34th M. Smoluchowski Symposium on Statistical Physics



Contribution ID: 18 Type: Flash talk

Collagen as Hopf's fibration

Wednesday, 29 September 2021 10:55 (5 minutes)

The shape of many biomolecules is similar to the Coxeter-Boerdijk (CB) helix, made of tetrahedra placed next to each other in such a way that the tetrahedrons are adjacent to each other with their faces. The helix CB is the solution to the maximum packing task. In the collagen helix, every third amino acid is glycine (Gly), and the entire chain contains primarily proline (Pro). Half of the Pro groups are hydroxyproline (Hyp). Each collagen chain contains the repeating components of Gly-X-Y, with X or Y being almost always Pro or Hyp. That is why a single chain of collagen itself is called proline. Proline is also found in biomolecules other than collagen. In collagen, the 3 proline (left-handed) helices fold into one right-handed helix.

The chain of tetrahedra of the CB helix can be detected in a polytope (3, 3, 5). It is the regular structure similar to the S3 hypersphere built of 600 tetrahedra. The 600-cell can be decomposed over 25 overlapping 24-cells. The vertices of this structure, numbering 120, are arranged 10 on the 12 non-intersecting great circles of the hypersphere. These great circles are Hopf S3 fibration fibers, each fiber having 10 vertices.

While the structure of biological molecules seems complicated, their topology turns out to be simple thanks to spatial considerations, as a result of which it turns out that mathematical helices have geometrical properties of biological helices.

Bibliography:

H.S.M. Coxeter, Regular Polytopes (3rd ed.) New York, Dover 1973.

N. S. Manton, Connections on discrete fibre bundles, Communications in Mathematical Physics 113, 341-351 (1987).

F. Sadoc and N. Rivier, Boerdijk-Coxeter helix and biological helices as quasicrystals, Mat.Sci. and Eng. Sp. Iss. Dec. 15, 397-400 (2000).

Primary author: Dr WOJNAR, Ryszard (Institute of Fundamental Technological Research PAS)

Presenter: Dr WOJNAR, Ryszard (Institute of Fundamental Technological Research PAS)

Session Classification: S7