New developments in relativistic dissipative hydrodynamics

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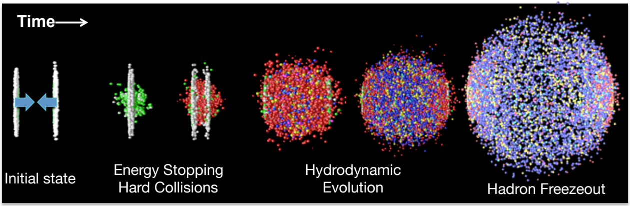
Outline

1.1 Standard model of heavy-ion collisions



T. K. Nayak, Lepton-Photon 2011 Conference

FIRST STAGE — HIGHLY OUT-OF EQUILIBRIUM ($0 < \tau_0 \lesssim 1$ fm)

- **initial conditions**, including fluctuations, reflect to large extent the distribution of matter in the colliding nuclei — Glauber model, works by A. Białas and W. Czyż
- **emission of hard probes**: heavy quarks, photons, jets
- **hydrodynamization stage** – the system becomes well described by equations of viscous hydrodynamics — crucial contributions from R. Janik and his collaborators

SECOND STAGE — HYDRODYNAMIC EXPANSION ($1\,\mathrm{fm} \lesssim \tau \lesssim 10\,\mathrm{fm}$)

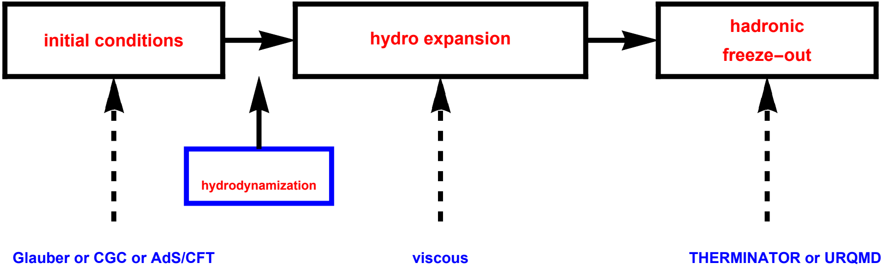
- expansion controlled by viscous hydrodynamics (effective description)
- **thermalization stage**
- **phase transition** from QGP to hadron gas takes place (encoded in the equation of state)
- **equilibrated hadron gas**

THIRD STAGE — FREEZE-OUT

- **freeze-out and free streaming of hadrons** ($10\,\mathrm{fm} \lesssim \tau$)

IN THIS TALK (except for the last part) EFFECTS OF FINITE BARYON NUMBER DENSITY ARE NEGLECTED

STANDARD MODEL (MODULES) of HEAVY–ION COLLISIONS



FLUCTUATIONS IN THE INITIAL STATE / EVENT–BY–EVENT HYDRO / FINAL–STATE FLUCTUATIONS

EQUATION OF STATE = lattice QCD

1 < VISCOSITY < 3 times the lower bound

Danielewicz and Gyulassy (quantum mechanics), Kovtun+Son+Starinets (AdS/CFT)

lower bound on the ratio of shear viscosity to entropy density $\eta/\mathcal{S} = 1/(4\pi)$

1.2 From perfect-fluid to viscous hydrodynamics

$T(x)$ and $u^\mu(x)$ are fundamental fluid variables

the relativistic perfect-fluid energy-momentum tensor is the most general symmetric tensor which can be expressed in terms of these variables without using derivatives

dynamics of the perfect fluid theory is provided by the conservation equations of the energy-momentum tensor, four equations for the four independent hydrodynamic fields – a self-consistent (hydrodynamic) theory

$$\partial_\mu T^{\mu\nu}_{\rm eq} = 0, \qquad T^{\mu\nu}_{\rm eq} = \mathcal{E} u^\mu u^\nu - \mathcal{P}_{\rm eq}(\mathcal{E}) \Delta^{\mu\nu}, \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \tag{1}$$

$$\mathcal{E}_{\rm eq}(T(x)) = \mathcal{E}(x), \qquad T^{\mu\nu}_{\rm eq}(x) u_\nu(x) = \mathcal{E}(x) u^\mu(x). \tag{2}$$

$$\text{local rest frame:} \quad u^\mu = (1,0,0,0) \quad \rightarrow \quad T^{\mu\nu}_{\rm eq} = \begin{bmatrix} \mathcal{E} & 0 & 0 & 0 \\ 0 & \mathcal{P}{\rm eq} & 0 & 0 \\ 0 & 0 & \mathcal{P}{\rm eq} & 0 \\ 0 & 0 & 0 & \mathcal{P}{\rm eq} \end{bmatrix} \tag{3}$$

DISSIPATION DOES NOT APPEAR! $u_\nu \partial_\mu T^{\mu\nu}_{\rm eq} = 0 \rightarrow \partial_\mu(\mathcal{S} u^\mu) = 0$

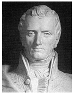
entropy conservation follows from the energy-momentum conservation and the form of the energy-momentum tensor

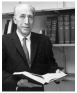
Navier-Stokes hydrodynamics

Claude-Louis Navier, 1785–1836, French engineer and physicist
Sir George Gabriel Stokes, 1819–1903, Irish physicist and mathematician

C. Eckart, Phys. Rev. 58 (1940) 919

L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Pergamon, New York, 1959






complete energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Pi^{\mu\nu} \tag{4}$$

where $\Pi^{\mu\nu} u_\nu = 0$, which corresponds to the Landau definition of the hydrodynamic flow u^μ

$$T^{\mu}_{\ \nu} u^\nu = \mathcal{E}\, u^\mu. \tag{5}$$

It proves useful to further decompose $\Pi^{\mu\nu}$ into two components,

$$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Pi \Delta^{\mu\nu}, \tag{6}$$

which introduces the **bulk viscous pressure** Π (the trace part of $\Pi^{\mu\nu}$) and the **shear stress tensor** $\pi^{\mu\nu}$ which is symmetric, $\pi^{\mu\nu} = \pi^{\nu\mu}$, traceless, $\pi^{\mu}_{\ \mu} = 0$, and orthogonal to u^μ, $\pi^{\mu\nu} u_\nu = 0$.

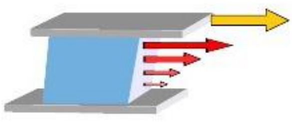
in the Navier-Stokes theory, the **bulk pressure** and **shear stress tensor** are given by the gradients of the flow vector

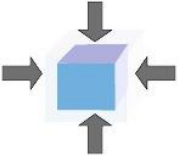
$$\Pi = -\zeta\,\partial_\mu u^\mu, \quad \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}. \qquad (7)$$

Here ζ and η are the bulk and shear viscosity coefficients, respectively, and $\sigma^{\mu\nu}$ is the shear flow tensor

shear viscosity η
$\Downarrow$
reaction to a change of shape

bulk viscosity ζ
$\Downarrow$
reaction to a change of volume





$$\pi^{\mu\nu}{}_{\text{Navier–Stokes}} = 2\eta\,\sigma^{\mu\nu}$$

$$\Pi_{\text{Navier–Stokes}} = -\zeta\theta$$

Navier-Stokes hydrodynamics

complete energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \pi^{\mu\nu} + \Pi\Delta^{\mu\nu} = T^{\mu\nu}_{\rm eq} + 2\eta\sigma^{\mu\nu} - \zeta\theta\Delta^{\mu\nu} \qquad (8)$$

again four equations for four unknowns

$$\partial_\mu T^{\mu\nu} = 0 \qquad (9)$$

1) THIS SCHEME DOES NOT WORK IN PRACTICE!

ACAUSAL BEHAVIOR + INSTABILITIES!

2) NEVERTHELESS, THE GRADIENT FORM (8) IS A GOOD APPROXIMATION

FOR SYSTEMS APPROACHING LOCAL EQUILIBRIUM

Great progress has been made in the last years to understand the hydrodynamic gradient expansion by

R. Janik, M. Spaliński , M. P. Heller, P. Witaszczyk and their collaborators

Israel-Stewart equations

$\Pi, \pi^{\mu\nu}$ promoted to new hydrodynamic variables!

W. Israel and J.M. Stewart, *Transient relativistic thermodynamics and kinetic theory*, Annals of Physics 118 (1979) 341

$$\dot{\Pi} + \frac{\Pi}{\tau_\Pi} = -\beta_\Pi \theta, \qquad \tau_\Pi \beta_\Pi = \zeta \tag{10}$$

$$\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} = 2\beta_\pi \sigma^{\mu\nu}, \qquad \tau_\pi \beta_\pi = 2\eta \tag{11}$$

1) HYDRODYNAMIC EQUATIONS DESCRIBE BOTH HYDRODYNAMIC AND NON-HYDRODYNAMIC MODES

perturbations $\sim \exp(-\omega_k t)$, hydro modes $\omega_k \to 0$ for $k \to 0$, nonhydro modes $\omega_k \to \text{const} \neq 0$ for $k \to 0$

2) HYDRODYNAMIC MODES CORRESPOND TO GENUINE HYDRODYNAMIC BEHAVIOR

3) NON-HYDRODYNAMIC MODES (TERMS) SHOULD BE TREATED AS REGULATORS OF THE THEORY

4) NON-HYDRODYNAMIC MODES GENERATE ENTROPY

Old Maxwell's idea?

J. Clerk Maxwell, On the Dynamical Theory of Gases, Phil. Trans. R. Soc. Lond. 147 (1867) 49-88, Eq. (143)

[49]

IV. *On the Dynamical Theory of Gases.* *By* J. Clerk Maxwell, *F.R.S. L. & E.*

Received May 16,—Read May 31, 1866.

86 MR. CLERK MAXWELL ON THE DYNAMICAL THEORY OF GASES.

and neglecting terms of the forms $\xi\eta$ and ξ^3 and $\xi\eta^2$ when not multiplied by the large coefficient k_1, we find by equations (75), (77), and (54),

$$\left.\begin{aligned} &\rho\frac{\partial}{\partial t}\beta(\xi^3+\xi\eta^2+\xi\zeta^2)+\beta\frac{d}{dx}\cdot\rho(\xi^4+\xi^2\eta^2+\xi^2\zeta^2)-\beta(\xi^2+\eta^2+\zeta^2)\frac{dp}{dx}-2\beta\xi^2\frac{dp}{dx} \\ &\quad=-3k_1\rho^2\mathrm{A}_2\beta\{\xi^3+\xi\eta^2+\xi\zeta^2\}. \end{aligned}\right\} \quad . \quad . \quad (143)$$

The first term of this equation may be neglected, as the rate of conduction will rapidly establish itself. The second term contains quantities of four dimensions in ξ, η, ζ,

C. Cattaneo, Sur une forme de l'équation de la chaleur éliminant le paradoxe d'une propagation instantanée, Comptes Rendus 247(4) (1958) 431.

1.3 Equation of state

Equation of state

in ultrarelativistic collisions (top RHIC and the LHC energies) we may neglect the baryon number

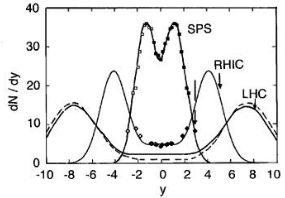
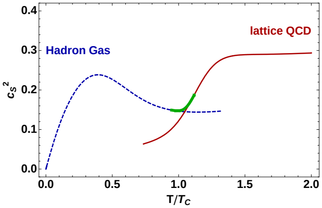


FIG. 1: Net-proton rapidity spectra in the Relativistic Diffusion Model (RDM), solid curves: Transition from the double-humped shape at SPS energies of $\sqrt{s_{NN}}$ = 17.3 GeV to a broad midrapidity valley in the three-sources model at RHIC (200 GeV) and LHC (5.52 TeV). See [11] for details.

R. Kuiper and G. Wolschin, Annalen Phys. 16, 67 (2007)



M. Chojnacki, WF, Acta Phys.Pol. B38 (2007) 3249

$$c_s^2 = \frac{\partial \mathcal{P}}{\partial \mathcal{E}}$$

$c_s^2 = \frac{1}{3}$ for conformal systems

$c_s^2 \to 0$ if $T \to T_{\rm critical}$ for the 1st order phase transition

1.4 Shear and bulk viscosities

Harmonic flows

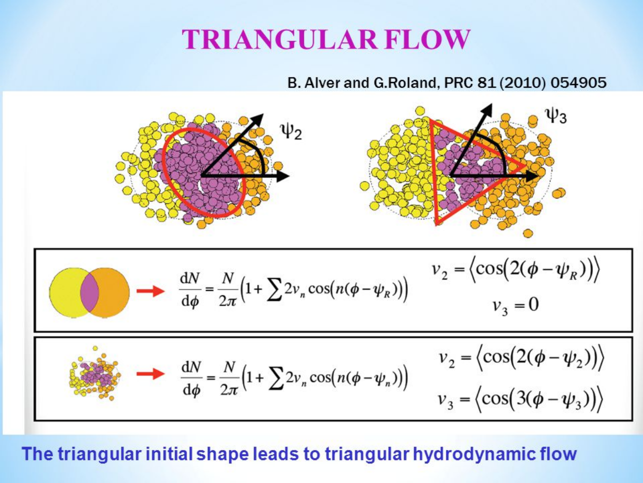
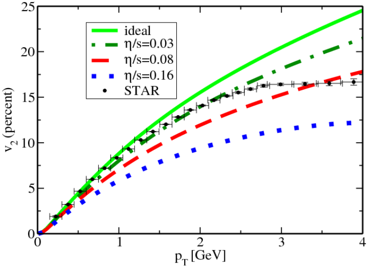


figure from L. Bravina's presentation at Quark Confinement and the Hadron Spectrum XI

Elliptic flow

shear viscosity affects elliptic flow

first principles tell us that one should use relativistic dissipative hydrodynamics, but better description of the data is also achieved with finite but small $\eta/\mathcal{S}$



P. Romatschke and U. Romatschke, PRL 99 (2007) 172301

E. Shuryak: small $\eta/\mathcal{S}$ means that QGP is strongly interacting, previous concepts of QGP hold at really asymptotic energies

QGP shear viscosity: large or small?

John Mainstone (Wikipedia)

Pitch

Start 1927

1^{st} drop 1938

8^{th} drop 2000

$\eta \sim 2\,10^8$ Pa s

$\sim 10^{11} \eta_{H_2O}$

Wikipedia: The ninth drop touched the eighth drop on 17 April 2014. However, it was still attached to the funnel. On 24 April 2014, Prof. White decided to replace the beaker holding the previous eight drops before the ninth drop fused to them. While the bell jar was being lifted, the wooden base wobbled and the ninth drop snapped away from the funnel.

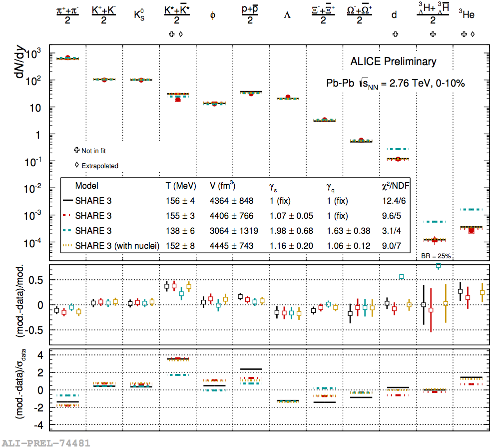
$$\eta_{\rm qgp} > \eta_{\rm pitch}$$

$$\eta_{\rm qgp} \sim 10^{11} \text{ Pa s}, \qquad (\eta/s)_{\rm qgp} < 3/(4\pi)\hbar \qquad \text{(from experiment)}$$

2 Freeze-out models

2.1 Thermal models for the ratios of hadronic abundances

Thermal fit to hadron multiplicity ratios



Model	T (MeV)	V (fm^3)	γ_s	γ_q	χ^2/NDF
SHARE 3	156 ± 4	4364 ± 848	1 (fix)	1 (fix)	12.4/6
SHARE 3	155 ± 3	4406 ± 766	1.07 ± 0.05	1 (fix)	9.6/5
SHARE 3	138 ± 6	3064 ± 1319	1.98 ± 0.68	1.63 ± 0.38	3.1/4
SHARE 3 (with nuclei)	152 ± 8	4445 ± 743	1.16 ± 0.20	1.06 ± 0.12	9.0/7

M. Floris, Nucl. Phys. A931 (2014) c103

P. Braun-Munzinger, D. Magestro, K. Redlich, J. Stachel

Hadron production in Au+Au collisions at RHIC, Phys.Lett. B518 (2001) 41

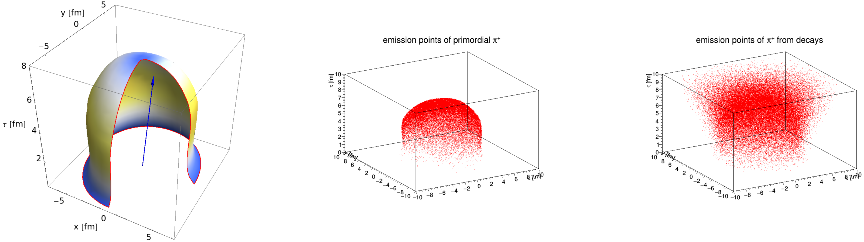
2.2 Single-freeze-out model/scenario

Cooper-Frye formula describing spectra of emitted particles (hadrons) on the freeze-out hyper surface $\Sigma_\mu(x)$

$$E_p \frac{dN}{d^3p} = \int d\Sigma_\mu(x) p^\mu f_{\rm eq}(x,p) \qquad (12)$$

basis for the thermal models — expansion “cancels” in the ratios

$$\frac{N_i}{N_j} = \frac{n_i^{\rm eq}(T,\mu) \int d\Sigma_\mu u^\mu}{n_j^{\rm eq}(T,\mu) \int d\Sigma_\mu u^\mu} = \frac{n_i^{\rm eq}(T,\mu)}{n_j^{\rm eq}(T,\mu)} \qquad (13)$$



“Monte-Carlo statistical hadronization in relativistic heavy-ion collisions” by R. Ryblewski, arXiv:1712.05213

W. Broniowski, WF, “Explanation of the RHIC p(T) spectra in a thermal model with expansion”, Phys. Rev. Lett. 87 (2001) 272302

chemical freeze-out (fixed ratios of abundances) = kinetic freeze-out (fixed spectra)

- SHARE: Statistical hadronization with resonances G. Torrieri, S. Steinke (Arizona U.), W. Broniowski (Cracow, INP), WF, J. Letessier, J. Rafelski (Arizona U.) Comput. Phys. Commun. 167 (2005) 229
- THERMINATOR: THERMal heavy-IoN generATOR A. Kisiel, T. Taluc (Warsaw U. of Tech.), W. Broniowski (Cracow, INP), WF, Comput. Phys. Commun. 174 (2006) 669
- THERMINATOR 2: THERMal heavy IoN generATOR 2 M. Chojnacki (Cracow, INP), Adam Kisiel (CERN & Warsaw U. of Tech.), WF, Wojciech Broniowski (Cracow, INP & Jan Kochanowski U.), Comput. Phys. Commun. 183 (2012) 746.

resonances important not only for the ratios but also for the spectra

theoretical basis -> virial expansion

open-source codes in heavy-ion physics

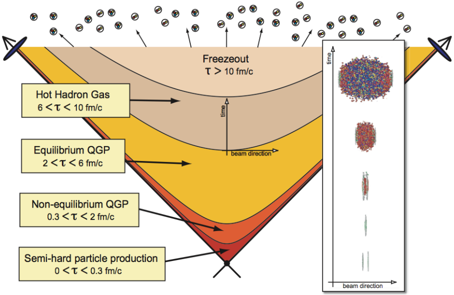
W. Broniowski, M. Chojnacki, WF, A. Kisiel, “Uniform description of soft hadronic observables’, Phys. Rev. Lett. 101 (2008) 022301

3 Anisotropic hydrodynamics

3.1 Problems of standard (IS) viscous hydrodynamics

Simplified space-time diagram

space-time diagram for a simplified, one dimensional and boost-invariant expansion



M. Strickland, Acta Phys.Polon. B45 (2014) 2355

evolution governed by the proper time $\tau = \sqrt{t^2 - z^2}$

Pressure anisotropy

space-time gradients in boost-invariant expansion increase the transverse pressure and decrease the longitudinal pressure

$$\mathcal{P}_T = \mathcal{P} + \frac{\pi}{2}, \quad \mathcal{P}_L = \mathcal{P} - \pi, \quad \pi = \frac{4\eta}{3\tau} \tag{14}$$

$$\left(\frac{\mathcal{P}_L}{\mathcal{P}_T}\right)_{\rm NS} = \frac{3\tau T - 16\bar{\eta}}{3\tau T + 8\bar{\eta}}, \quad \bar{\eta} = \frac{\eta}{\mathcal{S}}$$

using the AdS/CFT lower bound for viscosity, $\bar{\eta} = \frac{1}{4\pi}$

RHIC-like initial conditions, $T_0 = 400$ MeV at $\tau_0 = 0.5$ fm/c, $(\mathcal{P}_L/\mathcal{P}_T)_{\rm NS} \approx 0.50$
LHC-like initial conditions, $T_0 = 600$ MeV at $\tau_0 = 0.2$ fm/c, $(\mathcal{P}_L/\mathcal{P}_T)_{\rm NS} \approx 0.35$

3.2 Concept of aHydro

Thermodynamic & kinetic-theory formulations

Thermodynamic formulation

WF, R. Ryblewski

PRC 83, 034907 (2011), JPG 38 (2011) 015104

1. energy-momentum conservation $\partial_\mu T^{\mu\nu} = 0$
2. ansatz for the entropy source, e.g., $\partial(\sigma U^\mu) \propto (\lambda_\perp - \lambda_\parallel)^2/(\lambda_\perp \lambda_\parallel)$

Kinetic-theory formulation

M. Martinez, M. Strickland

NPA 848, 183 (2010), NPA 856, 68 (2011)

1. first moment of the Boltzmann equation = energy-momentum conservation
2. zeroth moment of the Boltzmann equation = specific form of the entropy source

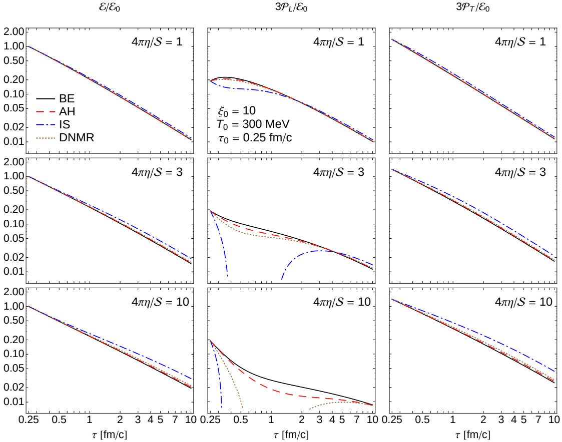
3. Generalized form of the equation of state based on the Romatschke-Strickland (RS) form

generalization of equilibrium/isotropic distributions, frequently used in the studies of anisotropic quark-gluon plasma (here as a modified Boltzmann distribution in the local rest frame)

$$f_{RS} = \exp\left(-\sqrt{\frac{p_\perp^2}{\lambda_\perp^2} + \frac{p_\parallel^2}{\lambda_\parallel^2}}\right) = \exp\left(-\frac{1}{\lambda_\perp}\sqrt{p_\perp^2 + x\, p_\parallel^2}\right) = \exp\left(-\frac{1}{\Lambda}\sqrt{p_\perp^2 + (1+\xi)\, p_\parallel^2}\right)$$

anisotropy parameter $x = 1 + \xi = \left(\frac{\lambda_\perp}{\lambda_\parallel}\right)^2$ and transverse-momentum scale $\lambda_\perp = \Lambda$

WF, R. Ryblewski, M. Strickland, Phys.Rev. C88 (2013) 024903, $m = 0$, boost-invariant, transversally homogeneous system, (0+1) case



aHydro being used and developed now by Heinz (Columbus, Ohio), Strickland (Kent, Ohio), Schaeffer (North Carolina), Rischke (Frankfurt), ... ; applied in other branches of physics, cold atoms, ...

4 Hydrodynamics with spin

4.1 Is QGP the most vortical fluid?

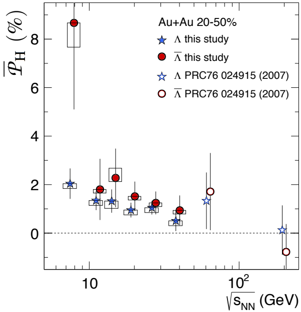
First positive measurements of Λ spin polarization

Non-central heavy-ion collisions create fireballs with large global angular momenta which may generate a spin polarization of the hot and dense matter in a way similar to the Einstein-de Haas and Barnett effects

Much effort has recently been invested in studies of polarization and spin dynamics of particles produced in high-energy nuclear collisions, both from the experimental and theoretical point of view

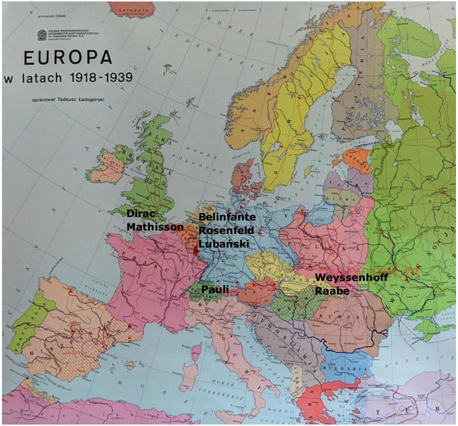
L. Adamczyk et al. (**STAR**), (2017), **Nature 548 (2017) 62-65**, arXiv:1701.06657 (nucl-ex)

Global Λ hyperon polarization in nuclear collisions: evidence for the most vortical fluid
www.sciencenews.org/article/smashing-gold-ions-creates-most-swirly-fluid-ever



4.2 Weyssenhoff's spinning fluid

The Weyssenhoff circle, years 1930s - 1940s



Jan Weyssenhoff
1889-1972

J. Weyssenhoff and A. Raabe, Acta Phys. Pol. 9 (1947) 7

Vol. IX (1947) Acta Physica Polonica Fasc. 1

RELATIVISTIC DYNAMICS OF SPIN-FLUIDS AND SPIN-PARTICLES*

By Jan WEYSSENHOFF and A. RAABE †, Institute of Theoretical Physics, Jagellonian University, Kraków **

1. Following up a train of thought inaugurated by Einstein and Gromer, Mathisson *(1, 1937)* and Lubański *(2, 1937)* deduced the equation of motion of a material particle endowed with spin from the general principles of the theory of relativity. Even for a free particle in Galileian domains these equations do not coincide with the Newtonian laws of motion; there remains an additional term depending on the internal angular momentum or spin of the particle, which raises the order of these differential equations to three. Mathisson did not notice the fact that for the above conditions his equations are equivalent to the equations which were previously found by Frenkel *(3, 1926)* for a spinning electron (if only their terms depending on the electromagnetic field are dropped). This is all the more excusable as Frenkel considered throughout the additional term in his equations as an infinitely small perturbation, and did not even mention the fact that they disagree with Newton's First Law for a free particle.

In the present paper we give a third method of obtaining the same equations by establishing first the laws of the dynamics of an incoherent spin-fluid and passing then to the limit. Strictly speaking, we obtain different and much simpler equations, which prove however to be equivalent to Frenkel's and Mathisson's. The simplification is due to the explicit introduction of the 4-vector of linear monumentum and energy G^{α}.

* Presented at a meeting of the Cracow Section of the Polish Physical Society on February 28, 1945. The main contents of this paper and the next one, as well as most of the results of the three following, were a subject of a lecture at a secret meeting of physicists at Prof. Pieńkowski's home in Warsaw, October 1942.

** Mr. Raabe was a highly gifted young physicist with whom I outlined in all its main features the contents of this paper and the next one in 1940/41 in Lwów. We tried to pursue our work in 1942 in Cracow, but unfortunately in June 1942 Mr. Raabe fell victim of a man-hunt in the streets of Cracow; he died four months later in the German concentration camp of Oświęcim. (J. W.)

1) conservation of energy and momentum with an asymmetric energy-momentum tensor

$$T^{\mu\nu}(x) = g^{\mu}(x)u^{\nu}(x), \quad \partial_\nu T^{\mu\nu}(x) = 0$$

u^μ is the four-velocity of the fluid element, while g^μ is the density of four-momentum with the notation $\partial_\nu(fu^\nu) \equiv Df$ we may write $Dg^\mu = 0$

2) conservation of total angular momentum $J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu}$ (orbital and spin parts)

$$L^{\lambda,\mu\nu}(x) = x^\mu T^{\nu\lambda}(x) - x^\nu T^{\mu\lambda}(x), \quad S^{\lambda,\mu\nu}(x) = s^{\mu\nu}(x)u^\lambda(x)$$

$s^{\mu\nu} = -s^{\nu\mu}$ describes the spin density

$$\partial_\lambda J^{\lambda,\mu\nu} = 0 \rightarrow Ds^{\mu\nu} = g^\mu u^\nu - g^\nu u^\mu$$

3) 10 equations for 13 unknown functions: g^μ, $s^{\mu\nu}$ and u^i ($i = 1, 2, 3$)
additional constraint has been adopted, the Frenkel (or Weyssenhoff) condition
$s^{\mu\nu}u_\mu = 0$

4.3 Spin hydrodynamics

revival of the ideas of spinning fluids

WF, B. Friman, A. Jaiswal, E. Speranza, Phys. Rev. C97 (2018) 041901 general concept of hydrodynamics with spin: conservation of energy, linear momentum, total angular momentum, and charge:

$$\partial_\mu T^{\mu\nu}[\beta^\alpha, \omega^{\alpha\beta}, \xi] = 0, \quad (15)$$

$$\partial_\lambda J^{\lambda,\mu\nu}[\beta^\alpha, \omega^{\alpha\beta}, \xi] = 0, \quad (16)$$

$$\partial_\mu j^\mu[\beta^\alpha, \omega^{\alpha\beta}, \xi] = 0. \quad (17)$$

Here $\beta^\alpha = u^\alpha/T$, $\xi = \mu/T$, where μ is the chemical potential

$\omega^{\alpha\beta}$ - new chemical potential connected with the angular momentum conservation
$J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu}$ - sum of the orbital and spin parts

$$\partial_\lambda J^{\lambda,\mu\nu} = 0 \iff \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu} \quad (18)$$

spin-orbit interaction, quantum energy-momentum tensors have asymetric parts
conservation of angular momentum for particle with spin is non-trivial

for s-wave dominated scattering, spin is separately conserved, one can use

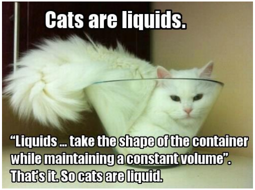
$$\partial_\lambda S^{\lambda,\mu\nu} = 0. \quad (19)$$

With spin-orbit interaction included:

- In global equilibrium, the spin chemical potential is not independent of (thermal) vorticity, $\omega_{\mu\nu} = -1/2\left(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu\right)$ = const.
- The form of local equilibrium for particles with spin is under discussion now [Becattini et al., Florence,...]
- Non-local effects in the Boltzmann equation for particles with spin [Rischke et al., Frankfurt,...]
- Different forms of the energy-momentum and spin tensors can be used – pseudo-gauge (in)dependence
- Inclusion of magnetic fields – spin magneto hydrodynamics is under development by many groups

5 Summary

- Golden era of heavy-ion collisions during the first runs of RHIC, 2000-2010
- Enormous progress in both experiment and theory
- More precise measurements at the LHC, lower energies studied at RHIC
- Great success of statistical methods



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