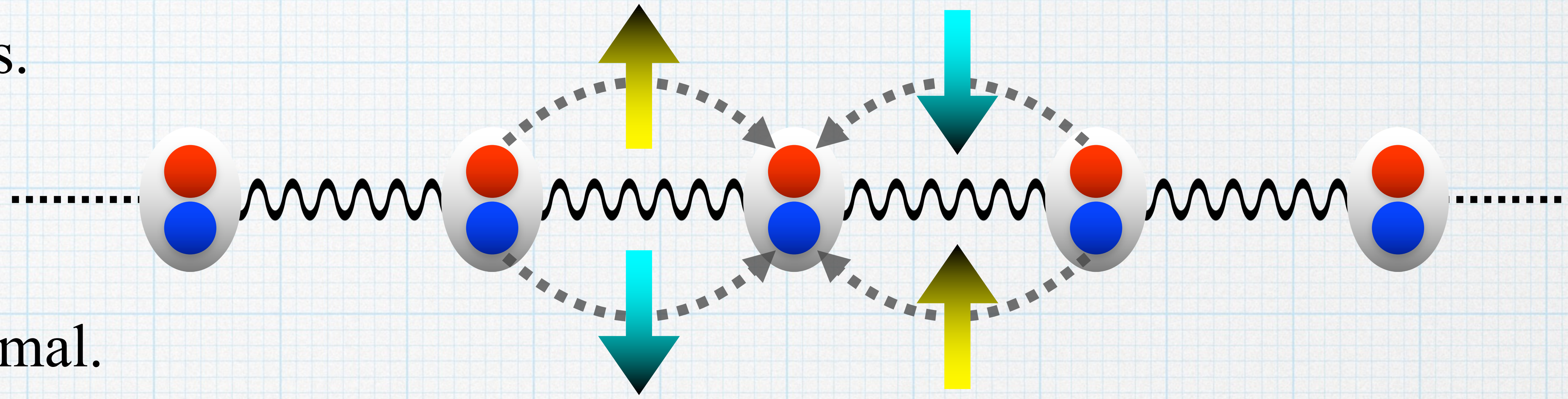
Bosonic Schwinger Model

1. Bosonic Schwinger model shows strong confining dynamics.

2. Trajectories of the bosons bends inwards.

3. As a result, asymptotic states are exotic and highly non-thermal.

4. These states are made of —
   i. Strongly correlated confined core that obeys area-law of entropy.
   ii. Almost thermal outer region (for lower masses) or vacuum (higher masses).



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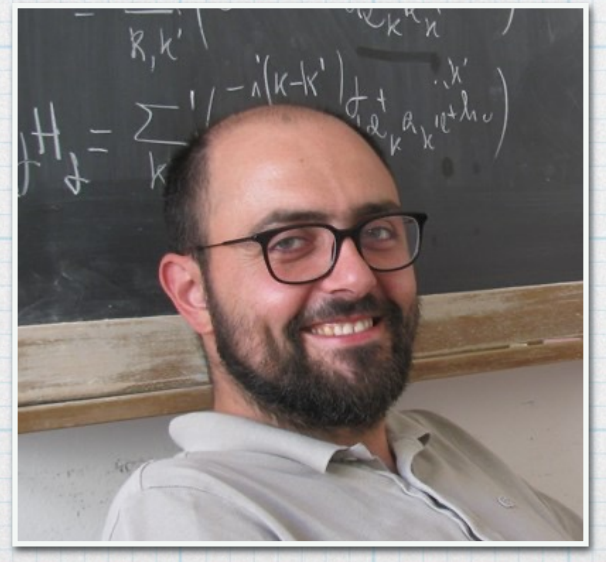

Abelian-Higgs Model

1. Higgs phase can be observed in 1+1D after lattice discretization.

2. Higgs phase is separated from the confined phase by a line of first order transition, a second order critical point, and then a smooth crossover.

3. The newly discovered critical point is very special with $c = 3/2$.

4. The origin of $c = 3/2$ can be explained by the Higgs