Quantum Information with Top Quarks

Y. Afik, JRMdN, EPJ Plus 136, 907 (2021)
Y. Afik, JRMdN, Quantum 6, 820 (2022)
Y. Afik, JRMdN, arXiv:2209.03969 (2022)

Juan Ramón Muñoz de Nova, Yoav Afik

Quantum Entanglement in High Energy Physics 2023, 10/05/2023

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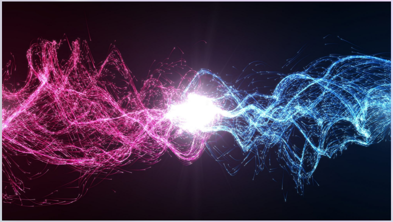
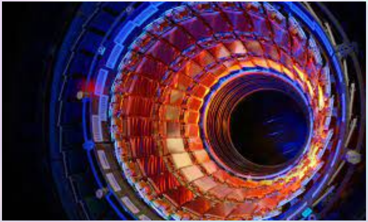
Motivation

- Standard Model is a Relativistic Quantum Field Theory = Special Relativity + Quantum Mechanics.
- Quantum Mechanics can be tested via Standard Model.
- Implementation of canonical techniques of Quantum Information $\rightarrow$ Quantum Information Theory at High-Energy Colliders.
- Highest-energy study at the frontier of the known Physics!
- Interest: Genuinely relativistic environment, exotic interactions and symmetries, fundamental nature...





Outline

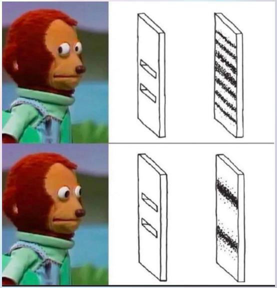
- Quantum Information Theory.
  - Quantum discord.
  - Entanglement.
  - Steering.
  - Bell nonlocality.
  - Quantum tomography.
- Top quark physics.
  - $t\bar{t}$ Kinematics.
  - $t\bar{t}$ Spin Quantum State.
  - LO QCD $t\bar{t}$ Production.
- Quantum Tops.
- Experimental Analysis.
- Conclusions and outlook.

Part I: Quantum Information Theory

Quantum vs. Classical

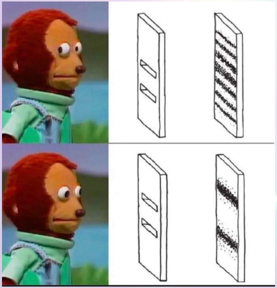
- Quantum Mechanics: Particles are in superposition of states $\rightarrow$ Probabilistic description of measurements.
- Classical Mechanics can also describe random outputs using classical probability distributions (noise, experimental variations...).
- Is there something genuinely quantum? Yes: Wave nature of quantum mechanics!

- Quantum Correlations=Correlations not accounted by classical theories.
  1. Quantum Discord
  2. Entanglement
  3. Steering
  4. Bell nonlocality



Quantum State

- Quantum descriptions:
  - **Pure state** $\rightarrow$ Wave function $\rightarrow$ Coherent mixture of quantum states $\rightarrow |\Psi\rangle = \sum_n \alpha_n \cdot |\phi_n\rangle$, α_n are amplitudes
  - **Mixed state** $\rightarrow$ Density matrix $\rightarrow$ Incoherent mixture of quantum states $\rightarrow \rho = \sum_n p_n \cdot |\phi_n\rangle \langle\phi_n|$, p_n are probabilities
- Density matrix: Most general quantum state.
- Classical descriptions accounted by density matrices.



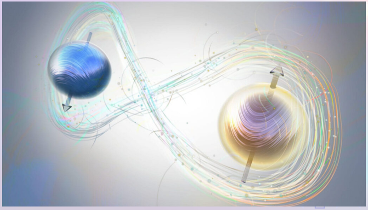
Qubits

- Qubit: Two-level quantum system $|\uparrow\rangle, |\downarrow\rangle \rightarrow$ Most simple quantum system.
- General density matrix (2×2) for 1 qubit $\rightarrow$ 3 parameters B_i:

$$\rho = \frac{1 + \sum_i B_i \sigma^i}{2}$$

- Two qubits $\rightarrow$ Most simple example of quantum correlations.
- General density matrix (4×4) for 2 qubits $\rightarrow$ 15 parameters $B_i^{\pm}, C_{ij}$

$$\rho = \frac{1 + \sum_i \left(B_i^+ \sigma^i \otimes 1 + B_i^- 1 \otimes \sigma^i\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$



Quantum Discord

- Classically, two equivalent expressions for mutual information of bipartite system A and B (Alice and Bob):

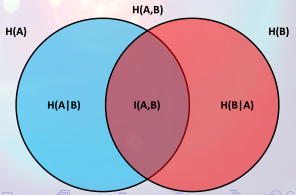
$$
\begin{aligned}
I(A,B) &= H(A) + H(B) - H(A,B) = H(A) - H(A|B) \\
H(A,B) &= -\sum_{x,y} p(x,y) \log_2 p(x,y) \\
H(A|B) &= \sum_{y} p(y) H(A|B=y)
\end{aligned}
$$

- Quantum mechanics can introduce a "discord" between both expressions:

$$\mathcal{D}(A,B) \equiv H(B) - H(A,B) + H(A|B) \neq 0$$

- Most basic form of quantum correlations!
- Quantum Discord is asymmetric $\mathcal{D}(A,B) \neq \mathcal{D}(B,A)$

Ollivier, Zurek PRL 88, 017901 (2001)



Quantum Discord: Two qubits

- How do we translate classical into quantum?

Quantum Discord: Two qubits

- How do we translate classical into quantum?
- Shannon entropy $\rightarrow$ Von Neumann entropy ($p_n \geq 0$, ρ eigenvalues)

$$H(A,B) \rightarrow H(\rho) = -\sum_n p_n \log_2 p_n$$

$$H(A) \rightarrow H(\rho_A),\ H(B) \rightarrow H(\rho_B),\ \rho_{A,B} = \mathrm{Tr}_{B,A}\rho$$

- Conditional probability $\rightarrow$ Conditional state $\rho_{A|B}$ = One-qubit state after Bob's spin measurement along $\hat{\mathbf{n}}$:

$$H(A|B) = p_{\hat{\mathbf{n}}} H(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} H(\rho_{-\hat{\mathbf{n}}})$$

$$\rho_{\hat{\mathbf{n}}} = \frac{\Pi^B_{\hat{\mathbf{n}}} \rho \Pi^B_{\hat{\mathbf{n}}}}{p_{\hat{\mathbf{n}}}} = \frac{1 + \mathbf{B}^+_{\hat{\mathbf{n}}} \cdot \sigma}{2},\ \mathbf{B}^+_{\hat{\mathbf{n}}} = \frac{\mathbf{B}^+ + \mathbf{C} \cdot \hat{\mathbf{n}}}{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^-},\ p_{\hat{\mathbf{n}}} = \frac{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^-}{2}$$

- Genuine quantumness $\rightarrow$ Minimization over all spin directions:

$$\mathcal{D}(A,B) = H(\rho_B) - H(\rho) + \min_{\hat{\mathbf{n}}} p_{\hat{\mathbf{n}}} H(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} H(\rho_{-\hat{\mathbf{n}}}) \neq 0$$

Entanglement

- **Entanglement**: Non-separability of a bipartite quantum state.
- Wave function:
  - Separability: $|\Psi\rangle = |\psi\rangle_A \otimes |\chi\rangle_B \rightarrow \langle O_A O_B\rangle = \langle O_A\rangle \langle O_B\rangle$
  - Correlation=Entanglement
- General quantum state:
  - Separability: $\rho = \sum_n p_n \rho_n^A \otimes \rho_n^B, \; \sum_n p_n = 1, \; p_n \geq 0$
  - Any classically correlated state (classical probability) is separable.
  - Correlation $\neq$ Entanglement!

Separable

Non-Separable

Entanglement: Two qubits

- Two qubits: Separability=Positive P-representation $P(\mathbf{n}_A, \mathbf{n}_B) \geq 0$:

$$\rho = \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \, P(\mathbf{n}_A, \mathbf{n}_B) \, |\mathbf{n}_A \mathbf{n}_B\rangle \langle \mathbf{n}_A \mathbf{n}_B| \,, \quad \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \, P(\mathbf{n}_A, \mathbf{n}_B) = 1$$

- Classical spins pointing at directions $\mathbf{n}_A, \mathbf{n}_B$!
- Separability=Purely classical correlations

$$C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle = \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \; P(\mathbf{n}_A, \mathbf{n}_B) n^i_A n^j_B$$

- Entanglement=NO probability distribution $\rightarrow$ Genuine non-classical!

Steering: Two qubits

- Measurements of Bob can "steer" quantum state of Alice.
- Steering: Original conception of Schrödinger of EPR paradox → Only well-defined in 2007! (Wiseman, Jones, Doherty, PRL 98, 140402 (2007))
- Alice post-measurement state described by local-hidden states:

$$\tilde{\rho}_{\hat{\mathbf{n}}} = \Pi^B_{\hat{\mathbf{n}}}\rho\Pi^B_{\hat{\mathbf{n}}} = \int d\lambda\ p(1|\hat{\mathbf{n}}\lambda)p(\lambda)\rho_B(\lambda)$$

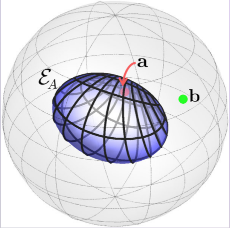
- If not, quantum state is **steerable**.

Steering: Two qubits

- Alice post-measurement state: same as for quantum discord.

$$\rho_{\hat{\mathbf{n}}} = \frac{\tilde{\rho}_{\hat{\mathbf{n}}}}{\mathrm{Tr}\tilde{\rho}_{\hat{\mathbf{n}}}} = \frac{1 + \mathbf{B}^{+}_{\hat{\mathbf{n}}} \cdot \sigma}{2}, \; \mathbf{B}^{+}_{\hat{\mathbf{n}}} = \frac{\mathbf{B}^{+} + \mathbf{C} \cdot \hat{\mathbf{n}}}{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^{-}}$$

- Set of conditional polarizations $\mathbf{B}^{+}_{\hat{\mathbf{n}}}$ describes an ellipsoid.
- Steering ellipsoid: Fundamental QI object, containing all information about the system.
- Similar for Bob → Steering: also asymmetric between Alice and Bob.



Jevtic, Pusey, Jennings, Rudolph
PRL 113, 020402 (2014)

Bell inequality: Two qubits

- Local realism: Joint Alice and Bob measurements M_A, M_B accounted by local hidden-variable model

$$p(a, b|M_A M_B) = \int d\lambda \; p(a|M_A\lambda)p(b|M_B\lambda)p(\lambda)$$

- Local realism holds if Bell inequality is satisfied. Two qubits $\rightarrow$ <u>**CHSH inequality**</u> ($\mathbf{a}_i, \mathbf{b}_i$ spin axes of measurements M_A, M_B)

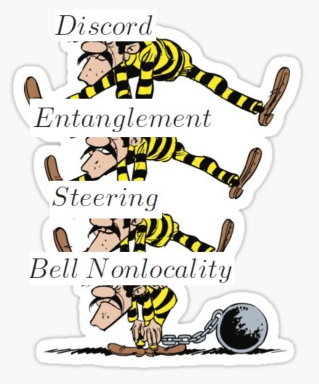
$$|\mathbf{a}_1^{\mathrm{T}}\mathbf{C}\,(\mathbf{b}_1 - \mathbf{b}_2) + \mathbf{a}_2^{\mathrm{T}}\mathbf{C}\,(\mathbf{b}_1 + \mathbf{b}_2)\,| \leq 2$$

- Stronger condition than entanglement $\rightarrow$ "Spooky action at distance"

Hierarchy of Quantum Correlations

- Steering and Discord can be asymmetric between Alice and Bob.
- Bell Nonlocality and Entanglement are always symmetric.
- Quantum Hierarchy:

$$\textit{Bell Nonlocality} \subset \textit{Steering} \subset \textit{Entanglement} \subset \textit{Discord}$$



Quantum Tomography: Two qubits

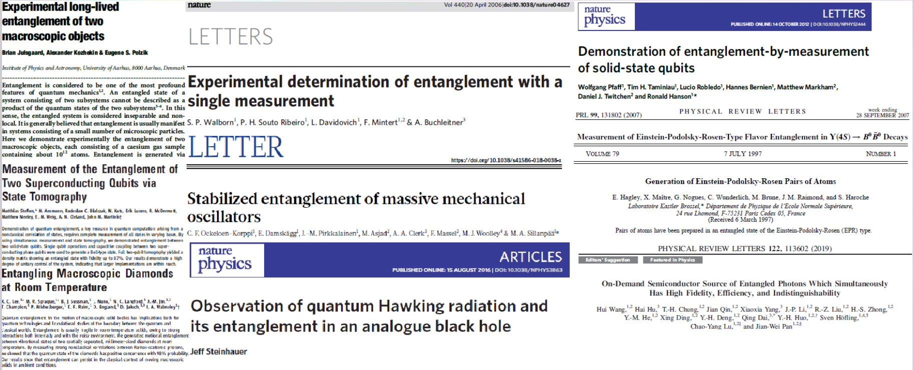
- **Quantum Tomography**: Reconstruction of quantum state from measurement of a set of observables.
- Quantum tomography $\rightarrow$ Measurement of ALL quantum correlations.
- Most general density matrices for $1, 2$ qubits:

$$\rho = \frac{1 + \sum_i B_i \sigma^i}{2}, \ \rho = \frac{1 + \sum_i \left(B_i^+ \sigma^i + B_i^- \bar{\sigma}^i\right) + \sum_{i,j} C_{ij} \sigma^i \bar{\sigma}^j}{4}$$

- One-qubit quantum tomography=Measurement of 3 parameters, polarization vector $\mathbf{B}$: $B_i = \langle \sigma^i \rangle$
- Two-qubit quantum tomography=Measurement of 15 parameters, polarization vectors $\mathbf{B}^{\pm}$ and correlation matrix $\mathbf{C}$:

$$B_i^+ = \langle \sigma^i \rangle, \ B_i^- = \langle \bar{\sigma}^i \rangle, \ C_{ij} = \langle \sigma^i \bar{\sigma}^j \rangle$$

- All these concepts have been observed in a wide variety of systems



Experimental long-lived entanglement of two macroscopic objects

Brian Julsgaard, Alexander Kozhekin & Eugene S. Polzik

Institute of Physics and Astronomy, University of Aarhus, 8000 Aarhus, Denmark

Entanglement is considered to be one of the most profound features of quantum mechanics[1,2]. An entangled state of a system consisting of two subsystems cannot be described as a product of the quantum states of the two subsystems[3–6]. In this sense, the entangled system is considered inseparable and non-local. It is generally believed that entanglement is usually manifest in systems consisting of a small number of microscopic particles. Here we demonstrate experimentally the entanglement of two macroscopic objects, each consisting of a caesium gas sample containing about 10^{12} atoms. Entanglement is generated via

Measurement of the Entanglement of Two Superconducting Qubits via State Tomography

Matthias Steffen,* M. Ansmann, Radoslaw C. Bialczak, N. Katz, Erik Lucero, R. McDermott, Matthew Neeley, E. M. Weig, A. N. Cleland, John M. Martinis†

Demonstration of quantum entanglement, a key resource in quantum computation arising from a nonclassical correlation of states, requires complete measurement of all states in varying bases. By using simultaneous measurement and state tomography, we demonstrated entanglement between two solid-state qubits. Single qubit operations and capacitive coupling between two superconducting phase qubits were used to generate a Bell-type state. Full two-qubit tomography yielded a density matrix showing an entangled state with fidelity up to 87%. Our results demonstrate a high degree of unitary control of the system, indicating that larger implementations are within reach.

Entangling Macroscopic Diamonds at Room Temperature

K. C. Lee,[1*] M. R. Sprague,[1*] B. J. Sussman,[2] J. Nunn,[1] N. K. Langford,[1] X.-M. Jin,[1,3] T. Champion,[1] P. Michelberger,[1] K. F. Reim,[1] D. England,[1] D. Jaksch,[1,3] I. A. Walmsley[1]†

Quantum entanglement in the motion of macroscopic solid bodies has implications both for quantum technologies and foundational studies of the boundary between the quantum and classical worlds. Entanglement is usually fragile in room-temperature solids, owing to strong interactions both internally and with the noisy environment. We generated motional entanglement between vibrational states of two spatially separated, millimeter-sized diamonds at room temperature. By measuring strong nonclassical correlations between Raman-scattered photons, we showed that the quantum state of the diamonds has positive concurrence with 98% probability. Our results show that entanglement can persist in the classical context of moving macroscopic solids in ambient conditions.

nature — Vol 440|20 April 2006|doi:10.1038/nature04627

LETTERS

Experimental determination of entanglement with a single measurement

S. P. Walborn[1], P. H. Souto Ribeiro[1], L. Davidovich[1], F. Mintert[1,2] & A. Buchleitner[3]

LETTER — https://doi.org/10.1038/s41586-018-0038-x

Stabilized entanglement of massive mechanical oscillators

C. F. Ockeloen-Korppi[1], E. Damskägg[1], J.-M. Pirkkalainen[1], M. Asjad[2], A. A. Clerk[3], F. Massel[2], M. J. Woolley[4] & M. A. Sillanpää[1]*

nature physics — ARTICLES — PUBLISHED ONLINE: 15 AUGUST 2016 | DOI: 10.1038/NPHYS3863

Observation of quantum Hawking radiation and its entanglement in an analogue black hole

Jeff Steinhauer

nature physics — LETTERS — PUBLISHED ONLINE: 14 OCTOBER 2012 | DOI: 10.1038/NPHYS2444

Demonstration of entanglement-by-measurement of solid-state qubits

Wolfgang Pfaff[1], Tim H. Taminiau[1], Lucio Robledo[1], Hannes Bernien[1], Matthew Markham[2], Daniel J. Twitchen[2] and Ronald Hanson[1]*

PRL 99, 131802 (2007) — PHYSICAL REVIEW LETTERS — week ending 28 SEPTEMBER 2007

Measurement of Einstein-Podolsky-Rosen-Type Flavor Entanglement in $\Upsilon(4S) \to B^0 \bar{B}^0$ Decays

VOLUME 79 — 7 JULY 1997 — NUMBER 1

Generation of Einstein-Podolsky-Rosen Pairs of Atoms

E. Hagley, X. Maître, G. Nogues, C. Wunderlich, M. Brune, J. M. Raimond, and S. Haroche
Laboratoire Kastler Brossel, Département de Physique de l'Ecole Normale Supérieure, 24 rue Lhomond, F-75231 Paris Cedex 05, France*
(Received 6 March 1997)

Pairs of atoms have been prepared in an entangled state of the Einstein-Podolsky-Rosen (EPR) type.

PHYSICAL REVIEW LETTERS 122, 113602 (2019)

Editors' Suggestion — Featured in Physics

On-Demand Semiconductor Source of Entangled Photons Which Simultaneously Has High Fidelity, Efficiency, and Indistinguishability

Hui Wang,[1,2] Hai Hu,[3] T.-H. Chung,[1,2] Jian Qin,[1,2] Xiaoxia Yang,[3] J.-P. Li,[1,2] R.-Z. Liu,[1,2] H.-S. Zhong,[1,2] Y.-M. He,[1,2] Xing Ding,[1,2] Y.-H. Deng,[1,2] Qing Dai,[3,*] Y.-H. Huo,[1,2,†] Sven Höfling,[1,4,5] Chao-Yang Lu,[1,2‡] and Jian-Wei Pan[1,2,§]

Quantum Experiments

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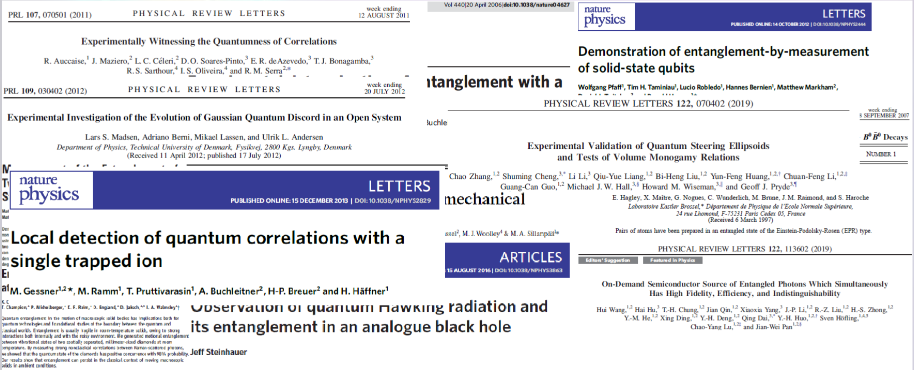

PRL 107, 070501 (2011) PHYSICAL REVIEW LETTERS week ending 12 AUGUST 2011

Experimentally Witnessing the Quantumness of Correlations

R. Auccaise,[1] J. Maziero,[2] L. C. Céleri,[2] D. O. Soares-Pinto,[3] E. R. deAzevedo,[3] T. J. Bonagamba,[3] R. S. Sarthour,[4] I. S. Oliveira,[4] and R. M. Serra[2,*]

PRL 109, 030402 (2012) PHYSICAL REVIEW LETTERS week ending 20 JULY 2012

Experimental Investigation of the Evolution of Gaussian Quantum Discord in an Open System

Lars S. Madsen, Adriano Berni, Mikael Lassen, and Ulrik L. Andersen
Department of Physics, Technical University of Denmark, Fysikvej, 2800 Kgs. Lyngby, Denmark
(Received 11 April 2012; published 17 July 2012)

nature physics LETTERS
PUBLISHED ONLINE: 15 DECEMBER 2013 | DOI: 10.1038/NPHYS2829

Local detection of quantum correlations with a single trapped ion

M. Gessner[1,2]★, M. Ramm[1], T. Pruttivarasin[1], A. Buchleitner[2], H-P. Breuer[2] and H. Häffner[1]

Vol 440|20 April 2006|doi:10.1038/nature04627

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PHYSICAL REVIEW LETTERS 122, 070402 (2019)

Experimental Validation of Quantum Steering Ellipsoids and Tests of Volume Monogamy Relations

Chao Zhang,[1,2] Shuming Cheng,[3,*] Li Li,[3] Qiu-Yue Liang,[1,2] Bi-Heng Liu,[1,2] Yun-Feng Huang,[1,2,†] Chuan-Feng Li,[1,2,‡] Guang-Can Guo,[1,2] Michael J. W. Hall,[3,§] Howard M. Wiseman,[3,||] and Geoff J. Pryde[3,¶]

week ending 8 SEPTEMBER 2007

$B^0\bar{B}^0$ Decays

NUMBER 1

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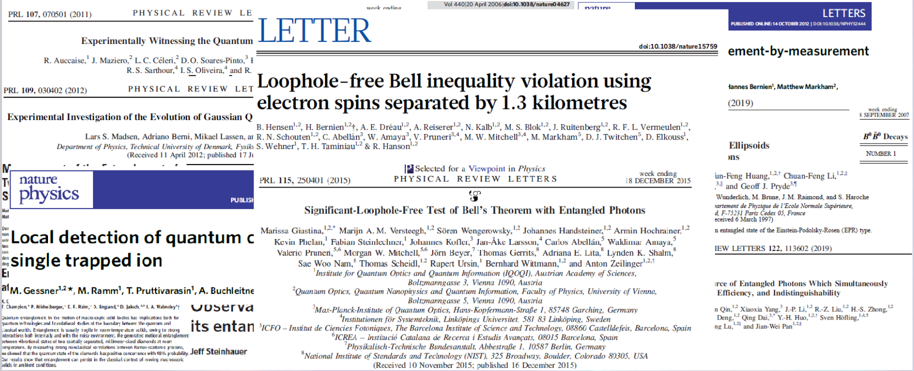

PRL 107, 070501 (2011) PHYSICAL REVIEW LE

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Observa
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LETTER

doi:10.1038/nature15759

Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres

B. Hensen[1,2], H. Bernien[1,2]†, A. E. Dréau[1,2], A. Reiserer[1,2], N. Kalb[1,2], M. S. Blok[1,2], J. Ruitenberg[1,2], R. F. L. Vermeulen[1,2], R. N. Schouten[1,2], C. Abellán[3], W. Amaya[3], V. Pruneri[3,4], M. W. Mitchell[3,4], M. Markham[5], D. J. Twitchen[5], D. Elkouss[1], S. Wehner[1], T. H. Taminiau[1,2] & R. Hanson[1,2]

Selected for a Viewpoint in Physics

PRL 115, 250401 (2015) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2015

Significant-Loophole-Free Test of Bell's Theorem with Entangled Photons

Marissa Giustina,[1,2,*] Marijn A. M. Versteegh,[1,2] Sören Wengerowsky,[1,2] Johannes Handsteiner,[1,2] Armin Hochrainer,[1,2] Kevin Phelan,[1] Fabian Steinlechner,[1] Johannes Kofler,[3] Jan-Åke Larsson,[4] Carlos Abellán,[5] Waldimar Amaya,[5] Valerio Pruneri,[5,6] Morgan W. Mitchell,[5,6] Jörn Beyer,[7] Thomas Gerrits,[8] Adriana E. Lita,[8] Lynden K. Shalm,[8] Sae Woo Nam,[8] Thomas Scheidl,[1,2] Rupert Ursin,[1] Bernhard Wittmann,[1,2] and Anton Zeilinger[1,2,†]
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LETTERS
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ement-by-measurement

Hannes Bernien[1], Matthew Markham[2],

(2019)

week ending 8 SEPTEMBER 2007

$B^0\bar{B}^0$ Decays

NUMBER 1

Ellipsoids

un-Feng Huang,[1,2,†] Chuan-Feng Li,[1,2,‡] and Geoff J. Pryde[3,¶]

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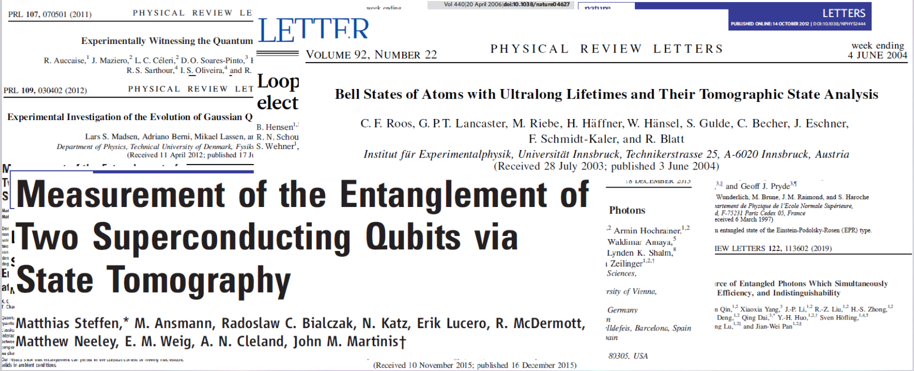

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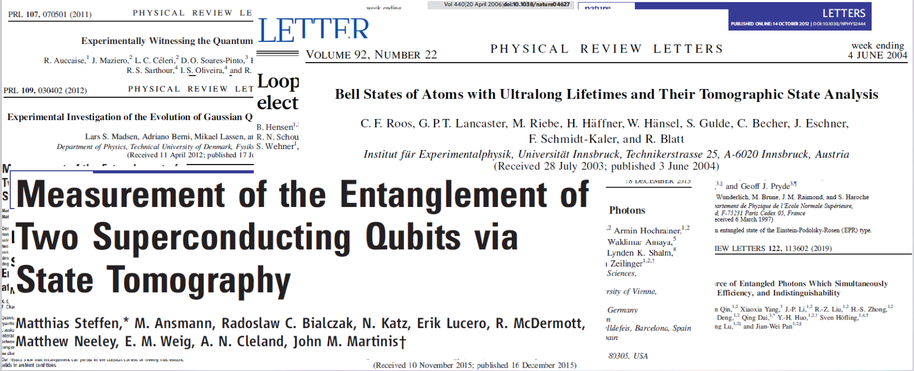


- Testing quantum mechanics in any new system is highly interesting by itself!
- Entanglement has never been measured between a pair of quarks :(

Part II: Top Quark Physics

Who Top Quarks?

- Top quark is the most massive fundamental particle known to exist ($m_t c^2 \approx 173$ GeV).
- First discovered by the D0 and CDF collaborations at the Tevatron in 1995.
- Top quarks produced in top-antitop ($t\bar{t}$) pairs through QCD or Electroweak processes.

Why Top Quarks?

- Large Width $\Gamma_t \sim 1$ GeV $\rightarrow$ Very short lifetime $\tau = 1/\Gamma_t \sim 10^{-25}\mathrm{s}$
- Tops decay before
  1. Hadronisation $\sim 10^{-23}\mathrm{s}$.
  2. Spin-decorrelation $\sim 10^{-21}\mathrm{s}$.
- $\rightarrow$ NO DECOHERENCE OR RANDOMIZATION!
- Rotational invariance in $t\bar{t}$ rest frames $\rightarrow$ $t\bar{t}$ spins measured from decay products.
- Measurements by D0 and CDF (Tevatron), ATLAS and CMS (LHC) $\rightarrow$ Well-established technique!

Top pair kinematics

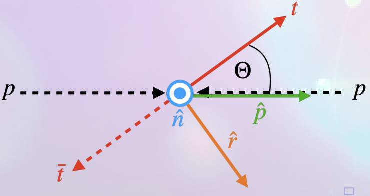
- $t\bar{t}$ pair kinematically described by invariant mass $M_{t\bar{t}}$ and top direction $\hat{k}$ in c.m. frame

$$\begin{aligned} k_t^\mu &= (k_t^0, \mathbf{k}), k_{\bar{t}}^\mu = (k_t^0, -\mathbf{k}) \\ M_{t\bar{t}}^2 &\equiv s_{t\bar{t}} \equiv (k_t + k_{\bar{t}})^2 \end{aligned}$$

- Invariant mass is simply related to top c. m. velocity β

$$M_{t\bar{t}} = \frac{2m_t}{\sqrt{1-\beta^2}} \to \beta = 0 \to M_{t\bar{t}} = 2m_t$$

- Threshold production: $M_{t\bar{t}} = 2m_t \approx 346$ GeV



Top pair Quantum State

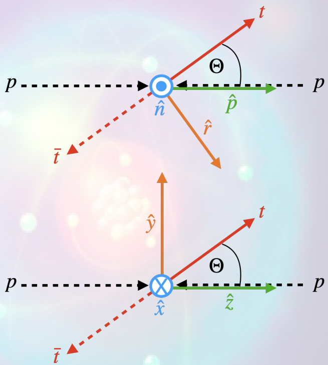
- How to translate HEP features to Quantum Information language?
- $t\bar{t}$ spins described by production spin density matrix $R(M_{t\bar{t}}, \hat{k})$:

$$R = \tilde{A} + \sum_i \left(\tilde{B}_i^+ \sigma^i + \tilde{B}_i^- \bar{\sigma}^i \right) + \sum_{i,j} \tilde{C}_{ij} \sigma^i \bar{\sigma}^j$$

- Quantum state in experiment: Momentum measurements+Average over events $\rightarrow$ Genuine density-matrix description!
- Proper spin density matrix $\rho(M_{t\bar{t}}, \hat{k}) = \dfrac{R(M_{t\bar{t}}, \hat{k})}{\mathrm{tr}\left[R(M_{t\bar{t}}, \hat{k})\right]}$

Basis Selection

- Different basis for computing spin polarization and spin correlations characterizing the quantum state.
- Helicity basis: $\{\hat{k}, \hat{r}, \hat{n}\}$:
  - $\hat{k}$ - top direction in $t\bar{t}$ c.m. frame.
  - $\hat{p}$ - beam direction ($\cos\Theta = \hat{k}\cdot\hat{p}$).
  - $\hat{r} = (\hat{p} - \cos\Theta\hat{k})/\sin\Theta$.
  - $\hat{n} = \hat{r} \times \hat{k}$.
  - Study of individual $t\bar{t}$ production with **fixed energy and direction**.
- Beam basis: $\{\hat{x}, \hat{y}, \hat{z}\}$:
  - $\hat{z}$ along beam axis.
  - $\hat{x}, \hat{y}$ transverse directions to beam.
  - Fixed in space: no change with $\hat{k}$.
  - Study of **total integrated quantum state**.

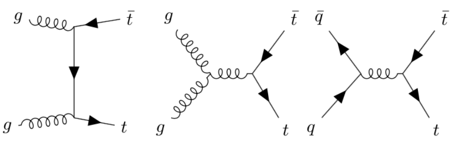


LO QCD Elementary Process

- Illustrative example: QCD analytical LO calculation.
  - Analytical results.
  - NLO corrections are small.
  - Building blocks of actual high-energy processes.
- Most elementary QCD processes:

$$\begin{aligned} q+\bar{q} &\to t+\bar{t}, \;\; q=u,d\ldots \\ g+g &\to t+\bar{t} \end{aligned}$$

- Each initial state $I=q\bar{q},gg$ gives rise to quantum state $\rho^I(M_{t\bar{t}},\hat{k})$



LO QCD Realistic

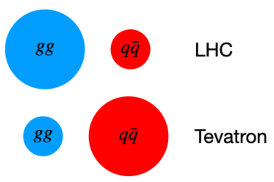
- No free quarks or gluons → Hadrons: Bound states of quarks and gluons (partons).
- LHC, Tevatron: pp, $p\bar{p}$ collisions at high c.m. energies $\sqrt{s}$.

$$\begin{array}{lcll} p+p & \rightarrow & \ldots \rightarrow t+\bar{t} & \text{LHC} \\ p+\bar{p} & \rightarrow & \ldots \rightarrow t+\bar{t} & \text{Tevatron} \end{array}$$

- Quantum state depends now on c.m. energy $\sqrt{s}$:

$$\rho(M_{t\bar{t}}, \hat{k}) = \sum_{I=q\bar{q},gg} w_I(M_{t\bar{t}}, \sqrt{s})\rho^I(M_{t\bar{t}}, \hat{k})$$

- Total QCD process: *Incoherent* sum of elementary QCD processes with probability w_I.
- QCD Input: $w_I(M_{t\bar{t}}, \sqrt{s}), \rho^I(M_{t\bar{t}}, \hat{k}) \rightarrow$ QI Output: Textbook problem of *convex sum* of quantum states!



LO QCD Quantum State

- Most general 2-qubit density matrix (15 parameters):

$$\rho(M_{t\bar{t}}, \hat{k}) = \frac{1 + \sum_i \left(B_i^+ \sigma^i + B_i^- \bar{\sigma}^i\right) + \sum_{i,j} C_{ij} \sigma^i \bar{\sigma}^j}{4}$$

- Standard Model $\rightarrow \mathbf{B}^+ = \mathbf{B}^-$, $\mathbf{C}^{\mathrm{T}} = \mathbf{C}$
- LO QCD $\rightarrow$
  1. $\rho(M_{t\bar{t}}, \hat{k})$ is a T-state (unpolarized) $\rightarrow \mathbf{B}^{\pm} = 0$
  2. Spin along $\hat{n}$ is uncorrelated to other directions
- Only 4 parameters in SM LO QCD: $C_{kk}, C_{rr}, C_{nn}, C_{kr}$

$$\mathbf{B}^{\pm} = 0, \;\; \mathbf{C} = \begin{bmatrix} C_{kk} & C_{kr} & 0 \\ C_{kr} & C_{rr} & 0 \\ 0 & 0 & C_{nn} \end{bmatrix}$$

Part III: Quantum Tops

Two-qubit Quantum Criteria

- In general, evaluation of all quantum correlations is a complicated problem (discord and steering).
- However, due to the simple form of $\rho(M_{t\bar{t}}, \hat{k})$ in SM LO QCD:

1. Quantum Discord: Analytical (T-states).
2. Entanglement: Concurrence $0 \leq \mathcal{C}[\rho] \leq 1$, $\mathcal{C}[\rho] > 0$ iff ρ entangled:

$$\mathcal{C}[\rho] = \max(\Delta, 0), \ \Delta \equiv \frac{-C_{nn} + |C_{kk} + C_{rr}| - 1}{2}$$

3. Steerability *iff* $\int \mathrm{d}\hat{\mathbf{n}} \sqrt{\hat{\mathbf{n}}^{\mathrm{T}}\mathbf{C}^{\mathrm{T}}\mathbf{C}\hat{\mathbf{n}}} > 2\pi$ (T-state).
4. CHSH violation *iff* $\mu_1 + \mu_2 > 1$ ($\mu_{1,2}$ largest eigenvalues of $\mathbf{C}^{\mathrm{T}}\mathbf{C}$).

$t\bar{t}$ Quantum Correlations

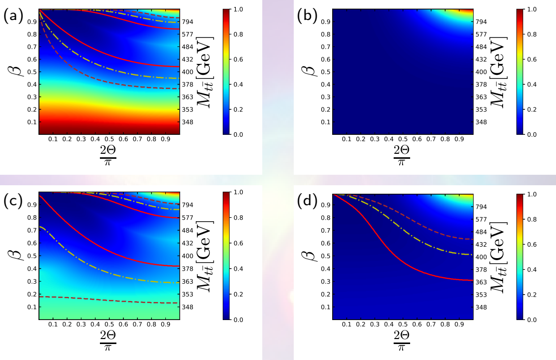
a) $gg \to t\bar{t}$ Discord.

b) $q\bar{q} \to t\bar{t}$ Discord.

c) LHC $\rho(M_{t\bar{t}}, \hat{k})$ Discord.

d) Tevatron $\rho(M_{t\bar{t}}, \hat{k})$ Discord.

- Solid red, dashed-dotted yellow, dashed brown: Critical boundaries of entanglement, steerability, and Bell nonlocality $\to$ Quantum Hierarchy respected!



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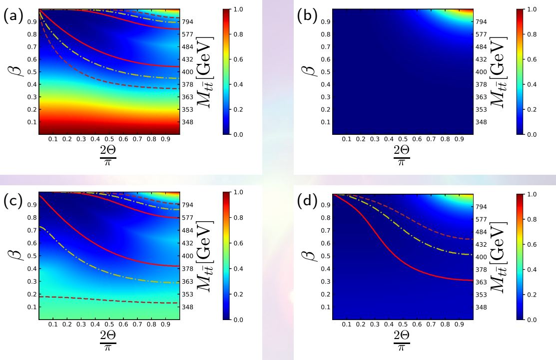


Full picture of quantum correlations in $t\bar{t}$.

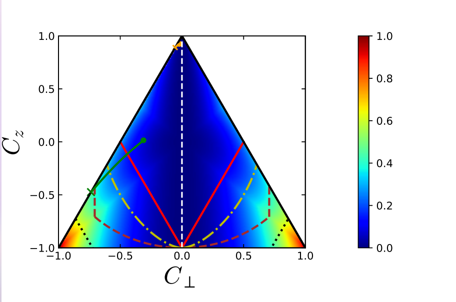
Total Quantum State

- Realistic measurement: Average over many different processes.
- Total quantum state: Events in window $[2m_t, M_{t\bar{t}}]$

$$\rho(M_{t\bar{t}}) \equiv \frac{1}{\sigma(M_{t\bar{t}})} \int_{2m_t}^{M_{t\bar{t}}} \mathrm{d}M \int \mathrm{d}\Omega \, \frac{\mathrm{d}\sigma}{\mathrm{d}M\mathrm{d}\Omega} \rho(M, \hat{k})$$

- Intuitively: Total quantum state=Sum of $t\bar{t}$ quantum states weighted with the differential cross-section.

- Rotational invariance around beam axis → Correlation matrix diagonal in beam basis $C_{ij} = C_i \delta_{ij}$, $C_x = C_y = C_\perp$ → 2D dependence on $C_\perp, C_z$.
- Green: LHC.
  Orange: Tevatron.
- Cross: $\beta = 0$; Circle: $\beta = 1$.



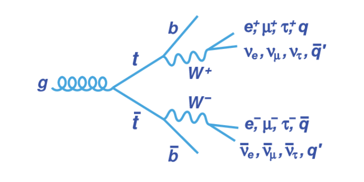
Part IV: Experimental Analysis

Top pair Quantum Tomography

- $\rho(M_{t\bar{t}}) \to$ Two qubit quantum state $\to$ Quantum tomography = Measurement of spin polarizations and spin correlations.
- Spin polarizations $\mathbf{B}^{\pm}$ and spin correlation matrix $\mathbf{C}$ extracted from cross-section $\sigma_{\ell\bar{\ell}}$ of dileptonic decay

$$\frac{1}{\sigma_{\ell\bar{\ell}}}\frac{\mathrm{d}\sigma_{\ell\bar{\ell}}}{\mathrm{d}\Omega_+\mathrm{d}\Omega_-} = \frac{1}{(4\pi)^2}\left[1 + \mathbf{B}^+ \cdot \hat{\ell}_+ - \mathbf{B}^- \cdot \hat{\ell}_- - \hat{\ell}_+ \cdot \mathbf{C} \cdot \hat{\ell}_-\right]$$

- $\hat{\ell}_{\pm}$: lepton directions in each top (antitop) rest frames.

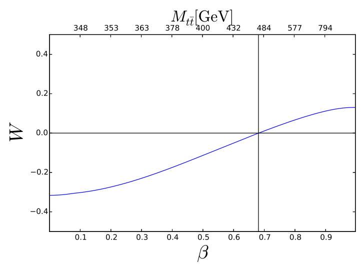


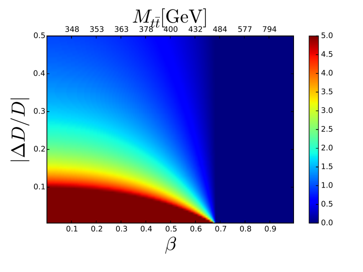
Entanglement in $t\bar{t}$ production at LHC $\sqrt{s} = 13$ TeV

- Entanglement witness
  $W = D + 1/3 < 0$, $D \equiv \mathrm{tr}\,\mathbf{C}/3 \rightarrow$
  Entanglement only close to threshold.
- D directly measurable from decay cross-sections:

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$

- Entanglement detection from one single magnitude! $\rightarrow$ No need for Quantum Tomography!
- High-statistical significance!
- Entanglement also available at high-p_T: Fabbrichesi, Floreanini, Panizzo, PRL 127, 161801 (2021), Severi, Boschi, Maltoni, Sioli, EPJC 82, 285 (2022)





Recent Related Measurement

- Recently, D was measured with no selection on $M_{t\bar{t}}$ by CMS.
- Results:
  $D = -0.237 \pm 0.011 > -1/3$;
  $\Delta D/D = 4.6\%$.
- No evidence of quantum entanglement.
  $\Rightarrow$ **We need a dedicated analysis!**

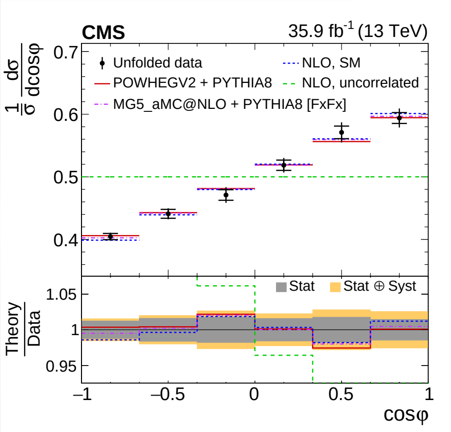


Figure: Distribution of $\cos\varphi$. Figure is from Phys. Rev. D 100, 072002.

Discord and Steering

- Normalized dileptonic cross-section → Angular probability distribution:

$$p(\hat{\ell}_+, \hat{\ell}_-) = \frac{1}{\sigma_{\ell\bar{\ell}}} \frac{\mathrm{d}\sigma_{\ell\bar{\ell}}}{\mathrm{d}\Omega_+ \mathrm{d}\Omega_-} = \frac{1 + \mathbf{B}^+ \cdot \hat{\ell}_+ - \mathbf{B}^- \cdot \hat{\ell}_- - \hat{\ell}_+ \cdot \mathbf{C} \cdot \hat{\ell}_-}{(4\pi)^2}$$

- Direct one-qubit tomography of $\rho_{A,B}, \rho_{\hat{\mathbf{n}}}$ from Bloch vectors $\mathbf{B}^\pm, \mathbf{B}^\pm_{\hat{\mathbf{n}}}$:

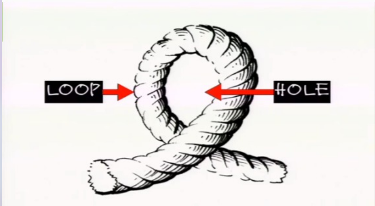
$$p(\hat{\ell}_\pm) = \int \mathrm{d}\Omega_\mp \; p(\hat{\ell}_+, \hat{\ell}_-) = \frac{1 \pm \mathbf{B}^\pm \cdot \hat{\ell}_\pm}{4\pi}$$

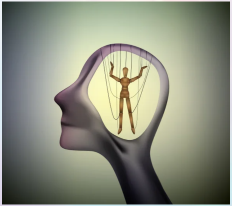
$$p(\hat{\ell}_\pm | \hat{\ell}_\mp = \mp\hat{\mathbf{n}}) = \frac{p(\hat{\ell}_\pm, \hat{\ell}_\mp = \mp\hat{\mathbf{n}})}{p(\hat{\ell}_\mp = \mp\hat{\mathbf{n}})} = \frac{1 \pm \mathbf{B}^+_{\hat{\mathbf{n}}} \cdot \hat{\ell}_\pm}{4\pi}$$

- Actual discord → Evaluated from minimization over $\hat{\mathbf{n}}$.
- Measurement of $\mathbf{B}^\pm_{\hat{\mathbf{n}}}$ → Reconstruction of $t, \bar{t}$ steering ellipsoids.
- Highly-challenging measurements in conventional setups → Natural implementation in colliders!

Bell Test Loopholes in a Collider Experiment

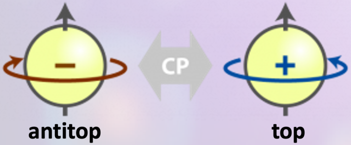
- Loopholes: Experimental tests of Bell's inequality may not fulfill all hypotheses of Bell's theorem.
- Collider experiment:
  1. Free-will loophole: Spin measurement directions should be free, independent from hidden-variables. $\rightarrow$ Not even single-detection events from Alice and Bob!
  2. Detection loophole: Only a subset of events selected for measurement $\rightarrow$ Bias!
- Quite natural: Colliders were not designed to test Bell's Inequality!





New Physics Witnesses

- Approximate CP-invariance of Standard Model $\rightarrow \mathbf{C} = \mathbf{C}^{\mathrm{T}}$, $\mathbf{B}^{+} = \mathbf{B}^{-}$ $\rightarrow$ Symmetric discord and steering!
- Therefore: Discord and/or Steering asymmetry $\rightarrow$ New Physics!
- New physics witnesses: Symmetry protected observables by SM, only non-zero for New Physics:
  - $\Delta\mathcal{D}_{t\bar{t}} \equiv \mathcal{D}_t - \mathcal{D}_{\bar{t}}$
  - Asymmetries in ellipsoid centers and/or semiaxes.
- No SM contribution to New Physics witnesses!



Conclusions and outlook

- Quantum Information theory $\longleftrightarrow$ High-Energy Physics. Interdisciplinary, huge potential and great interest!
- QI perspective:
  1. Highest-energy observation of entanglement ever!
  2. Genuinely relativistic, exotic symmetries and interactions, fundamental nature $\rightarrow$ Frontier of known Physics!
  3. Highly-demanding measurements naturally implemented at LHC.
- HEP perspective:
  1. Quantum Tomography: Novel experimental tool.
  2. QI techniques can inspire new approaches for searching New Physics:
     - Aoude, Madge, Maltoni, Mantani, PRD (2022).
     - Severi, Vryonidou, JHEP (2023).
     - Fabbrichesi, Floreanini, Gabrielli, EPJC (2023).
- Extension to e^+e^- colliders: Spin of initial state can be controlled! $\rightarrow$ Manipulation of qubits? Quantum gates?
- Adaptation to τ leptons, qutrits $W^{\pm}, Z^0$.

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     - Fabbrichesi, Floreanini, Gabrielli, EPJC (2023).
- The first measurements of entanglement between a pair of top-quarks are ongoing within ATLAS and CMS.
- **The results are expected to be public soon - stay tuned!**

Thank You